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# Uplifting Runaways and the Tadpole Problem

with I. Bena, E. Dudas, and M. Graña [arXiv:1809.06861],
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# de Sitter vacua in string theory

Three-step procedure [Kachru, Kallosh, Linde, Trivedi '03]:

1. warped IIB with CS-moduli stabilized by three-form fluxes including a region with strong warping [Giddings, Kachru, Polchinski '01] described by the Klebanov Strassler throat [Klebanov, Strassler '00]
   $\rightarrow$ large hierarchy of scales

2. Stabilize Kähler moduli by non-perturbative effects
   $\rightarrow$ supersymmetric AdS-vacuum

3. Supersymmetry breaking by an $\overline{D3}$-brane at the bottom of the throat
   $\rightarrow$ exponentially suppressed uplift to dS due to strong warping

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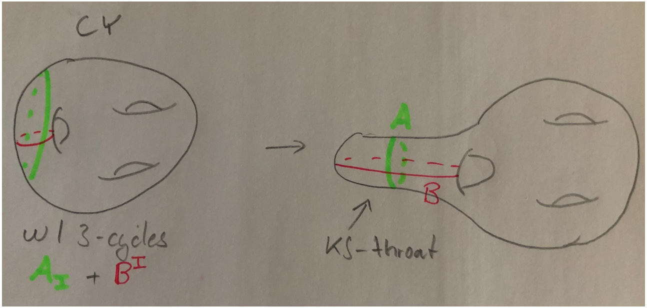
# Warped CY

- Metric: $\mathrm{d}s_{10}^2 = e^{2A}\mathrm{d}s_4^2 + e^{-2A}\mathrm{d}s_{CY_3}^2$
- Fluxes fix the sizes of the 3-cycles: $\int_{A_I} F_3 = M^I, \int_{B^I} H_3 = K_I$




- Choose a configuration such that one cycle is exponentially large.

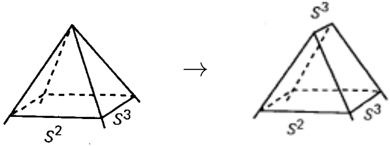
→ Klebanov-Strassler throat.

# Deformed conifold

- In the region of high warping, the six-dimensional geometry is given by the deformed conifold.
- embedding of the deformed conifold into $\mathbb{C}^4$:

$$\sum_{a=1}^{4} z_a^4 = S\,.$$

- Replace the singularity of the conifold ($S = 0$) by a $S^3$ of size $|S|$

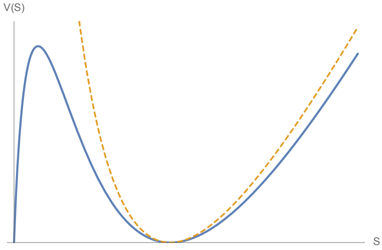



[Candelas, Ossa '89]

- $S$ is a complex structure modulus of the deformed conifold.

# Potential for $S$

- Fluxes $M$ and $K$ along the two three-cycles of the conifold generate a potential $V_{KS}(S)$ [Douglas, Shelton, Torroba '07, '08]:




(dotted without warping effects)

- (Supersymmetric) minimum at $s_{KS} = \Lambda_0^3 \exp\left(-\frac{2\pi K}{g_s M}\right)$.
- Relative warp factor: $\Lambda_0/\Lambda_{IR} \sim |s_{KS}|^{\frac{1}{3}}$.

$\rightarrow$ Large hierarchy for suitable values of $K$, $M$, and $g_s$ [Giddings et al. '01].

# Mass of $S$

- The mass of $S$ at the minimum $s_{KS}$ can be computed by

$$m_S^2 \equiv \frac{1}{M_{pl}^2} G^{S\bar{S}} \partial_S \partial_{\bar{S}} V \Big|_{S=s_{KS}}$$

- Including the effects of the warping we find:

$$m_S^2 \sim \frac{s_{KS}^{2/3}}{\alpha'^2}$$

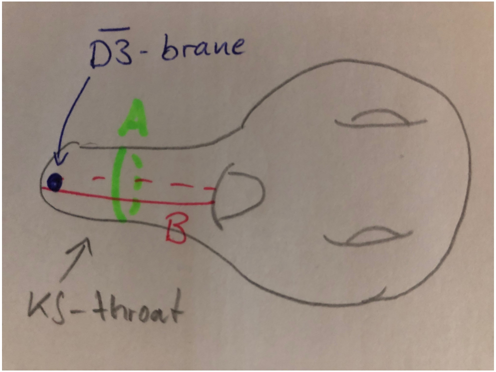
$\rightarrow$ If $s_{KS}$ is exponentially small, $S$ becomes exponentially light.

$\rightarrow$ $S$ cannot be integrated out before uplifting with an anti-brane.

Comparison with Kähler moduli masses: [Blumenhagen, Kläwer, Schlechter '19]

# $\overline{D3}$-brane in the KS throat

- Place an anti-D3 brane at the bottom of the throat




- Positive contribution to the energy $\rightarrow$ uplift to de Sitter

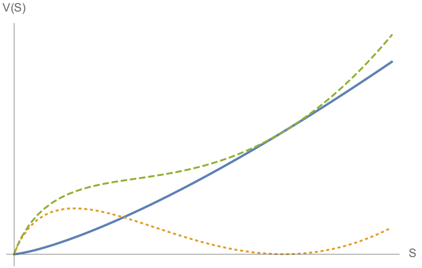
# $\overline{D3}$-brane in the KS throat

- The $\overline{D3}$-brane gives a contribution to the potential:

$$V_{\overline{D3}}(S) \propto e^{4A} \propto \frac{|S|^{4/3}}{g_s(\alpha' M)^2}$$

with $e^{4A}$ the warp factor of the Klebanov-Strassler solution.

- Plot of the potential:




(dotted lines represent the KS potential and their superposition)

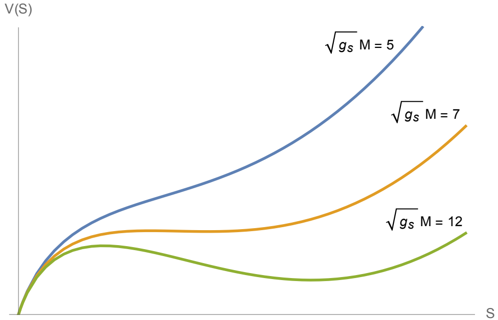
# Stability with one $\overline{D3}$-brane

- A stable minimum of $V_{KS} + V_{\overline{D3}}$ with $S > 0$ exists iff

$$g_s M^2 > M_{min}^2 \quad \text{with} \quad M_{min} = \frac{8}{3}\sqrt{\pi c' c''} \approx 6.8\,.$$

(see also [Blumenhagen et al. '19])

- Superposition of the potentials:



# Klebanov-Strassler black holes

- Recently: Numerical construction of a KS black hole [A. Buchel '18]
- Holographically dual to a theory with spontaneously broken chiral symmetry at finite temperature.
- Exists only if its energy density is below a critical value:

$$\mathcal{E} < \mathcal{E}_{\chi SB}$$

- For higher energies there exist only Klebanov-Tseytlin black holes on the singular conifold with $S = 0$ and no chiral symmetry breaking.
- Translate $\mathcal{E}$ into $\overline{D3}$-units [Bena, Buchel, SL '19]:

$$g_s M^2 > \gamma_{BH}^2 N_{\overline{D3}} \qquad \text{with} \qquad \gamma_{BH} \approx 4.16$$

$\rightarrow$ Same functional form as our analytic bound.

# Implications on the maximal hierarchy

- Warping creates a hierarchy of scales

$$h = \ln \frac{\Lambda_0}{\Lambda_{IR}} = \frac{2\pi K}{3 g_S M}$$

- Tadpole cancellation:

$$M^I K_I + Q_3^{loc} = 0\,,$$

  where $Q_3^{loc}$ is the D3-charge of localized sources.

- Stability of the KS throat + tadpole cancellation:

$$h = \frac{2\pi}{3}\frac{MK}{g_s M^2} < \frac{2\pi}{3}\frac{\left|Q_3^{loc}\right|}{M_{min}^2} \approx 0.045 \times \left|Q_3^{loc}\right|$$

- KKLT requires $h \gtrsim 20 \quad \Rightarrow \quad \left|Q_3^{loc}\right| \gtrsim 500$

# Tadpole cancellation in IIB

- For CY orientifolds with O3-planes and D3-branes:

$$Q_3^{loc} = N_{D3} - \frac{1}{4} N_{03}$$

  - Largest number of O3-planes: $T^6/\mathbb{Z}_2$: $Q_3^{loc} \leq 32$

$\rightarrow$ No large hierarchy possible.

- O7-planes and D7-branes:

$$Q_3^{loc} = \frac{1}{24}\chi(D7) + \frac{1}{6}\chi(O7) - (\text{gauge})$$

  - $\chi$ : Euler number of the 4-cycles wrapped by the D7s/O7s.

$\rightarrow$ Large tadpole possible, but D7-moduli need to be stabilized.

# Tadpole cancellation in F-theory

- Tadpole cancelation for F-theory on a Calabi-Yau four-fold $CY_4$ with four-form flux $G$:

$$N_{D3} + \frac{1}{2}\int G \wedge G = \frac{\chi(CY_4)}{24}$$

- $\chi(CY_4)$: Euler number of the CY $\rightarrow$ can be very large (largest know example [Klemm et al. '97]: $\chi = 1\,820\,448 = 24 \cdot 75\,852$)

- But: Large $\chi$ implies a lot of moduli:

$$\chi(CY_4) = 6(8 + h^{1,1} + h^{3,1} - h^{2,1})$$

- $h^{3,1}$: complex structure of $CY_4$ $\rightarrow$ must be stabilized by flux:

$$\int G \wedge G = \mathcal{O}(h^{3,1}) \quad ?$$

# The Tadpole Conjecture

- Any choice of four-form flux which stabilizes all $h^{3,1}$ complex structure moduli satisfies

$$\frac{1}{2}\int G \wedge G \geq \alpha h^{3,1}$$

  with $\alpha \sim \mathcal{O}(1)$.

- Implications (for $h^{1,1}$ small):
  - $\alpha < \frac{1}{4}$: Moduli stabilization generically possible.
  - $\alpha = \frac{1}{4}$: Moduli stabilization possible but no large hierarchies.
  - $\alpha > \frac{1}{4}$: Moduli stabilization not possible in the large $h^{3,1}$ regime.

# Can this be true?

- Generic four-form flux $G$: F-term condition:

$$D_i W = 0$$

$\rightarrow$ $h^{3,1}$ equations to stabilize $h^{3,1}$ moduli.

- But: Are these conditions always all independent, in particular when $\int G \wedge G \ll h^{3,1}$ ?

- Very difficult to answer (in the large $h^{3,1}$ regime).

$\rightarrow$ Explicit example: $K3 \times K3$.

# $K3 \times K3$ as a toy model

- [Aspinwall, Kallosh '05]:
  All complex structure moduli can be stabilized within the tadpole bound (and all Kähler moduli by instanton effects).

- Moreover:
  $$\frac{\chi(K3 \times K3)}{24} = 24$$
  too small for our purposes.

- [Braun et al. '08]:
  *All* (Kähler + complex structure) moduli can be stabilized by fluxes (*"114 equations to stabilize 114 moduli"*).

- Here: Can we stabilize all moduli within the tadpole bound?

# Moduli stabilization on $K3 \times \widetilde{K3}$

- Flux matrix: expand $G \in H^2(K3, \mathbb{Z}) \times H^2(\widetilde{K3}, \mathbb{Z})$:

$$G = G^{IJ} \alpha_I \wedge \tilde{\alpha}_J \,, \qquad \alpha_I \in H^2(K3, \mathbb{Z}) \,, \quad \tilde{\alpha}_J \in H^2(\widetilde{K3}, \mathbb{Z})$$

- [Braun et al. '08]: Moduli stabilization can be described in terms of

$$N^I{}_J = G^{IK} d_{KL} G^{ML} \tilde{d}_{LI} \,, \qquad \text{with} \qquad d_{IJ} = \int \alpha_I \wedge \alpha_J$$

  - Minkowski vacuum: $N^I{}_J$ is diagonalizable with non-negative eigenvalues $\{a_1, a_2, a_3, b_1, \ldots, b_{19}\}$ ($a_i$: positive-norm eigenvectors, $b_i$: negative-norm eigenvectors)
  - No flat directions: The sets $\{a_1, a_2, a_3\}$ and $\{b_1, \ldots, b_{19}\}$ are pair-wise distinct.
  - $b_i = 0$: either flat direction or singularity.
  - Tadpole: $\int G \wedge G = \mathrm{tr}(N)$

# Moduli stabilization on $K3 \times \widetilde{K3}$

- Use genetic algorithms to search for integer matrices $G^{IJ}$ which minimize $\mathrm{tr}(GdG^T d)$.

- See Johan's talk for more details on the search ...

- Our result:

$$\frac{1}{2}\int G \wedge G \geq 30$$

unless the potential has flat directions or $K3 \times K3$ becomes singular.

$\rightarrow$ No moduli stabilization for smooth $K3 \times K3$.

# Conclusions

- With a large hierarchy the KS-modulus becomes exponentially light.
- One $\overline{D3}$ makes a Klebanov-Strassler throat unstable unless

$$g_s M^2 > \gamma^2 N_{\overline{D3}}$$

- Tadpole-cancellation: Constraints on the hierarchy.
- Moduli stabilization with fluxes for many moduli?

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# Thank You!