Kähler moduli stabilization from 10d

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Based on **ArXiv:1908.01785** (JHEP),
in collaboration with **I. Bena, M. Graña and A. Retolaza**.

Plan for the talk

Intro: the three-step KKLT[1]. proposal for de Sitter in IIB

1. Start with a GKP $\mathcal{N}=1$ compactification:
   - Geometry: $\text{Mink}_4 \times$ *conf.* CY_3.
   - 3-form fluxes fix the complex structure moduli.

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2. Include (NP) gaugino condensation on a stack of D7-branes wrapping internal 4-cycle.
   - In the 4d EFT, this leads to a SUSY AdS$_4$ solution.
   - All moduli fixed + small (< 0) cosmological constant.

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3. Lift this Λ to (small) positive values by some source of positive energy:
   - $\overline{\text{D3}}$-branes at the bottom of some throat.

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Main motivation

Step 2: non-perturbative effects

Fix Kähler moduli ($\rho = i\sigma$) + get $\mathcal{N} = 1$ AdS_4

Usually, this is described in the 4d EFT...

$$W_{\text{total}} = W_0 + W_{\text{NP}} \approx W_0 + A \exp\left(ia\rho\right),$$

with W_0 from fluxes. Assuming $K = -3 \log\left[-i(\rho - \bar{\rho})\right]$ one gets a susy AdS_4 solution with

$$W_0 = -Ae^{-a\sigma_*}\left(1 + \frac{2}{3}a\sigma_*\right).$$

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Usually, this is described in the 4d EFT...

- Can we understand this from 10 dimensions ?
- How does the internal geometry look like ?
- Is there a *geometrization* of the gaugino condensate ?
- Is this a particular case of a more general procedure ?
- Can this help for describing the uplift to dS ? (*)

Other recent papers with related goals

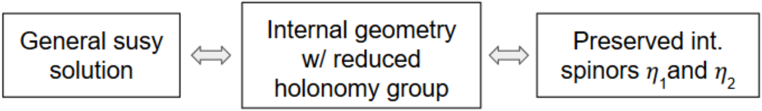
- [Moritz, Retolaza & Westphal '17].
- [Hamada, Hebecker, Shiu & Soler '18 (x2) and '19].
- [Kallosh '19].
- [Gautason, Van Hemelryck, Van Riet & Venken '19].
- [Carta, Moritz & Westphal '19].
- [Kachru, Kim, McAllister & Zimet '19].

GCG for type II susy compactifications

Consider a compactification with

$$ds_{10}^2 = e^{2A(y)} g_{\mu\nu}(x) dx^\mu dx^\nu + h_{mn}(y) dy^m dy^n .$$

Then,



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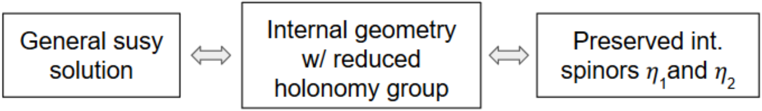

such that the susy generators are

$$\epsilon_1 = \zeta_+ \otimes \eta^1_+ + \zeta_- \otimes \eta^1_- \ , \ \epsilon_2 = \zeta_+ \otimes \eta^2_\mp + \zeta_- \otimes \eta^2_\pm$$

where the upper (lower) sign is for type IIA (IIB), and

$$2\nabla_\nu \zeta_- = \pm \mu \gamma_\nu \zeta_+ \ , \ \Lambda_{4d} = -3|\mu|^2 .$$

GCG for type II susy compactifications

In general: SU(3)×SU(3) structure, described by the polyforms[2]

$$\Psi_\pm \equiv -\frac{8i}{||\eta||^2} \sum_p \frac{1}{p!} \eta_\pm^{2\dagger} \Gamma_{m_1 \dots m_p} \eta_+^1 \, dy^{m_1} \wedge \dots \wedge dy^{m_p} .$$

[2] [Hitchin '02, Gualtieri '04, Graña et al '05-'06]

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The susy conditions (zero $\delta\psi_M$ and $\delta\lambda$) read (SF)

$$\begin{aligned} d_H\left(e^{3A-\phi}\Psi_2\right) &= 2i\mu e^{2A-\phi}\mathrm{Im}\,\Psi_1 \\ d_H\left(e^{2A-\phi}\mathrm{Im}\,\Psi_1\right) &= 0 \\ d_H\left(e^{4A-\phi}\mathrm{Re}\,\Psi_1\right) &= 3e^{3A-\phi}\mathrm{Re}\,(\bar\mu\Psi_2) + e^{4A} *_6 F \end{aligned}$$

Here

$$d_H = d - H\wedge\ ,\ \Psi_1 = \Psi_\mp\ ,\ \Psi_2 = \Psi_\pm.$$

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GCG: the 10d superpotential

$$\begin{aligned} d_H\left(e^{3A-\phi}\Psi_2\right) &= 2i\mu e^{2A-\phi}\mathrm{Im}\,\Psi_1 \qquad (*) \\ d_H\left(e^{2A-\phi}\mathrm{Im}\,\Psi_1\right) &= 0 \\ d_H\left(e^{4A-\phi}\mathrm{Re}\,\Psi_1\right) &= 3e^{3A-\phi}\mathrm{Re}\,(\bar{\mu}\Psi_2) + e^{4A} *_6 F \end{aligned}$$

These can be obtained as F- and D-flatness for[3]

$$W_{10\text{d}} = \int_{M_6} \langle Z, dT \rangle,$$

with $\mu \sim \langle W_{10\text{d}} \rangle$. The holomorphic fields are

$$Z = e^{3A-\phi} e^B \Psi_2 \, , \qquad T = e^B (C + i e^{-\phi} \mathrm{Re}\,\Psi_1).$$

[3] [Koerber & Martucci '07-'08]

GCG: examples with SU(3) structure

Here $\eta^2_+ = -ie^{i\theta}\eta^1_+$ and in type IIB

$$\Psi_2 = \Psi_- = ie^{-i\theta}\Omega\ ,\ \Psi_1 = \Psi_+ = e^{-i\theta}\exp(-iJ).$$

For D3/D7 susy: $\theta = 0$, For D5 susy: $\theta = \pi/2$.

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For D3/D7 susy: $\theta = 0$, For D5 susy: $\theta = \pi/2$.

while for type IIA (D6 susy) Ψ_1 and Ψ_2 exchange roles. Also,

$$\begin{aligned} W_{\mathrm{D3/D7}} &= -\int_{M_6} \Omega \wedge G_3, \\ W_{\mathrm{D5}} &= -\int_{M_6} \Omega \wedge \left(F_3 + ie^{-\phi} dJ\right) \\ W_{\mathrm{D6}} &= \int_{M_6} (J - iB) \wedge \left(F_4 + id[e^{-\phi}\mathrm{Re}\,\Omega]\right). \end{aligned}$$

These describe the usual CY and conformal CY compactifications.

Including NP effects[4]

Let us add a D-brane wrapping an internal cycle Σ.

- **Calibrated** cycle $\Rightarrow$ susy WV theory $\approx \mathcal{N}=1$ SYM $\Rightarrow$ we assume $\langle\lambda\lambda\rangle \neq 0$ in the IR from strong coupling effects.

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But

$$S_{Dp}|_{\mathcal{F}^2} = -\frac{1}{8\pi}\int_\Sigma e^{-\phi}\mathrm{Re}\,\Psi_1 \int d^4x\sqrt{-g}\,\mathrm{Tr}\,\mathcal{F}^2 + \text{WZ term},$$

giving a holomorphic coupling

$$\tau = \int_\Sigma (C + ie^{-\phi}\mathrm{Re}\,\Psi_1)|_\Sigma = \int_{M_6}\langle T, \delta^{9-p}[\Sigma]\rangle.$$

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Take $W_{10\mathrm{d}} \to W^{(0)}_{10\mathrm{d}} + W_{\mathrm{NP}} \sim W^{(0)}_{10\mathrm{d}} + A\exp\left[-a\tau(T)\right]$.

$\Rightarrow$ This will modify (at least) the 1st susy condition.

[4] [Koerber & Martucci '07, Dymarsky & Martucci '10]

Including NP effects: consequences

The new susy condition is

$$d_H\left(e^{3A-\phi}\Psi_2\right) = 2i\mu e^{2A-\phi}\mathrm{Im}\,\Psi_1 + 2i\langle S\rangle\delta^{9-p}[\Sigma].$$

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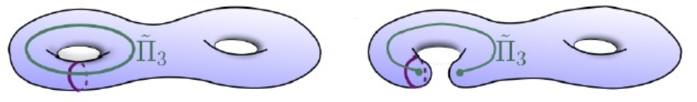
Consequences for $g_{\mu\nu} \equiv$ Mink$_4$

We see that $\delta^{9-p}(\Sigma)$ – the **Poincaré-dual** of the cycle Σ – becomes trivial in co-homology.

$\Rightarrow \Sigma$ becomes trivial itself: it shrinks![a]



This is a **geometric transition**, where the localized source disappears, leaving only flux.

[a] [García-Valdecasas Tenreiro & Uranga '17]

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Consequences for $g_{\mu\nu} \equiv \mathrm{AdS}_4$

For $\mu \neq 0$,

- **0-form comp:** $\mathrm{Im}\,\Psi_1 = 0$ restricts the relative phase of $\eta_{1,2}$. For SU(3) structure this only leaves D3/D7 susy or "static SU(2)".
- **(9-p)-form comp:** Due to the CC term $\delta^{9-p}(\Sigma)$ is not trivial anymore

  $\Rightarrow$ this stops the cycle from shrinking!

$\Rightarrow$ we find a 10d **qualitative** manifestation of σ stabilization.

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Consequences for $g_{\mu\nu} \equiv \mathrm{AdS}_4$

For D7-branes $\Rightarrow$ $9-p=2$ (and $\mathrm{Im}\,\Psi_1$ not localized)

$\Rightarrow$ We need a 1-form in Ψ_2

$\Rightarrow$ SU(3) structure can not be enough (*). We need a more general structure (geometry) $\subset$ SU(3)$\times$SU(3).

We propose a dynamical SU(2) structure in order to

- distinguish $\|$ and $\perp$ directions w.r.t to Σ_4.
- re-obtain the conf. CY geometry at long distances.

Including NP effects: an observation

By defining the generalized flux[5]

$$G = F + ie^{-4A} d_H \left(e^{4A-\phi} \mathrm{Re}\, \Psi_1 \right)$$

the 3rd susy Eq. for $\mu = 0$ can be recast as an ISD condition

$$\boxed{\tilde{*}_6 G = iG}$$

- SU(3) str. and D3/D7 susy: the usual for $G_3 = F_3 + ie^{-\phi} H_3$.

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- SU(3) str. and D3/D7 susy: the usual for $G_3 = F_3 + ie^{-\phi}H_3$.
- the CC term and (probably) also the $\langle\lambda\lambda\rangle$ give new contributions

$\Rightarrow$ IASD "flux"[6].

[5] [Lüst, Marchesano, Martucci & Tsimpis '08]
[6] [Baumann et al '10, Dymarsky & Martucci '10]

Including NP effects: remarks and questions

- At least w.r.t. to this equation, the procedure is very general: it is not restricted to the KKLT scenario and D7-branes/E3-instantons. Can we find other examples?
- Can we say something at the **quantitative** level?
- For a KKLT-like situation, can we recover the critical value σ_*?

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[7] [Heidenreich , McAllister & Torroba ' 10]

Dynamic SU(2) structure...

One generalizes the set of inv. tensors

$$\{J, \Omega\} \longrightarrow \{\Theta, J_2, \Omega_2\}$$

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The angle φ is position-dependent and characterizes the angle between η_1 and η_2: the deviation from SU(3). Thus,

$$\begin{aligned}\Psi_+ &= e^{\frac{1}{2}\Theta\wedge\bar{\Theta}}\wedge\left[\cos\varphi\left(1-\frac{1}{2}J_2^2\right)+\sin\varphi\,\mathrm{Im}\,\Omega_2 - iJ_2\right],\\ \Psi_- &= \Theta\wedge\left[\sin\varphi\left(1-\frac{1}{2}J_2^2\right)-\cos\varphi\,\mathrm{Im}\,\Omega_2 + i\mathrm{Re}\,\Omega_2\right].\end{aligned}$$

... and $\mathbb{P}^2$-cone geometry

We take a non-compact example: the Σ_4 at the bottom of the resolution of

$$\mathbb{C}_3/\mathbb{Z}_3 : \ z^i \sim e^{2\pi i/3} z^i \ , \ u^{1,2} = \frac{z^{1,2}}{z^3} \ , \quad z = \frac{1}{3}(z^3)^2,$$

with radial coordinate

$$\rho^2 = (3|z|)^{2/3} \left(1 + |u_1|^2 + |u_2|^2\right) \ , \ r = \rho^3/3 \rightarrow_{\text{near } \Sigma_4} r_z .$$

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The cycle is at z=0, and we have the susy conditions

$$\begin{aligned} d\left[e^{3A-\phi} \sin\varphi \, \Theta\right] &= 2i\mu e^{2A-\phi} (\cos\varphi \, J_1 + J_2) - 2i\langle S\rangle \delta^2[z] , \\ d\left[e^{2A-\phi} (\cos\varphi \, J_1 + J_2)\right] &= 0 . \end{aligned}$$

[Loc. term missing before]

Solution: regimes and Matching

Near-Σ_4 region:

- everything depends on $r \approx |z|$.
- at first order the δ-term wins: we can approximate $\mu = 0$.

$$\Theta = F(r)\frac{dz}{z} \text{ and susy cond.} \Rightarrow F(r)\sin\varphi = c = \langle\lambda\lambda\rangle$$

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Now extend the definition far away:

$$\Theta = r\,\frac{dz}{z} \to \frac{\partial r^2}{r}.$$

Then $d\left[\sin\varphi\,\Theta\right] = 2i\mu\left(\cos\varphi\,J_1 + J_2\right)$ implies

$$F(r)\sin\varphi = \mu e^{2L_2(r)}$$

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Matching: $\langle\lambda\lambda\rangle = \mu e^{2l_2}$

Re-interpreting the matching condition

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Now we can use

$$\mu = e^{K/2} W_{\mathrm{KKLT}} = \frac{W_0 + W_{\mathrm{NP}}}{(2\sigma)^{3/2}},$$

together with[8]

$$W_{\mathrm{NP}} \sim \langle\lambda\lambda\rangle \sim A e^{-a\sigma_*} \quad \text{and} \quad e^{2l_2} \propto \sqrt{2\sigma}.$$

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$\Rightarrow$ the matching condition implies

$$W_0 = -A e^{-a\sigma_*}\,(1 + \#\, a\, \sigma_*)$$

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Conclusions and outlook

By trying to include the $\langle\lambda\lambda\rangle$ superpotential in 10d and studying its back-reaction we argued that

- the presence of the CC term stops the shrinking of the cycle wrapped by the branes.
- for D7's SU(3) structure is not enough to describe the internal geometry.
- this 10d description of Kähler moduli stabilization can be made precise by matching solutions near and far away from the branes.

- Can we find a complete solution?
- Can we compute the constant #? Is it model dependent?
- Can we use this to describe other small susy AdS_4 solutions?
- Can we use this say something new about the uplift do dS?

Thank you for your attention! Any questions?