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東京大学 THE UNIVERSITY OF TOKYO
JICFuS
RIKEN NISHINA CENTER

Prevailing triaxial shapes in exotic and heavy nuclei

Takaharu Otsuka

Y. Tsunoda[*], Y. Utsuno[%], N. Shimizu[#], T. Abe[+], H. Ueno[+], N. Tsunoda[*], K. Takayanagi[¥],
T. Suzuki[a], S. Yoshida[b]

[*]CNS (U. Tokyo) , [%]JAEA, [#]Tsukuba, [+]RIKEN, [¥]Sophia U., [a]Nihon U., [b]Utsunomiya U.

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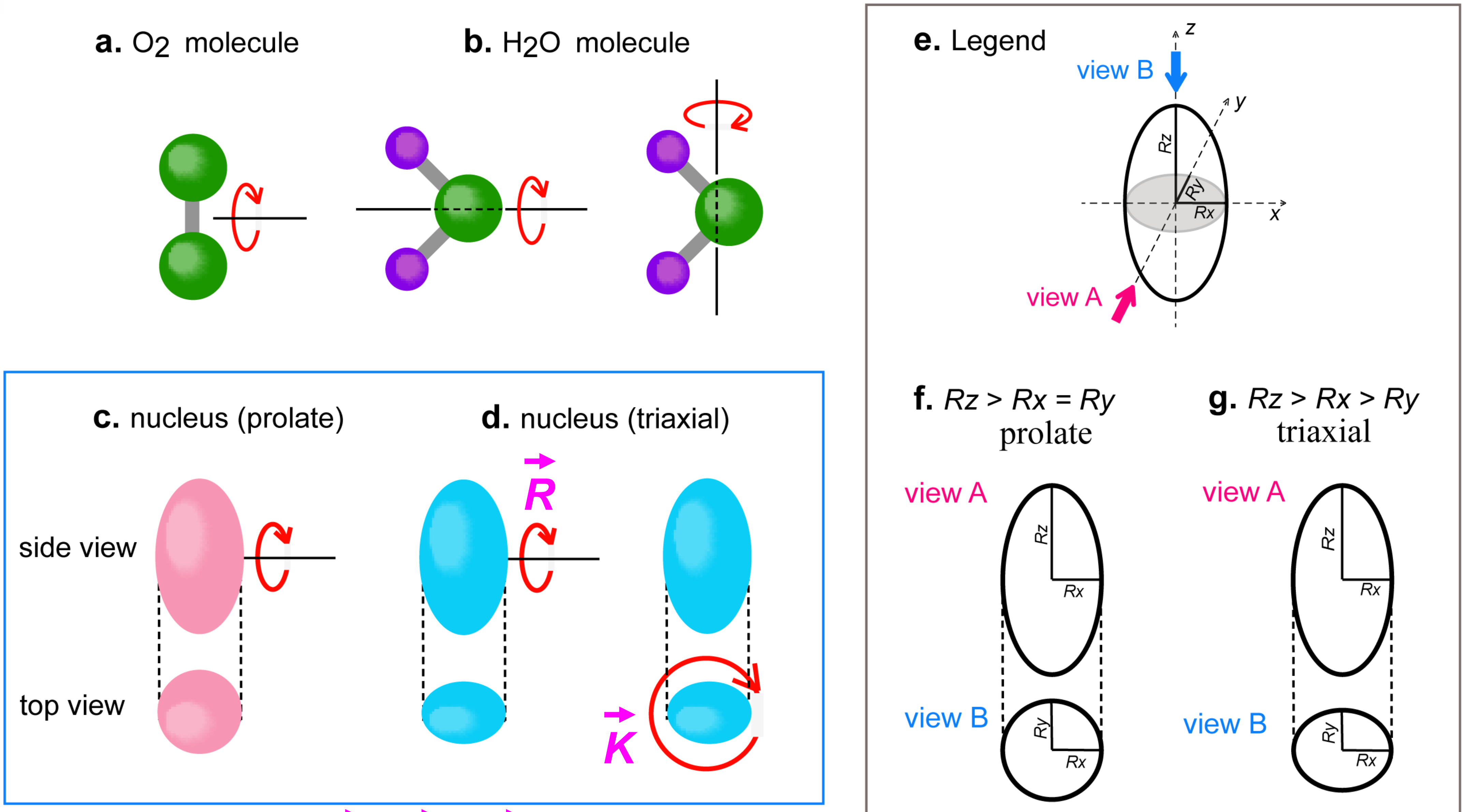
Molecular and nuclear rotations

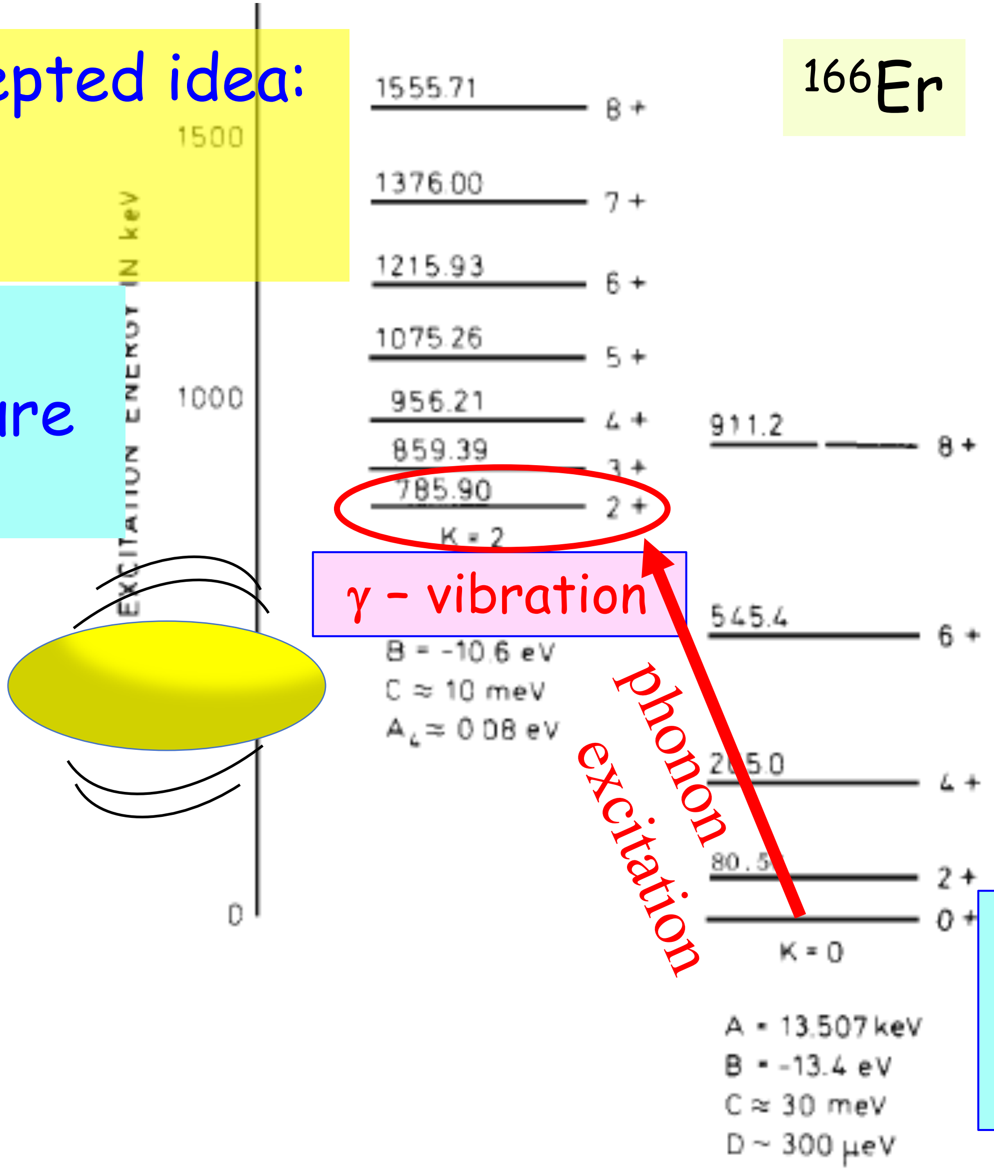


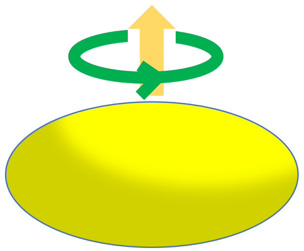
$\vec{J} = \vec{R} + \vec{K}$ *K can be approximately conserved*

Traditionally accepted idea:
Preponderance of prolate shape

Aage Bohr
Novel Prize Lecture
(1975)

Aage N. Bohr, 1922-2009
Nobel Foundation archive





Axially symmetric prolate ellipsoid (equilibrium)

textbook example:
原子核 by M. Nogami

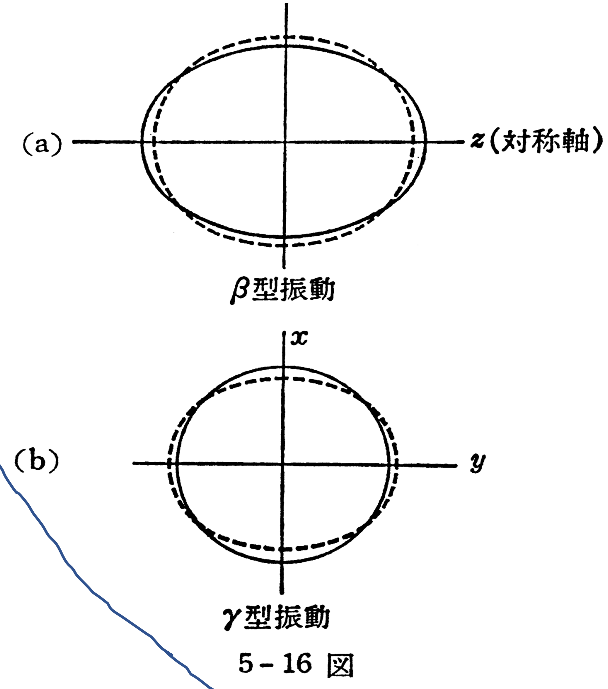


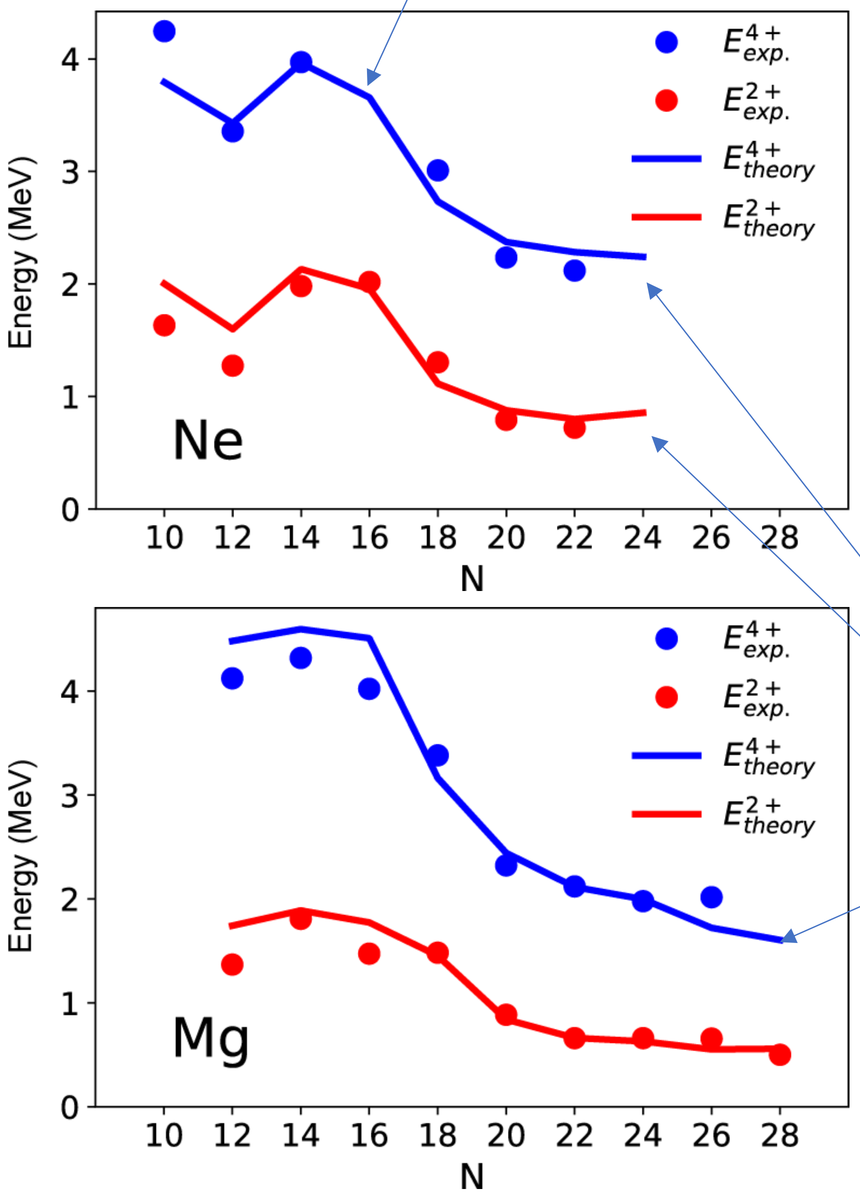
Fig. 9. Rotational bands in ^{166}Er. The figure is from (35) and is based on the experimental data by Reich and Cline (75). The bands are labelled by the component K of the total angular momentum with respect to the symmetry axis. The K = 2 band appears to represent the excitation of a mode of quadrupole vibrations involving deviations from axial symmetry in the nuclear shape.

I will discuss what shapes appear in exotic nuclei and in deformed heavy nuclei, by utilizing contemporary shell model calculations.

→ Although the nuclear shape is a classical subject (~70 years old), there seem to be surprises brought in by radioactive isotope science, and the traditional view may be superseded ...

We start with shapes near driplines predicted by shell model calculations with *ab-initio* effective *NN* interaction, EEdf1.

2^+ and 4^+ level systematics of Ne and Mg isotopes up to driplines



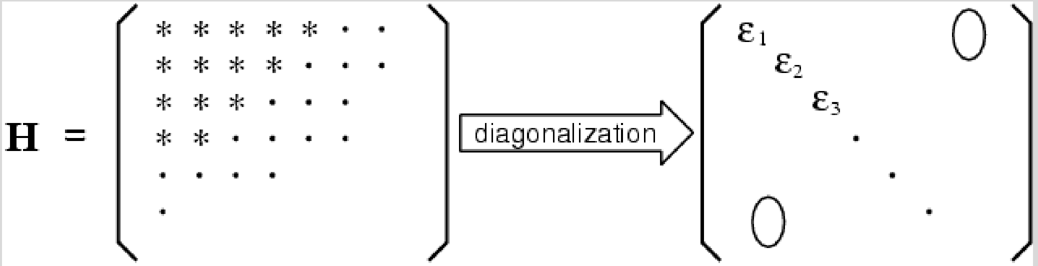
The EEdf1 Hamiltonian appears to be reasonable up to N~28 for Z=9-12.

No magicity of N=20 shows up, whereas those of N=14 and 16 do in both experiment and theory.

Levels may not exist as bound states, because their energies are above the threshold of neutron emission.

Monte Carlo shell model (MCSM)

More than 10^{15} (in our model spaces)



Conventional Shell Model

all Slater determinants

Direct diagonalization is infeasible in large model space

about 100

Monte Carlo Shell Model

bases important for a specific eigenstate

In MCSM, we diagonalize small Hamiltonian matrix constructed from MCSM bases

Generated to minimize eigenenergies using stochastic and variational methods

$$|\Psi_N\rangle = \sum_{n=1}^{N} \sum_{K=-J}^{J} f_{n,K}^{(N)} P_{MK}^{J\pi} |\psi_n\rangle$$

eigenstate

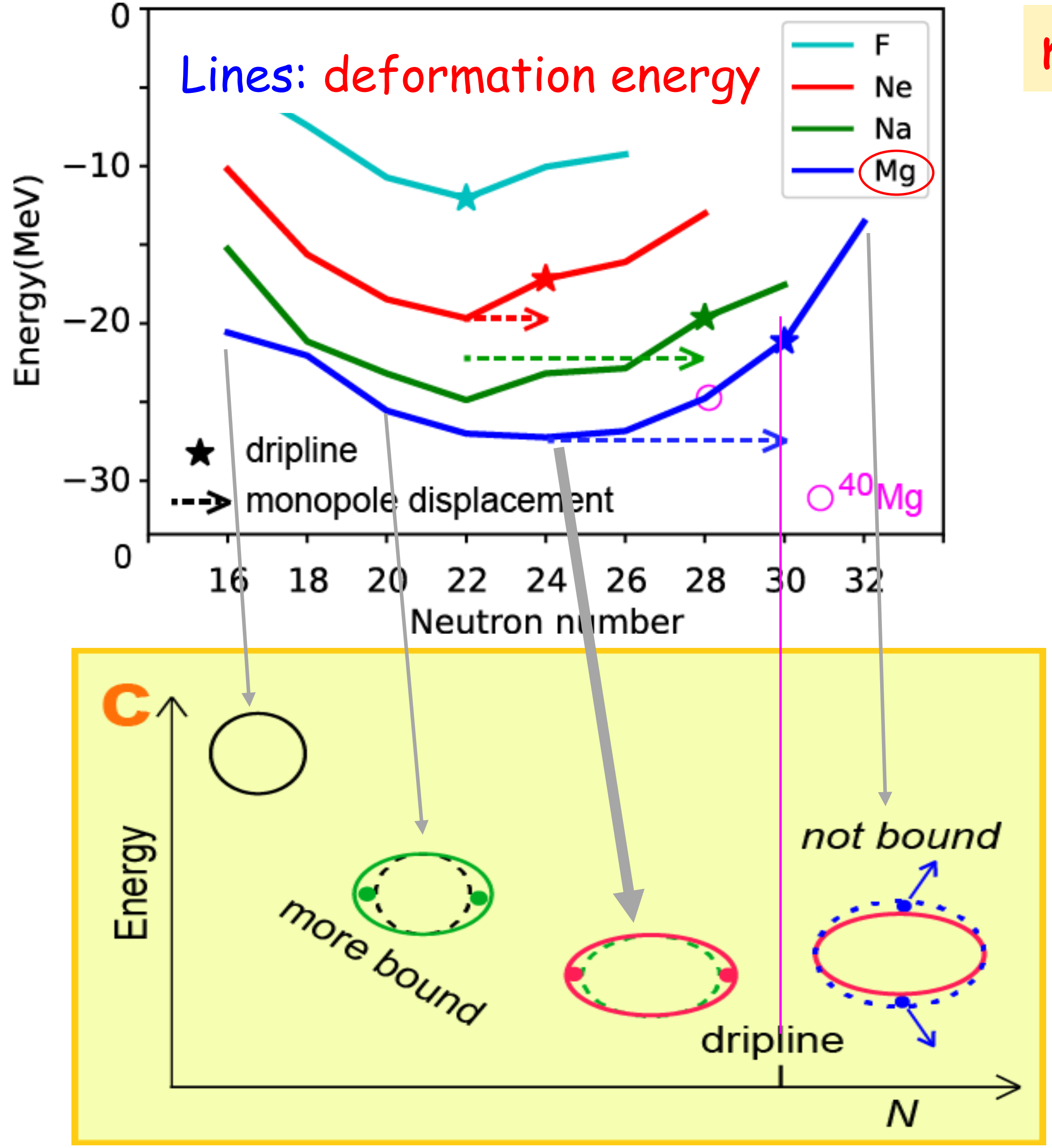
Slater determinant

$$|\psi_n\rangle = \prod_k \left(\sum_l D_{lk}^{(n)} c_l^\dagger \right) |-\rangle$$

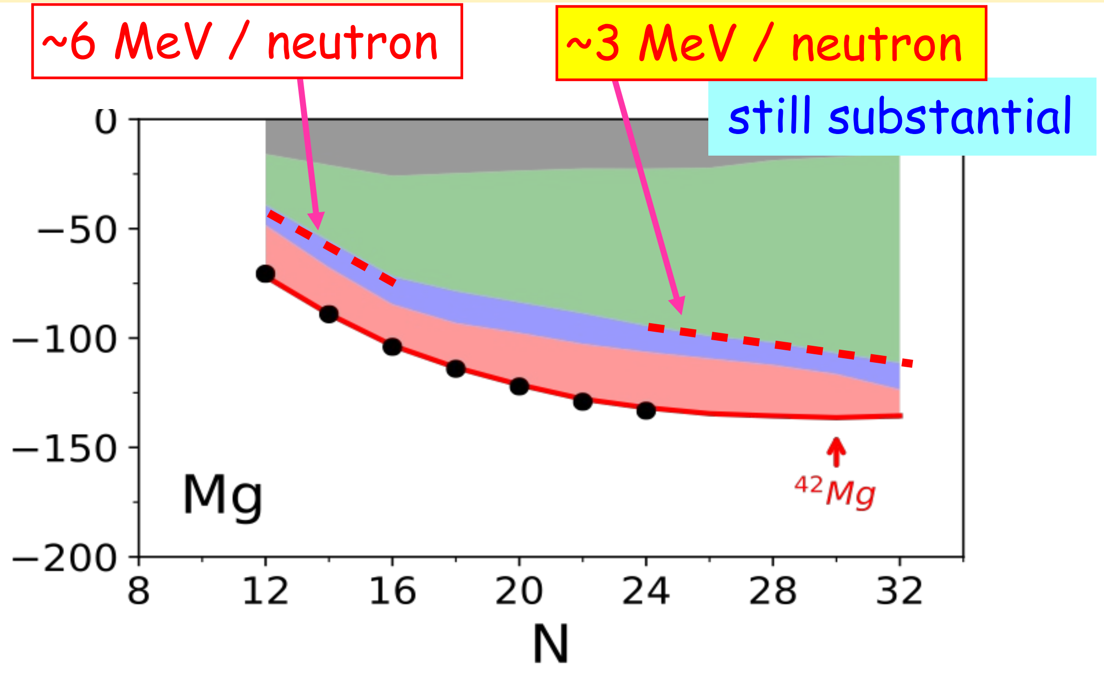
angular-momentum, parity projection

T. Otsuka *et al.*, PPNP47, 319 (2001)

Two driving forces: example from Mg isotopes



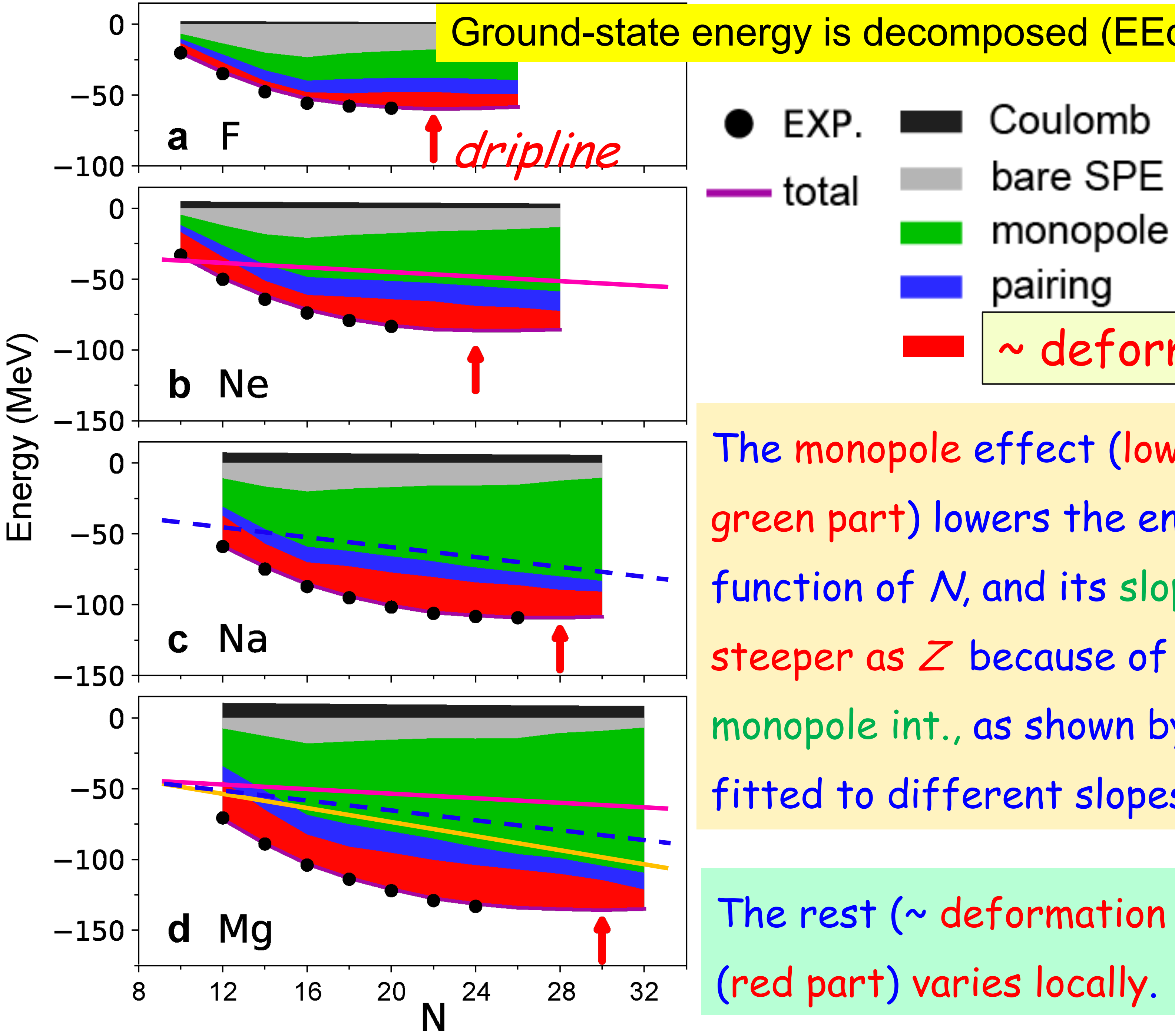
monopole effect = S.P.E. + monopole int.



The deformation effect is saturated (or maximized) at N=24.

The monopole effects compensate the decrease of deformation energy, and pushes the dripline away (dashed arrows).

Ground-state energy is decomposed (EEdf1 int.)



monopole effects

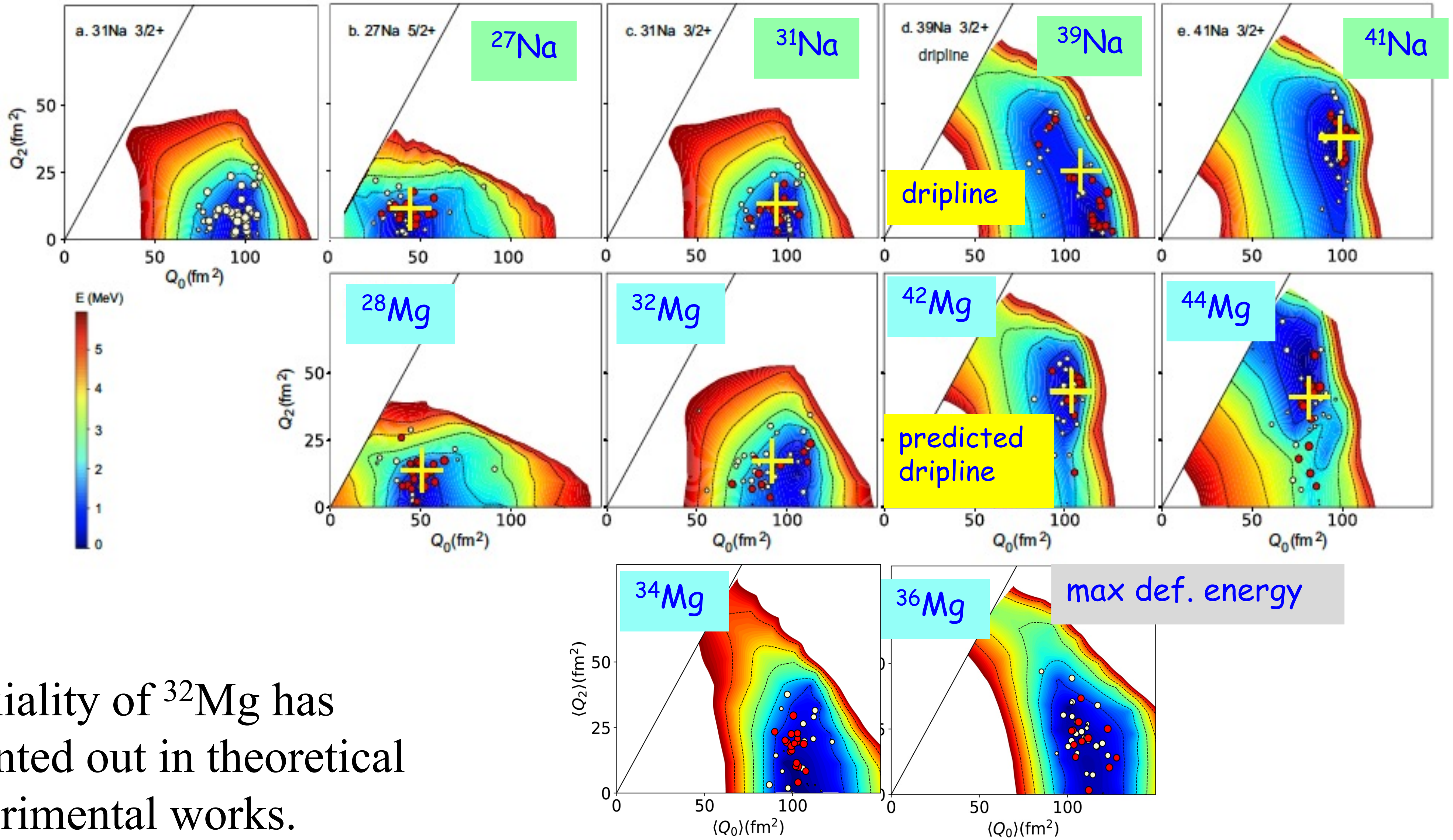
~ deformation effects

The monopole effect (lower edge of green part) lowers the energy as a function of *N*, and its slope becomes steeper as *Z* because of the p-n monopole int., as shown by three lines fitted to different slopes.

The rest (~ deformation energy effect) (red part) varies locally.

Growing triaxiality towards driplines

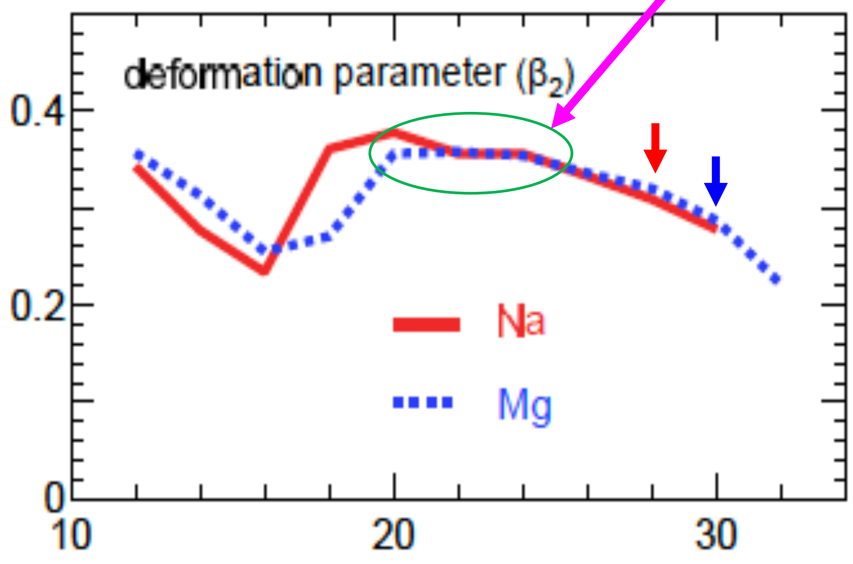
T-plots on the PES



The triaxiality of ^{32}Mg has been pointed out in theoretical and experimental works.

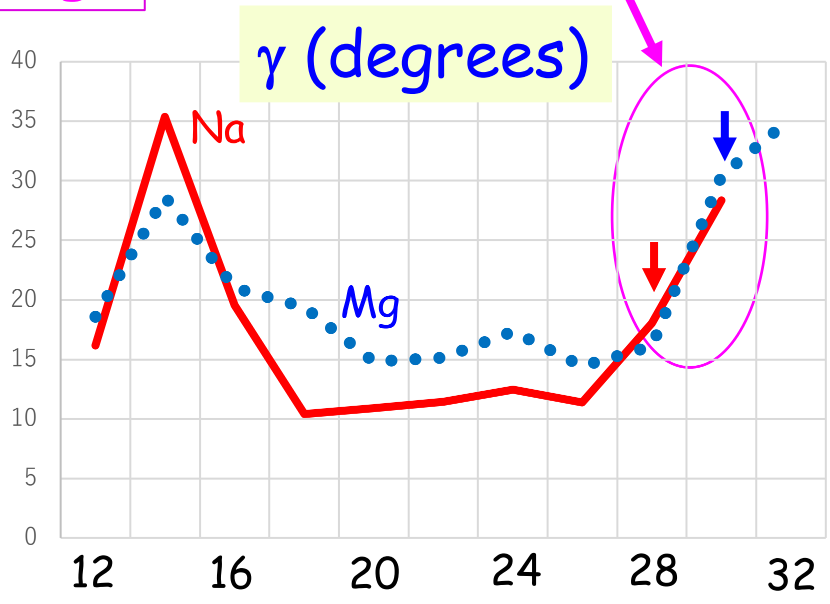
Deformation parameters, β and γ , extracted from T-plot

maximum deformation (Mg)



Neutron number (N)

growing triaxiality



Neutron number (N)

Mg dripline is shifted away due to the monopole interaction

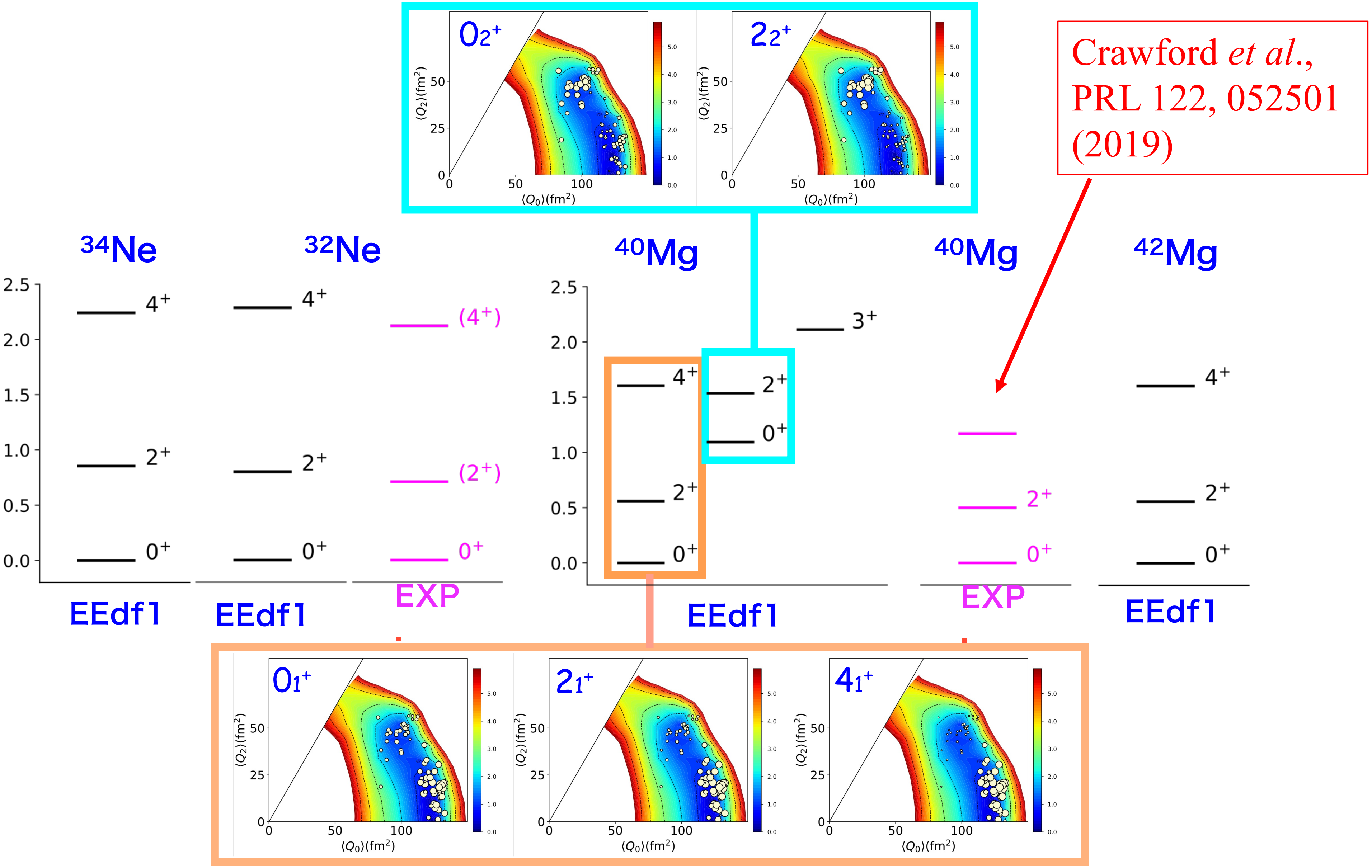
Na
Mg
driplines

$$\beta_2 = \sqrt{5/16\pi}\,\{(e+e'_p+e'_n)/e\}\,(4\pi/3R_0^2A^{5/3})\sqrt{(Q_0)^2+2(Q_2)^2}$$

$$\gamma = \arctan\{\sqrt{2}\,Q_2/Q_0\}$$

Shape coexistence near dripline

because of the weakening of the ground-band deformation ?



This part is published in

nature

Article | Published: 04 November 2020

The impact of nuclear shape on the emergence of the neutron dripline

Naofumi Tsunoda, Takaharu Otsuka, Kazuo Takayanagi, Noritaka Shimizu, Toshio Suzuki, Yutaka Utsuno, Sota Yoshida & Hideki Ueno

Nature **587**, 66–71(2020) | Cite this article

"Moments and radii of exotic Na and Mg isotopes"

TO, N. Shimizu and Y. Tsunoda

Phys. Rev. C, 105, 014319 (2022)

From a global viewpoint, this mechanism can be interpreted :

physics

Physics 2022, 4, 258-285.
https://doi.org/10.3390/physics4010018

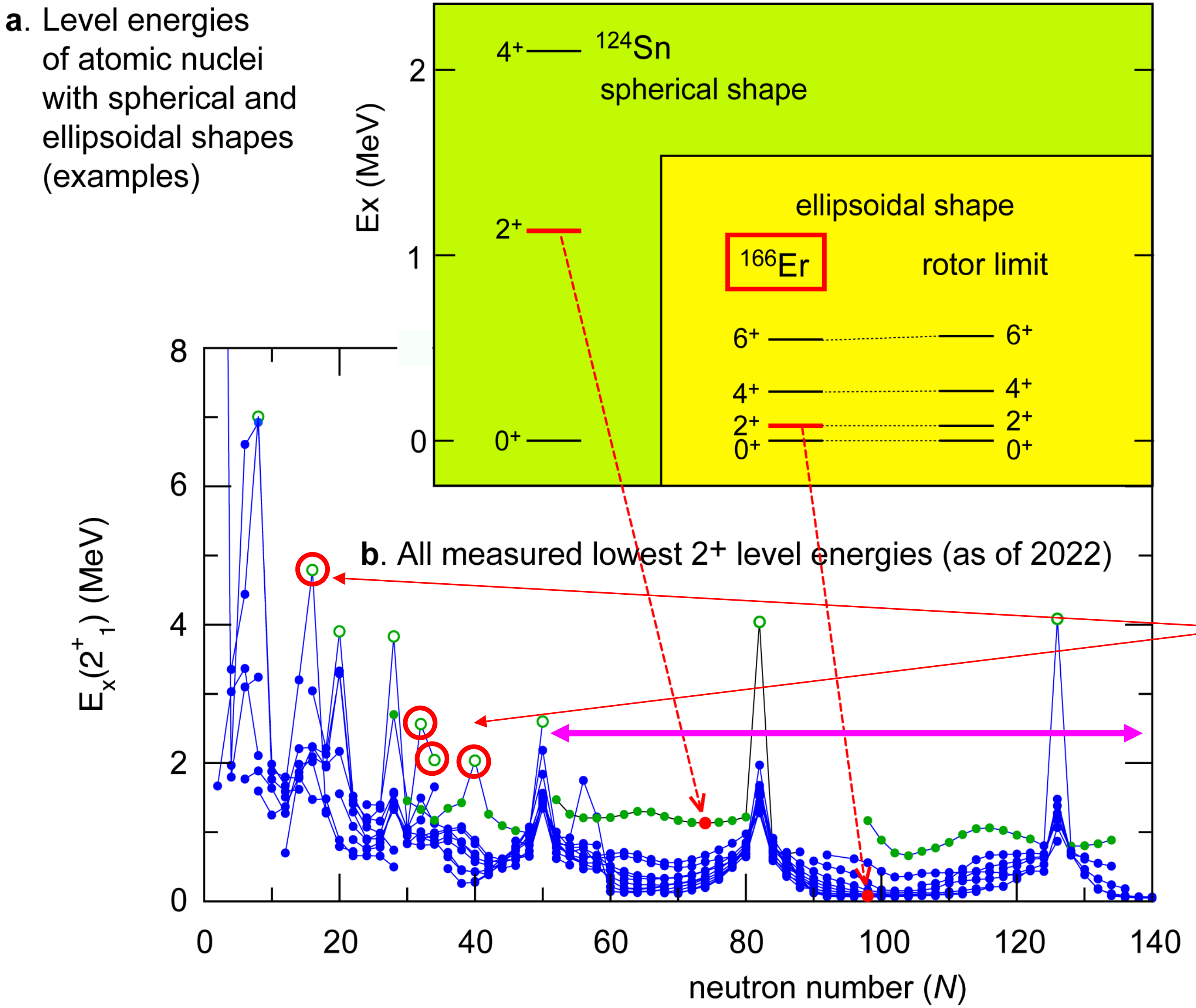
MDPI

Review

Emerging Concepts in Nuclear Structure Based on the Shell Model

Takaharu Otsuka [1,2,3]

Excitation energies of the lowest 2^+ states of even-even nuclei



- ○ magic nuclei
- ● semi-magic nuclei
- ● other nuclei

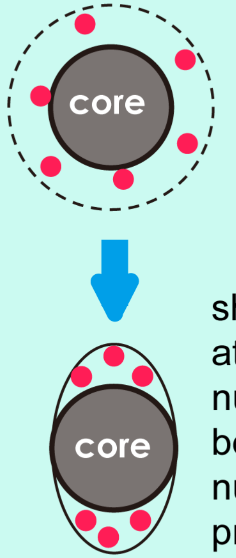
In exotic nuclei, the shell evolution due to tensor + central monopole interactions produce new magic numbers shown by ○ (N= 16, 32, 34, 40), which are absent in Mayer-Jensen model.

Question:
What happens in heavy nuclei

Revisit with Monte Carlo Shell Model

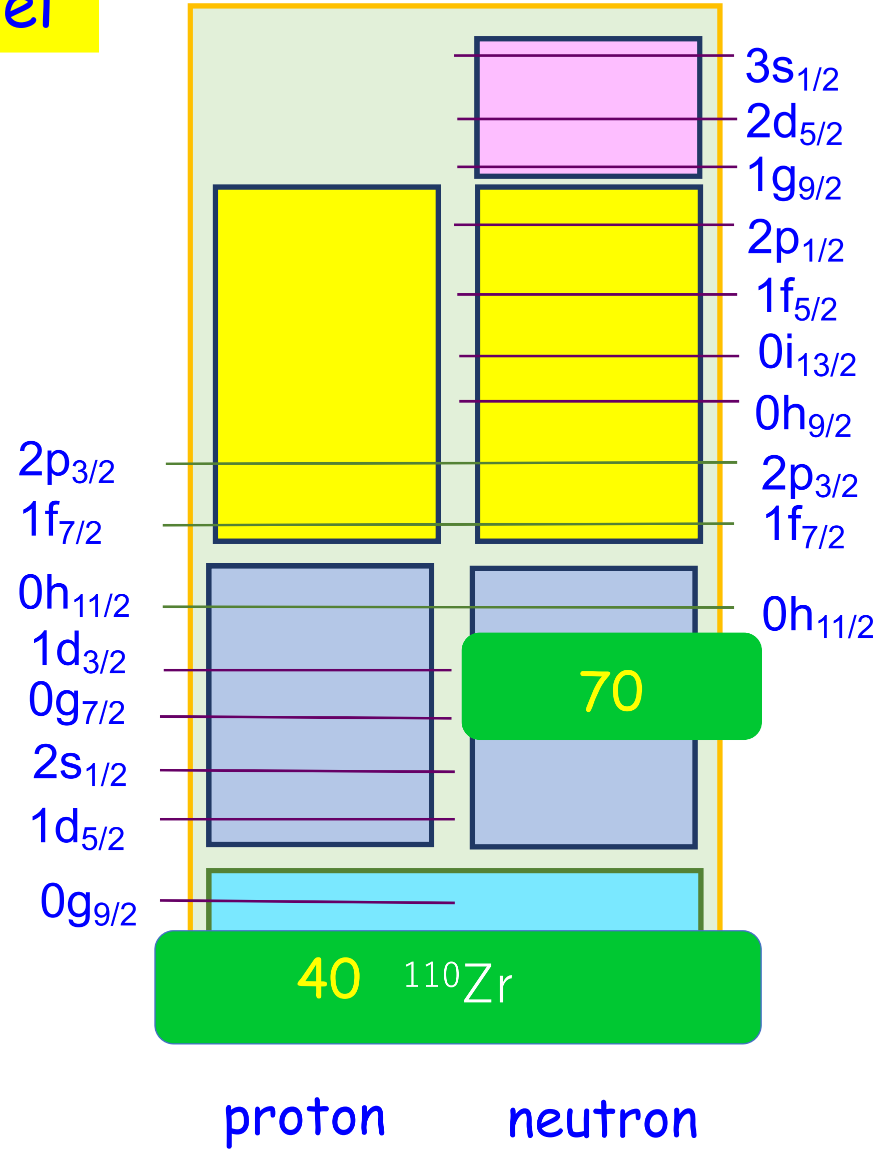
Effective interaction:
G-matrix* + V_{MU}

* Brown, PRL 85, 5300 (2000)



Nucleons are excited fully within this model space (no truncation)

We performed Monte Carlo Shell Model (MCSM) calculations, where the largest case corresponds to the diagonalization of 3.9 x 10^{31} dimension matrix.
Its recent extension, Quasiparticle Vacua Shell Model (QVSM)* is used, for most of the calculations to be shown. * Shimizu *et al*, PRC 103, 014312 (2021)

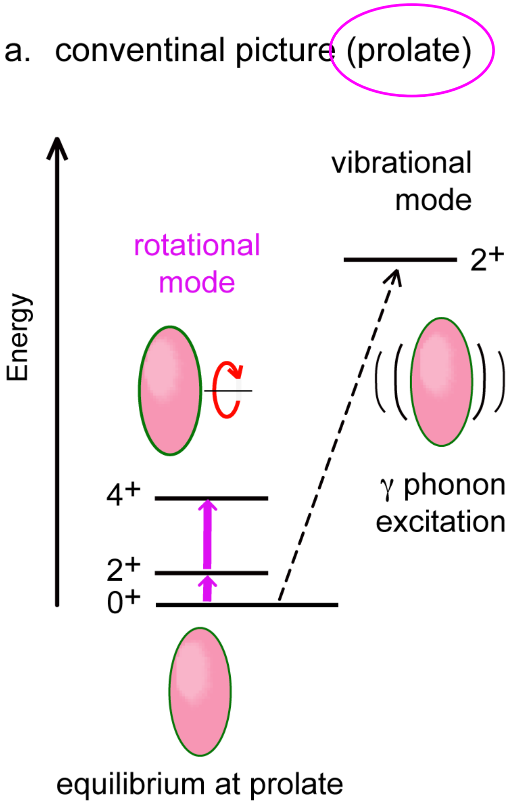


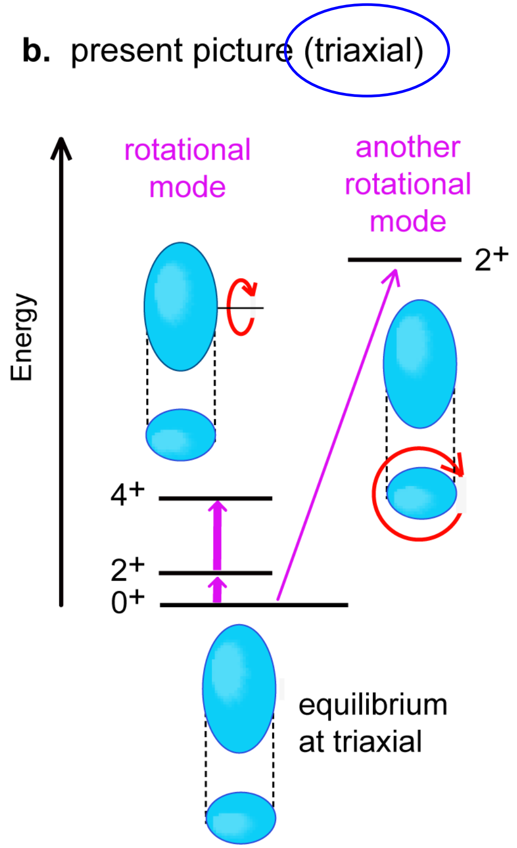
V_{MU} : same interaction for the description of shell evolution in exotic nuclei

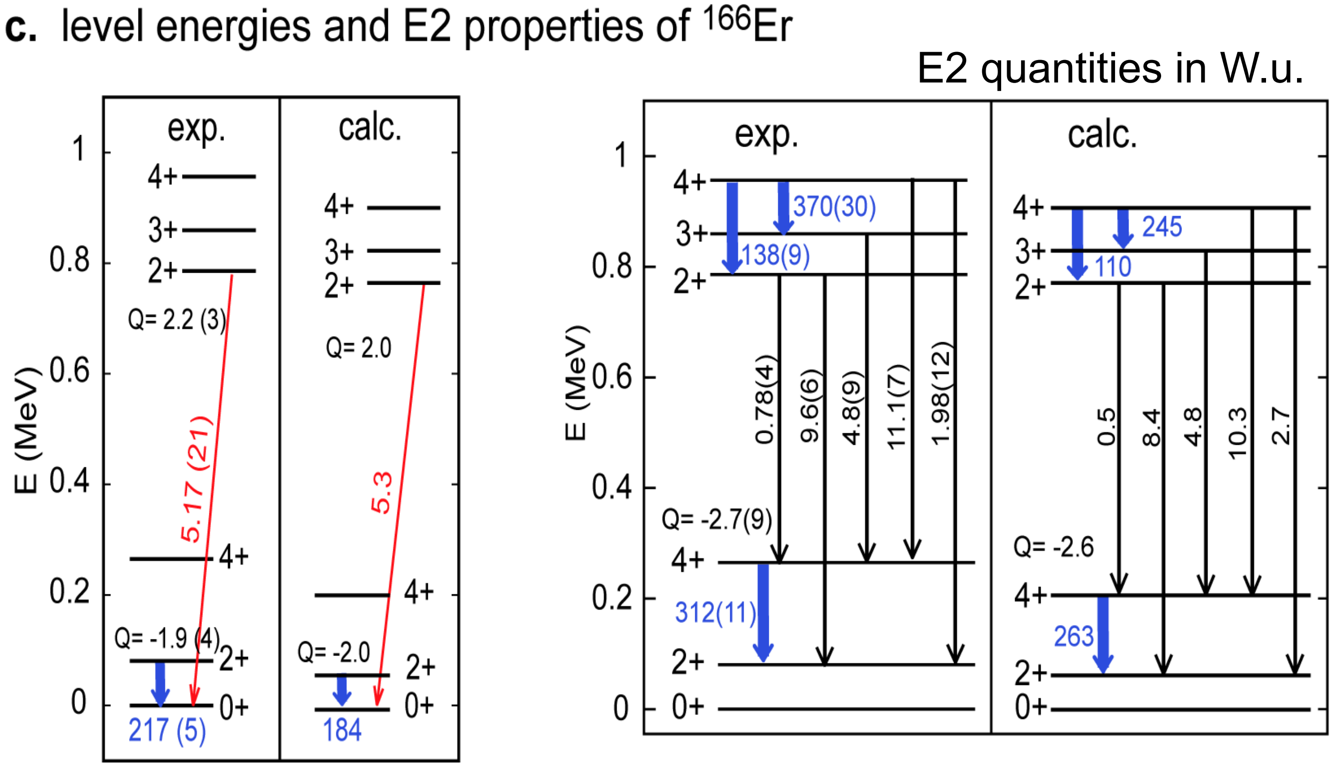
Aage Bohr's picture

Our picture

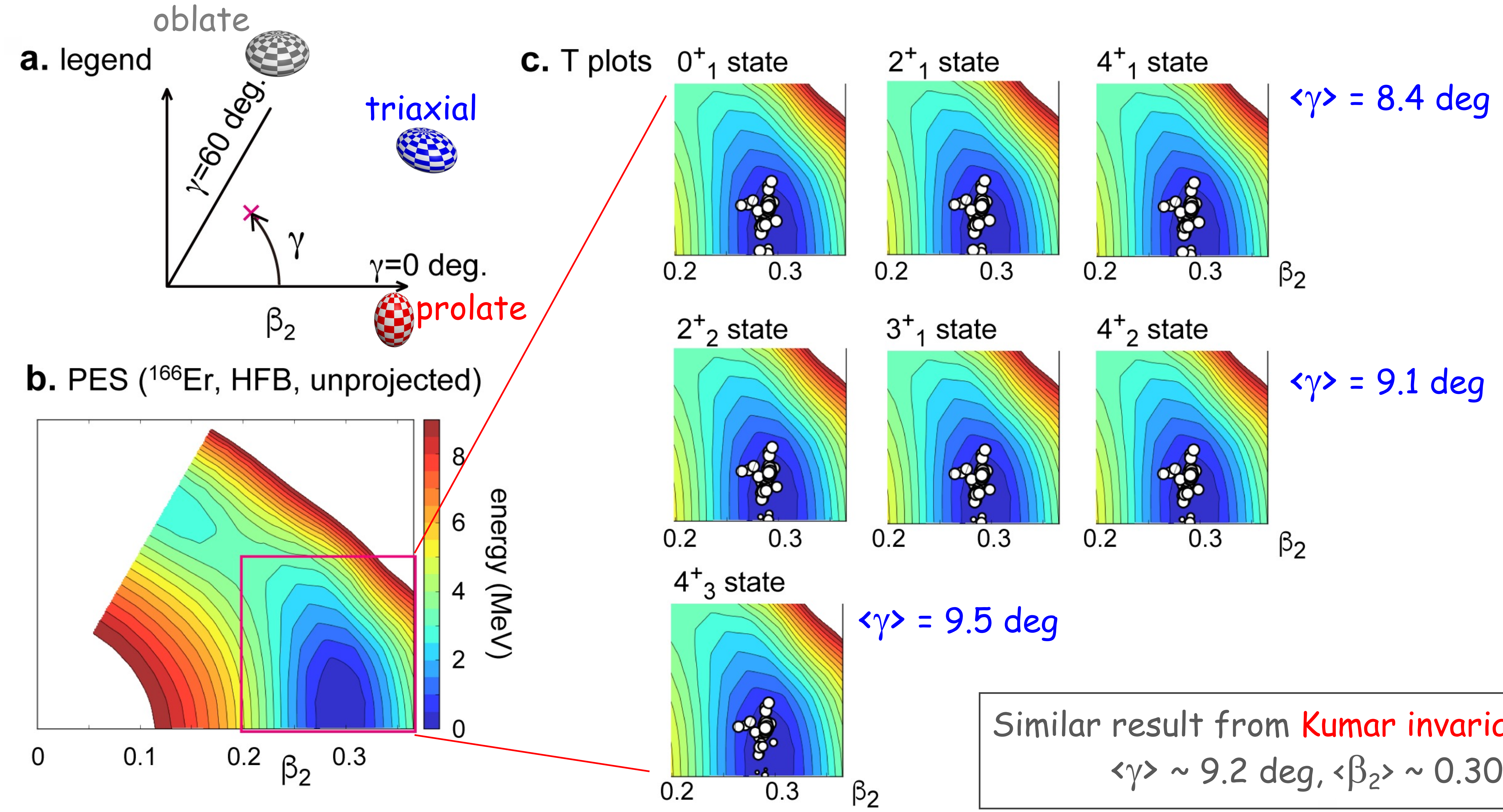
Result of MCSM calculation







PES and T-plot of the ground and lowest states of ^{166}Er



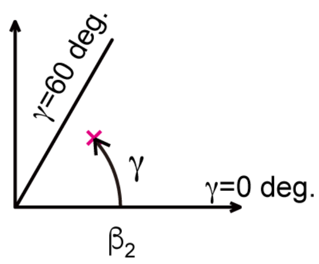
Similar result from Kumar invariant

$\langle\gamma\rangle \sim 9.2$ deg, $\langle\beta_2\rangle \sim 0.30$

What provides such triaxial shapes
in ground and low-lying state.

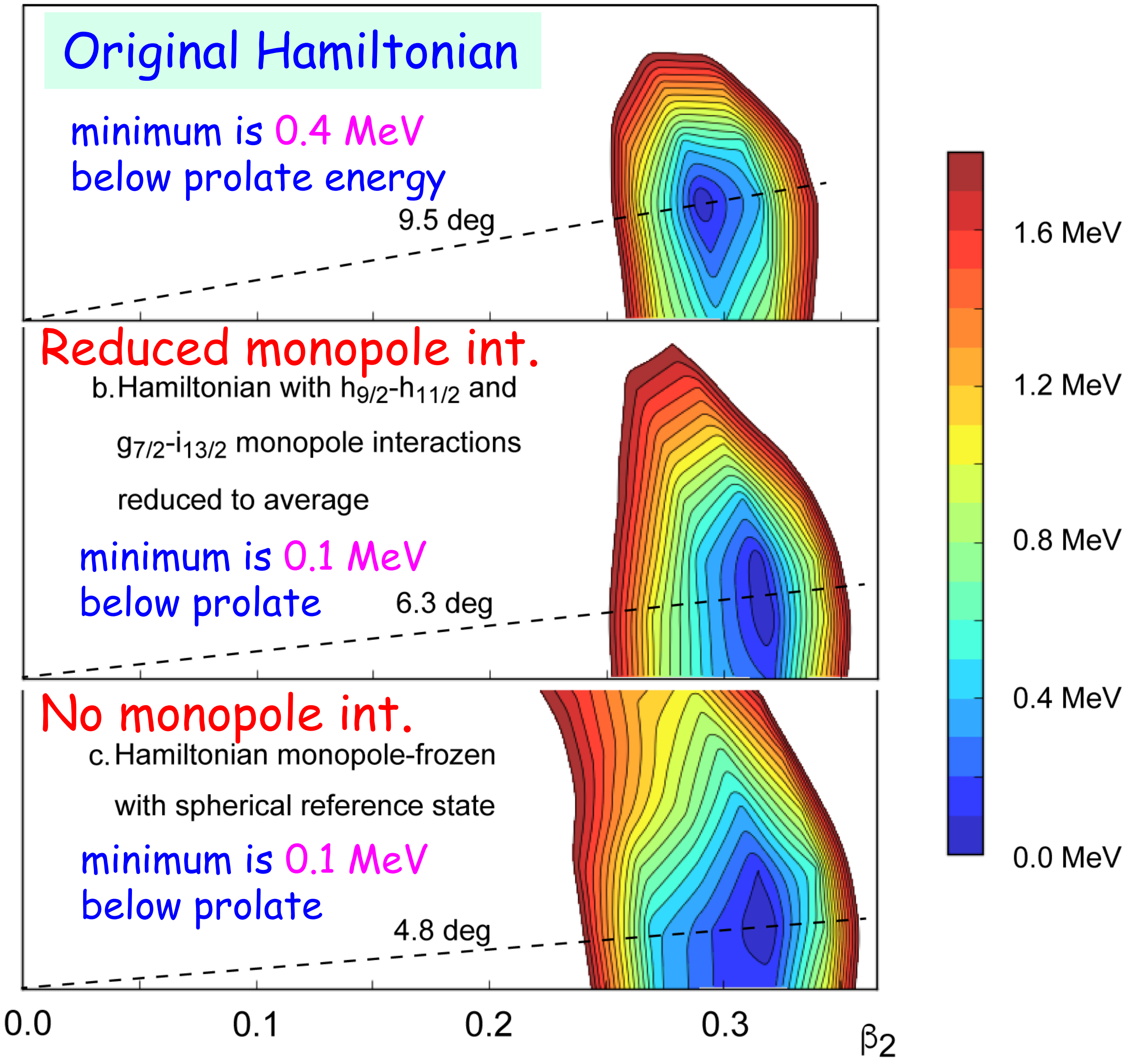
Monopole interaction

Monopole-interaction effects seen in projected PES near the minimum: refined contour plots

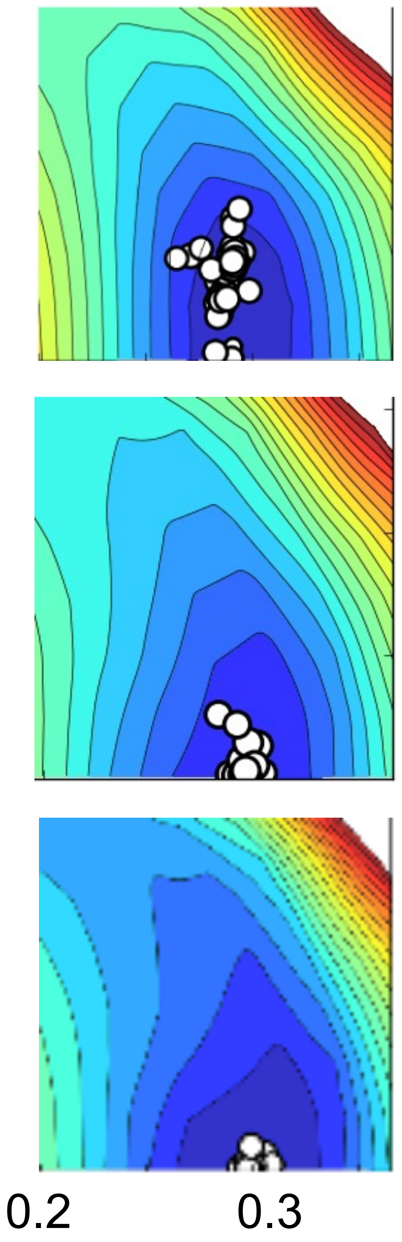


two most attractive monopole interactions $h_{11/2}$-$h_{9/2}$ and $g_{7/2}$-$i_{13/2}$ are weakened to average value

monopole interactions are replaced by constant SPEs assessed for spherical reference state (Monopole-Frozen)

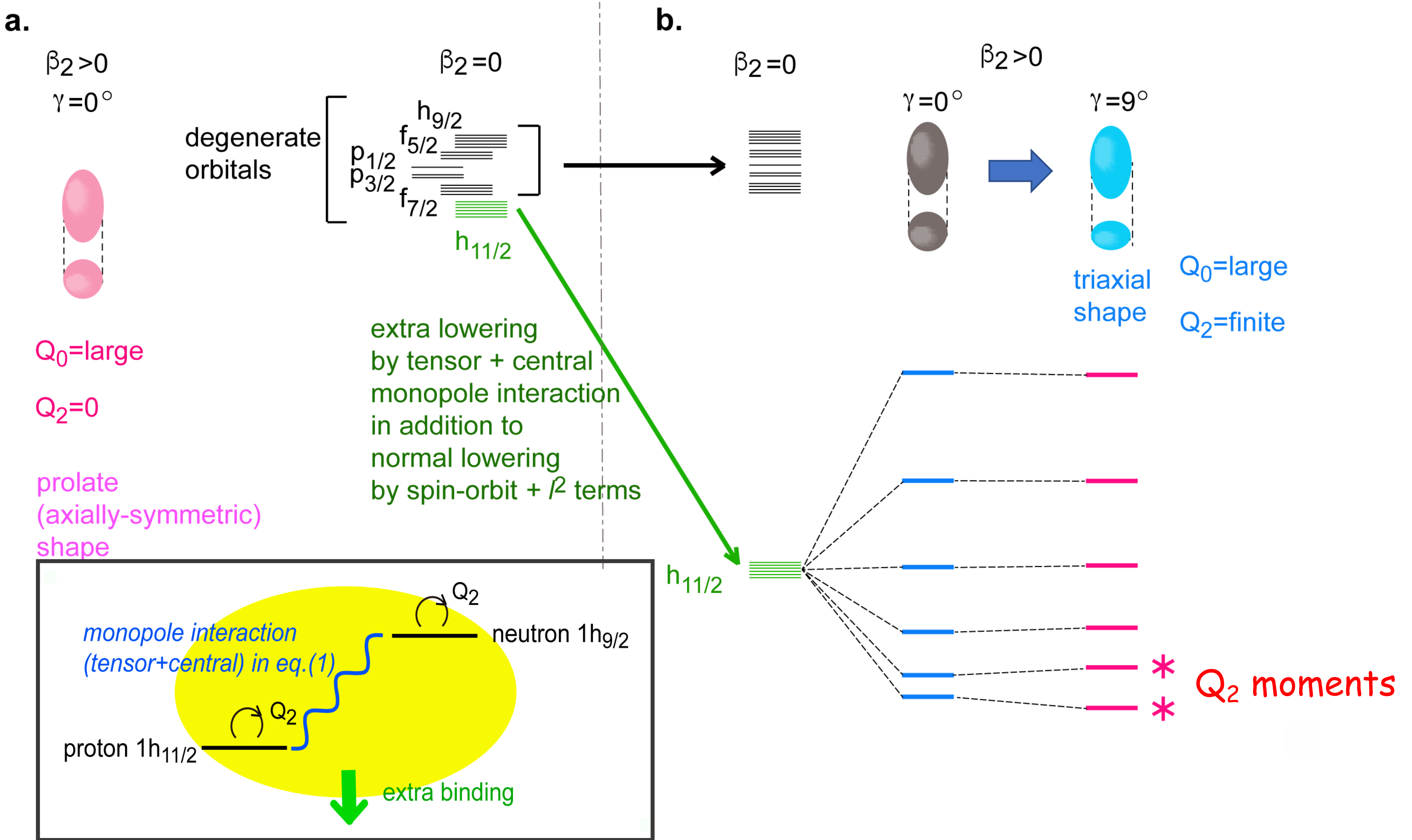


T plot of 0^+_1 state on unprojected PES



Prolate shape produced by many single-particle orbitals

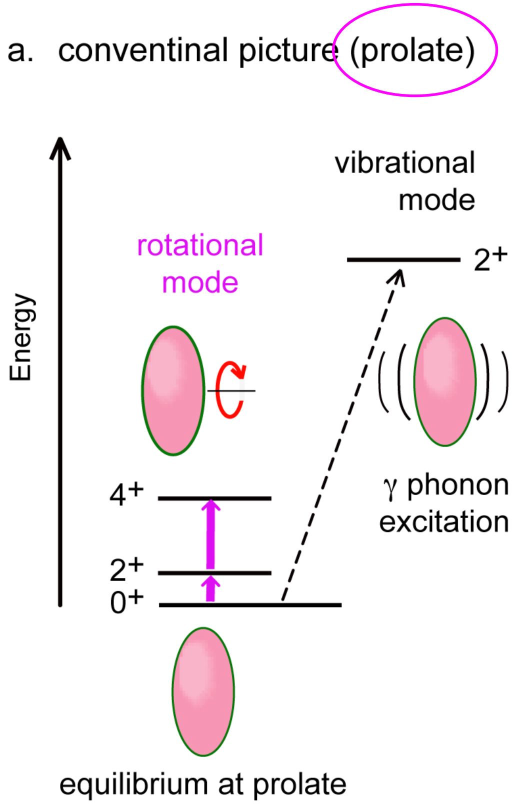
Triaxial shape produced by large-j single-particle orbital lowered by the monopole int.

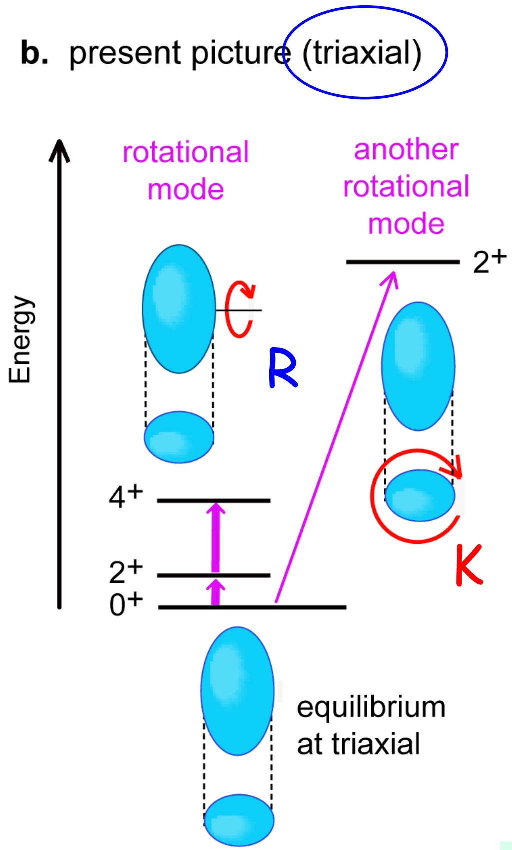


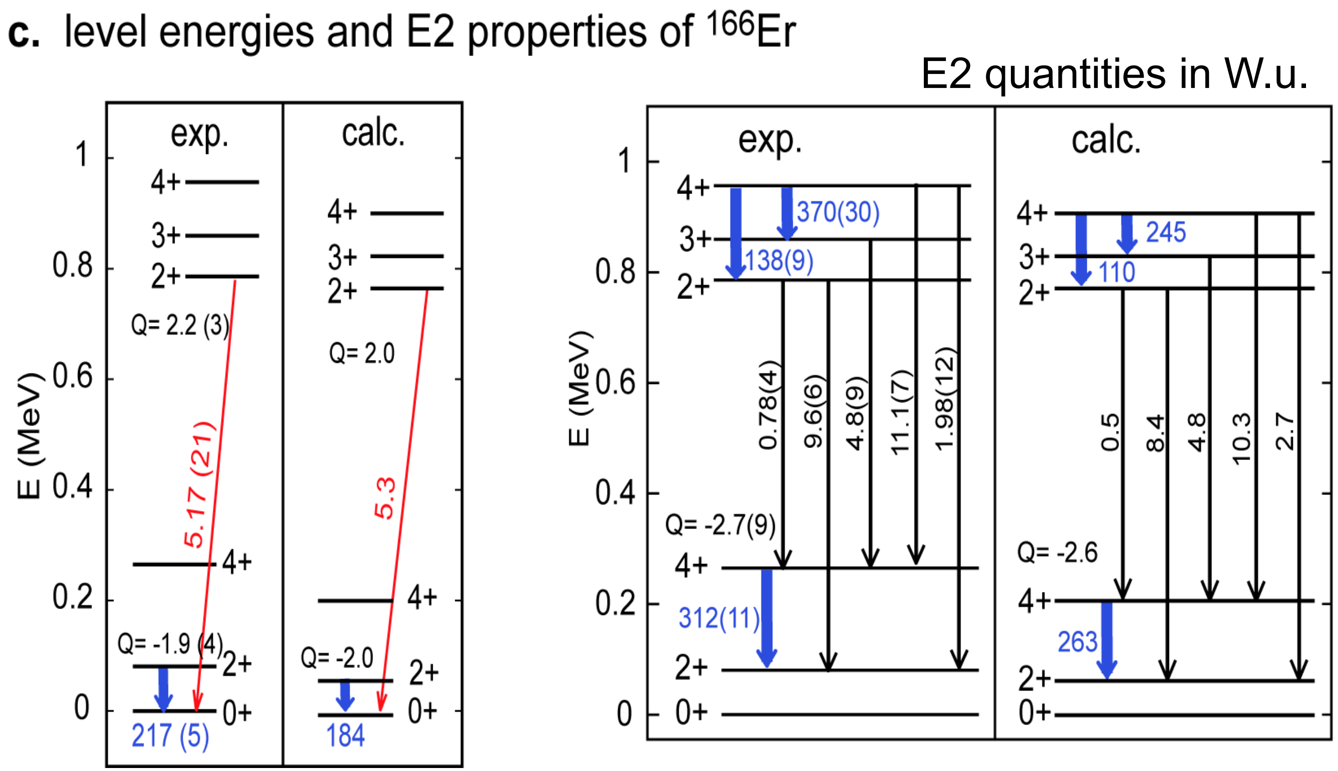
Aage Bohr's picture

Our picture

Result of MCSM calculation







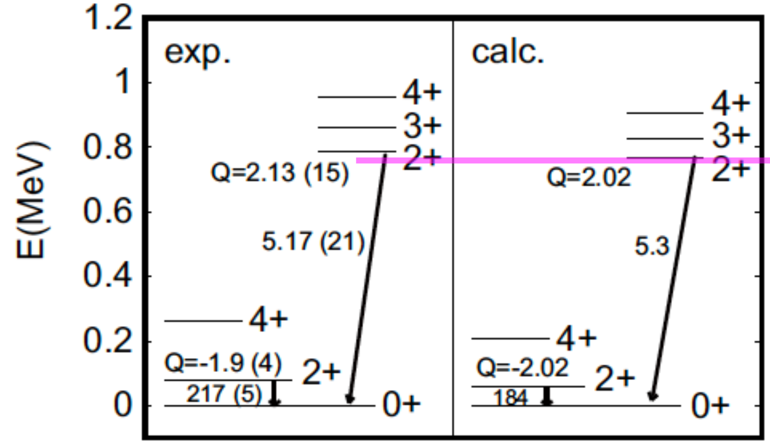
Why is the 2^+_2 level so low ?

Much higher in the Davydov model

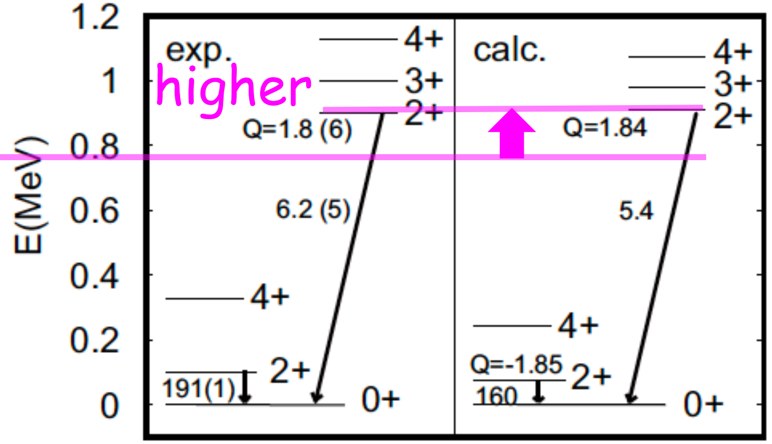
high rigidity for R rotation
lower rigidity for K rotation
-> "stretching" lowers 2^+_2 level by ~0.5 MeV
(Rigid rotor model of Davydov fails)
The value of γ changes by ~ 1 degree.

Variations as Z and/or N changes (examples)

^{166}Er (Z=68, N=98) γ =8.4 deg

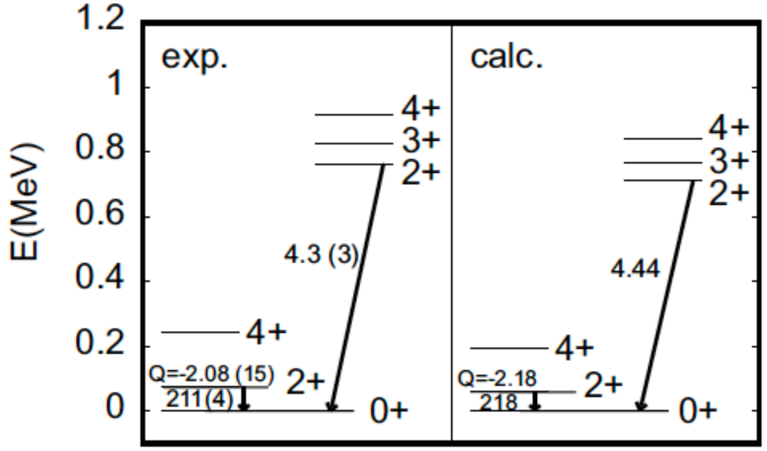


^{162}Er (Z=68, N=94)

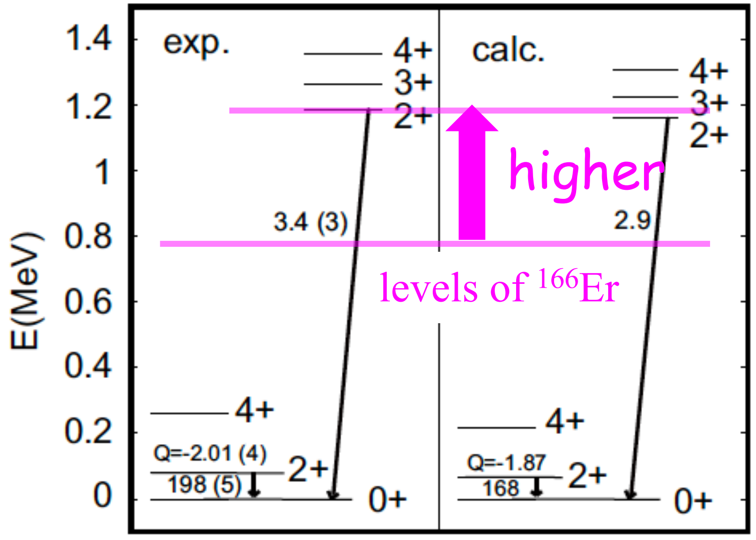


no major change

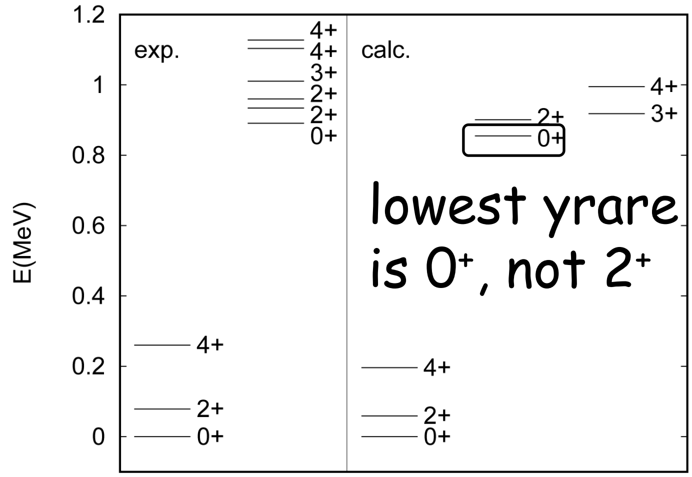
^{164}Dy (Z=66, N=98) γ =7.4 deg



^{158}Gd (Z=64, N=94) γ =5.9 deg



^{170}Er (not triaxial)



~ prolate

Coulex exp. showed consistent γ values

Cline et al. (1986)

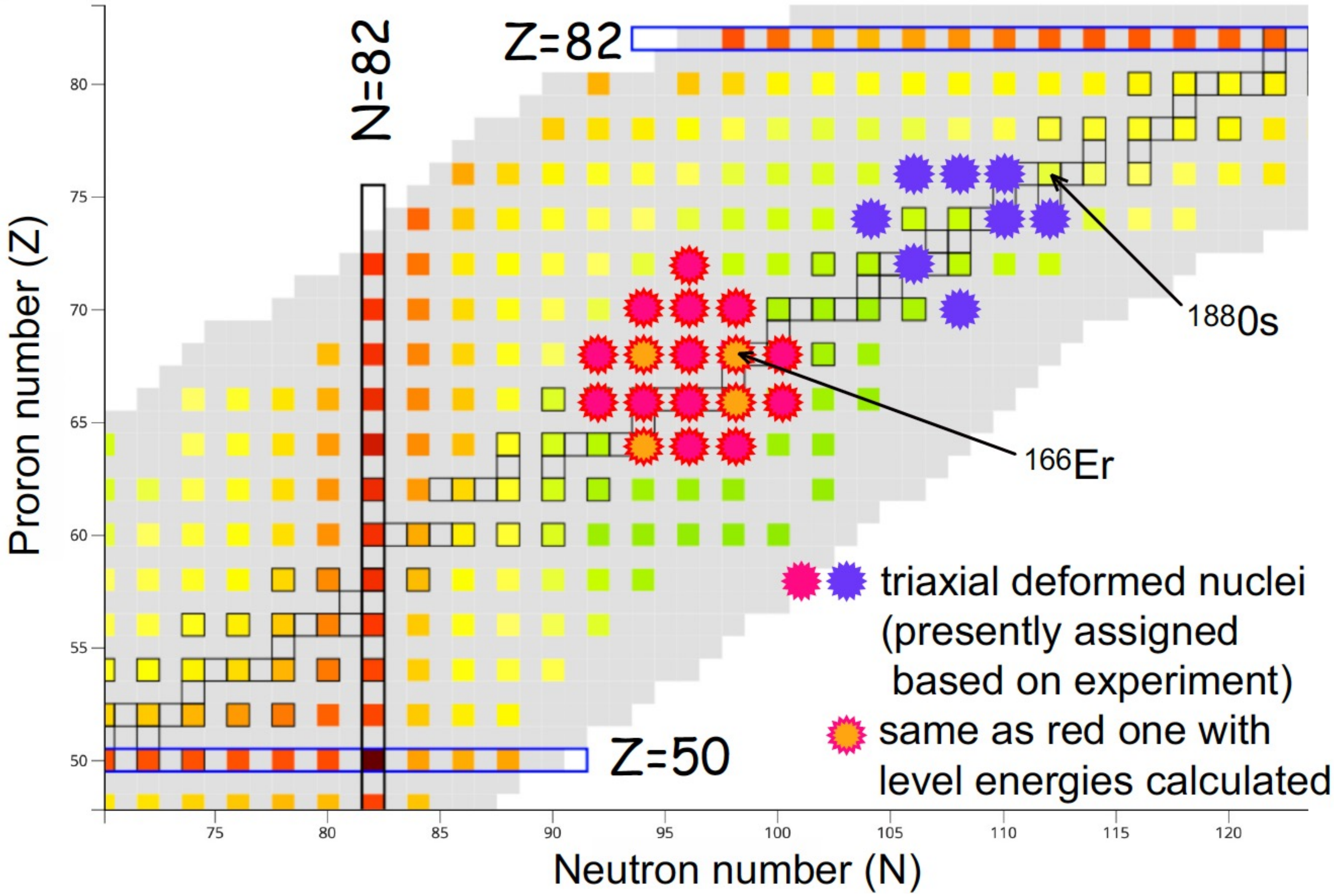
Fahlander et al. (1990)

Werner et al. (2005),

for which natural interpretation is the triaxiality in the ground states.

17 triaxially strongly deformed nuclei around ^{166}Er

($Ex(2^+_2) < Ex(0^+_2)$; there can be more)

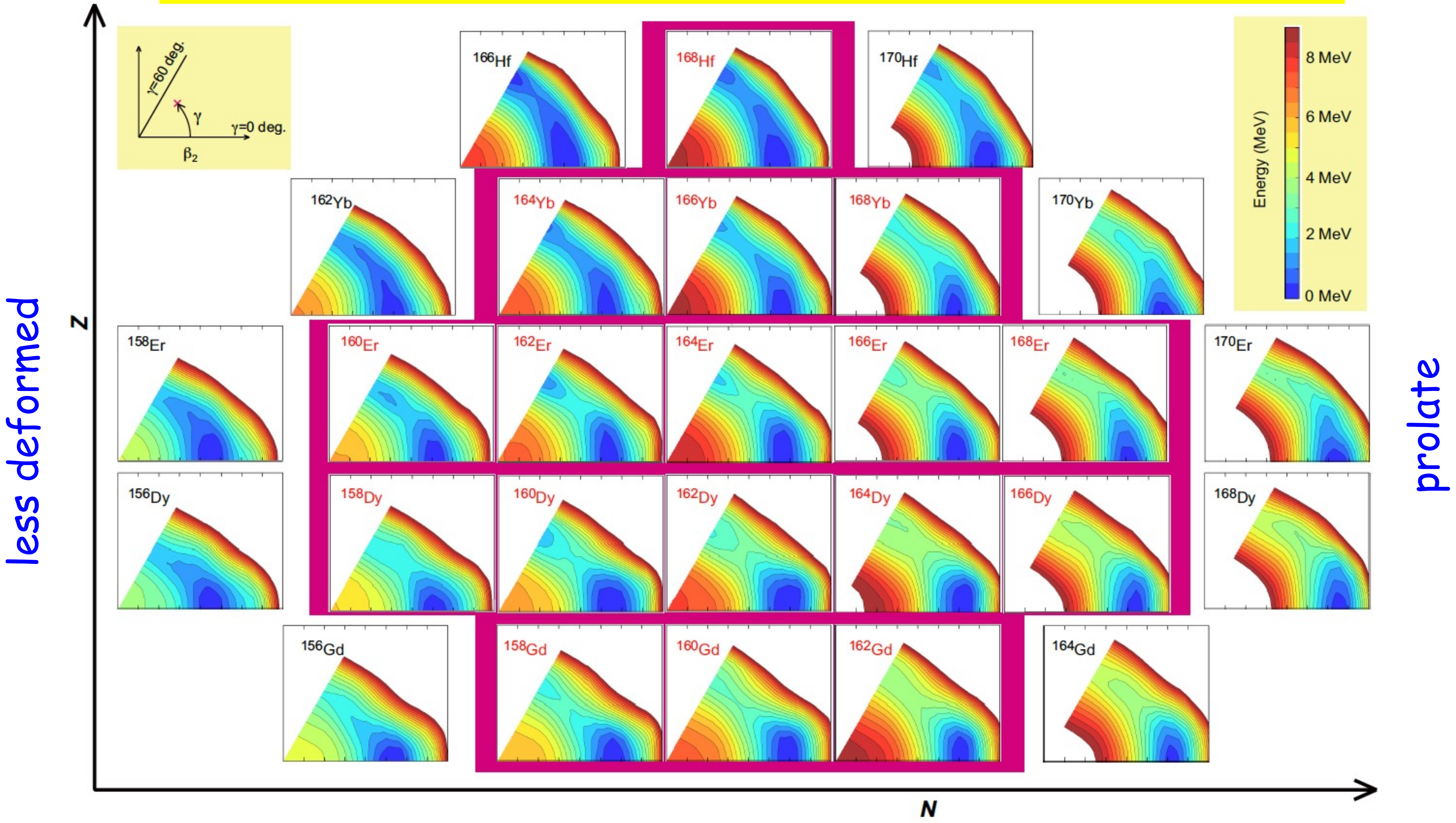


Besides existing Coulex data, could we observe their shapes by *Relativistic Heavy-Ion Collisions* at *LHC* ?
(*cf*: Giacalone *et al*.)

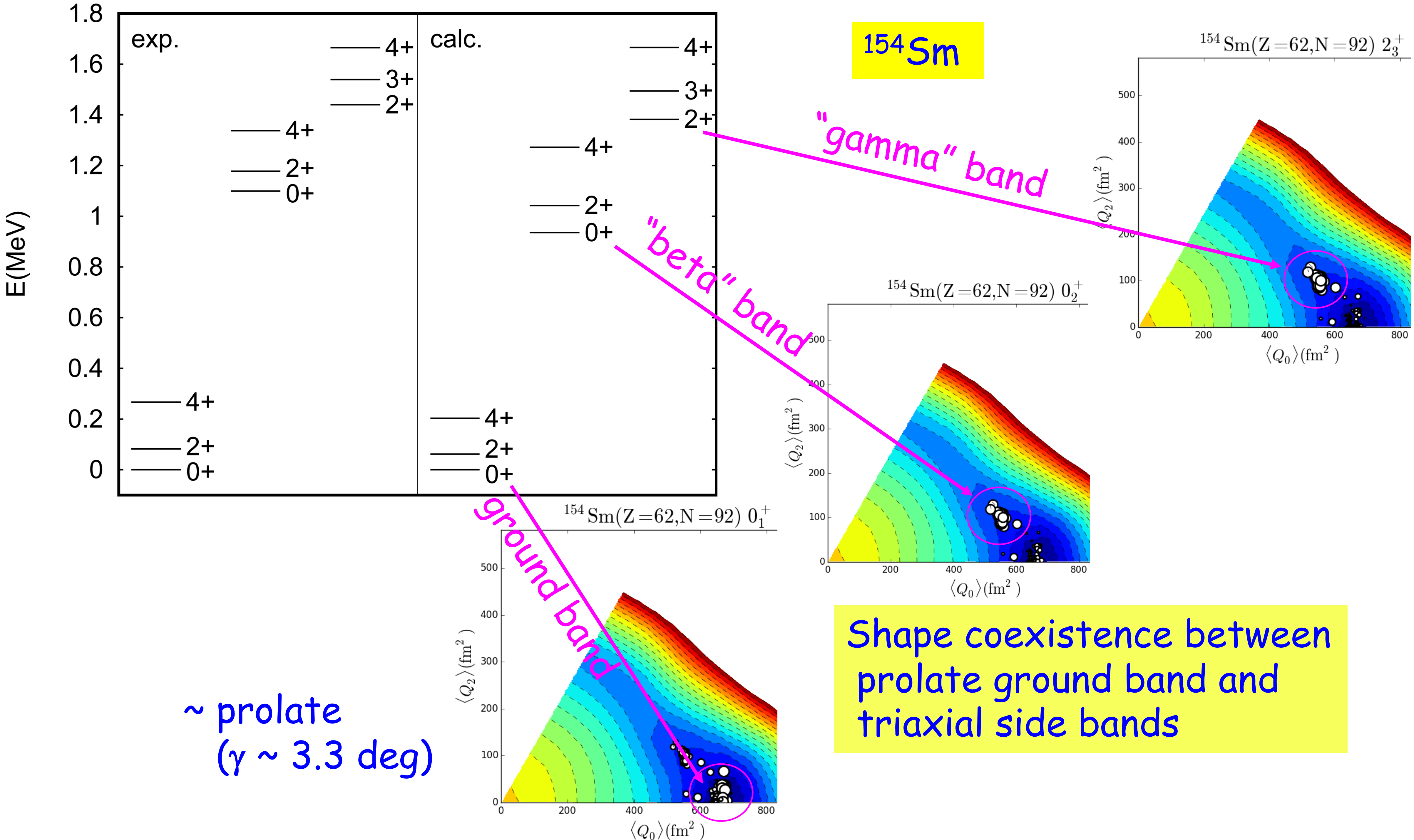
Extended scissors mode (rolling mode) is another possibility *to be studied in HIγS and RCNP.*

Hyper nuclei (with Λ particle) are another possibility in J-Lab and JPARC.

PES of 17 triaxial deformed nuclei as well as their neighbors



^{154}Sm



~ prolate
(γ ~ 3.3 deg)

Shape coexistence between prolate ground band and triaxial side bands

Summary

The shell-model calculation is now feasible up to rotational bands of heavy nuclei, exhibiting some new features involving nuclear forces.

The dripline mechanism due to monopole-quadrupole interplay is found. Evolving triaxial shapes towards driplines and shape coexistence (like ^{40}Mg) are shown.

The majority of heavy deformed nuclei have been considered to be (axially-symmetric) prolate (*a la* A. Bohr). This textbook view seems to be superseded by the prevailing triaxiality due to the central + tensor forces which are responsible for the shell evolution. No gamma vibration is obtained. Thus, RI-beam physics unveils a hidden general feature of the shapes of stable and exotic nuclei. Triaxiality is associated with large-*j* orbitals, like $h_{11/2}$, which suggests impacts on superheavies and fissions as well as superdeformation.

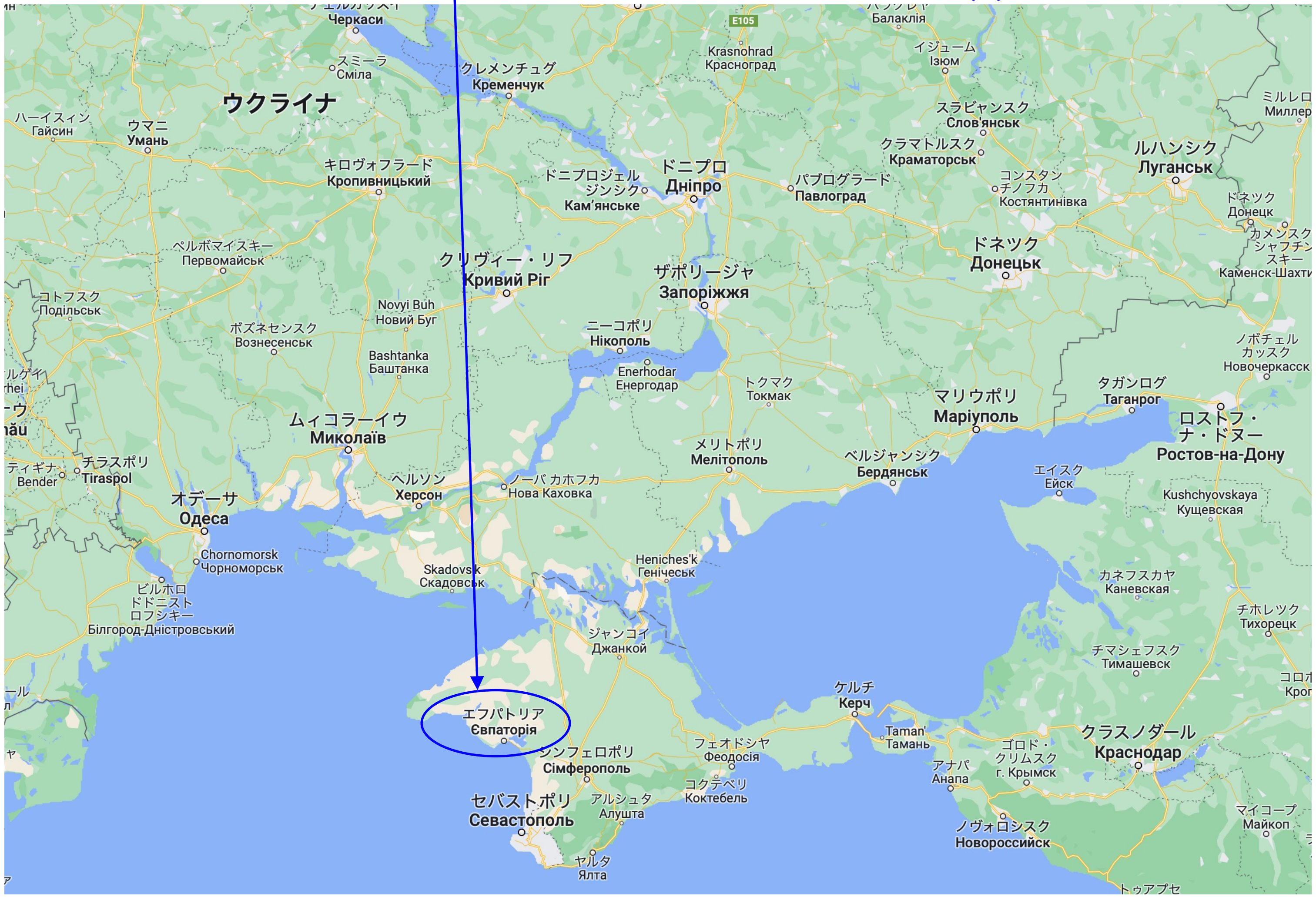
Prolate ground state arises in some nuclei (^{154}Sm) with shape coexistence with triaxial bands.

Known Coulex data obtained around 1990s are supportive of the present idea, but were not addressed this way. (The shell evolution by tensor force was not known before 2005.)

Davydov et al. suggested triaxiality in many nuclei, which appears to be correct, although their rigid-rotor model turned out to be not precise enough.

Alexander S. Davydov, (Ukrainian, 1912 – 1993), suggested triaxiality of nuclear shapes and derived the features resulting from the rotation of triaxial objects. He did not present the underlying mechanism, or the rigid-rotor model may not be too good. Nevertheless, his contributions deserve more appreciation.

from Wikipedia



Google map

E N D

Thank you for your attention

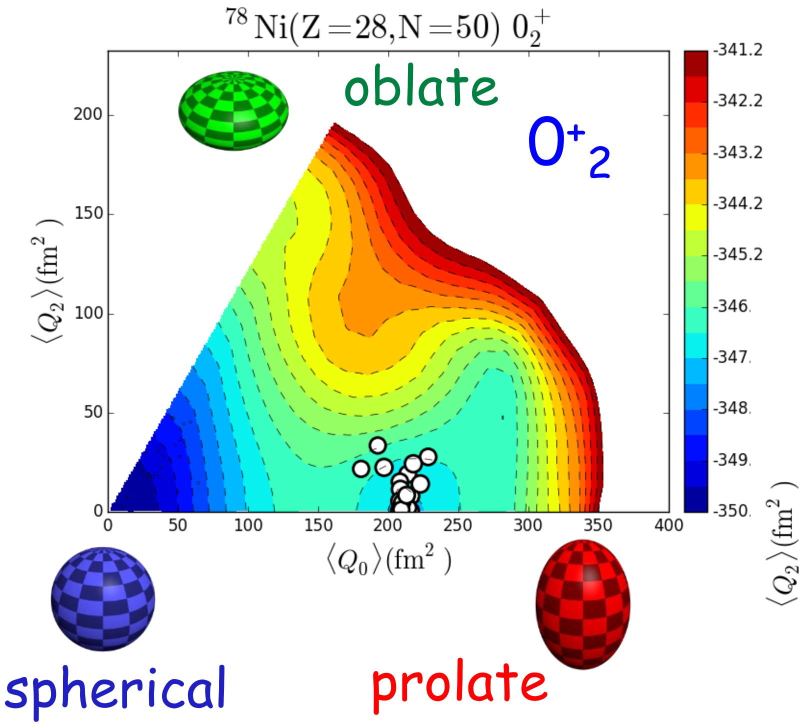
Identification of nuclear shape by T-plot of MCSM

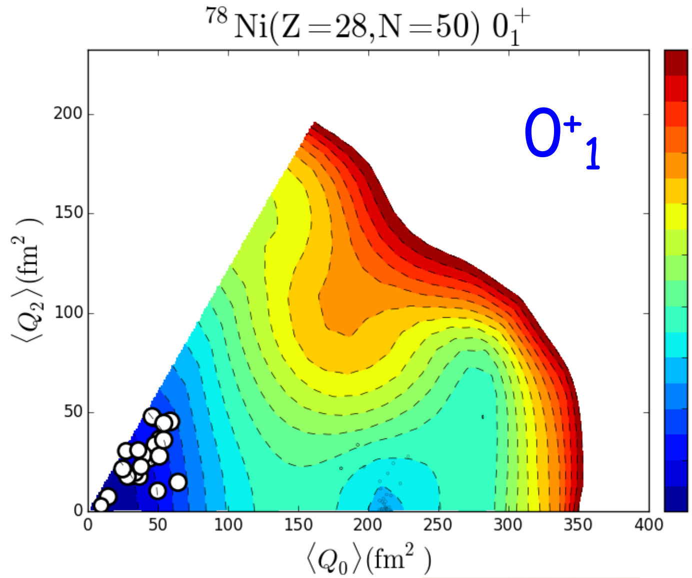
- Location of circle: shape
  quadrupole deformation of unprojected MCSM basis vector

- Area of circle: importance
  overlap probability between each projected basis vector and the eigen wave function

- Potential energy surface (PES) is calculated by Constrained HF for the same interaction

T-plot of 0^+ states of ^{78}Ni (Z=28, N=50)





angular-momentum, parity projection → $P^{J\pi}$; Slater determinant → $|\psi_n\rangle$

$$|\Psi\rangle = \sum_n f_n P^{J\pi} |\psi_n\rangle$$

MCSM eigen wave function | MCSM basis vector

Y. Tsunoda, *et al.*
PRC 89, 031301 (R) (2014)

The tensor and central forces produce the shell evolution

→ one of the major subjects of RI-beam facilities for exotic nuclei

This word, shell evolution, did not exist before 2004.

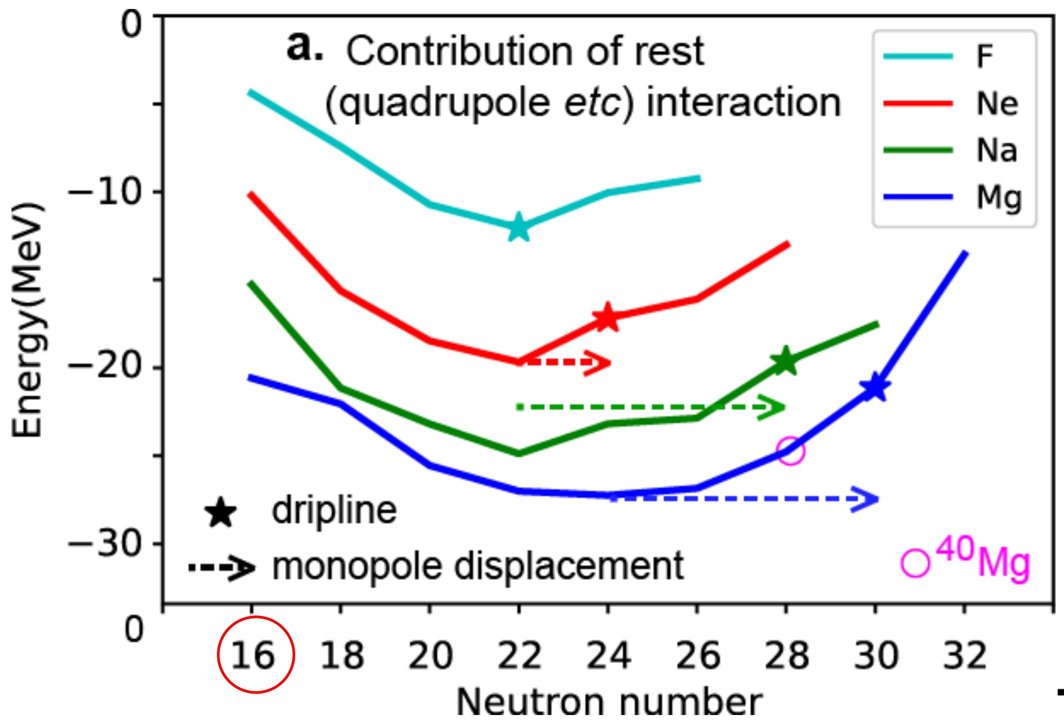
"Shell evolution"* :	0 hit in Google Scholar in	2003
	1	2004
	~140 hits/year	~2021

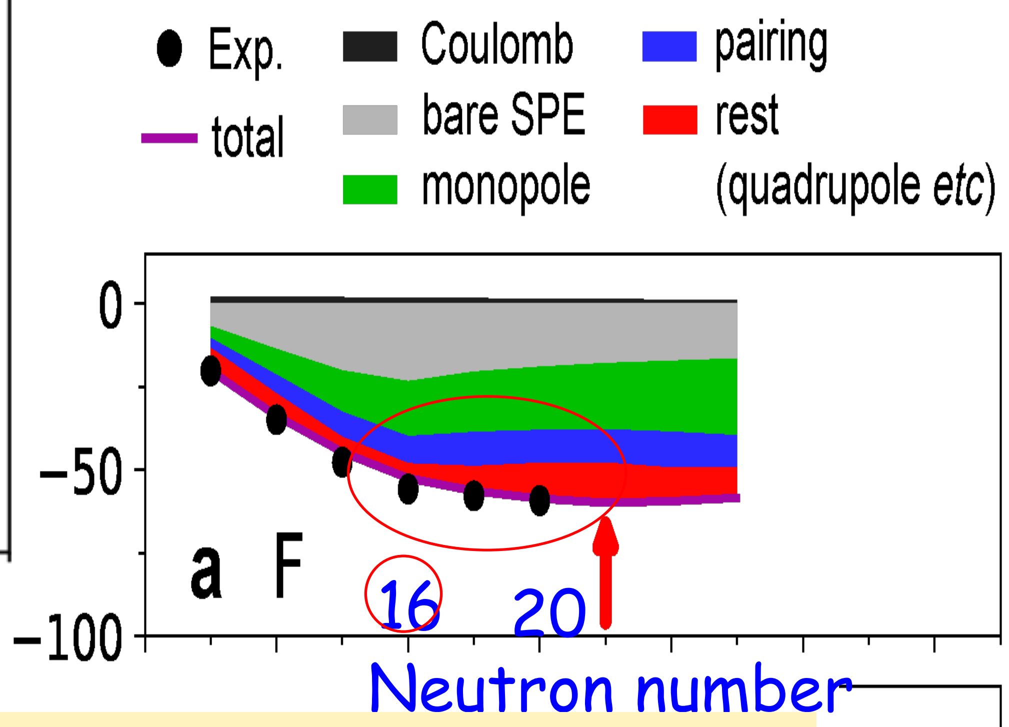
*Combined with "atomic nuclei", to avoid biology, … .

*Type II shell evolution, an extension, is included.

We now find notable effects on the shapes from the same origin

Dripline of F isotopes





Monopole effect (edge of green part) becomes weaker for N > 16 in F isotopes.   It even decreases (see gray edge).

If there were no "rest" (~ quadrupole deformation) effect (red part), the dripline would be at N = 16, which is the same as oxygen isotopes.

Loose binding phenomena may be seen, in contrast to Ne, Na or Mg.

Most advanced methodology in the MCSM is used

- (Ordinary) MCSM: superposed Slater determinants with angular momentum and parity projections
- QVSM(Quasiparticle Vacua Shell Model): superposed quasiparticle vacua with number, angular momentum, and parity projections
- Pairing correlations over many single-particle orbitals are already incorporated in each basis vector because of its BCS-type character

quasiparticle vacuum → $|\phi\rangle = \prod_p (u_p + v_p a_p^\dagger a_{\bar{p}}^\dagger)|-\rangle$ ← core (vacuum)

PHYSICAL REVIEW C **103**, 014312 (2021)

Variational approach with the superposition of the symmetry-restored quasiparticle vacua for nuclear shell-model calculations

Noritaka Shimizu,[1,*] Yusuke Tsunoda,[1] Yutaka Utsuno,[2,1] and Takaharu Otsuka[3,4,2]

The QVSM code was fully used, but huge computer resources were still needed.

Monopole interactions are the key

for Central force

Stronger attraction between single-particle orbits of similar radial wave functions

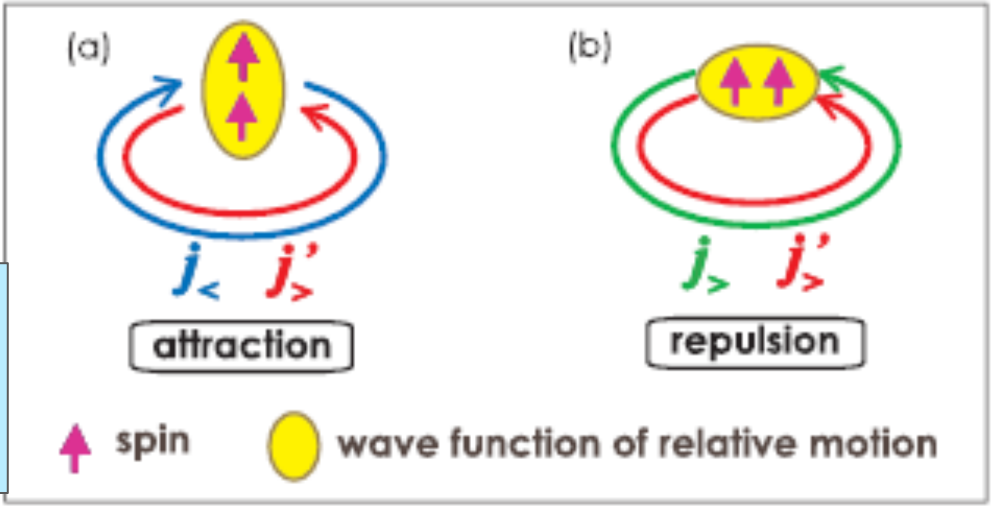
ex.: $f_{7/2} - f_{5/2}$, $g_{9/2} - h_{11/2}$ *cf: Federman-Pittel (1977)*

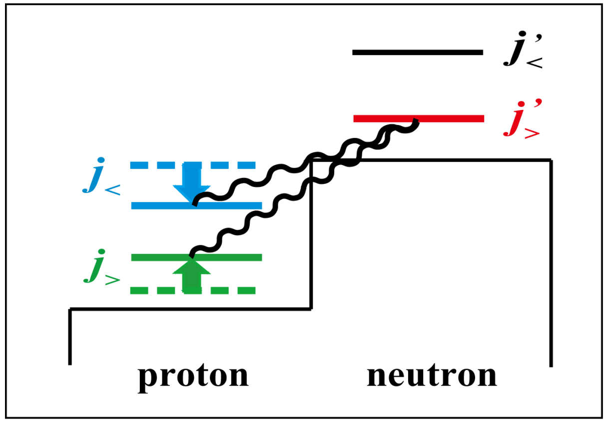
for Tensor force

(long-range part, or 1π, 2π exchange)

$j_> = l + ½$

$j_< = l - ½$





The combination of these two creates new magic numbers N=32, 34, transition from Zr to Sn isotopes, $h_{11/2}$-$g_{7/2}$ splitting in Sb isotopes, *etc*

for Three-nucleon force (Δ-hole) : overall repulsive effect

Bohr & Mottelson, Nuclear Structure II, 1975

Two possibilities 1. Vibrational mode (most likely preferred)

2. Equilibrium shape deviating from axial symmetry

Interpretation of the $K\pi = 2+$ excitation ^{166}Er

The low-energy and large $E2$-matrix element for exciting the $K=2$ band suggests that we are dealing with a collective mode involving deviations of the nuclear shape from axial symmetry. (The $B(E2)$ value for exciting the $K=2, I=2$ state is about $28 B_W(E2)$, which is 14 times the appropriate single-particle unit (see p. 549).) Such a collective mode could have the character of a vibration around an axially symmetric equilibrium or might be associated with an equilibrium shape deviating from axial symmetry.

shape coexistence

Nobel lecture by A. Bohr (1975)

Only the possibility 1. was mentioned for ^{166}Er.

The dominance of axially-symmetric shapes has been one of the textbook items.

Kumar, K. and Baranger, M.
Nuclear deformations in the pairing-plus-quadrupole model (III).
Static nuclear shapes in the rare-earth region. Nucl. Phys. A 1968, 110, 529–554.

This paper presents statements such as

While most of the deformed nuclei are found to be prolate,

and

The preponderance of axially symmetric shapes (prolate or oblate)

Bes, D.R. and Sorensen, R.A.
The Pairing-Plus-Quadrupole Model.
In Advances in Nuclear Physics; Ed. by Baranger, M. and Vogt, E, (Plenum Press, New York, NY, USA, 1969)

This conclusion is correct, as far as the Pairing + Quadrupole Model is adopted.
However it may change, if the interaction differs from this one.

Questions were raised from experimental viewpoints ….

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THE EUROPEAN PHYSICAL JOURNAL A

Review

"Stiff" deformed nuclei, configuration dependent pairing and the β and γ degrees of freedom

J.F. Sharpey-Schafer[1,a], R.A. Bark[2], S.P. Bvumbi[3], T.R.S. Dinoko[4], and S.N.T. Majola[5,b]

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J. Phys. G: Nucl. Part. Phys. **27** (2001) R1–R22   www.iop.org/Journals/jg   PII: S0954-3899(01)18337-4

TOPICAL REVIEW

Characterization of the β vibration and 0_2^+ states in deformed nuclei

P E Garrett

And from empirical approaches….

P. Boutachkov, A. Aprahamian, Y. Sun, J.A. Sheikh & S. Frauendorf

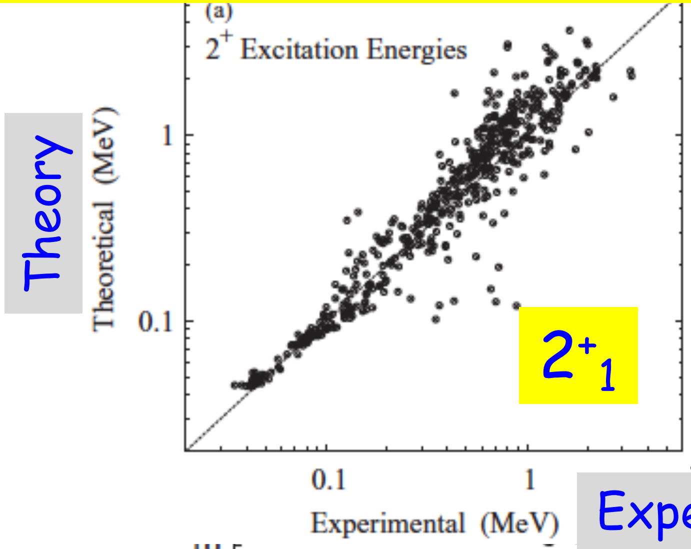
The European Physical Journal A - Hadrons and Nuclei **15**, 455–458 (2002)

Furthermore, there have been microscopic approaches also, where the description of excited bands are still a challenge.

Gogny -> 5DCH
(Bohr-Hamiltonian)

Delaroche et al.,
PR C 81, 014303 (2010)

Correlations between theoretical & experimental values (scale: logarithmic)



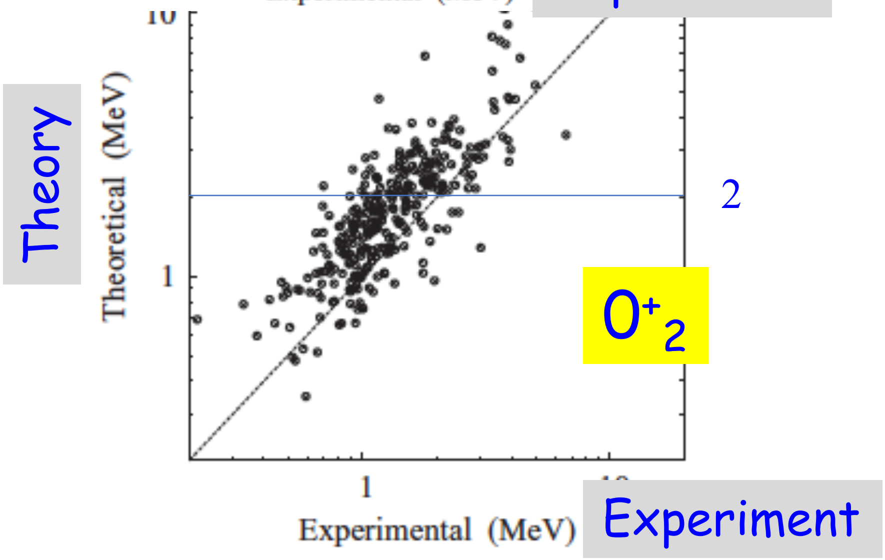


FIG. 20. Excitation energy of the 0_2^+ state compared with experiment [24].

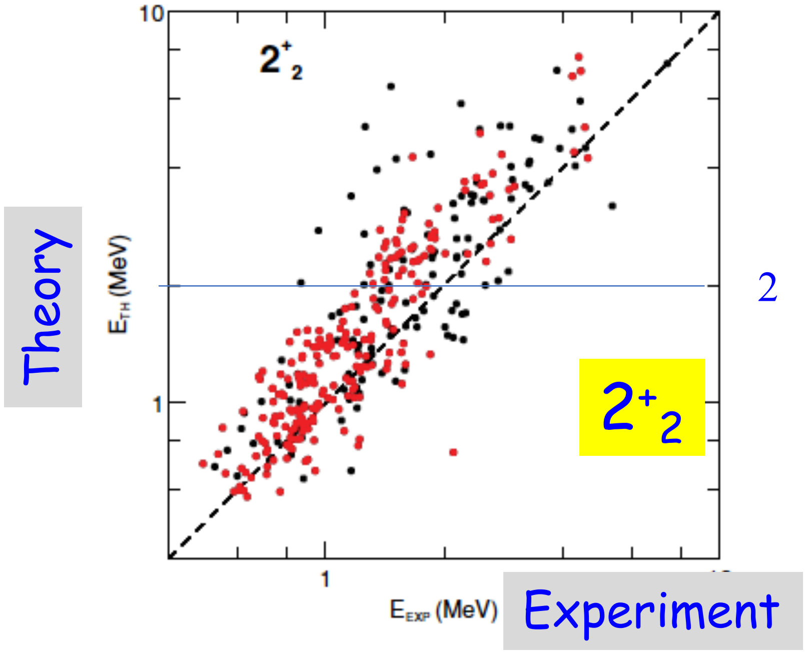


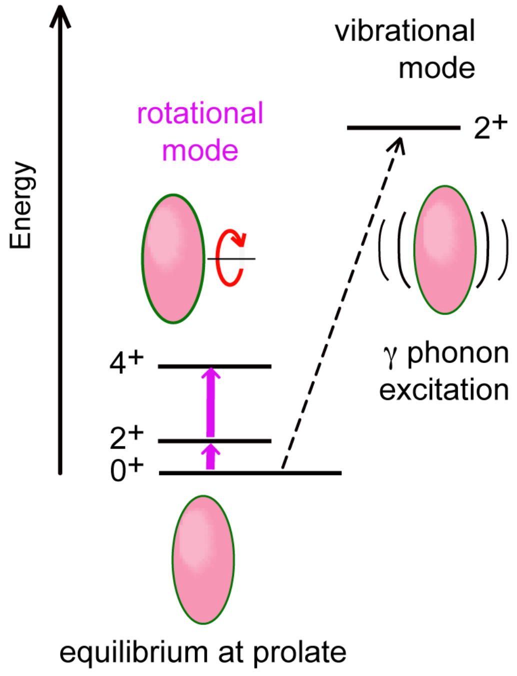
FIG. 19. (Color online) Excitation energy of the second $J = 2$ excitation, comparing 352 nuclei. Experimental data are from Ref. [24]. The 2_γ^+ levels are marked with red color.

Aage Bohr's picture

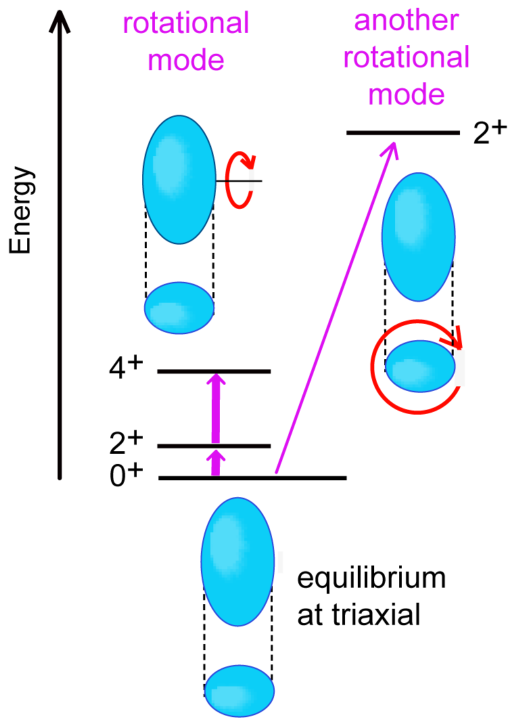
Our picture

Result of MCSM calculation

a. conventinal picture (prolate)

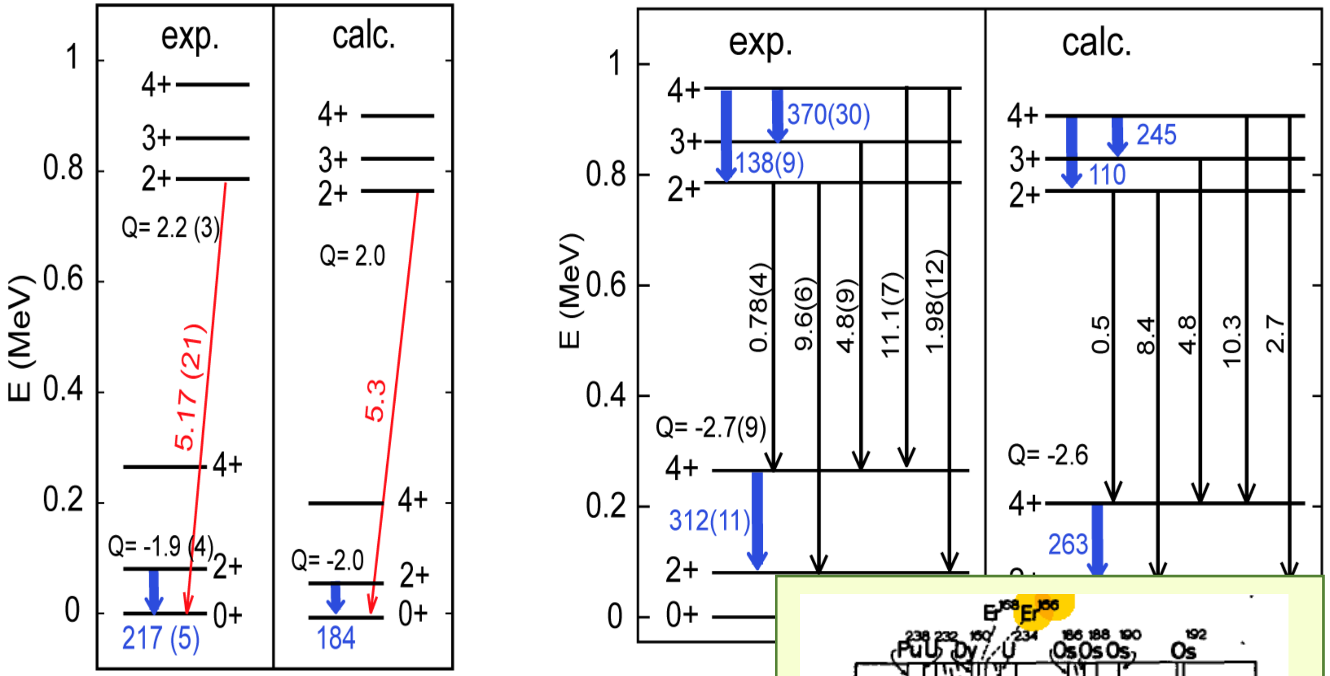


b. present picture (triaxial)



c. level energies and E2 properties of ^{166}Er

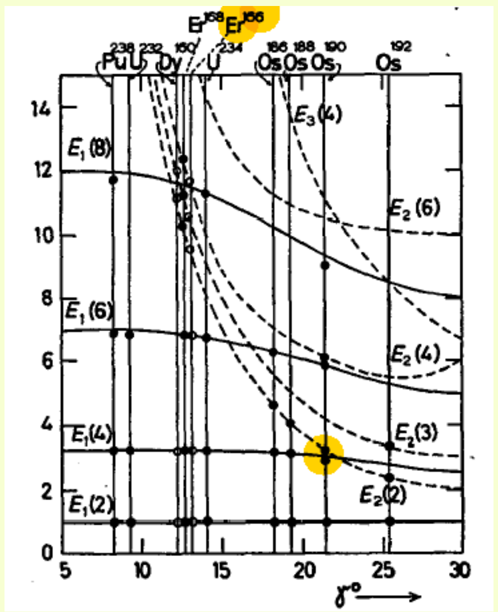
E2 quantities in W.u.



Davydov model

$2^+_2/2^+_1$ energy ratio
→ γ ~ 13 deg.

B(E2) $2^+_2/2^+_1$ ratio
→ γ ~ 9 deg.



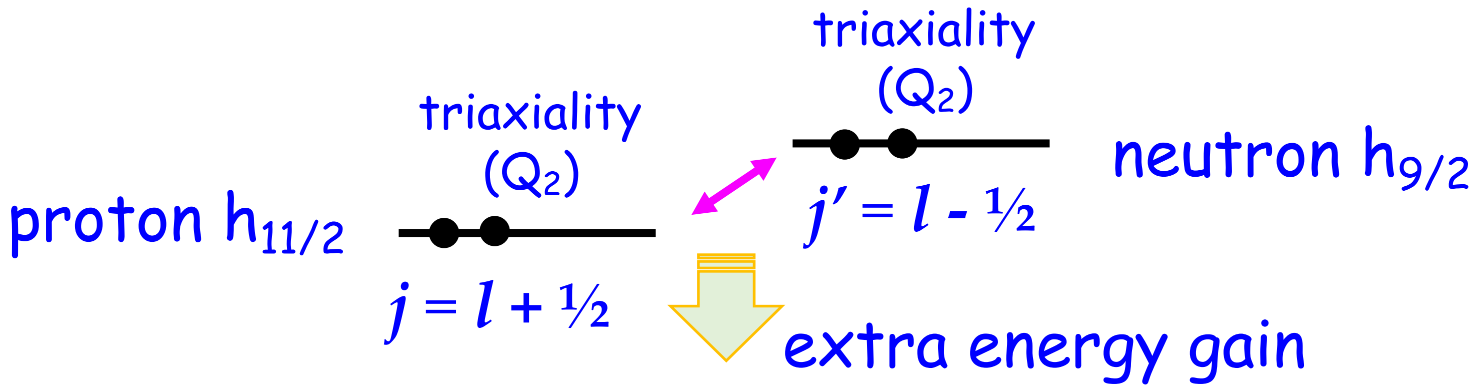
Davydov, A.S., Filippov, G.F., Rotational states in even atomic nuclei, Nucl. Phys. **8**, 237 (1958).
Davydov, A.S., Rostovsky, V.S., Relative transition probabilities between rotational levels of non-axial nuclei. Nucl. Phys. **12**, 58 (1959).

Which monopole interactions are relevant to triaxiality

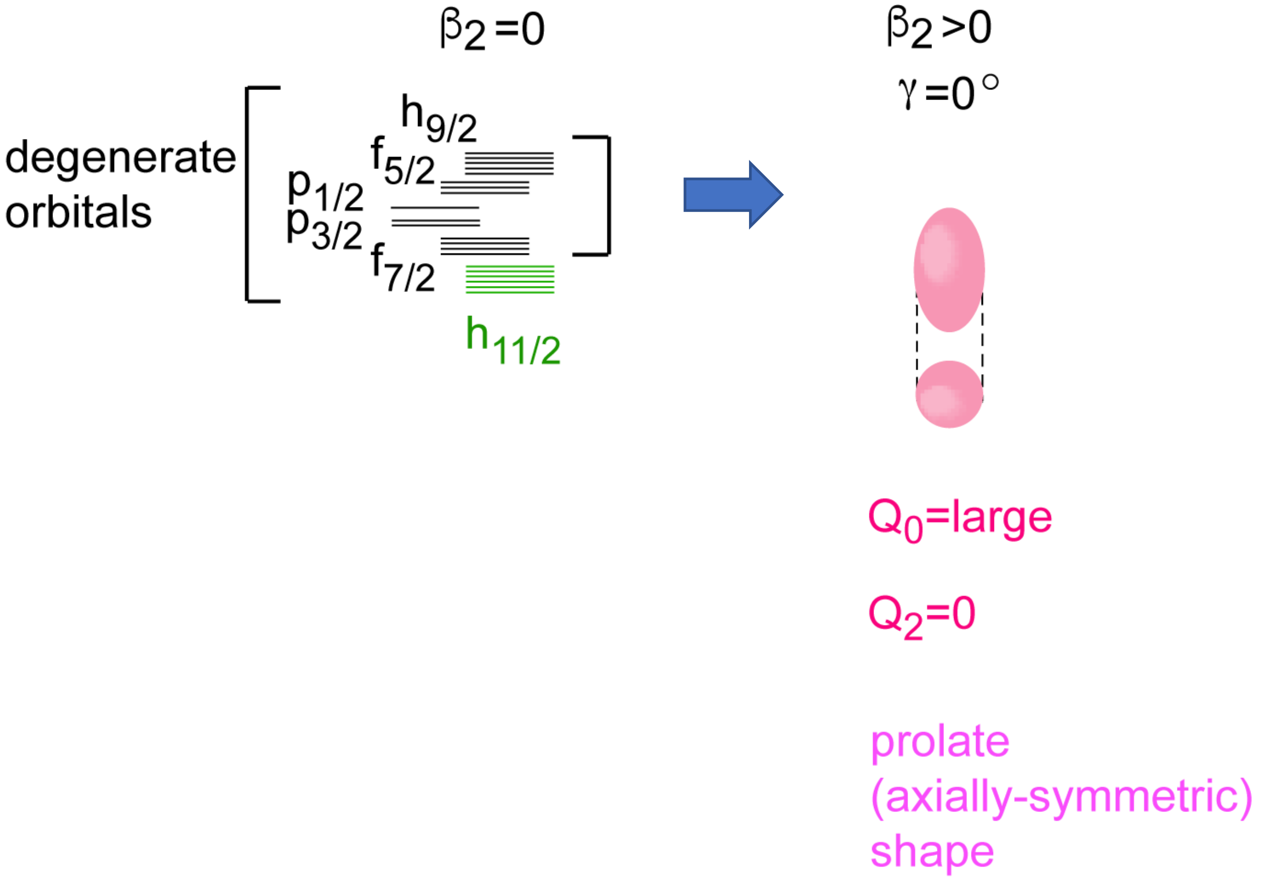
A substantial effect can be expected from the following correlation

A single-particle orbit with large j (e.g. $h_{11/2}$) can produce sizable triaxiality (i.e., Q_2), if the number of particles in the orbit is appropriate.

Proton single-particle orbit with large j_p and neutron orbit with large j_n are coupled by the monopole interactions of the central and tensor forces, as it occurs in the shell evolution.



Closely lying single-particle orbits of the same parity → axial symmetry



Multi-axis rotation is always exciting !

Ayumu Hirano,
Gold medalist, 2022 Olympics

from NHK