

Andy Sproles, ORNL

Microscopic models of nuclear structure at scale

Wouter Ryssens, G. Grams, M. Bender and S. Goriely

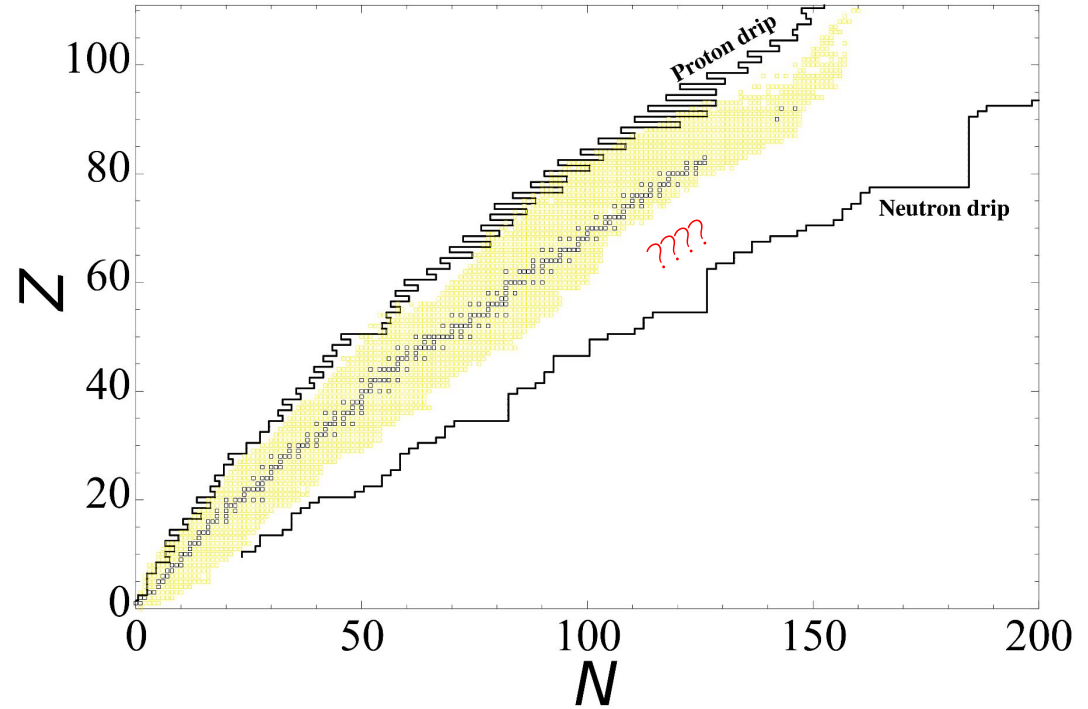
5th of June 2023



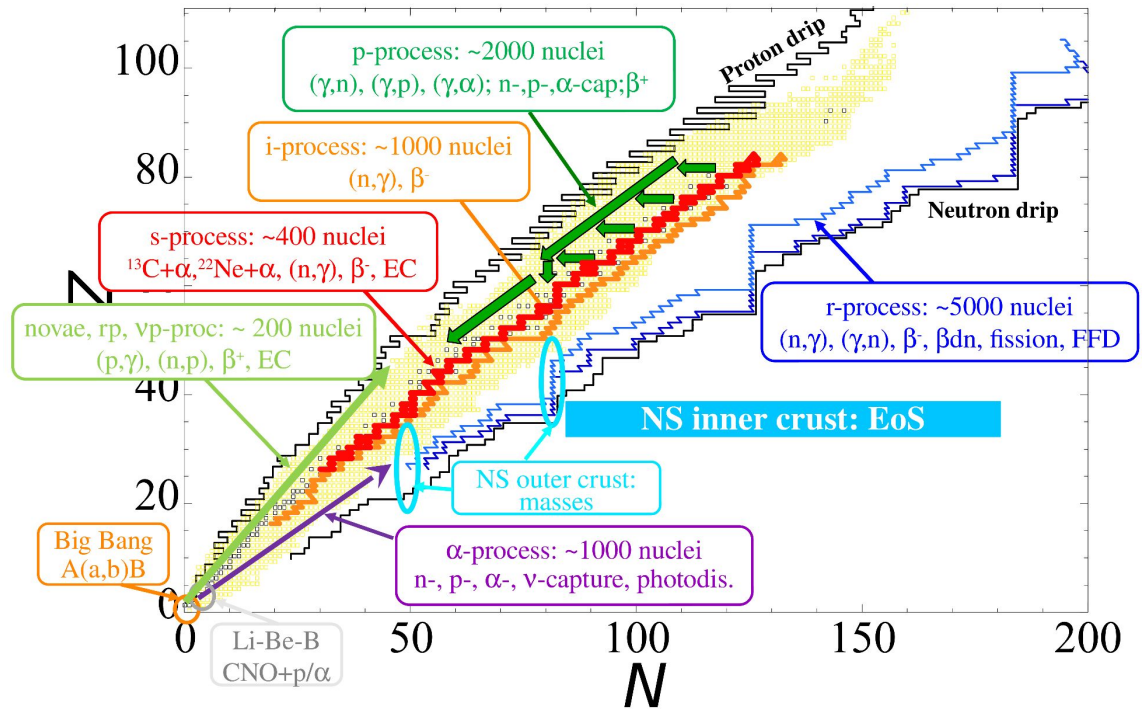
wryssens.com

wryssens@ulb.be

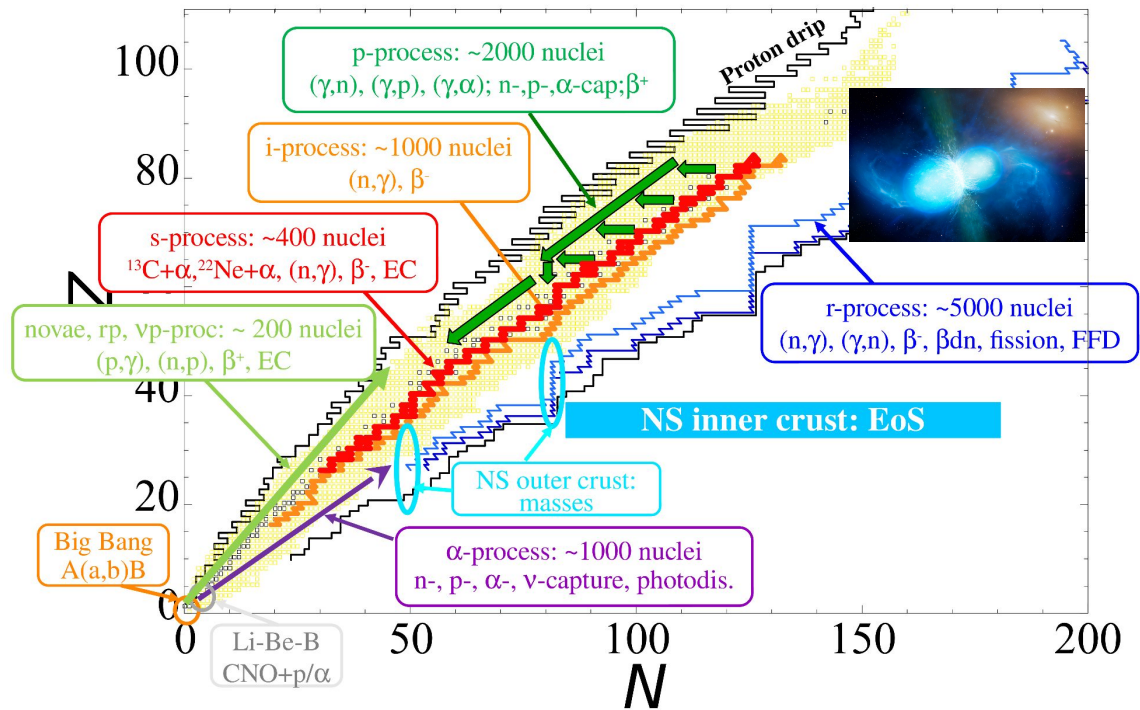
The nuclear chart...



The nuclear chart and the processes traversing it



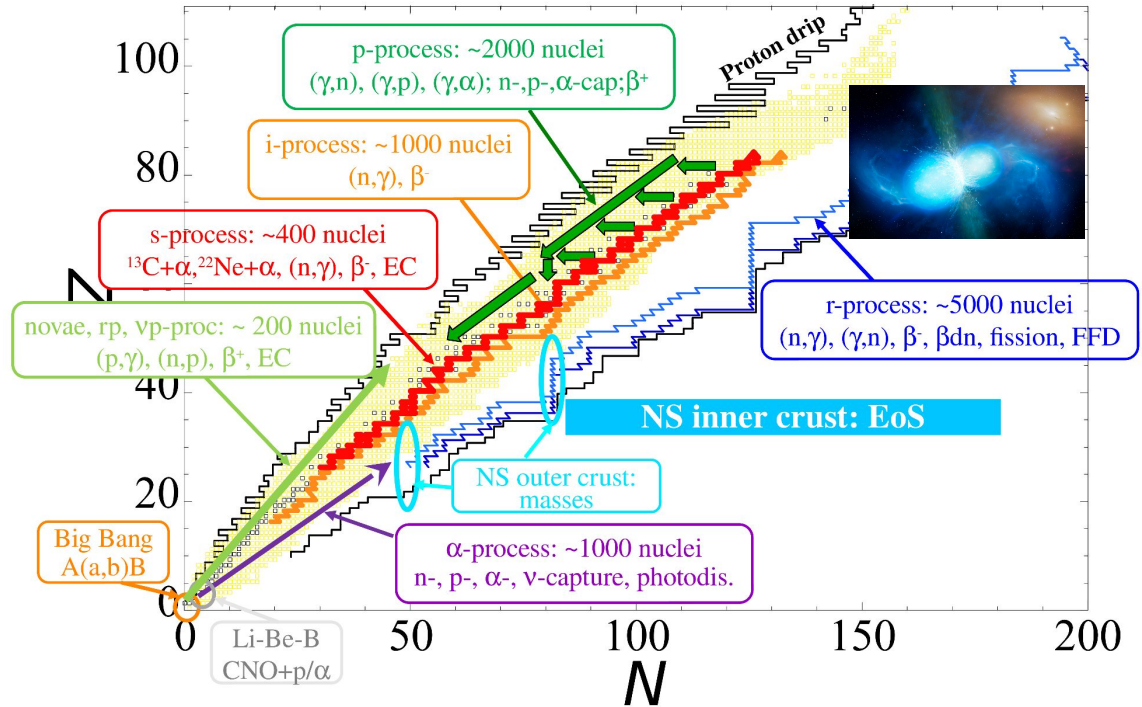
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Extrapolations in

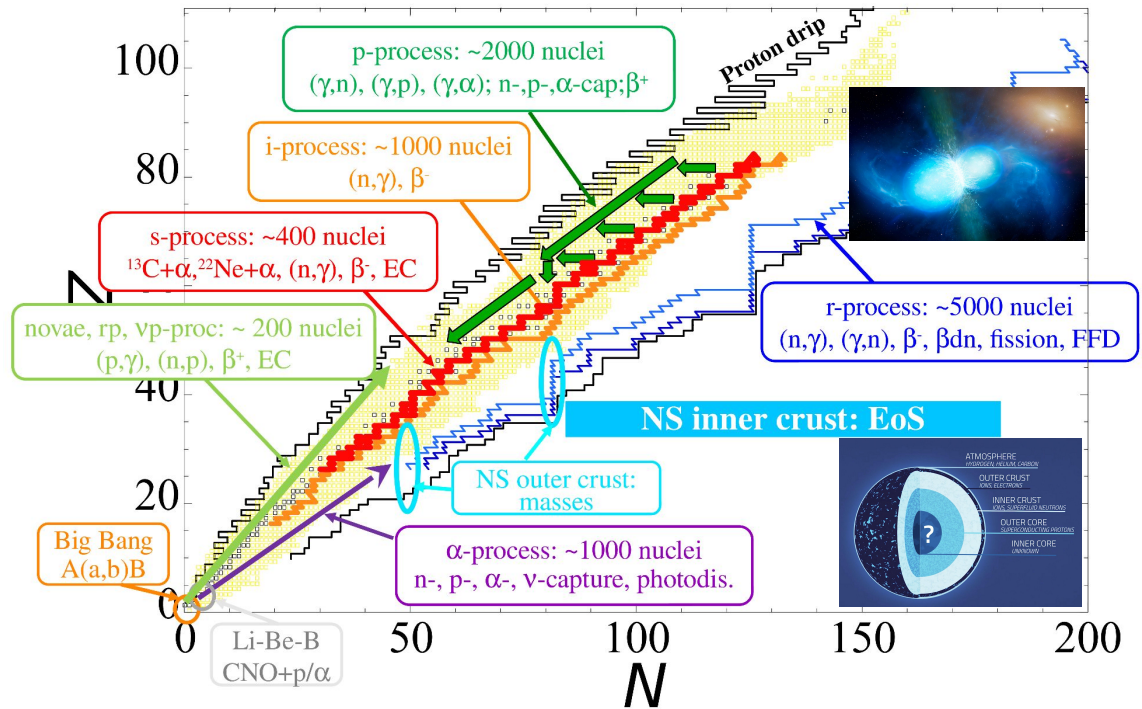
- nucleon number
- energy
- temperature
- density
-



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Extrapolations in

- nucleon number
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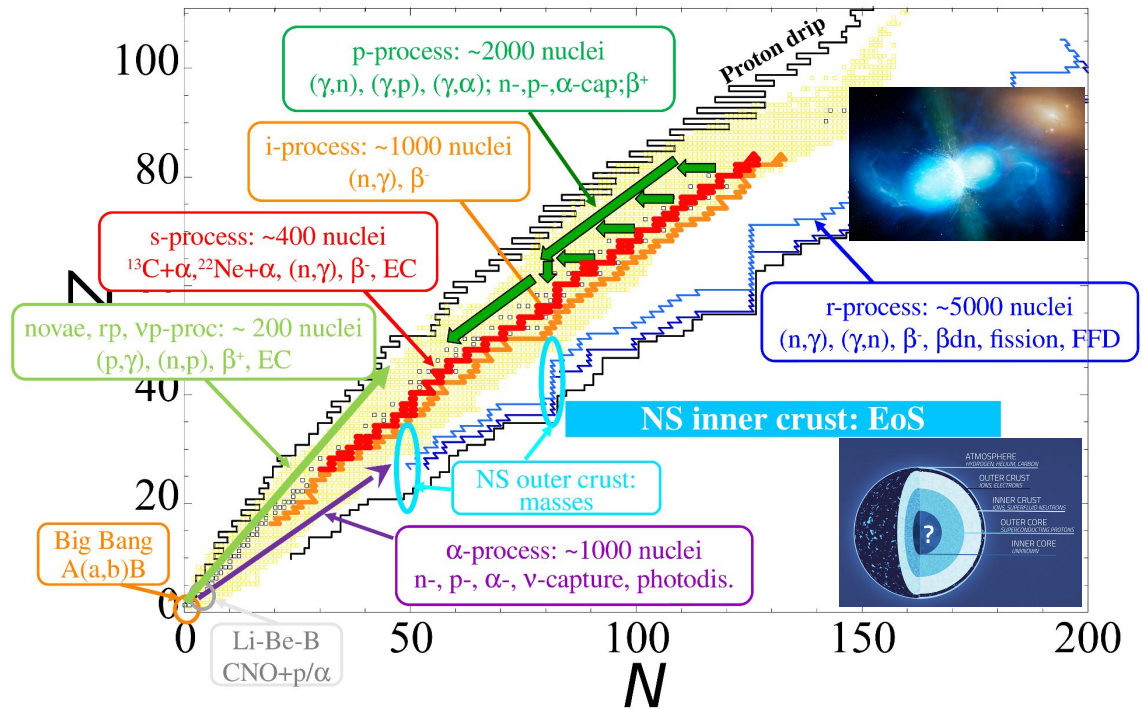
The nuclear chart and the processes traversing it

Extrapolations in

- nucleon number
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and all of that for

- **~7000** nuclei
- **many** reactions



The nuclear chart and the processes traversing it

Extrapolations in

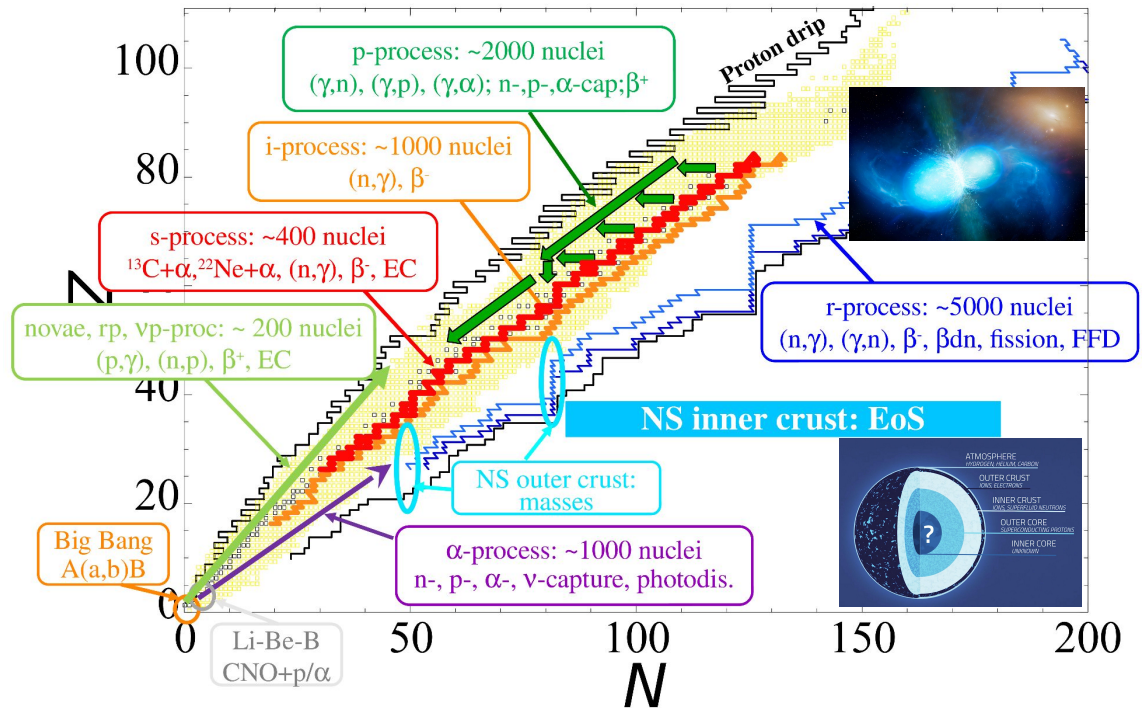
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and all of that for

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what we need is models that should be

1. predictive....
2. but also complete




Skyrme **E**nergy **D**ensity **F**unctionals (**EDFs**)

$$E \sim \int d^3r \left[C^\rho \rho(\mathbf{r})\rho(\mathbf{r}) + C^\tau \tau(\mathbf{r})\rho(\mathbf{r}) + \dots \right]$$



Skyrme **E**nergy **D**ensity **F**unctionals (**EDFs**)

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Local densities and currents of a wavefunction



Skyrme **E**nergy **D**ensity **F**unctionals (**EDFs**)

Coupling constants (\sim **25** parameters) fitted to data

The diagram shows the Skyrme Energy Density Functional equation:
$$E \sim \int d^3r \left[C^\rho \rho(\mathbf{r})\rho(\mathbf{r}) + C^\tau \tau(\mathbf{r})\rho(\mathbf{r}) + \dots \right]$$
 Annotations include: a blue line connecting the coupling constants C^ρ and C^τ to the text 'Coupling constants (~ 25 parameters) fitted to data'; and a red bracket under the density terms $\rho(\mathbf{r})\rho(\mathbf{r})$ and $\tau(\mathbf{r})\rho(\mathbf{r})$ pointing to the text 'Local densities and currents of a wavefunction'.

Local densities and currents of a wavefunction

Skyrme **E**nergy **D**ensity **F**unctionals (**EDFs**)

Energy

Coupling constants (~ 25 parameters) fitted to data

$$E \sim \int d^3r \left[C^\rho \rho(\mathbf{r})\rho(\mathbf{r}) + C^\tau \tau(\mathbf{r})\rho(\mathbf{r}) + \dots \right]$$

Local densities and currents of a wavefunction

The diagram shows the Skyrme Energy Density Functional equation. The energy E is enclosed in a pink box and labeled 'Energy' with a pink line. The integrand consists of two terms: $C^\rho \rho(\mathbf{r})\rho(\mathbf{r})$ and $C^\tau \tau(\mathbf{r})\rho(\mathbf{r})$. The coupling constants C^ρ and C^τ are enclosed in blue boxes, and the local densities $\rho(\mathbf{r})$ and $\tau(\mathbf{r})$ are enclosed in red boxes. A blue line connects the blue boxes to the text 'Coupling constants (~ 25 parameters) fitted to data'. A red line connects the red boxes to the text 'Local densities and currents of a wavefunction'.

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Local densities and currents of a wavefunction

The diagram illustrates the Skyrme Energy Density Functional (EDF) equation. The energy E is represented by a pink box. The coupling constants C^ρ and C^τ are highlighted with blue boxes. The local densities and currents $\rho(\mathbf{r})\rho(\mathbf{r})$ and $\tau(\mathbf{r})\rho(\mathbf{r})$ are highlighted with red boxes. The equation is shown as $E \sim \int d^3r [C^\rho \rho(\mathbf{r})\rho(\mathbf{r}) + C^\tau \tau(\mathbf{r})\rho(\mathbf{r}) + \dots]$. Annotations with lines point from the text labels to the corresponding parts of the equation.

Strong points

- wavefunctions with individual nucleons
- based on “in-medium” N-N interaction
- many observables accessible
- Feasible for ~ 7000 nuclei

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Local densities and currents of a wavefunction

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Strong points

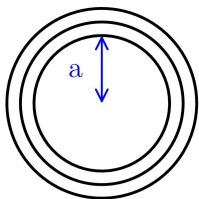
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How to move forward?

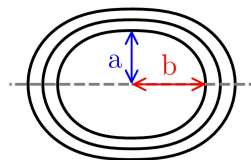
1. search for a “better” EDF form
2. include more experimental information
3. include more physics in the wavefunction

Large-scale models in 1-2 dimensions

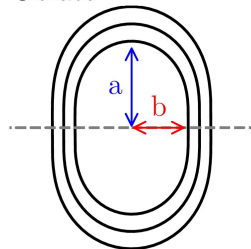
Spherical



Prolate



Oblate



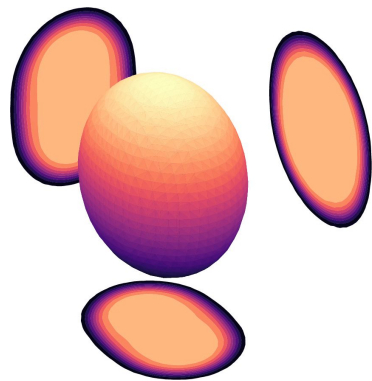
One DOF: β_{20}

Nuclear deformation

- larger variational space
- shape DOF characterized by multipole moments
- capture correlations at modest CPU cost
- intuitive interpretation

Large-scale models in 1-2-**3** dimensions

β_{20}, β_{22} or β_2, γ



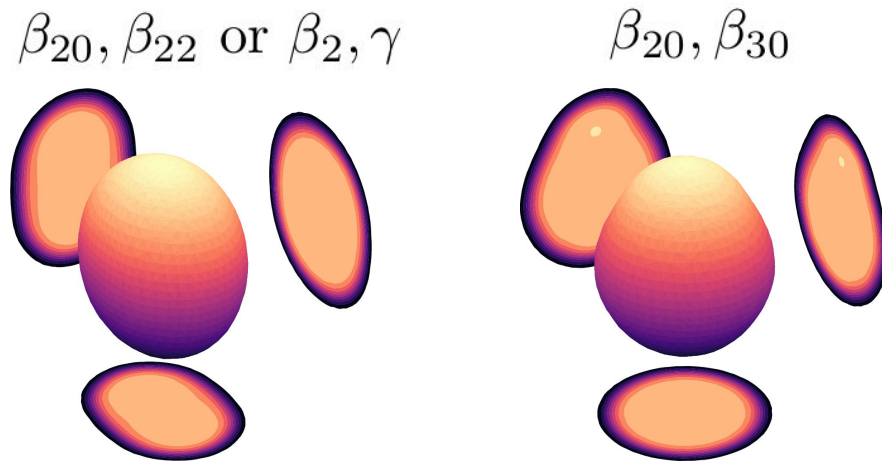
Symmetry breaking leads to deformation

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- shape DOF characterized by multipole moments
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More general configurations

- triaxial shapes

Large-scale models in 1-2-**3** dimensions



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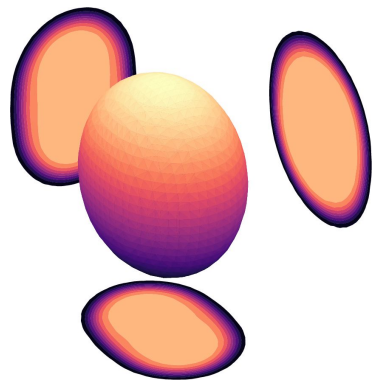
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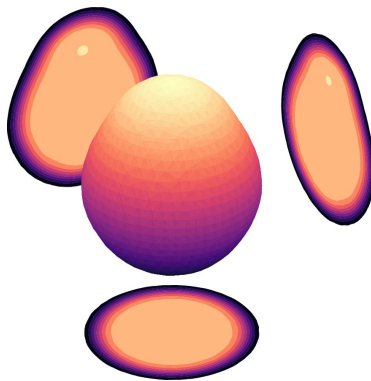
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- reflection asymmetry

Large-scale models in 1-2-3 dimensions

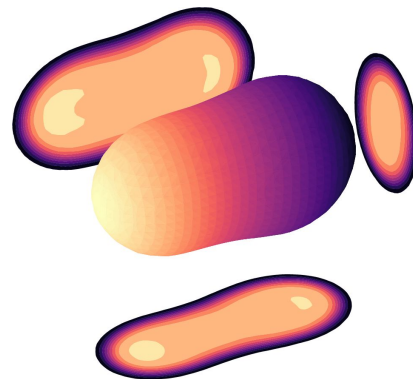
β_{20}, β_{22} or β_2, γ



β_{20}, β_{30}



β_{20}, β_{22} **and** β_{30}



Symmetry breaking leads to deformation

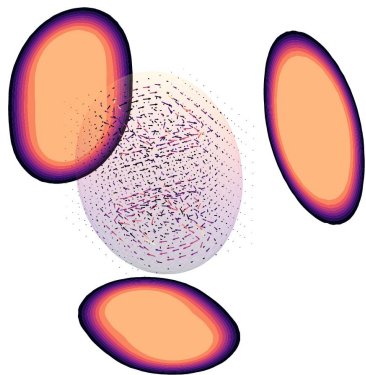
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More general configurations

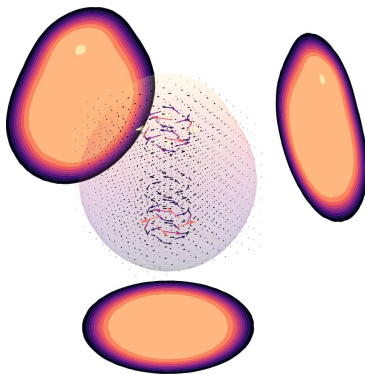
- triaxial shapes
- reflection asymmetry
- elongated shapes

Large-scale models in 1-2-**3** dimensions

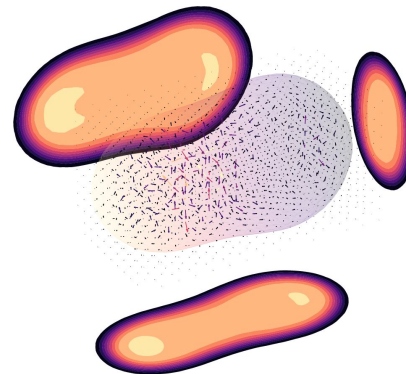
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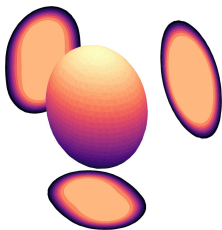
More general configurations

- triaxial shapes
- reflection asymmetry
- elongated shapes
- spin densities and currents

Brussels-Skyrme-on-a-Grid: BSkG

BSkG1 (2021)

- fitted to 2457 masses
- fitted to 884 charge radii
- includes triaxial deformation



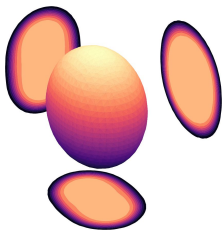
BSkG1: G. Scamps et al., EPJA **57**, 333 (2021).
BSkG2: W. Ryssens et al., EPJA **58**, 246 (2022).
W. Ryssens et al., EPJA **59**, 96 (2023).
BSkG3: G. Grams et al., in preparation.

Rms σ	BSkG1	BSkG2	BSkG3
Masses [MeV]	0.741		
Radii [fm]	0.024		
	-	-	-

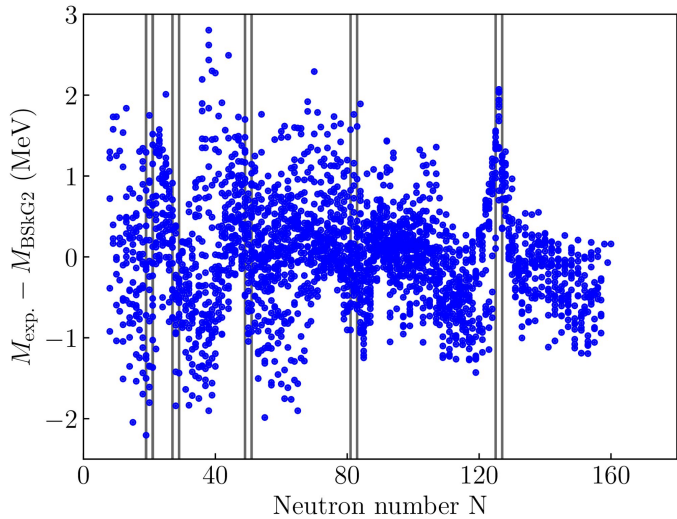
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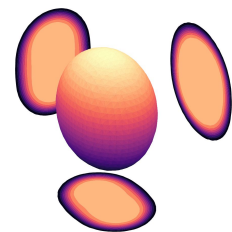


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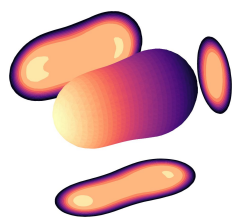
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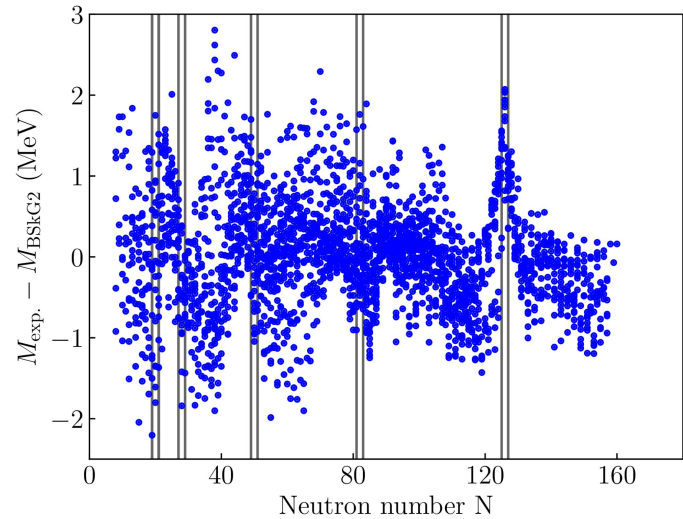


BSkG2 (2022)

- fitted to 45 fission barriers
- includes spins, currents,...



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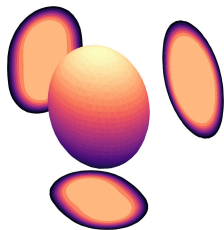


Rms σ	BSkG1	BSkG2	BSkG3
Masses [MeV]	0.741	0.678	
Radii [fm]	0.024	0.027	
Prim. barriers [MeV]	0.88	0.44	
Secon. barriers [MeV]	0.87	0.47	
Fission isomers [MeV]	1.0	0.49	

Brussels-Skyrme-on-a-Grid: BSkG

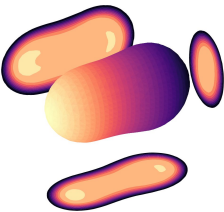
BSkG1 (2021)

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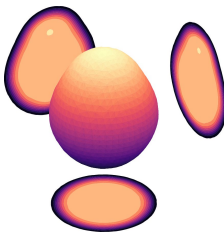
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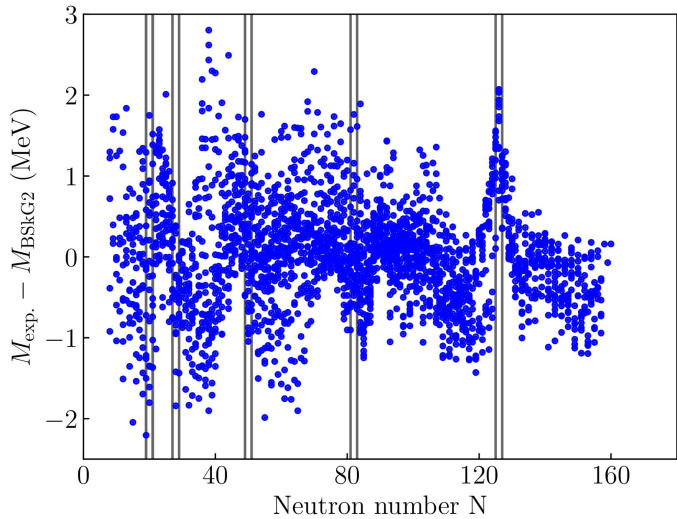


BSkG3 (2023)

- larger max. neutron star mass
- includes octupole deformation

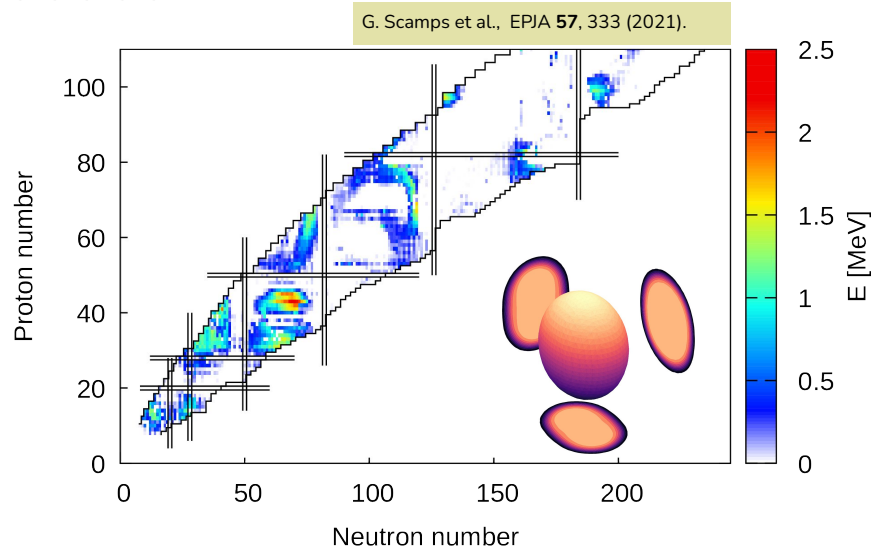


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Prim. barriers [MeV]	0.88	0.44	0.33
Secon. barriers [MeV]	0.87	0.47	0.51
Fission isomers [MeV]	1.0	0.49	0.34
Max. NS mass [M_\odot]	1.8	1.8	2.3

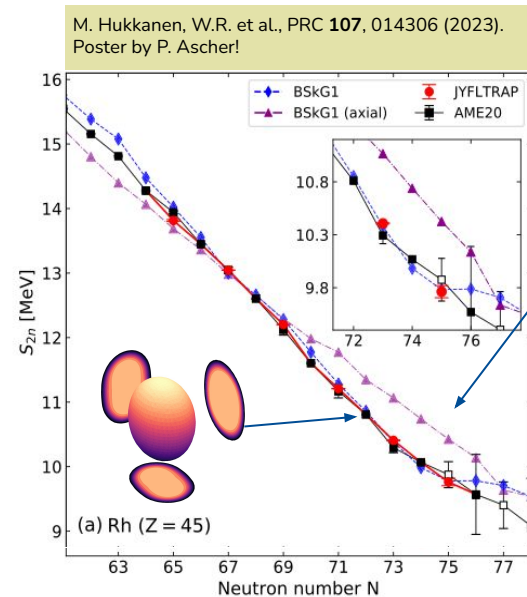
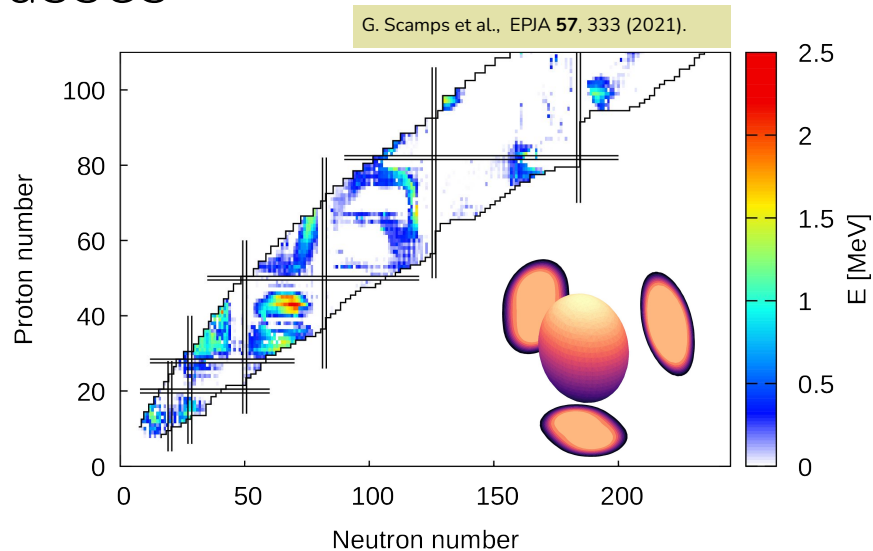
Masses



Triaxial deformation

- many nuclei are affected
- effects up to 2.5 MeV near $Z \sim 44$

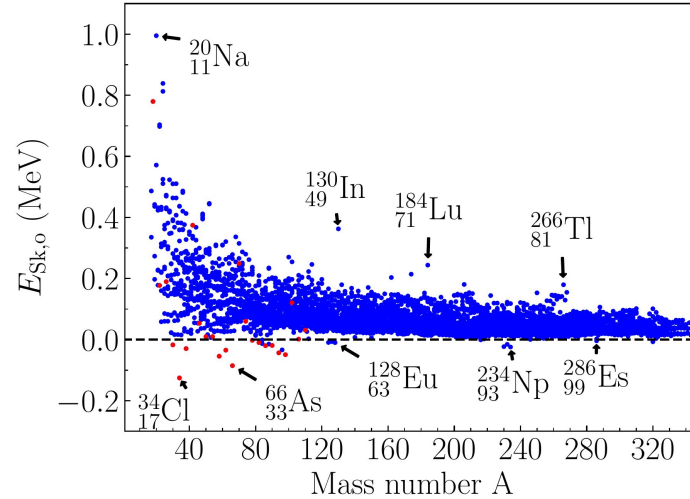
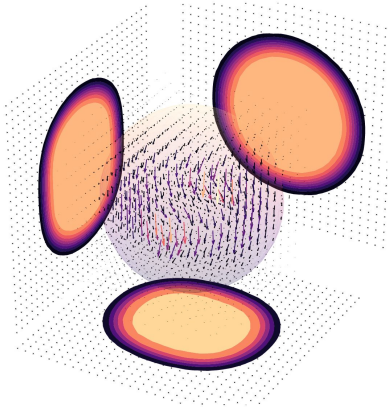
Masses



Triaxial deformation

- many nuclei are affected
- effects up to 2.5 MeV near $Z \sim 44$
- does help reproduce trends, e.g. Rh

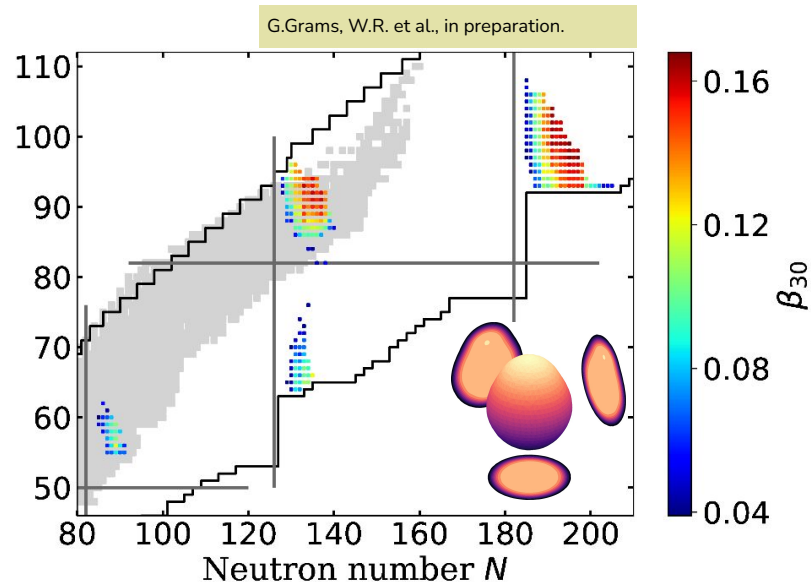
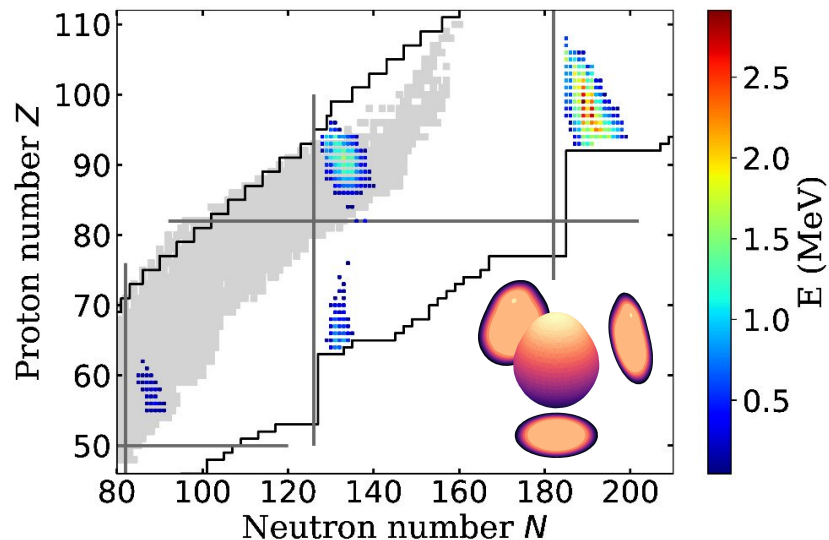
Masses



Time-odd terms

- small impact on the masses
- globally repulsive
- first time checked on this scale!
- first step towards other observables

Masses



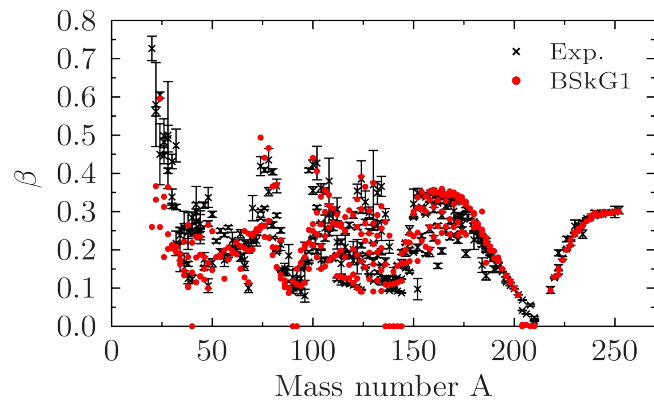
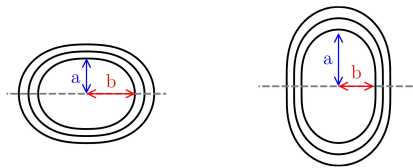
Reflection asymmetry

- small number of known nuclei affected
- Near $N=184$:
 - large effect up to 2.5 MeV
 - dripline modified
 - fission properties modified

Deformations

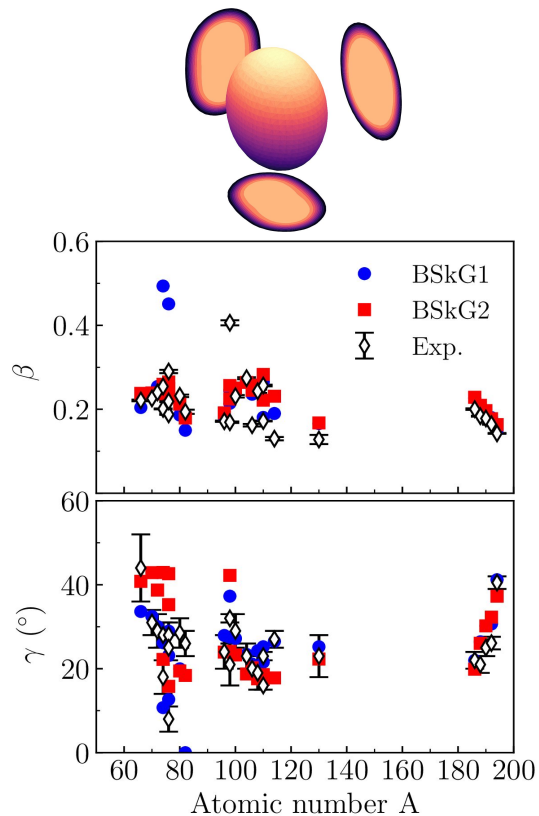
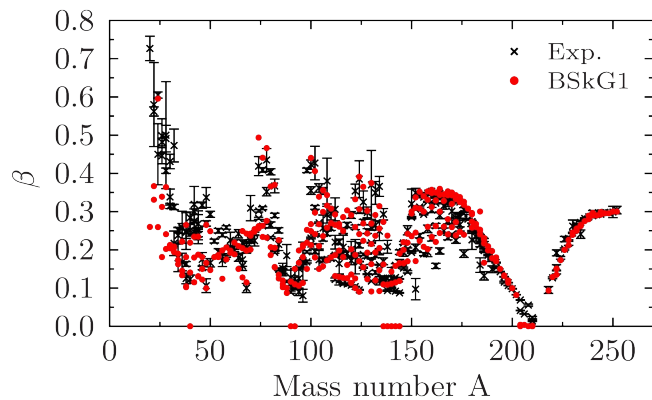
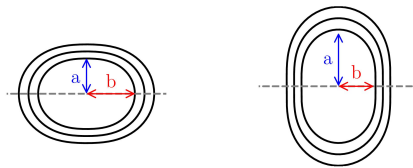
Deformations

“Ordinary” quadrupole deformation



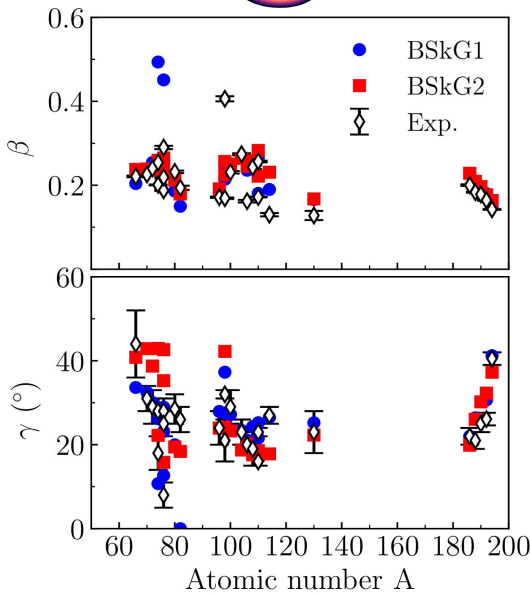
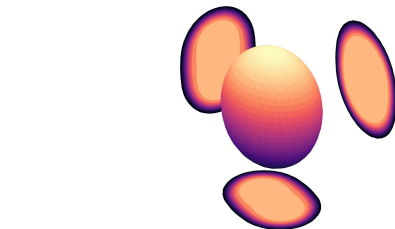
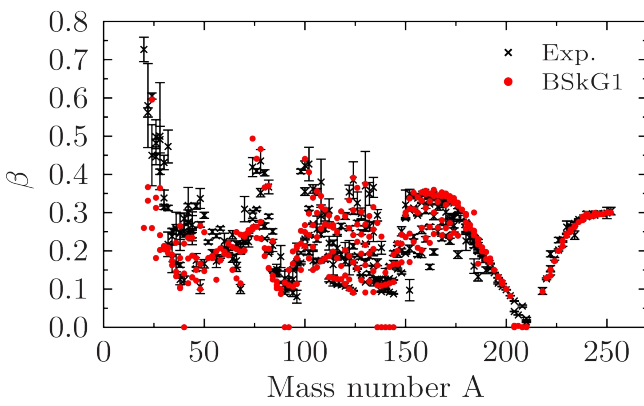
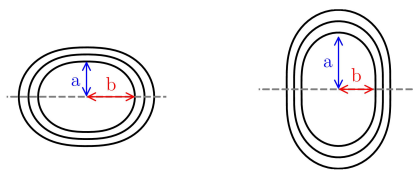
Deformations

“Ordinary” quadrupole deformation ... and triaxial deformation ...

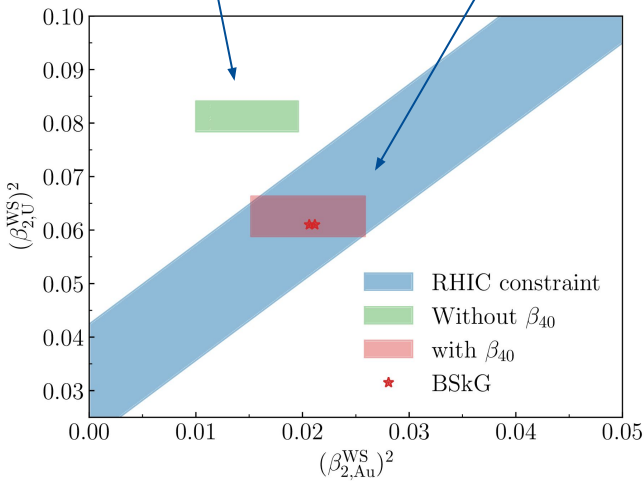


Deformations

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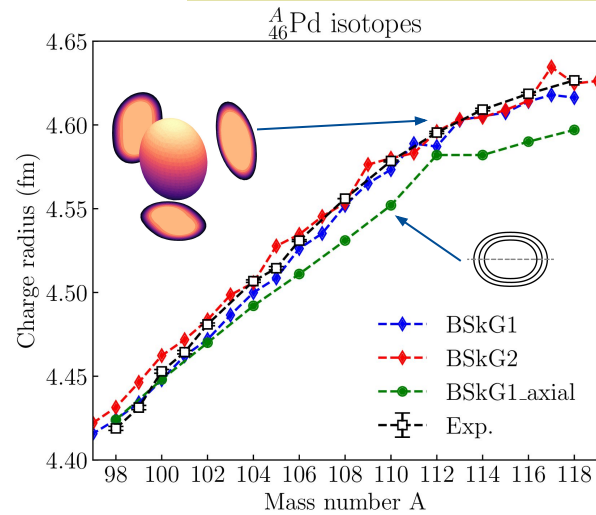


... and even hexadecapole!



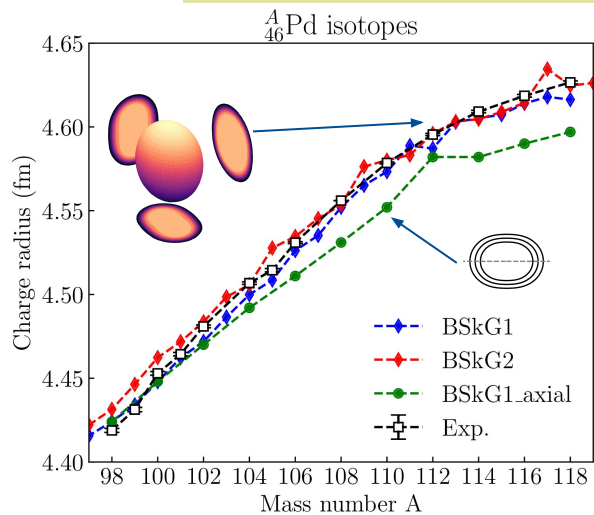
Radii

S. Geldhof, PRL **128**, 152501 (2022).
Talk by S. Geldhof on Thursday!

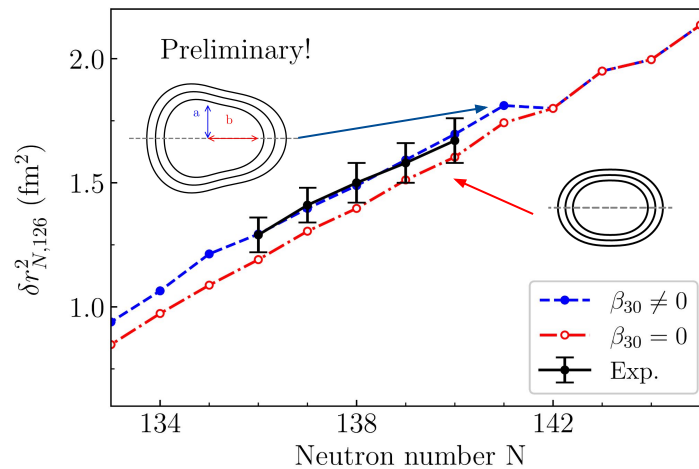


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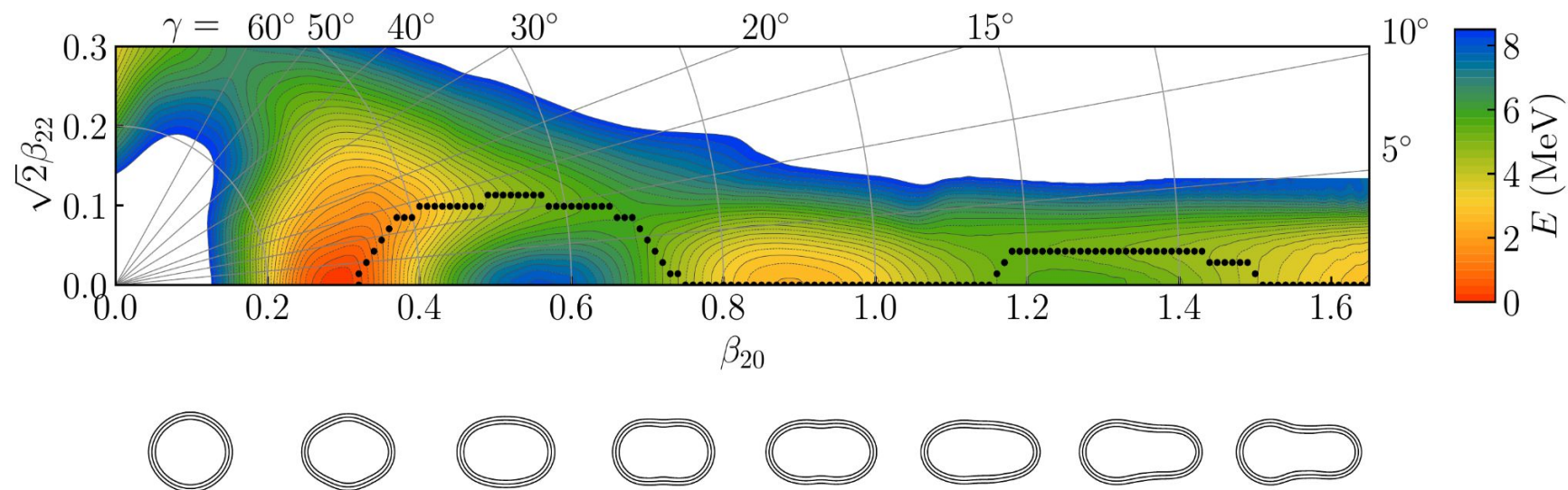
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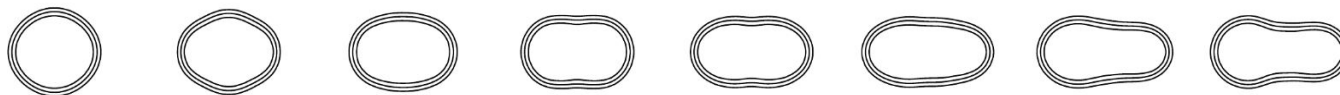
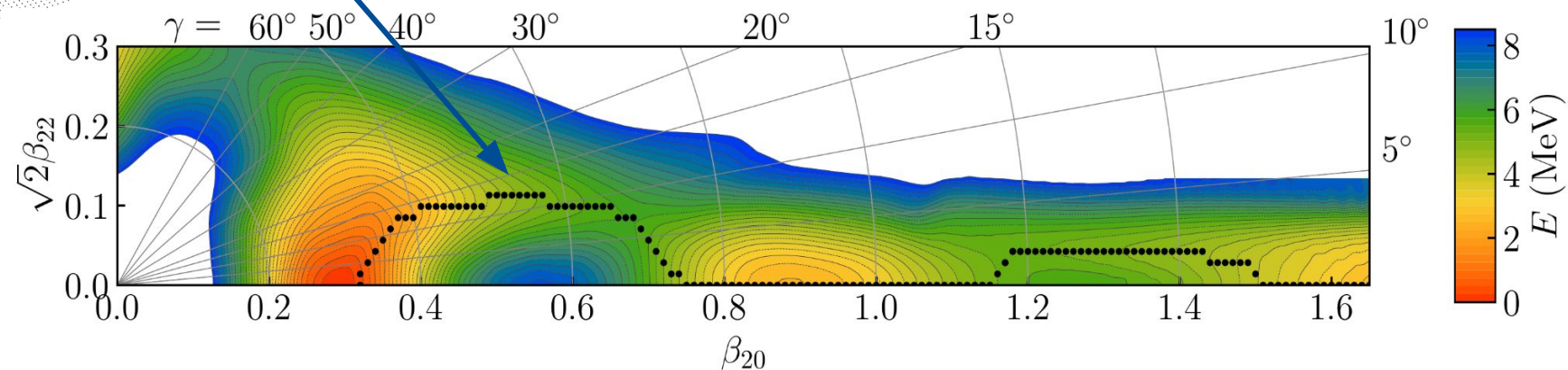
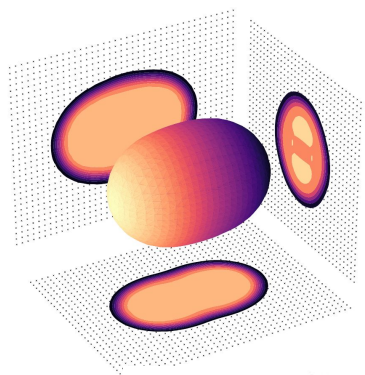
E. Verstraelen, PRC 100, 044321 (2019)



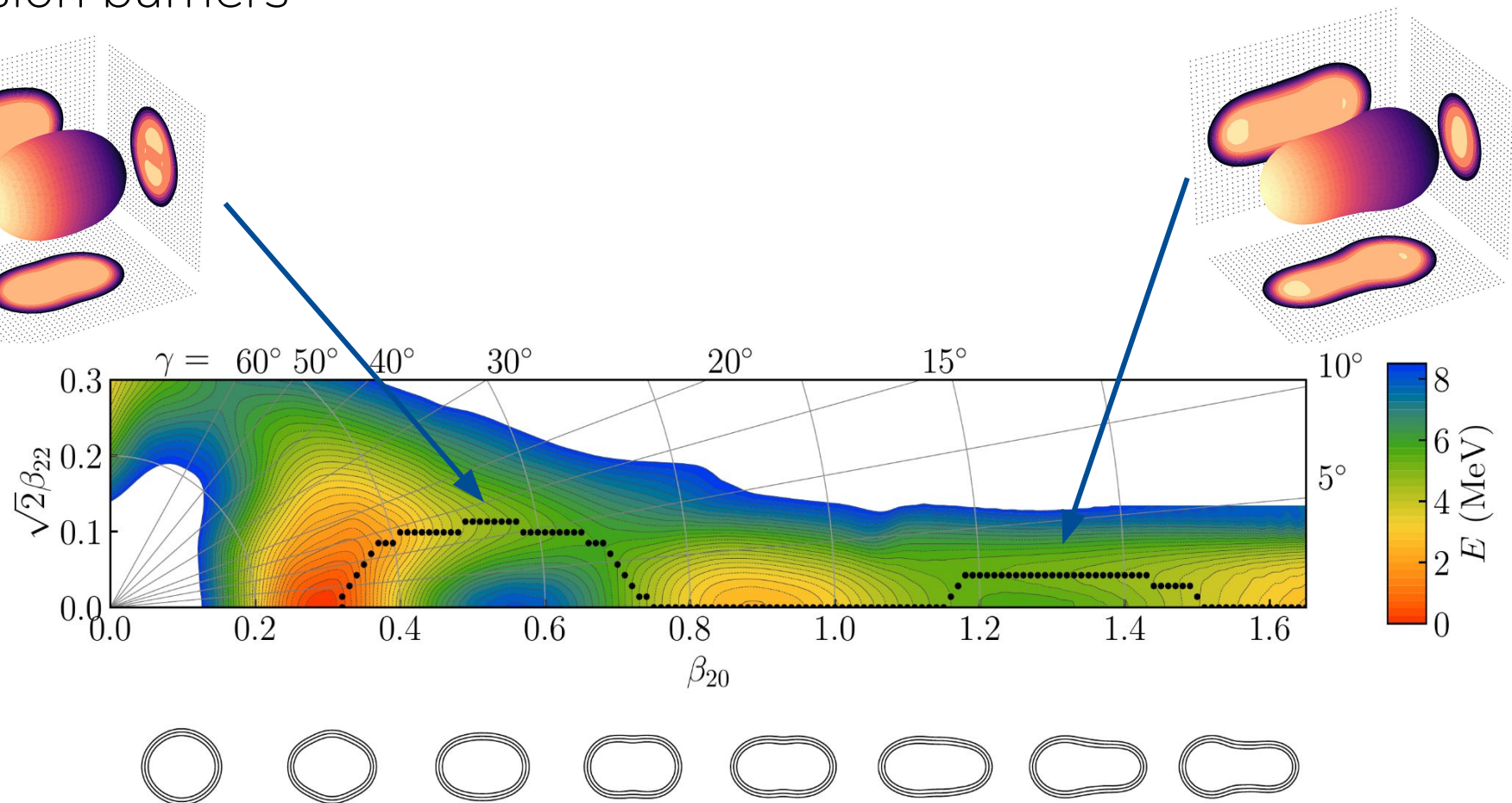
Fission barriers

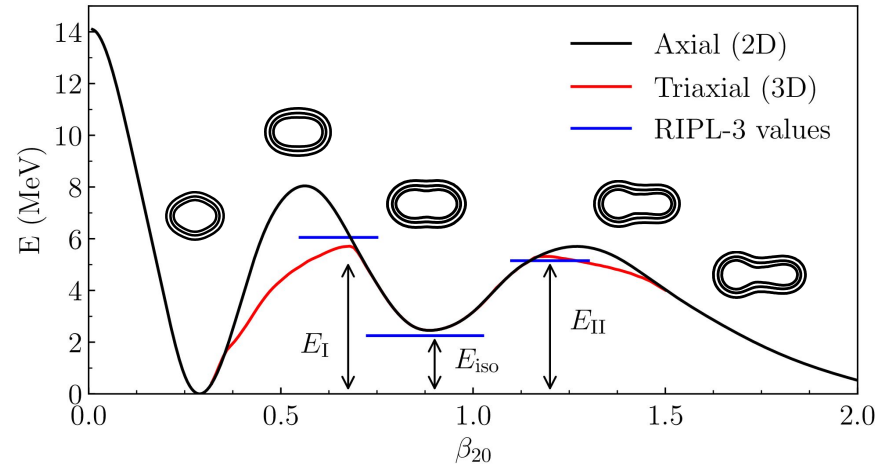


Fission barriers

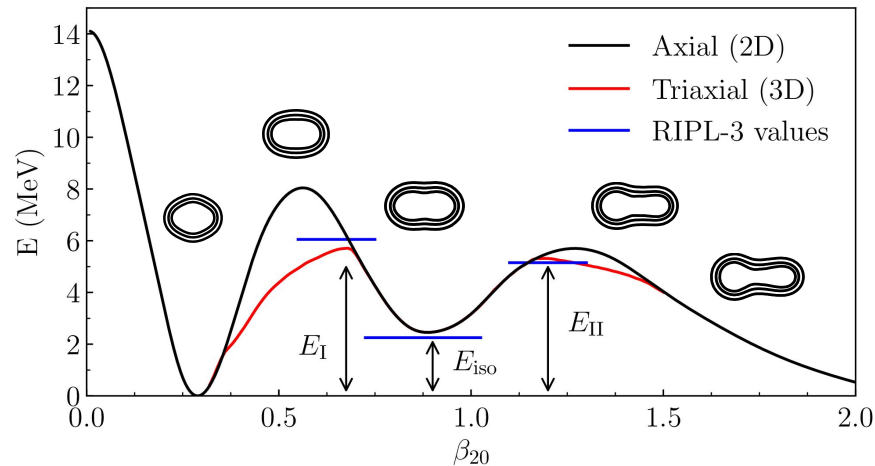


Fission barriers





Fission



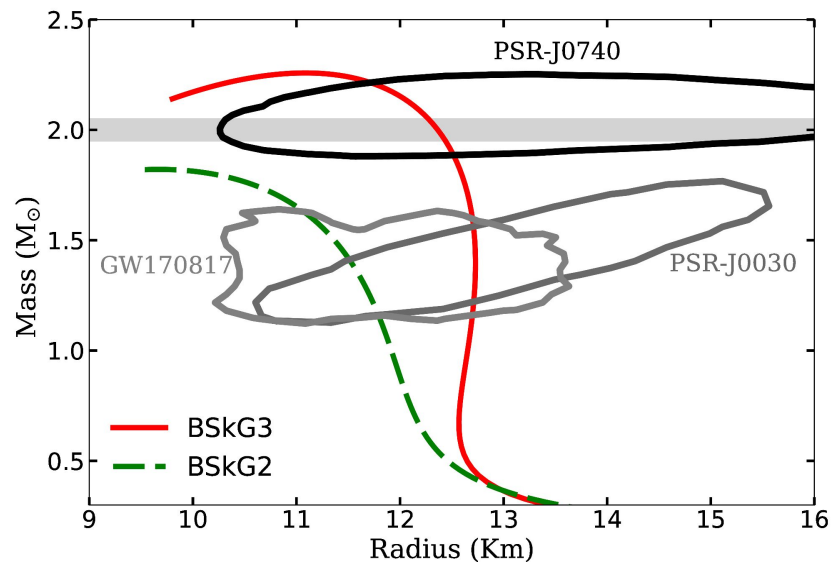
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Fission properties of 45 actinide nuclei

- includes odd-A and odd-odds
- all inner barriers exploit triaxiality
- all outer barriers exploit
 - octupole deformation
 - triaxial deformation

Neutron stars

G. Grams, W.R. et al., in preparation.

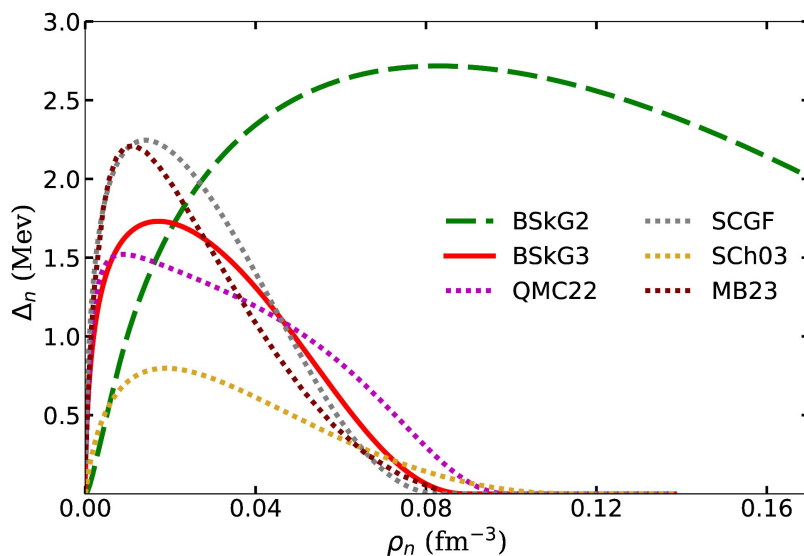
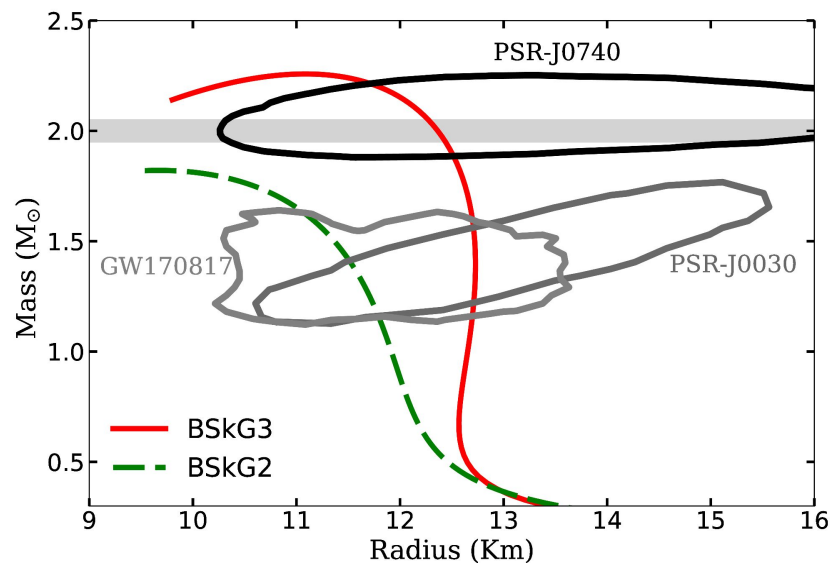


More realistic NS predictions:

- higher maximum mass
 - compatible with NICER
 - compatible with LIGO-VIRGO

Neutron stars

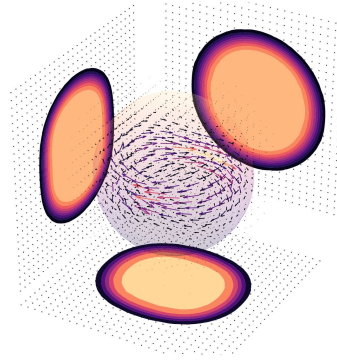
G. Grams, W.R. et al., in preparation.



More realistic NS predictions:

- higher maximum mass
 - compatible with NICER
 - compatible with LIGO-VIRGO
- realistic pairing properties in INM
 - constrained to advanced calculations
- but not at the cost of finite nuclei!

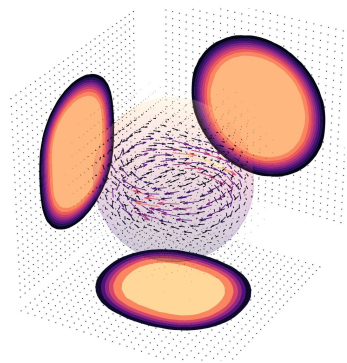
Additional observables



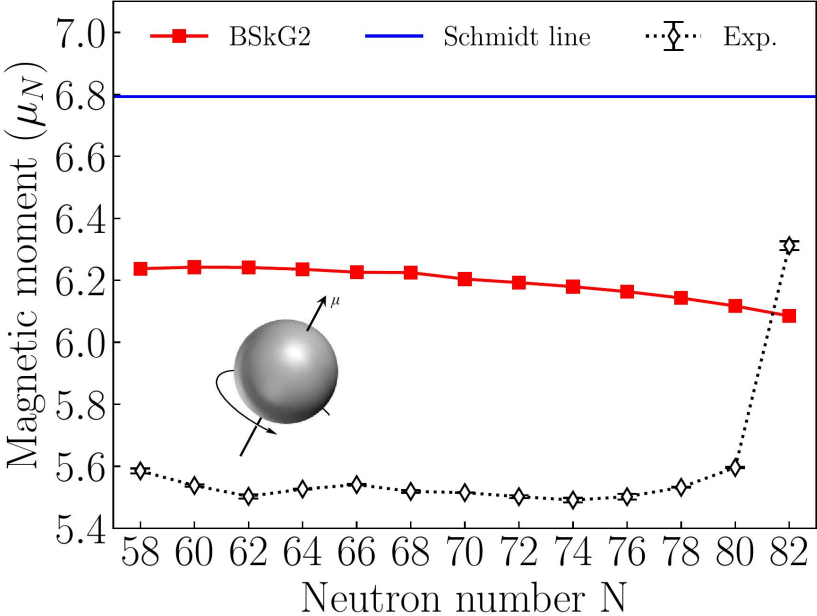
A. R. Vernon et al., Nature 607, **260** (2022),
J. Eberz et al., NPA **464**, 9 (1987).
J.Y. Zeng et al. PRC **50**, 1388 (1994)

Additional observables

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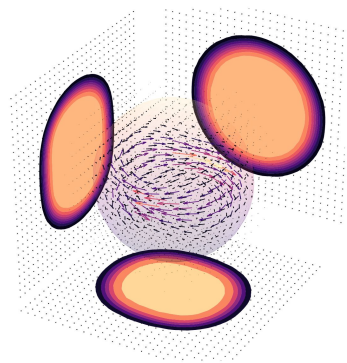


$J^\pi = 9/2^+$ states in In isotopes

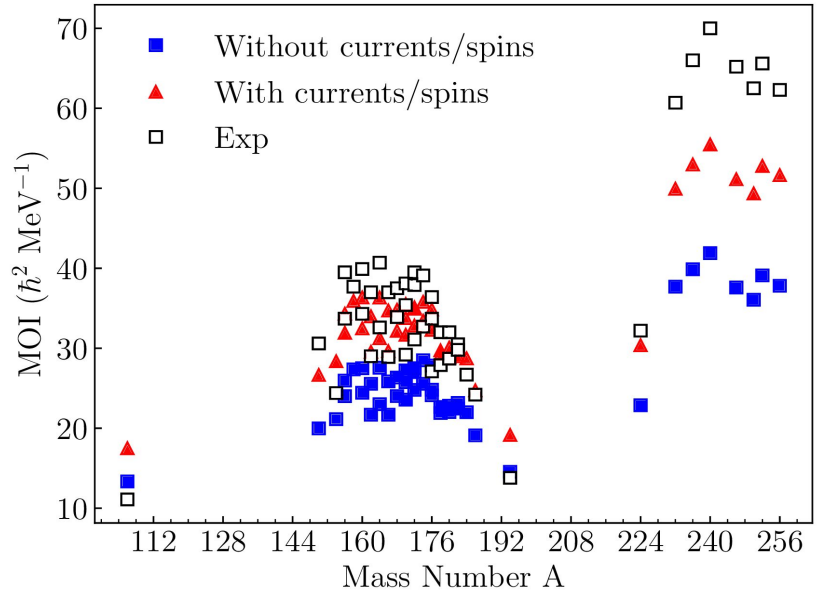
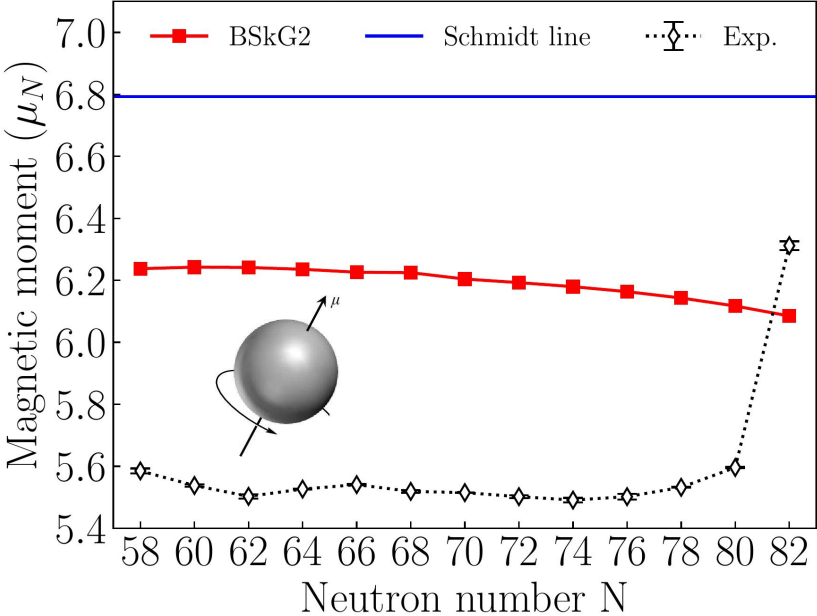


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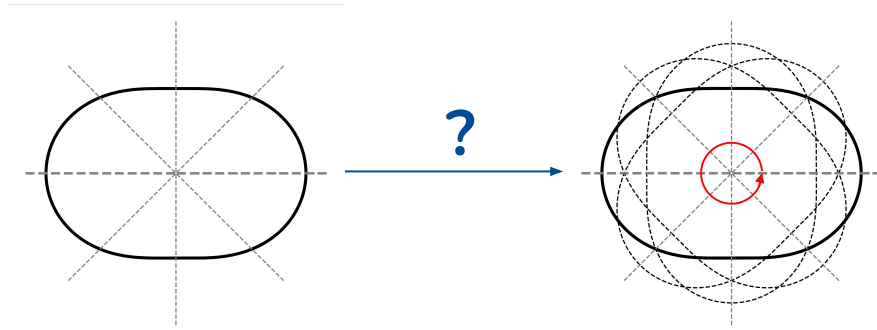
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$J^\pi = 9/2^+$ states in In isotopes



Challenges: less phenomenology

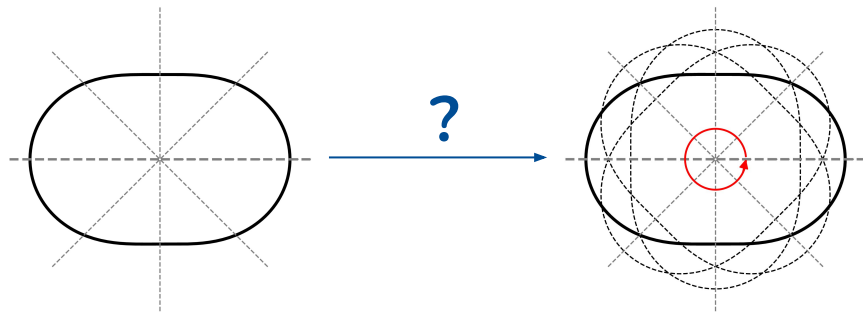


Leave the mean-field picture behind

- techniques exist
- ... but remain extremely costly

Challenges: less phenomenology

$$E \sim \int d^3r \left[C^\rho \rho(\mathbf{r})\rho(\mathbf{r}) + C^\tau \tau(\mathbf{r})\rho(\mathbf{r}) + \textcolor{red}{?} \dots \right]$$



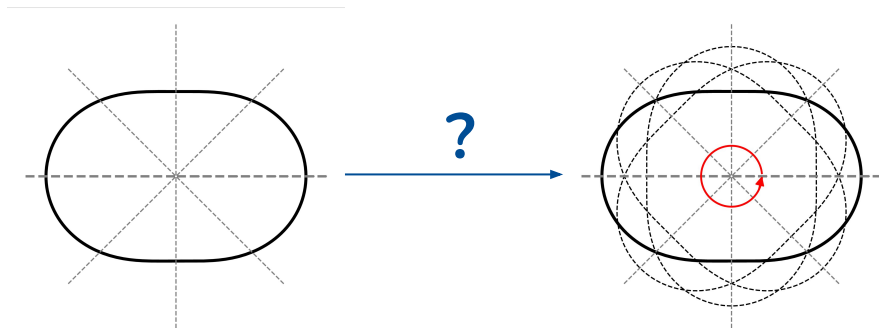
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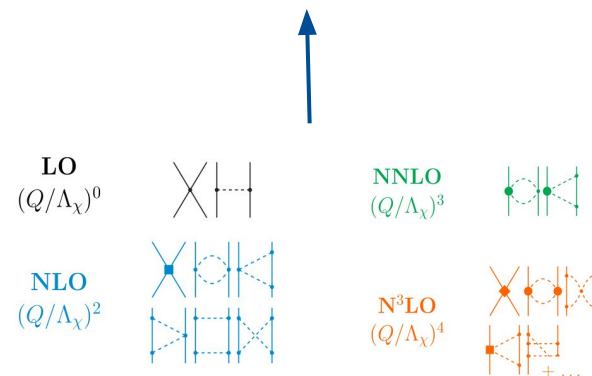
A “well-founded” functional

- traditional EDF forms are wearing out
- ways for **systematic** construction?
- ... perhaps by linking with ab initio?

Challenges: less phenomenology



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Adapted from H. Hergert, Front. Phys. 8:379 (2020).

Leave the mean-field picture behind

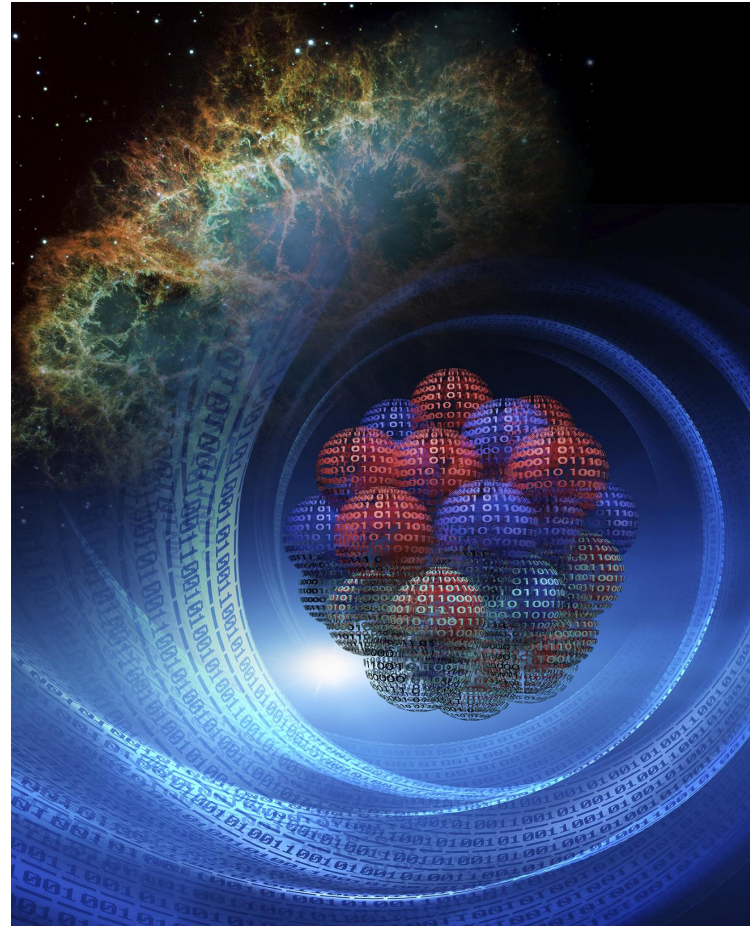
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A “well-founded” functional

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Conclusion

We build large-scale, microscopic models for (astro) applications.

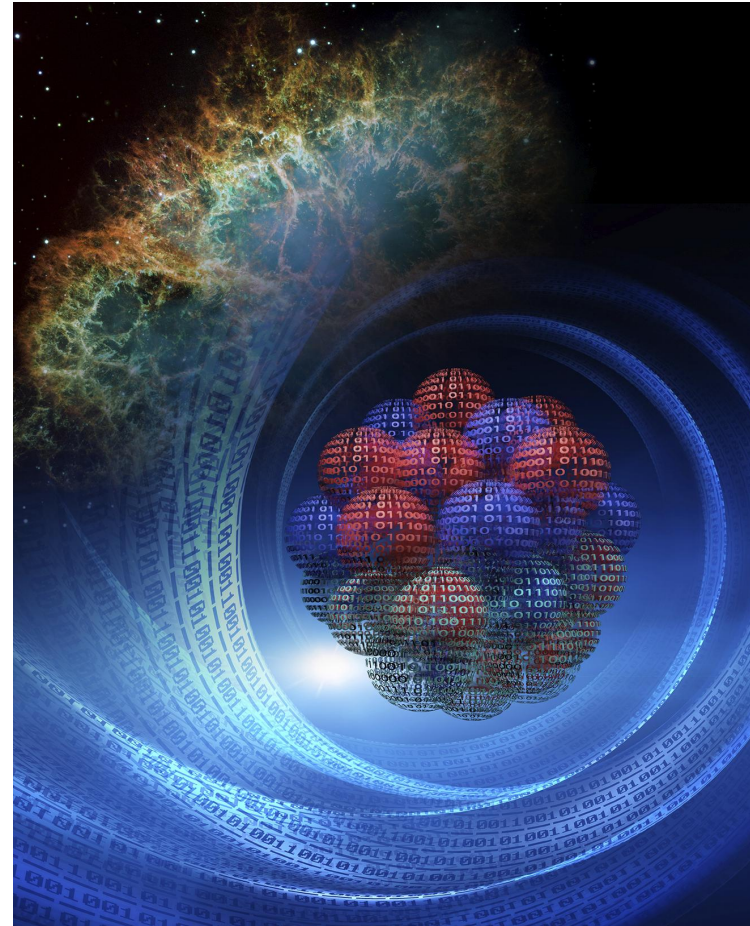


Conclusion

We build large-scale, microscopic models for (astro) applications.

Large-scale = thousands of nuclei and many observables.

Microscopic = simple wave functions yet complex symmetry breaking.



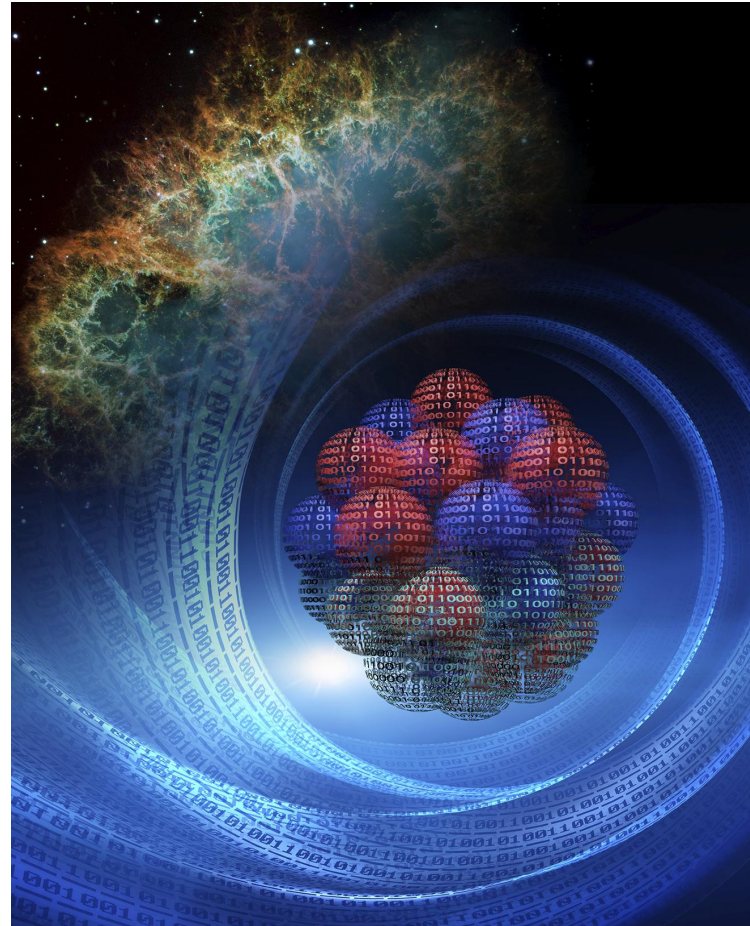
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BSkG1 and BSkG2 are pretty good, but....



Conclusion

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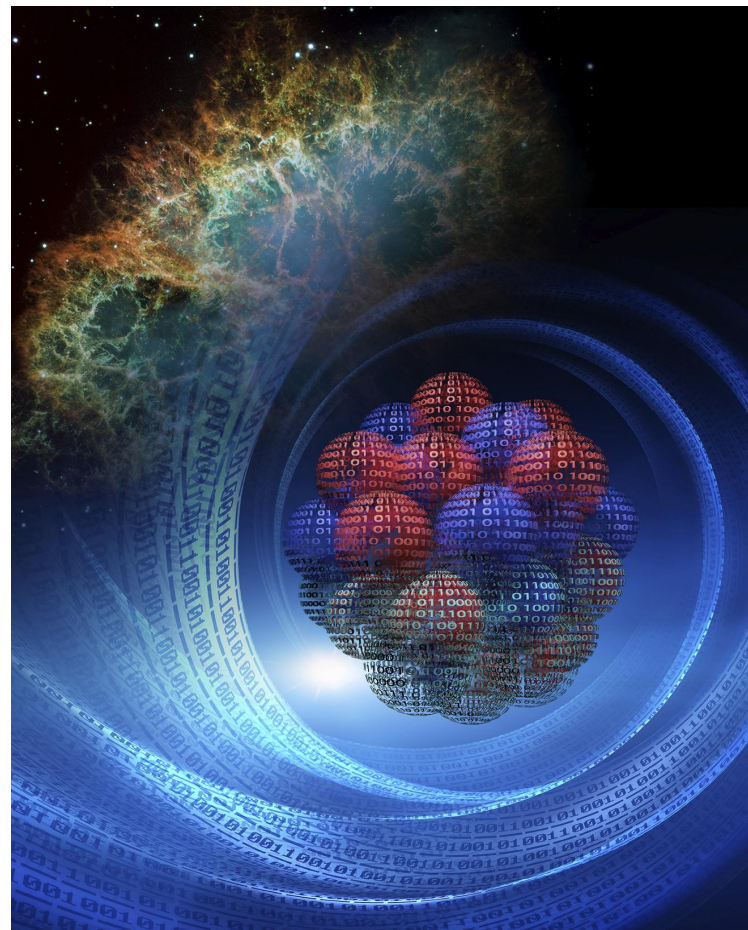
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BSkG1 and BSkG2 are pretty good, but....

BSkG3

- global inclusion of
 - triaxial deformation
 - time-reversal breaking
 - octupole deformation
- competitive reproduction of masses and charge radii
- best on the market for **fission** properties
- consistent with astrophysical observations



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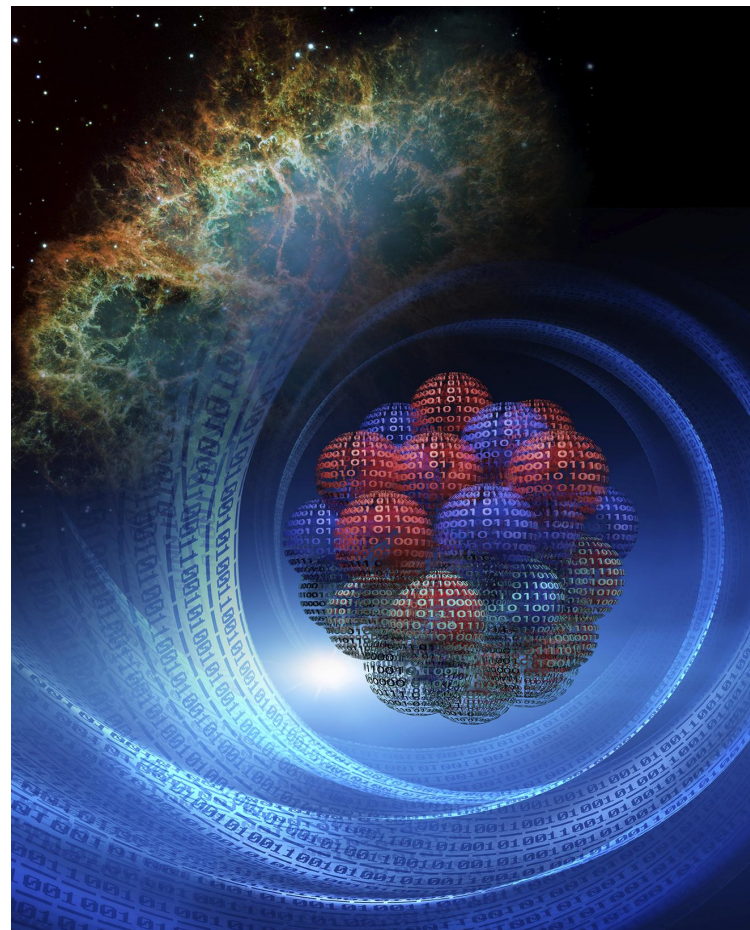
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- best on the market for **fission** properties
- consistent with astrophysical observations

Coming up from the Brussels group:

- all BSkG3 data
- detailed study of ground state densities
- large-scale **fission** and **level density** calculations
- unified Equation of State for neutron star applications



Thank you for...

..... all the wonderful work!



S. Goriely

G. Grams

N. Chamel

N. Shchepochin



M. Bender

J. Bonnard



G. Scamps



M. Hukkanen

M. Stryczyk

A. Kankainen



P. Ascher

S. Grévy



E. Verstraeten

T. Cocolios

P. Van Duppen



**UNIVERSITÄT
HEIDELBERG**
ZUKUNFT
SEIT 1386

G. Giacalone



B. Schenke

C. Shen



S. Hilaire

..... the computing time!



..... the funding!

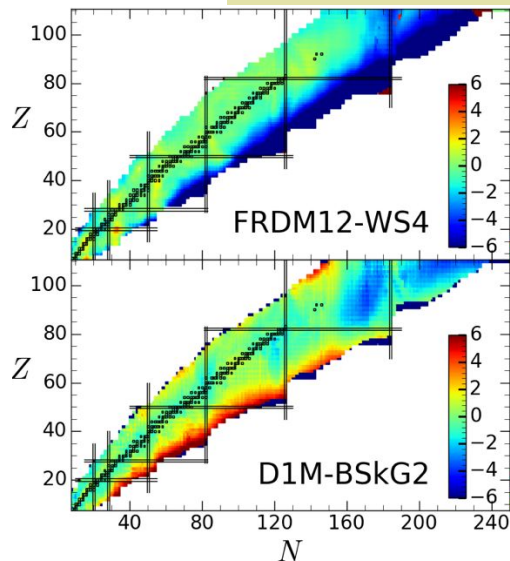


..... your attention!

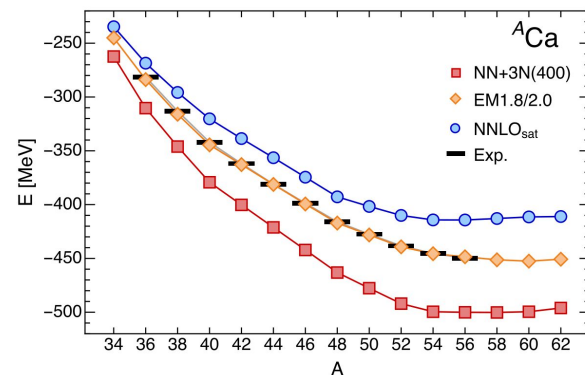
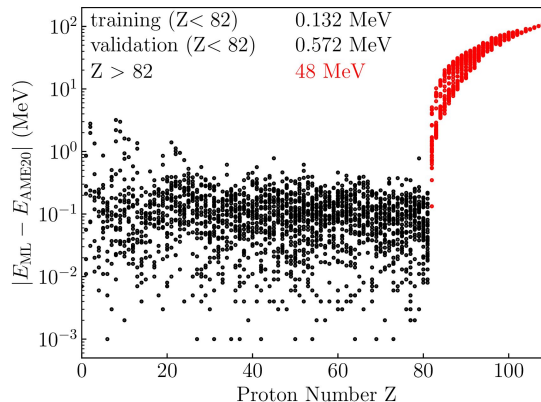
Bonus!

Interlude: why do we do these complex things?

S. Goriely, EPJA 59, 16 (2023).



G. Grams, W.R. et al., in preparation



Mic-mac approaches?

- ✓ competitive in rms
- ✓ multiple observables
- ✗ comparatively unstable
- ✗ no link mic. \leftrightarrow mac.

Machine learning?

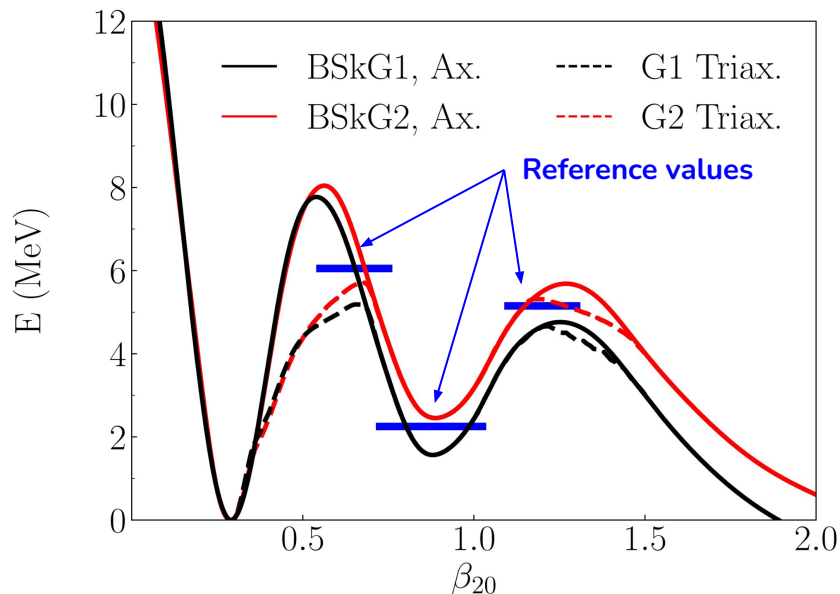
- ✓ absolute champion in rms
- ✓ ridiculously easy
- ✗ thousands (?) of parameters
- ✗ single observable

Ab Initio?

- ✓ error quantification
- ✓ “truly” microscopic
- ✓ multiple observables
- ✗ infeasible at scale (for now)
- ✗ not competitive on rms (for now)

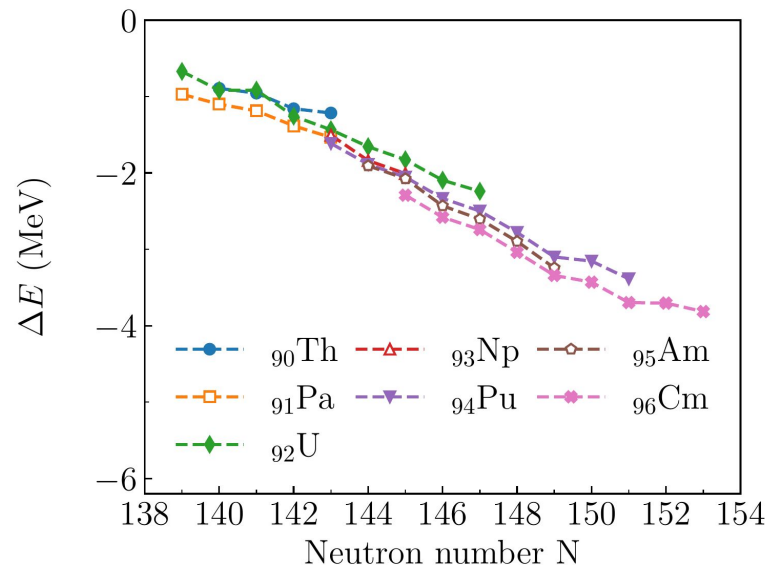
Triaxiality has a **large** effect on barriers

Reference values from R. Capote et al., Nuclear Data Sheets **110**, 3107 (2009).



Triaxial deformation for ^{240}Pu

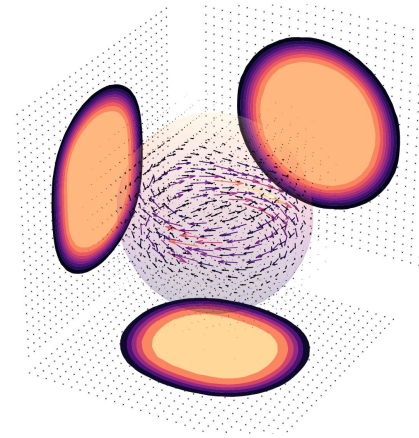
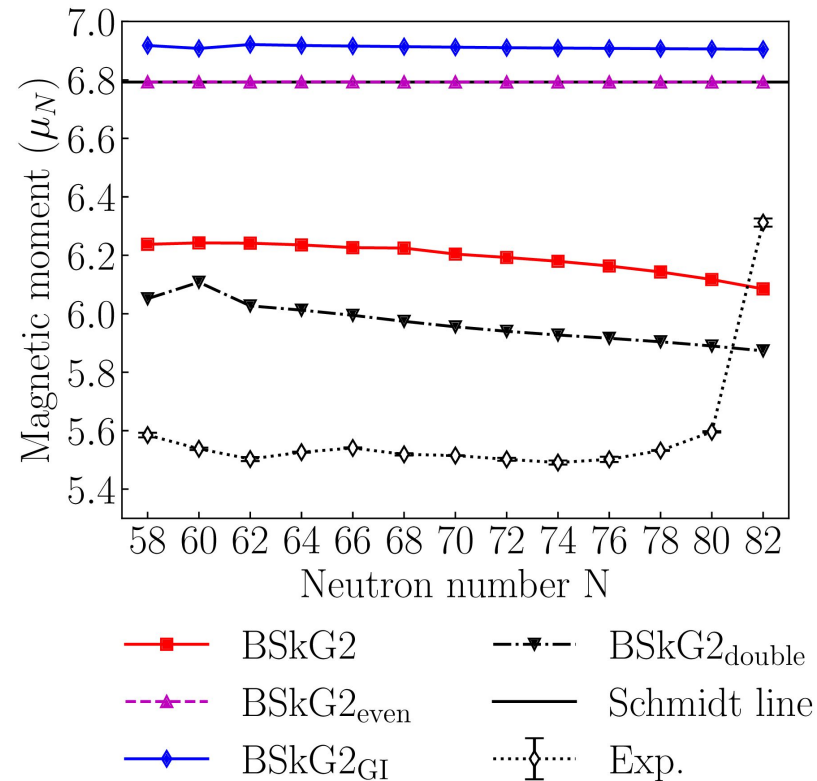
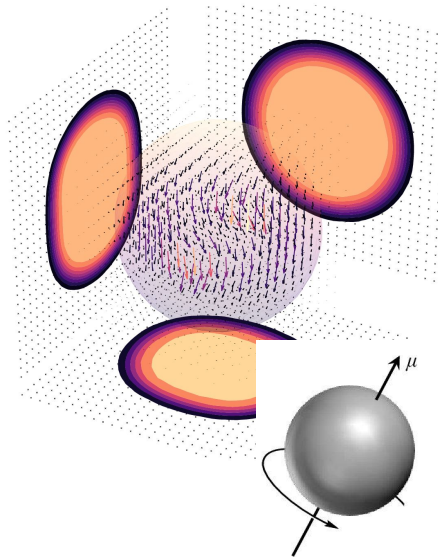
- Large effect on inner barrier
- No effect on isomers
- Modest effect on outer barrier



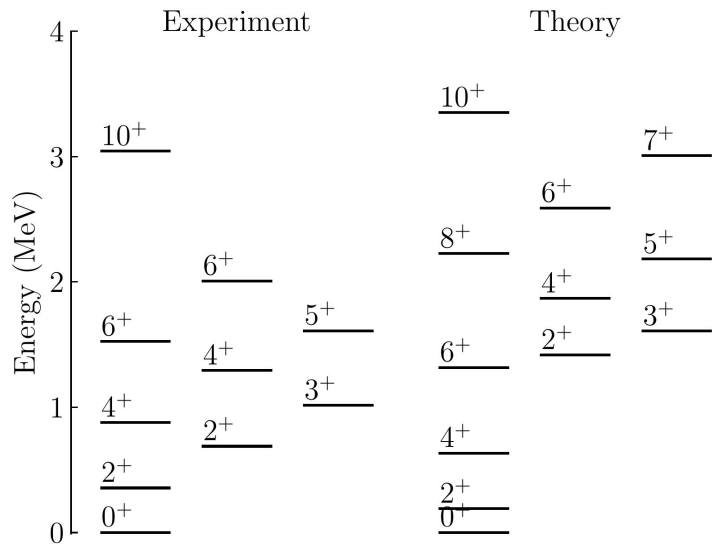
Triaxial deformation for actinides

- Larger effects with growing N
- reminder: $\sigma(\text{fission}) < 0.5 \text{ MeV}$
- what other regions does it affect?

Magnetic moments

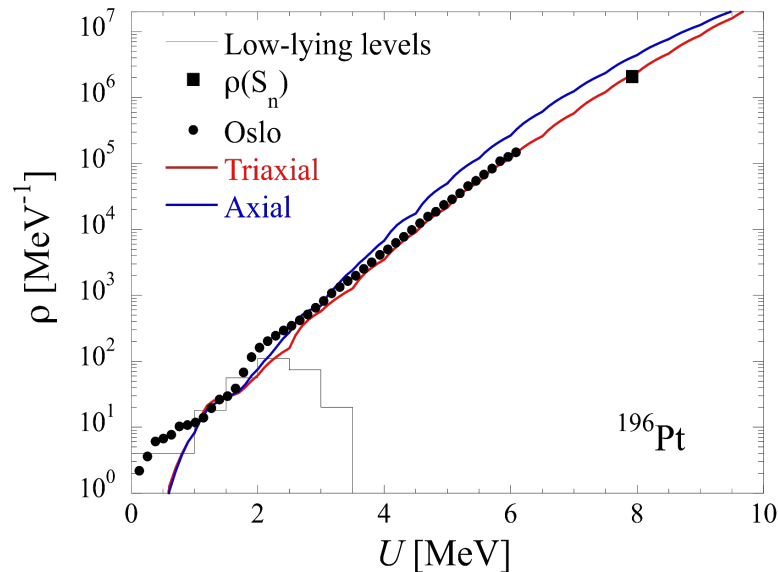


What is the effect on **nuclear level densities**?



Broken symmetries impact NLDs

- **axial** rotors give rise to **sparse** spectra
- **triaxial** rotors have **dense** spectra
- simple models for collective effects



Level densities with BSkG2

F. Giacoppo et al., PRC 90, 054330 (2014).

- not always higher level density
- but a **different energy dependence!**
- systematic calculations underway