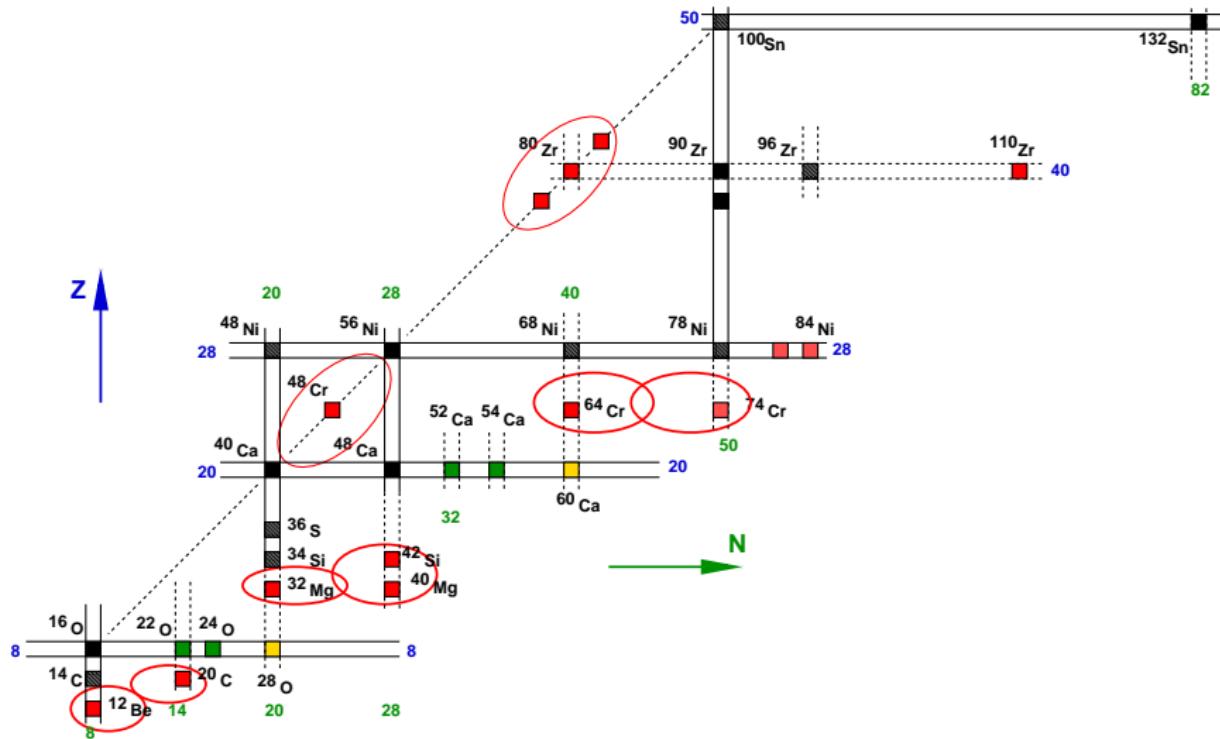


Island of Inversion at the N=Z line

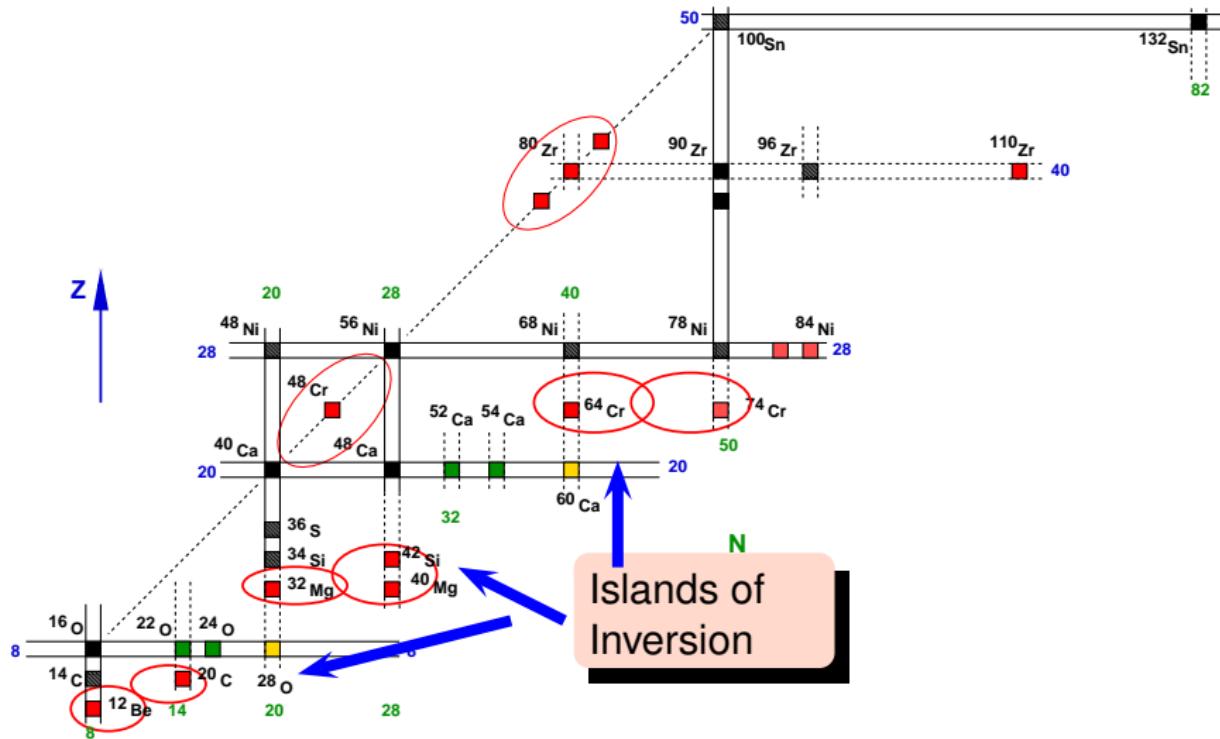
D. D. Dao, F. Nowacki, A. Poves



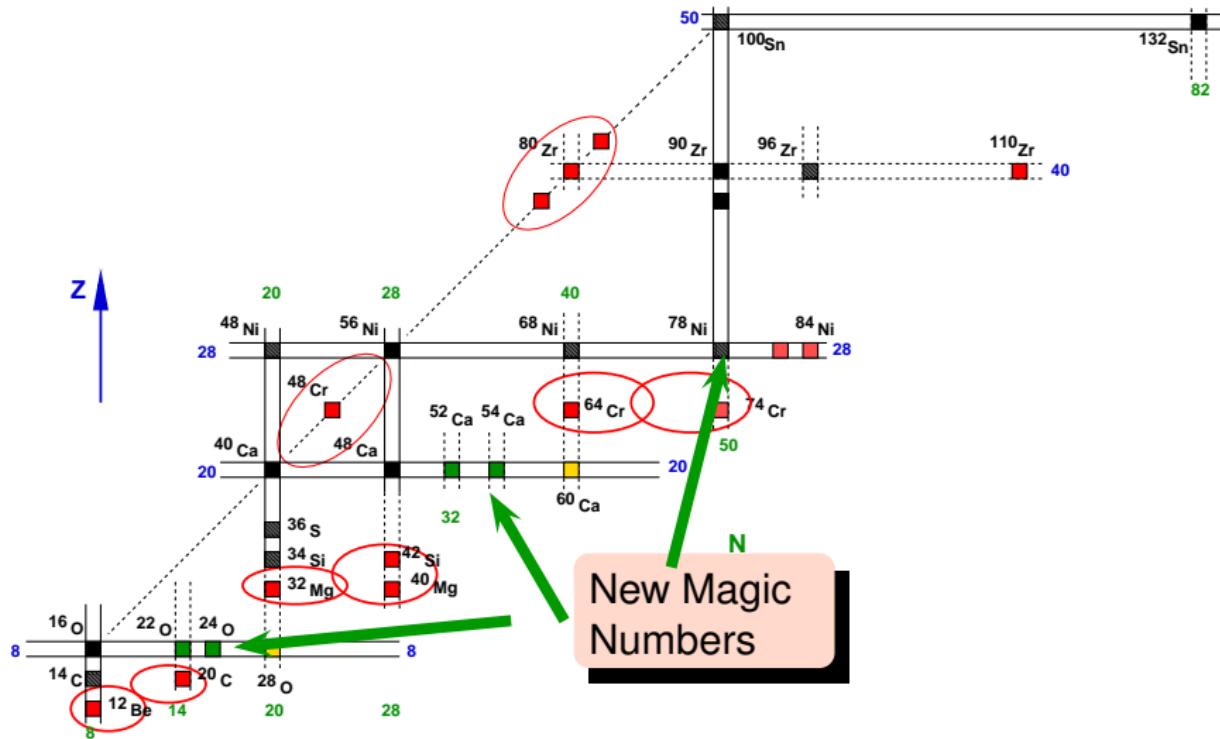
Landscape of medium mass nuclei



Landscape of medium mass nuclei



Landscape of medium mass nuclei



Landscape of medium mass nuclei

UNDERSTANDING REGULARITIES for both SPHERICAL and DEFORMED systems

132Sn
82

- New Magic Numbers: ^{24}O , ^{48}Ni , ^{54}Ca , ^{78}Ni , ^{100}Sn
- Vanishing of shell closures: ^{12}Be , ^{32}Mg , ^{42}Si , ^{64}Cr , ^{80}Zr ...
- Island of deformation around $A \sim 32$, $A \sim 64$
- Low-lying dipole excitations in Ne, Ni isotopes

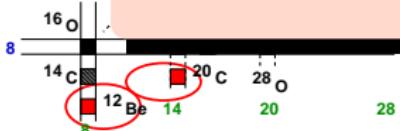
Z

- Variety of phenomena dictated by shell structure
- Close connection between collective behaviour and underlying shell structure
-

$$\mathcal{H} = \mathcal{H}_m + \mathcal{H}_{\mathcal{M}}$$

Interplay between

- Monopole field (spherical mean field)
- Multipole correlations (pairing, Q.Q, ...)



Development of deformation at N=8,20,40,70

F. Nowacki, A. Obertelli and A. Poves

Progress in Particle and Nuclear Physics 120 (2021) 103866

- Magic numbers are associated to energy gaps in the spherical mean field. Therefore, to promote particles above the Fermi levels costs energy
- However some intruders configurations can overwhelm their loss of monopole energy with their huge gain in correlation energy
- Several examples of this phenomenon exist in stable magic nuclei (as in ^{40}Ca nucleus) in the form of coexisting spherical, deformed and superdeformed states in a very narrow energy range
- At the very neutron rich or very proton rich edges, the T=0 and T=1 channels of the effective nuclear interaction weight very differently than they do at the stability line. Therefore the effective single particle structure may suffer important changes, leading in some cases to the vanishing of established shell closures or to the appearance of new ones

Fig. 40. Schematic view of the valence spaces at N = 8, 20, 40 and 70. The intruder configurations that develop quadrupole collectivity are highlighted.

Development of deformation at N=8,20,40,70

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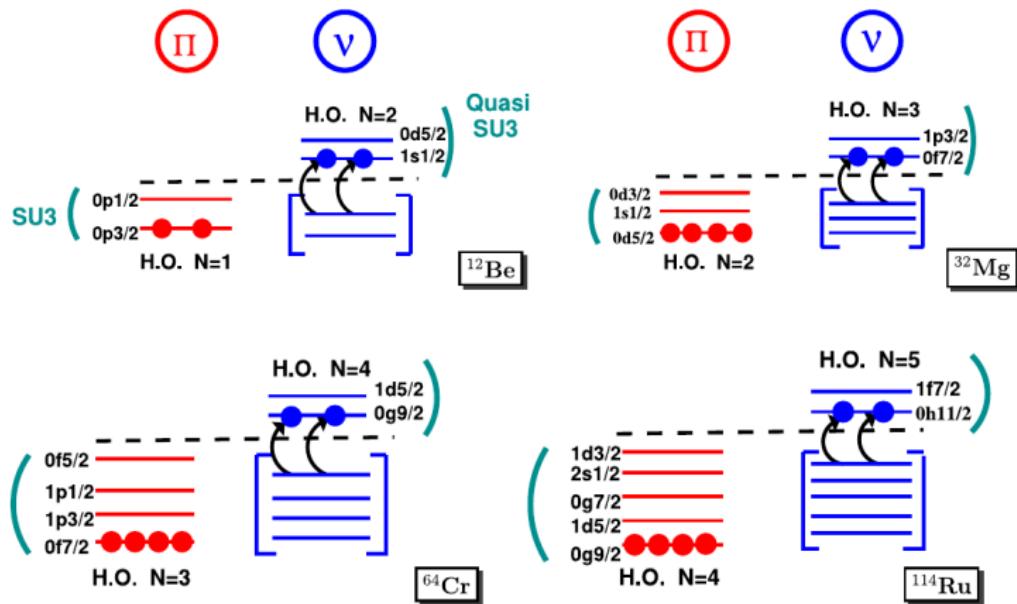
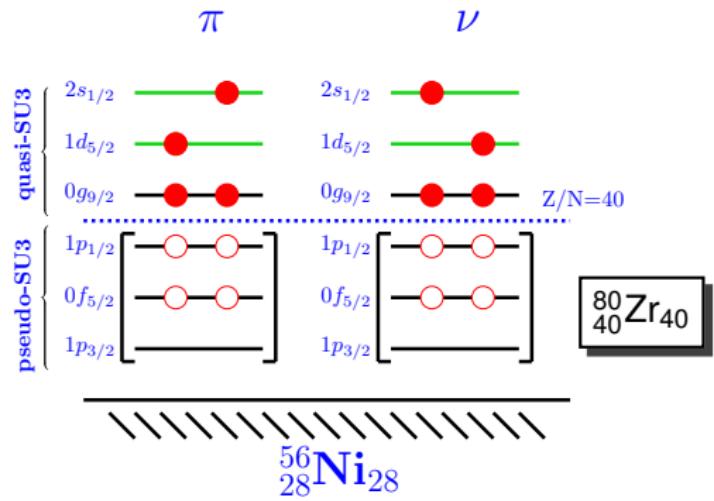


Fig. 40. Schematic view of the valence spaces at $N = 8, 20, 40$ and 70 . The intruder configurations that develop quadrupole collectivity are highlighted.

H.O. vs Spin-Orbit shell closure at N=Z



- p shell: ^{16}O
spherical/doubly magic
- sd shell: ^{40}Ca
spherical/doubly magic
- pf shell: ^{80}Zr
deformed nucleus

- Low-lying states in H.O. N=Z=8: CS , 4p4h, 8p8h
- Low-lying states in H.O. N=Z=20: CS , 4p4h, 8p8h
- Low-lying states in H.O. N=Z=40: ???

Island of Inversion at the N=Z line

Strongly deformed states at $N = Z$:

- Configuration mixing in ^{72}Kr
- Most deformed cases for ^{76}Sr , ^{80}Zr
- Shape transition between ^{84}Mo and ^{86}Mo

NSCL/GRETINA Experiment

R.D.O. Llewellyn *et al.*, Phys. Rev. Lett. **124**, 152501 (2020)

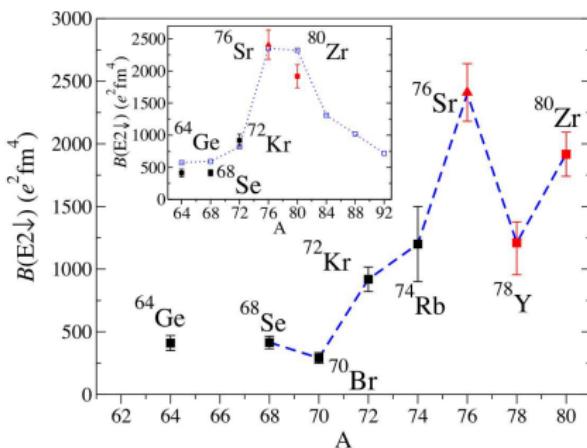
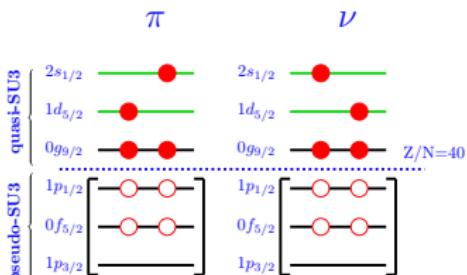


FIG. 3. Schematics of the $B(E2\downarrow)$ values for the $N = Z$ nuclei



- ZBM3 valence space:
extension of JUN45
to pseudo-SU3 + Quasi-SU3
- New effective interaction
(Realistic TBME
+ Monopole “3N” constraints”)
- SM + DNO-SM for most deformed cases

Discrete Non-Orthogonal Shell Model

Generator Coordinate Method: $|\Psi_{\text{eff}}\rangle = \sum_i f_i |\Phi_i\rangle$

1) Deformed Hartree-Fock (HF) Slater determinants

2) Restoration of rotational symmetry

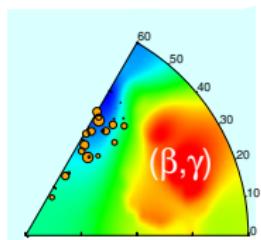
3) Mixing of shapes:

$$|\Psi_{\text{eff}}\rangle = \text{shape}_1 + \text{shape}_2 + \text{shape}_3 + \dots$$

Intrinsic/Laboratory Description

- **Deformation structure of nuclear states:** $\{J_\alpha^\pi\}$, $q = (\beta, \gamma)$

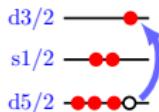
$$M_\alpha^{(J)}(q, K) = \sum_{q', K'} [\hat{N}^{1/2}]_{K' K}^{(J)}(q', q) f_\alpha^{(J)}(q', K')$$



- ◊ Probability of a configuration (β, γ) :

$$P_\alpha^{(J)}(q) = \sum_K |M_\alpha^{(J)}(q, K)|^2$$

- **particle-hole interpretation:**



M-scheme

- **K-quantum numbers:**

$$P_\alpha^{(J)}(K) = \sum_q |M_\alpha^{(J)}(q, K)|^2$$

Discrete Non-Orthogonal Shell Model

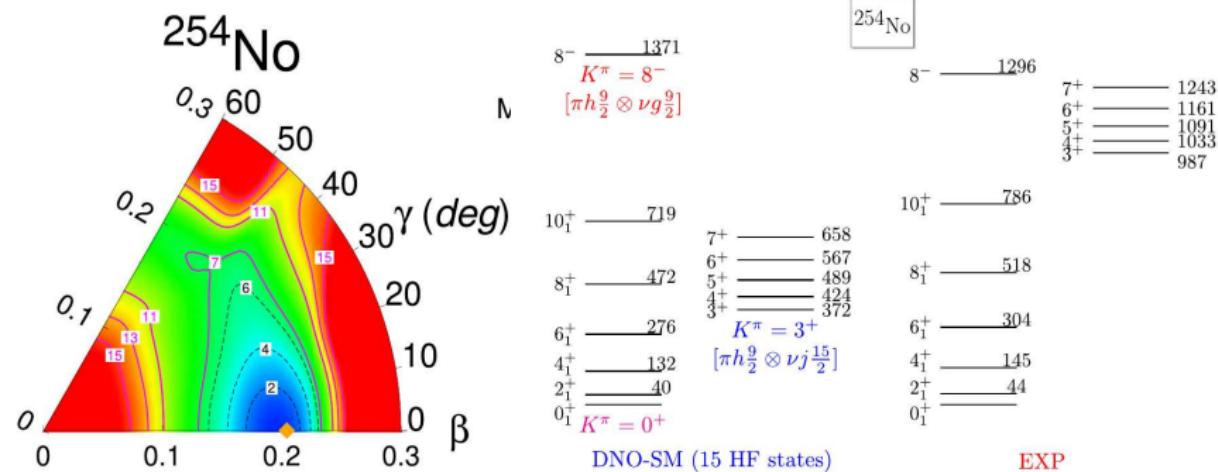
PHYSICAL REVIEW C 105, 054314 (2022)

Nuclear structure within a discrete nonorthogonal shell model approach: New frontiers

D. D. Dao[✉] and E. Nowacki[✉]

Université de Strasbourg, CNRS, IPHC UMR7178, 23 rue du Loess, F-67000 Strasbourg, France

(Received 8 March 2022; accepted 6 May 2022; published 23 May 2022)

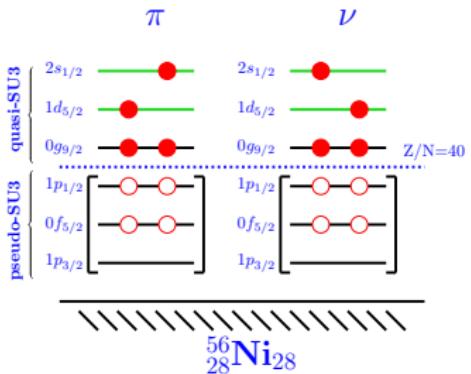
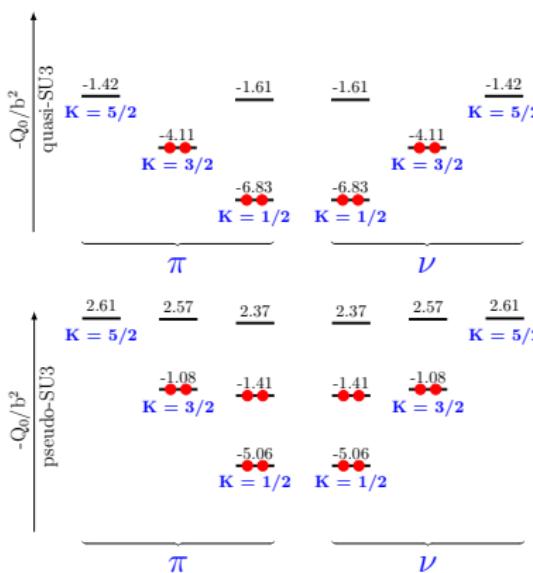


First “SM” calculations for superheavies !!!

Island of Inversion at the N=Z line

Strongly deformed states at $N = Z$

- Configuration mixing in ^{72}Kr
 - Most deformed cases for ^{76}Sr , ^{80}Zr
 - Shape transition between ^{84}Mo and ^{86}Mo
- NSCL/GRETINA Experiment**



nucleus	NpNh*	B(E2)(e ² .fm ⁴)			
		ZRP	PHF	Exp.	DNO-SM
^{76}Sr	4p-4h	924	806		
	8p-8h	2189	2101	2220	1847
	12p-12h	2316	2300		
^{80}Zr	4p-4h	587	637		
	8p-8h	1713	1509	1910	2325
	12p-12h	2663	2396		

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R.D.O. Llewellyn *et al.*, Phys. Rev. Lett. **124**, 152501 (2020)

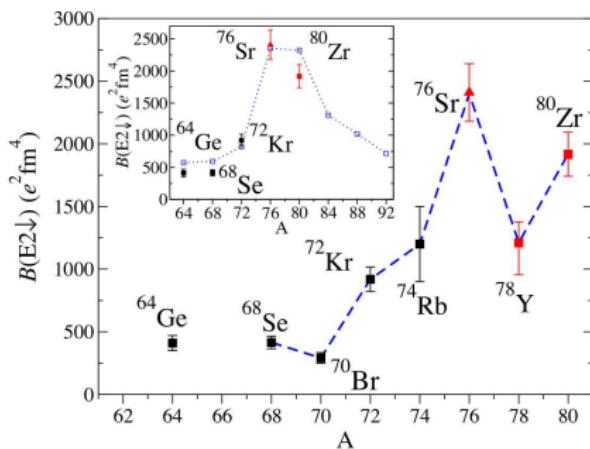
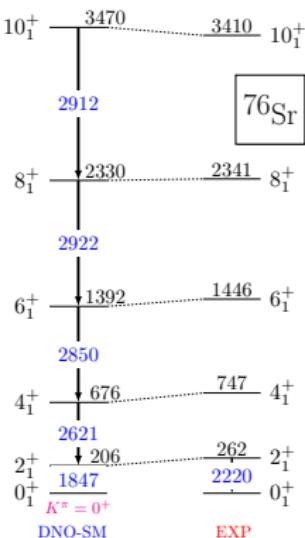
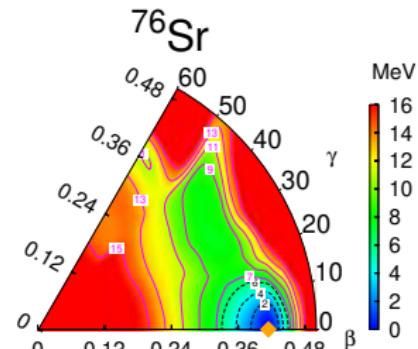


FIG. 3. Schematics of the $B(E2\downarrow)$ values for the $N = Z$ nuclei

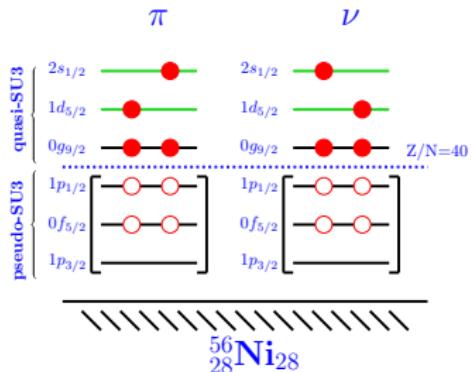
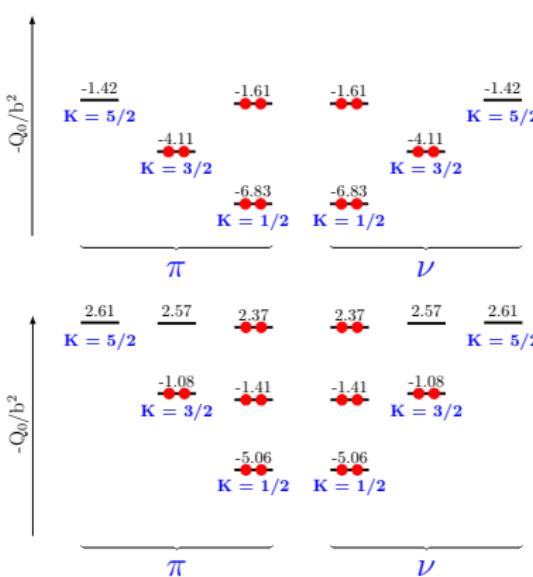


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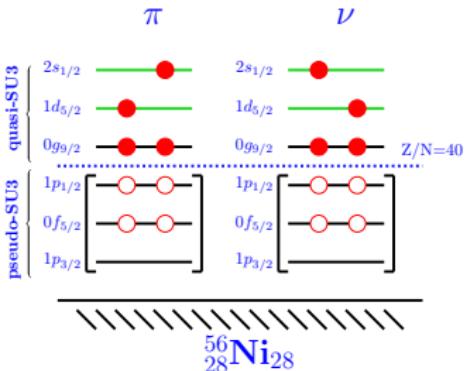
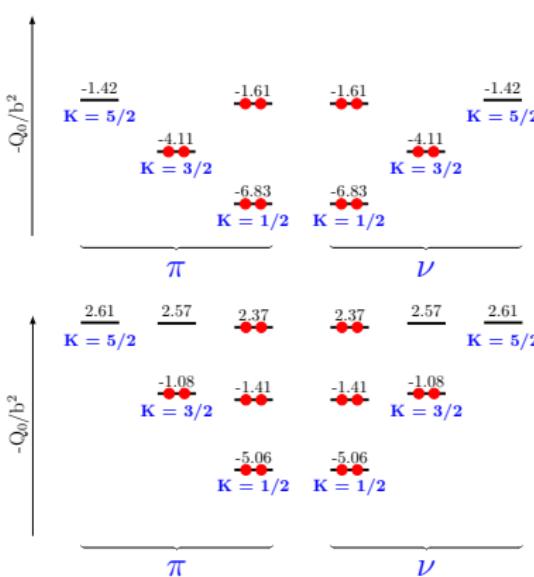
nucleus	Np-Nh*	B(E2)(e ² .fm ⁴)				SM	
		ZRP	PHF	Exp.	DNO-SM*		
^{84}Mo	4p-4h	1104	1193	1740^{+580}_{-730}	1765	-	
	8p-8h	1891	1732				
	0p-0h	542	196	$707(71)$	1184		
	2p-2h	1030	871				
^{86}Mo	4p-4h	1416	1179				
	6p-6h	1858	1655				

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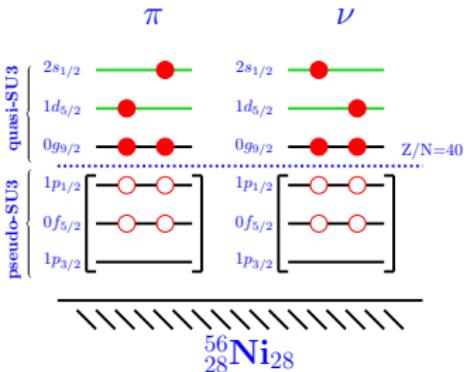
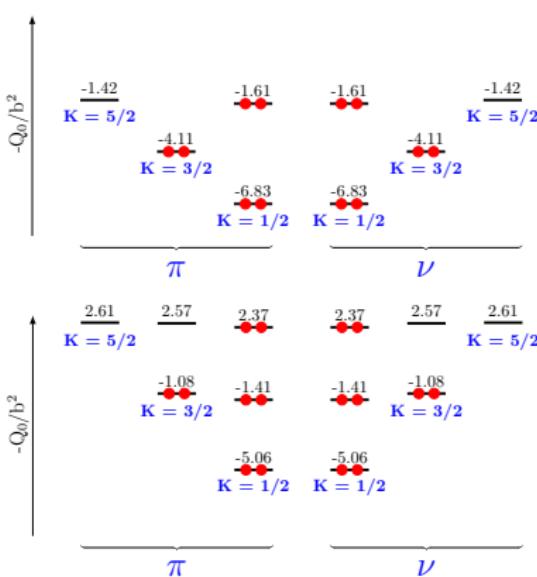
nucleus	Np-Nh*	B(E2)(e^2 .fm^4)			
		ZRP	PHF	Exp.	DNO-SM*
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^{86}Mo	4p-4h	1416	1179	707(71)	1184
	6p-6h	1858	1655		731

Island of Inversion at the N=Z line

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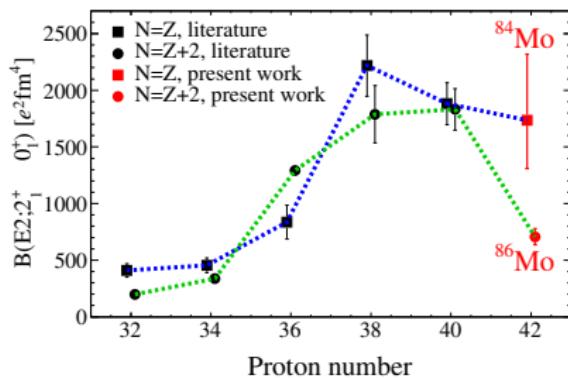
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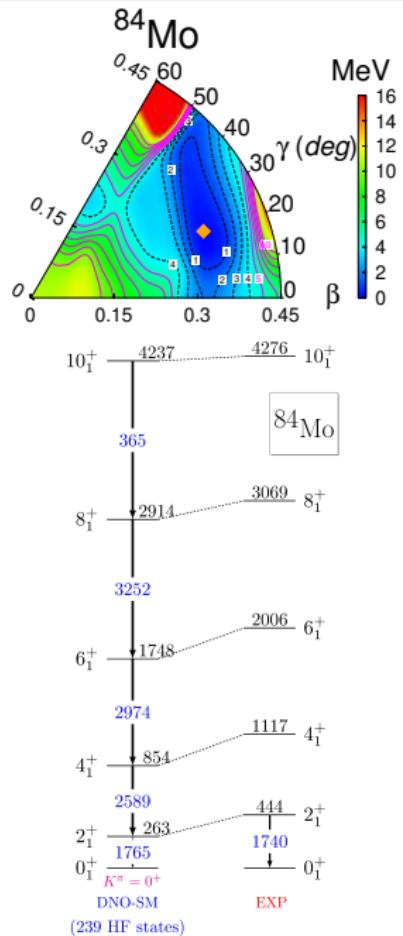
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NSCL/GRETINA Experiment



J. Ha, F. Recchia *et al.*, to be submitted

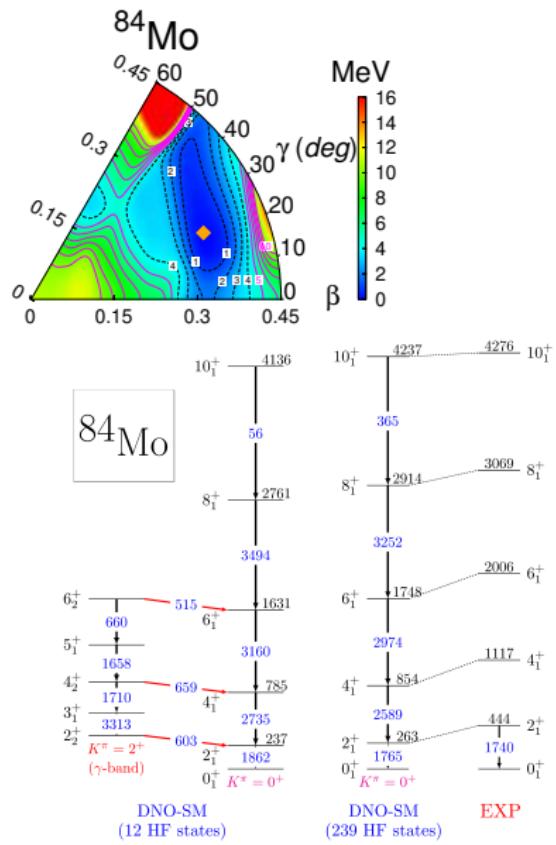
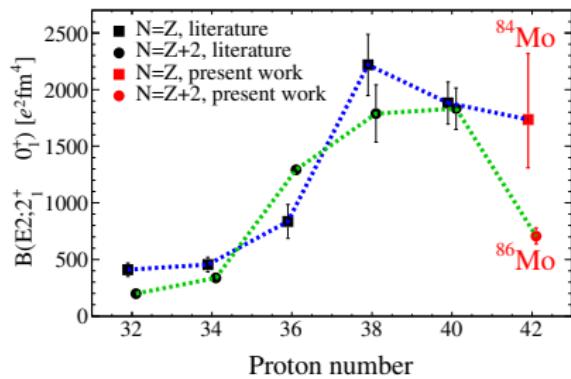


Island of Inversion at the N=Z line

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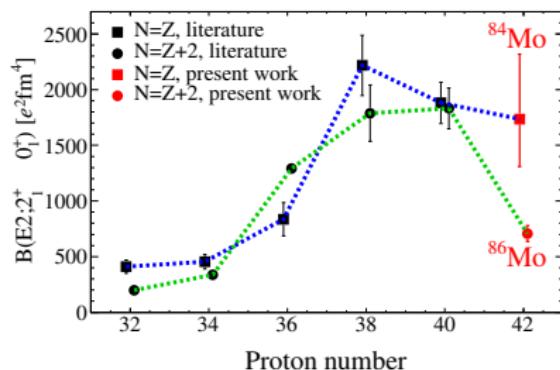
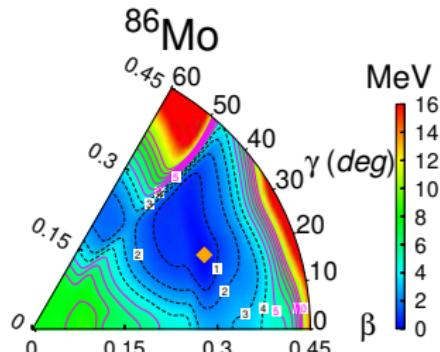


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NSCL/GRETINA Experiment



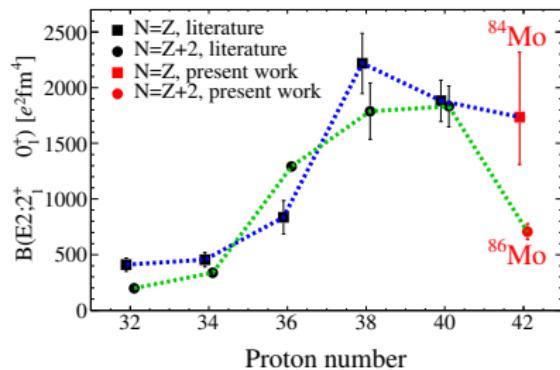
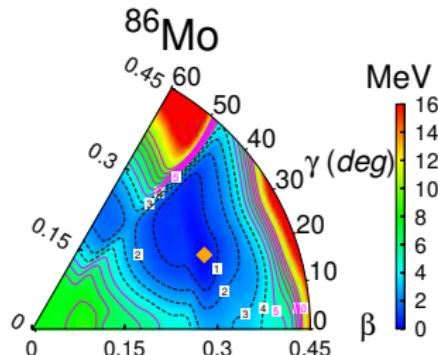
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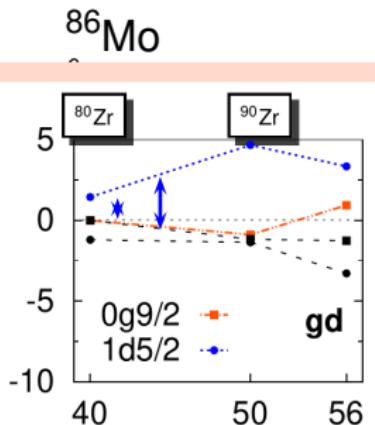
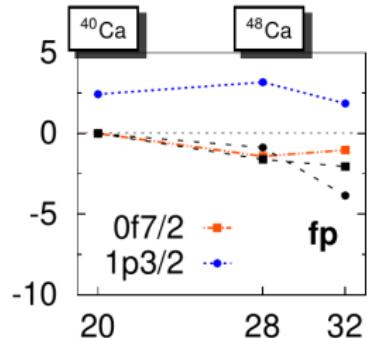
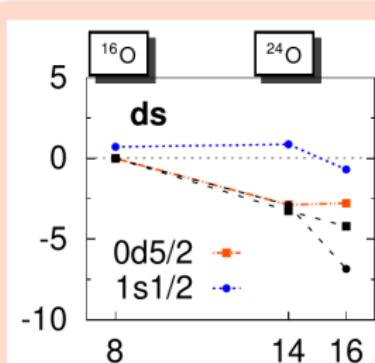
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Island of Inversion at the N=Z line

◇ Strongly deformed states at $N = Z$

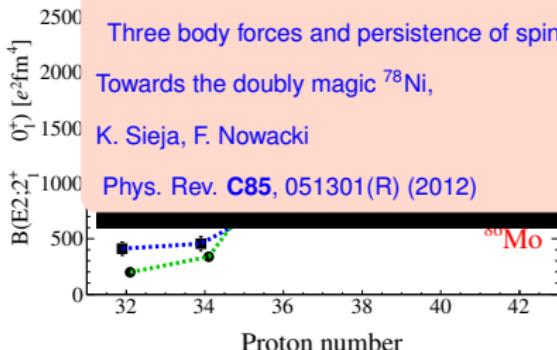


Three body forces and persistence of spin-orbit shell gaps in medium-mass nuclei:

Towards the doubly magic ^{78}Ni ,

K. Sieja, F. Nowacki

Phys. Rev. C85, 051301(R) (2012)



^{86}Mo	2p-2h	1030	871	1184	731	707(71)
	4p-4h	1416	1179			
	6p-6h	1858	1655			

Summary

- Monopole drift develops in all regions but the Interplay between correlations (pairing + quadrupole) and spherical mean-field (monopole field) determines the physics.
- New “island of inversion” or “island of deformation” present for neutron-rich systems show up also at N=Z line.
- Around A~ 80, an “island of enhanced collectivity” show very deformed rotors dominated by Many-particles-Many-holes configurations.
- Shape transition between ^{84}Mo and ^{86}Mo and first fingerprint of 3N forces in deformed systems

Summary

Special thanks to:

- J. Ha, S. Lenzi, F. Recchia
- J. Herzfeld-Nowacki

Discrete Non-Orthogonal Shell Model

Generator Coordinate Method: $|\Psi_{\text{eff}}\rangle = \sum_i f_i |\Phi_i\rangle$

- 1) Deformed Hartree-Fock (HF) Slater determinants
- 2) Restoration of rotational symmetry
- 3) Mixing of shapes:

$$|\Psi_{\text{eff}}\rangle = \text{shape}_1 + \text{shape}_2 + \text{shape}_3 + \dots$$

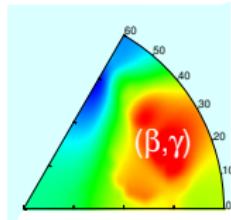
Basis Truncation Method



choice of relevant deformed Hartree-Fock states

- **E. Caurier's Minimization Technique:**

(E. Caurier, Proc. on GCM, BLG report **484** (1975))



- Based on the variational principle
- Minimization of the energy of given states $\{J^\pi\}$

- **Iterative procedure:**

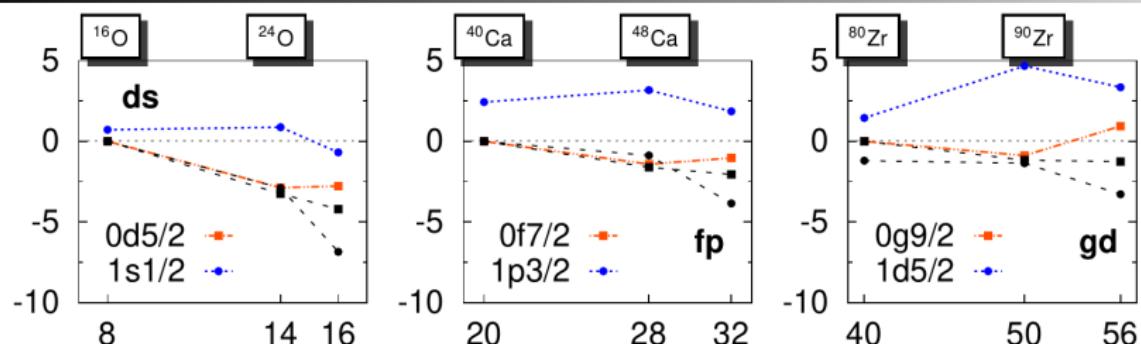
$$\Phi_1 \rightarrow (\Phi_1, \Phi_2) \rightarrow (\Phi_1, \Phi_2, \Phi_3) \dots$$

$N = 1$

$N = 2$

$N = 3$

Three-body forces in medium mass nuclei



- Evolution of the neutron effective single-particle energies with neutron filling in ds, fp, and gd shells
- “Universal” mechanism for the generation of T=1 spin-orbit shell closures
- Connection with 3N forces and ab-initio calculations:
 - “works” for Coupled-Cluster and to be checked in nickels
 - problems for “ab-initio” core shell-model

Three body forces and persistence of spin-orbit shell gaps in medium-mass nuclei: Towards the doubly magic ^{78}Ni
K. Sieja, F. Nowacki
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