Spin-independent couplings of WIMPs to nucleons

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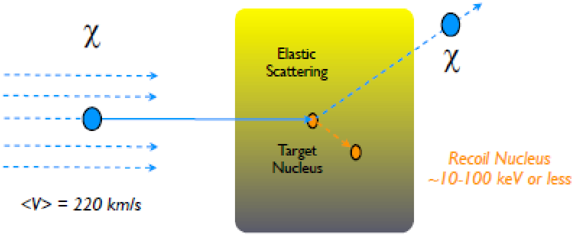
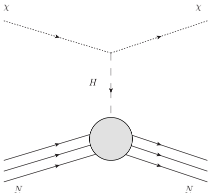
OCEVU "Lattice QCD enlightens DM"

(in progress/preliminary)

Direct WIMP dark matter detection



$$\downarrow$$



$$\mathcal{L}_{q\chi} = \sum_q \lambda_q^\Gamma [\bar{q}\Gamma q][\bar{\chi}\Gamma\chi] \longrightarrow \mathcal{L}_{N\chi} = \lambda_N^\Gamma [\bar{N}\Gamma N][\bar{\chi}\Gamma\chi]$$

Quark are confined within nucleons
$\rightarrow$ nonperturbative QCD tool

WIMP-nucleus spin-independent cross section

In low-E limit

$$\frac{d\sigma^{\rm SI}_{\chi\,{}^A_Z X}}{dq^2} = \frac{1}{\pi v^2}\left[Z f_p + (A-Z) f_n\right]^2\ |F_X(q^2)|^2$$

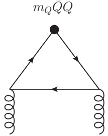
w/ $F_X(\vec{q}=0)=1$ nuclear FF and χN couplings $(N=p,n)$

$$\frac{f_N}{M_N} = \sum_{q=[ud],s} f_q^N \frac{\lambda_q}{m_q} + \sum_{Q=c,b,t} f_Q^N \frac{\lambda_Q}{m_Q}$$

such that ($f=u,d,s,c,b,t$ and $\langle N(\vec{p}\,')|N(\vec{p})\rangle = (2\pi)^3\delta^{(3)}(\vec{p}\,'-\vec{p})$)

$$f_{ud}^N M_N = \sigma_{\pi N} = m_{ud}\langle N|\bar{u}u+\bar{d}d|N\rangle, \qquad f_f^N M_N = \sigma_{fN} = m_f\langle N|\bar{f}f|N\rangle$$

For heavy $Q=c,b,t$ (Shifman et al '78)



$$\longrightarrow \qquad m_Q\,\bar{Q}Q = -\frac{1}{3}\frac{\alpha_s}{4\pi}G^2 + O\Big(\alpha_s, \frac{\mathcal{O}_6}{m_Q^2}\Big)$$

Heavy quark contributions

Then obtain f_Q^N in terms of f_q^N through to $M_N = \langle N|\theta^\mu{}_\mu|N\rangle$, w/

$$\theta^\mu{}_\mu = (1-2\gamma_m(\alpha_s))\left[\sum_{q=u,d,s} m_q\bar{q}q + \sum_{Q=c,b,t} m_Q\bar{Q}Q\right] + \frac{\beta(\alpha_s)}{2}G^2$$

Find, $Q=(c,)b,t$,

$$f_Q^N \equiv \frac{\langle N|m_Q\bar{Q}Q|N\rangle}{M_N} = -\frac{2}{3}\frac{\alpha_s}{\tilde{\beta}(\alpha_s)}\Big(1-\sum_{q=[ud],s} f_q^N\Big) + O\Big(\alpha_s, \frac{\Lambda^2_{\rm QCD}}{m_Q^2}\Big)$$

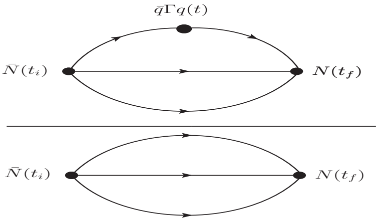
w/ $\tilde{\beta}(\alpha_s) = -\tilde{\beta}_0\alpha_s + O(\alpha_s)$ and $\tilde{\beta}_0 = \beta_0 + \frac{2}{3}N_Q = 11 - \frac{2}{3}N_q = 9$

- Need to measure or compute nonperturbative QCD quantities $f_q^N M_N = \sigma_{qN} = m_q\langle N|\bar{q}q|N\rangle$, $q=u,d,s(,c)$ $\rightarrow$ Lattice QCD
- If only t,b obtained w/ HQE, syst on $f^N_{(t/b)}$ is $\sim O(\Lambda^2_{\rm QCD}/m_b^2) \sim 0.005$, and if c is also, syst on $f^N_{(t/b/c)}$ is $\sim O(\Lambda^2_{\rm QCD}/m_c^2) \sim 0.06$

σ-terms from LQCD: matrix element (ME) method

$$\sigma_{\pi N} = m_{ud}\langle N|\bar{u}u + \bar{d}d|N\rangle \qquad\qquad \sigma_{fN} = m_f\langle N|\bar{f}f|N\rangle$$

Extract directly from time-dependence of 3-pt fns:



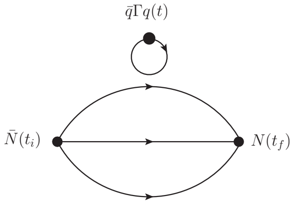
$$\xrightarrow{(t-t_i),(t_f-t)\to\infty} \qquad \langle N(\vec{0})|\bar{q}\Gamma q|N(\vec{0})\rangle$$

✓ Desired matrix element appears at leading order

✗ Must compute more noisy 3-pt fn

✗ Quark-disconnected contribution very challenging, though generally suppressed

✗ $m_q\bar{q}q$ renormalization challenging (Wilson fermions)

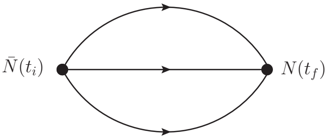


σ-terms from LQCD: Feynman-Hellmann (FH) method

Feynman-Hellmann (FH) theorem gives

$$\langle N|m_q\bar{q}q|N\rangle = m_q \left.\frac{\partial M_N}{\partial m_q}\right|_{m_q^{(\phi)}}$$

On lattice get M_X from time-dependence of 2pt-fn, e.g.:



$$\xrightarrow{(t-t_i)\to\infty} \quad \langle 0|N|N\rangle\langle N|\bar{N}|0\rangle \exp\{-M_N(t_f - t_i)\}$$

- ✓ Only simpler and less noisy 2pt-fn is needed
- ✓ No difficult quark-disconnected contributions
- ✓ No difficult renormalization
- ✗ m_{ud}-dependence not very large

$$M_N \simeq 939\,\mathrm{MeV} \simeq 895\,\mathrm{MeV} + O(m_{ud})$$

$\rightarrow$ must extract correction precisely

- ✗ m_s and even more so m_c dependences are even smaller around their physical values

Strategy of calculation

For f_q^N, $q = u, d, s$, use lattice QCD and Feynman-Hellmann theorem

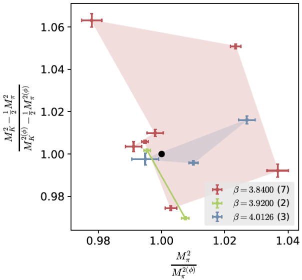
$$f_q^N M_N = \langle N|m_q\bar{q}q|N\rangle = m_q \left.\frac{\partial M_N}{\partial m_q}\right|_{m_q^{(\phi)}}$$

Method:

- Perform many high-statistics simulations with various m_q around physical values, various $a \lesssim 0.1\,\mathrm{fm}$ and various $L \gtrsim 8\,\mathrm{fm}$
- For each compute M_π ($\to m_{ud}$), $M_{K_\chi} = \sqrt{2M_K^2 - M_\pi^2}$ ($\to m_s$), M_{D_s} ($\to m_c$) and M_N ($\to \Lambda_{\mathrm{QCD}}$)
- Study dependence of m_q, $q = ud, s, c$ and M_N on M_π^2, $M_{K_\chi}^2$, M_{D_s}, a and L
- For each simulation determine a, $m_q^{(\phi)}$'s such that M_π, ...take their physical value in $a \to 0$ and $L \to \infty$ limit
- Compute, at physical point

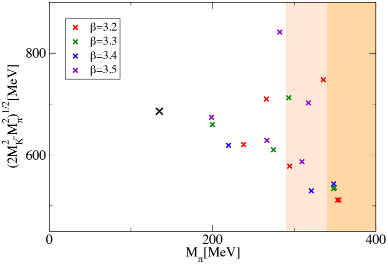
$$f_q^N = \sum_{P=\pi,K_\chi} \frac{\partial \ln M_P^2}{\partial \ln m_q} \frac{\partial \ln M_N}{\partial \ln M_P^2}$$

$$\frac{\partial \ln M_P^2}{\partial \ln m_q}$$



- $N_f = 2+1+1$ 4-stout staggered fermions on tree-level improved Symanzik gluons
- 3 $a \in [0.064, 0.095]\,\mathrm{fm}$ and $LM_\pi > 4$

$$\frac{\partial \ln M_N}{\partial \ln M_P^2}$$



- $N_f = 1+1+1+1$ 3-HEX clover fermions on tree-level improved Symanzik gluons
- 4 $a \in [0.064, 0.102]\,\mathrm{fm}$ and $LM_\pi > 12$

Improvements over BMWc, PRL '16

- ✓ Charm in sea
- ✓ $\gtrsim \times 100$ in statistics
- ✓ $\gtrsim \times 2$ lever arm in m_s
- ✓ Like PRL '16 FH in terms of quark and not meson masses
- ✗ No physical m_{ud} for $\frac{\partial \ln M_N}{\partial \ln M_P^2}$, but small enough and know M_N from experiment

Determination of Jacobian (preliminary)

- Jacobian:

$$\frac{\partial \ln M_P^2}{\partial \ln m_q}$$

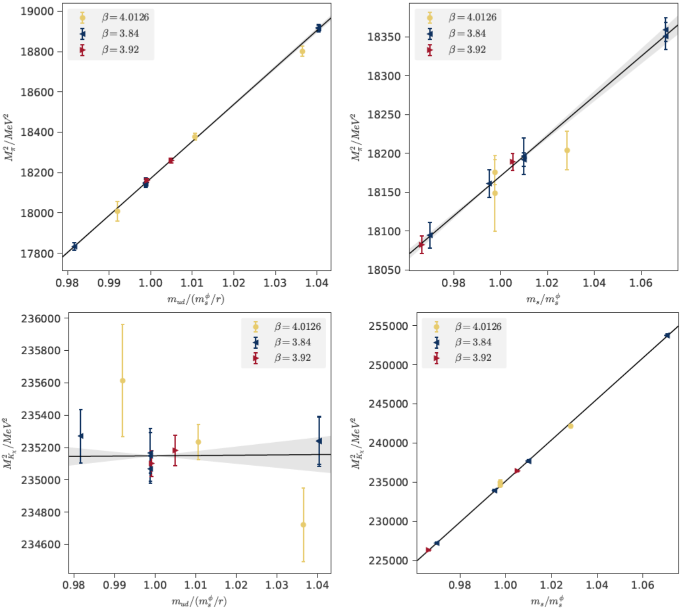
w/ $P = \pi, K_\chi$, $q = u, d, s$

- Two independent analyses give completely compatible results

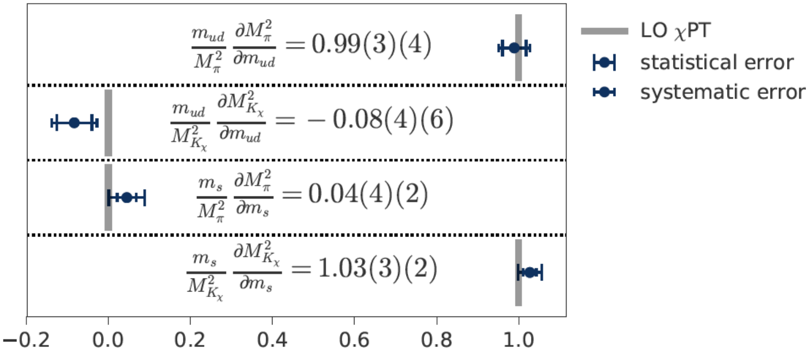
  (1) m_q vs M_π^2 & $M_{K_\chi}^2$

  (2) M_π^2 & $M_{K_\chi}^2$ vs m_q

- Here show (2)



Results for Jacobian (preliminary)

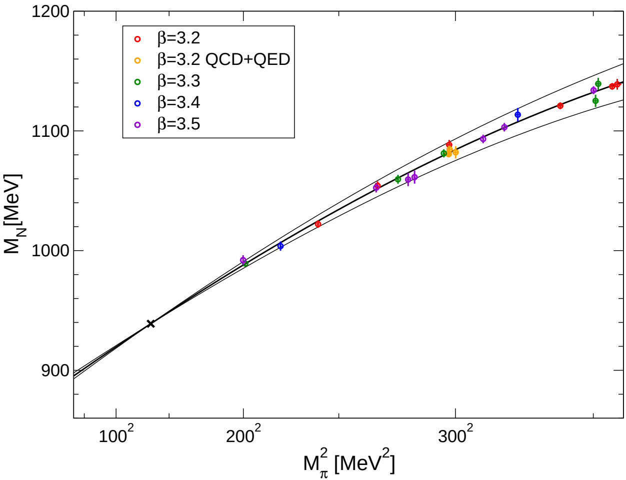


Side product:

$$\frac{m_s}{m_{ud}} = 27.29(33)(8)$$

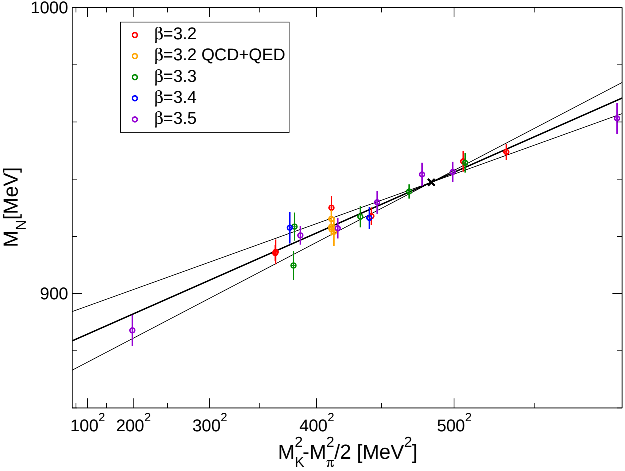
(see FLAG '19 $m_s/m_{ud} = 27.31(10)(10)$)

$M_\pi^2 \sim m_{ud}$ dependence of M_N (preliminary)



- Fit dependence of M_N on M_π^2, $M_{K_\chi}^2$, a, L
- Various polynomial, Padé and χPT ansätze
- $M_\pi < \{360, 420\}\,\mathrm{MeV}$
- Spread into systematic error

$M_{K_\chi} \sim m_s$ dependence of M_N (preliminary)



- Fit dependence of M_N on M_π^2, $M_{K_\chi}^2$, a, L
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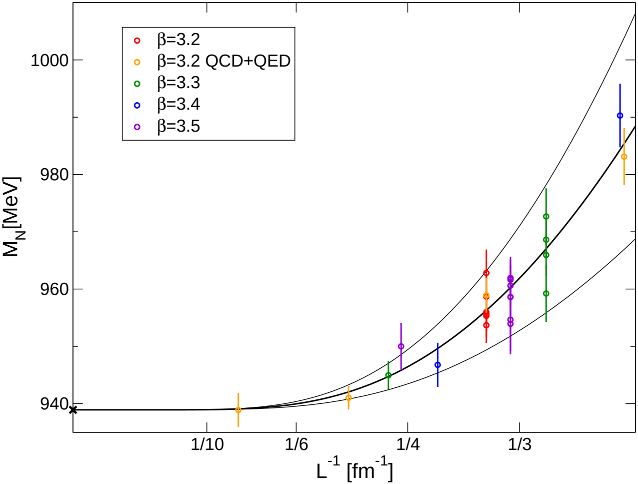
Finite-volume effects



- Correct for leading FV effects: $\frac{M_X(L)-M_X}{M_X} = cM_\pi^{1/2} L^{-3/2} e^{-M_\pi L}$
- $c = 35(13)(5)\text{GeV}^{-2}$ compatible with χPT expectation (Colangelo et. al. '10)

SU(2) isospin relations for $f_{u/d}^{p/n}$

[BMWc, PRL 116 (2016), see also Crivellin et al '14]

- Input: f_{ud}^N, $\Delta_{\rm QCD}M_N = 2.52(17)(24)\,\mathrm{MeV}$ (BMWc, Science '15) & $r \equiv m_u/m_d = 0.46(2)(2)$ (FLAG '17)

- SU(2) relations w/ $\delta m = m_d - m_u$

$$H = H_{\rm iso} + H_{\delta m}\,, \quad H_{\delta m} = \frac{\delta m}{2}\int d^3x\,(\bar{d}d - \bar{u}u)$$

$$\Delta_{\rm QCD}M_N = \delta m\langle p|\bar{u}u - \bar{d}d|p\rangle$$

lead to

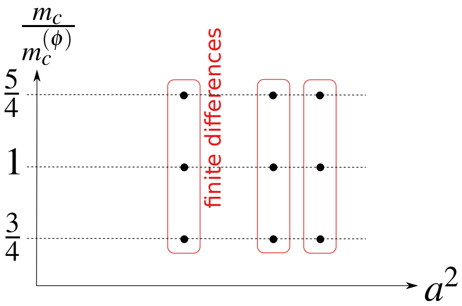
$$f_u^{p/n} = \left(\frac{r}{1+r}\right)f_{ud}^N \pm \frac{1}{2}\left(\frac{r}{1-r}\right)\frac{\Delta_{\rm QCD}M_N}{M_N} + O((\delta m)^2, m_{ud}\delta m)$$

$$f_d^{p/n} = \left(\frac{1}{1+r}\right)f_{ud}^N \mp \frac{1}{2}\left(\frac{1}{1-r}\right)\frac{\Delta_{\rm QCD}M_N}{M_N} + O((\delta m)^2, m_{ud}\delta m)$$

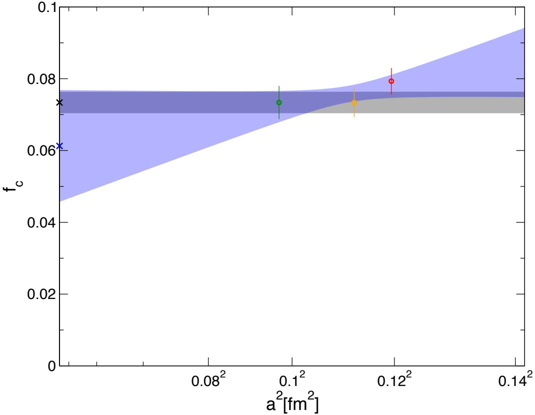
- Huge improvement on usual SU(3)-flavor approach

Strategy for charm sigma term

- 9 dedicated $N_f = 2 + 1 + 1$, 4-stout staggered fermion simulations on tree-level improved Symanzik gluons
- 3 $a \in [0.095, 0.118]\,\mathrm{fm}$ and $LM_\pi > 4$
- For each a, fix m_{ud} and m_s to physical values and consider 3 charm masses: $\frac{3}{4}m_c^\Phi$, m_c^Φ, $\frac{5}{4}m_c^\Phi$
- Get σ_{cN} through finite differences



Continuum extrapolation of f_{cN}

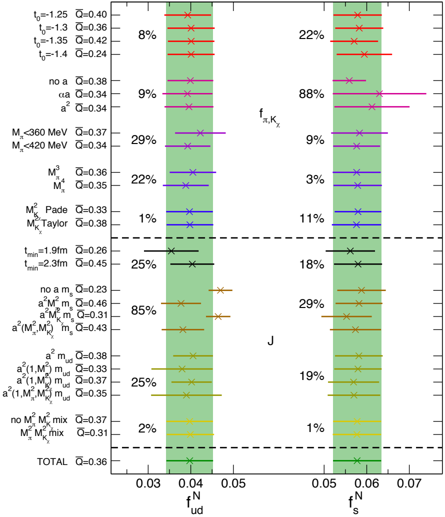


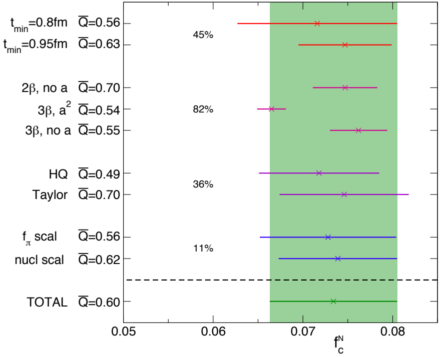
For systematic error estimate associated w/ $a \to 0$, use differences between:

- constant fit in a^2 to two finest lattice results
- linear fit in a^2 to all lattice results
- quadratic fit in a^2 to all lattice results

Sources of systematic error (preliminary)

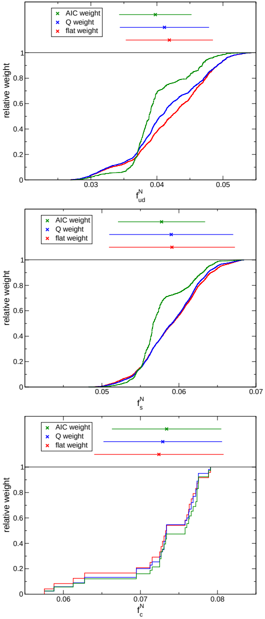
Use extended frequentist approach (BMWc '08, '15): perform large number of plausible analyses and use variation to get systematic error





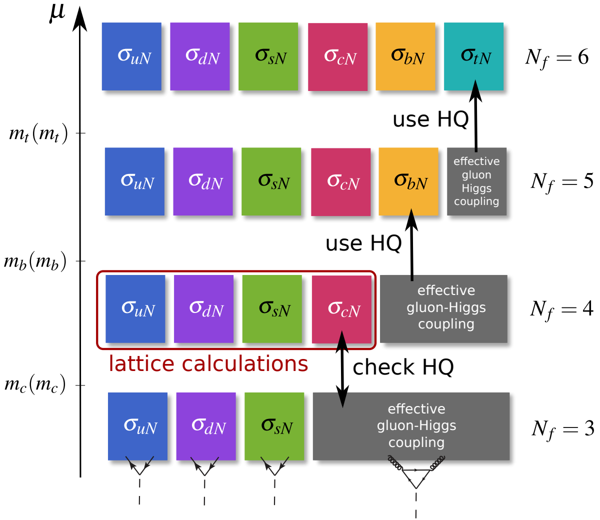
Systematic and statistical errors (preliminary)

- Perform large number of plausible analyses
- Combine into pdf using associated probability
- Mean over pdf gives central value
- Variance over pdf gives systematic error
- Different weights possible
- Crosscheck agreement
- Statistical error obtained from variance of central value over resampled samples

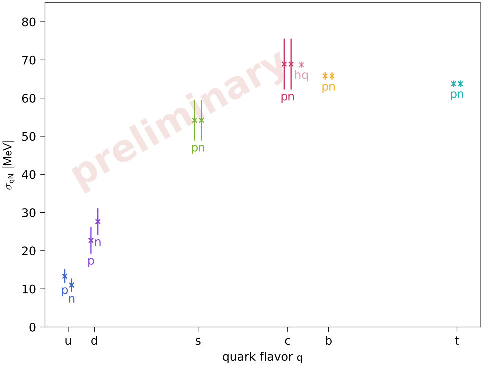


Heavy-quark sigma terms

Use HQ expansions of QCD w/ $N_f \leq 4$ to $O(\alpha_s^3)$ (Hill et al '15)

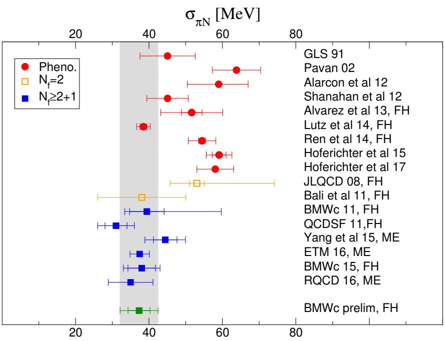


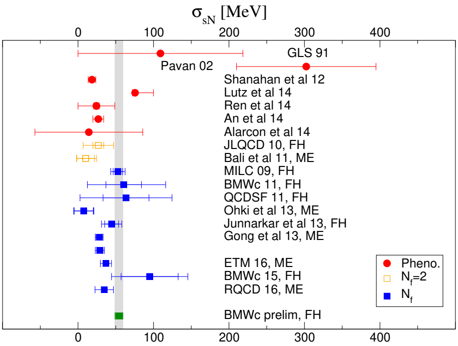
Results for all σ-terms (preliminary)

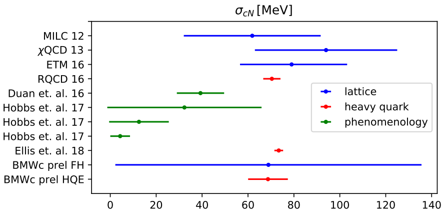


- All σ terms have total errors less than 15%
- $f_c^N - f_c^N|_{\text{HQ}} = 0.0002(45)(55)$ consistent with $O(\Lambda^2/m_c^2)$
- Low energy $N\!-\!h$ coupling is $f_{hN} \times M_N$ w/ $f_{hN} = \sum_{q=u,\cdots,t} f_q^N = 0.3090(58)(61)$
- $N\!-\!h$ coupling is 31% or nucleon mass insead of 100% for fundamental particles
- 87% (resp. 68%) of that coupling comes for sea s, c, b, t (resp. c, b, t) quarks

Comparison







- Compatible w/ earlier BMWc results
- Hugely improved σ_{sN}
- Tension w/ Hoferichter et. al. '15 & '17

BACKUP

Finite difference approximations for σ_{cN}

At fixed a, define:

$$\Delta^+ M_N = M_N(m_c = \frac{5}{4} m_c^{(\phi)}) - M_N(m_c = m_c^{(\phi)})$$
$$\Delta^- M_N = M_N(m_c = m_c^{(\phi)}) - M_N(m_c = \frac{3}{4} m_c^{(\phi)})$$

Combine in two ways:

- Standard finite difference formula (error: $O((\delta m_c/m_c)^2) = O(1/16)$)

$$\sigma_{cN} = m_c \frac{\partial M_N}{\partial m_c} = 2 \frac{\Delta^+ M_N + \Delta^- M_N}{M_N^{(\phi)}}$$

- Based on HQ behavior (error: $O((\sigma_{cN}/M_N^{(\phi)})^3) = O(3 \times 10^{-4})$)

$$\sigma_{cN} = \frac{1}{\log \frac{5}{4} \log \frac{4}{3} \log \frac{5}{3}} \left(\log^2 \frac{4}{3} \Delta^+ M_N + \log^2 \frac{5}{4} \Delta^- M_N \right)$$

Preliminary results

	Nucleon		Individual p and n
f_{ud}^N	0.0398(32)(44)	f_u^p	0.0142(12)(15)
f_s^N	0.0577(46)(33)	f_d^p	0.0242(22)(30)
f_c^N	0.0734(45)(55)	f_u^n	0.0117(11)(15)
f_b^N	0.0701(7)(8)	f_d^n	0.0294(22)(30)
f_t^N	0.0679(6)(7)		

Side product: M_N in SU(2) and SU(3) chiral limits

$$M_{N\chi}^{\mathrm{SU(2)}} = 895(2)(4)\,\mathrm{MeV} \qquad M_{N\chi}^{\mathrm{SU(3)}} = 845(5)(6)\,\mathrm{MeV}$$