



Revisiting velocity distribution uncertainties (application to capture by the Sun)

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V. Bertin, E. Nezri

[[arXiv:1906.11674](#)]

Lacroix, Nunez-Castineyra, Stref, Lavalley, Nezri in prep
Nunez-Castineyra, Nezri, Devriendt, Teyssier in prep

Marseille, October 2019

Three main ways to approach $f(v)$

- Take the standard halo model (SHM)

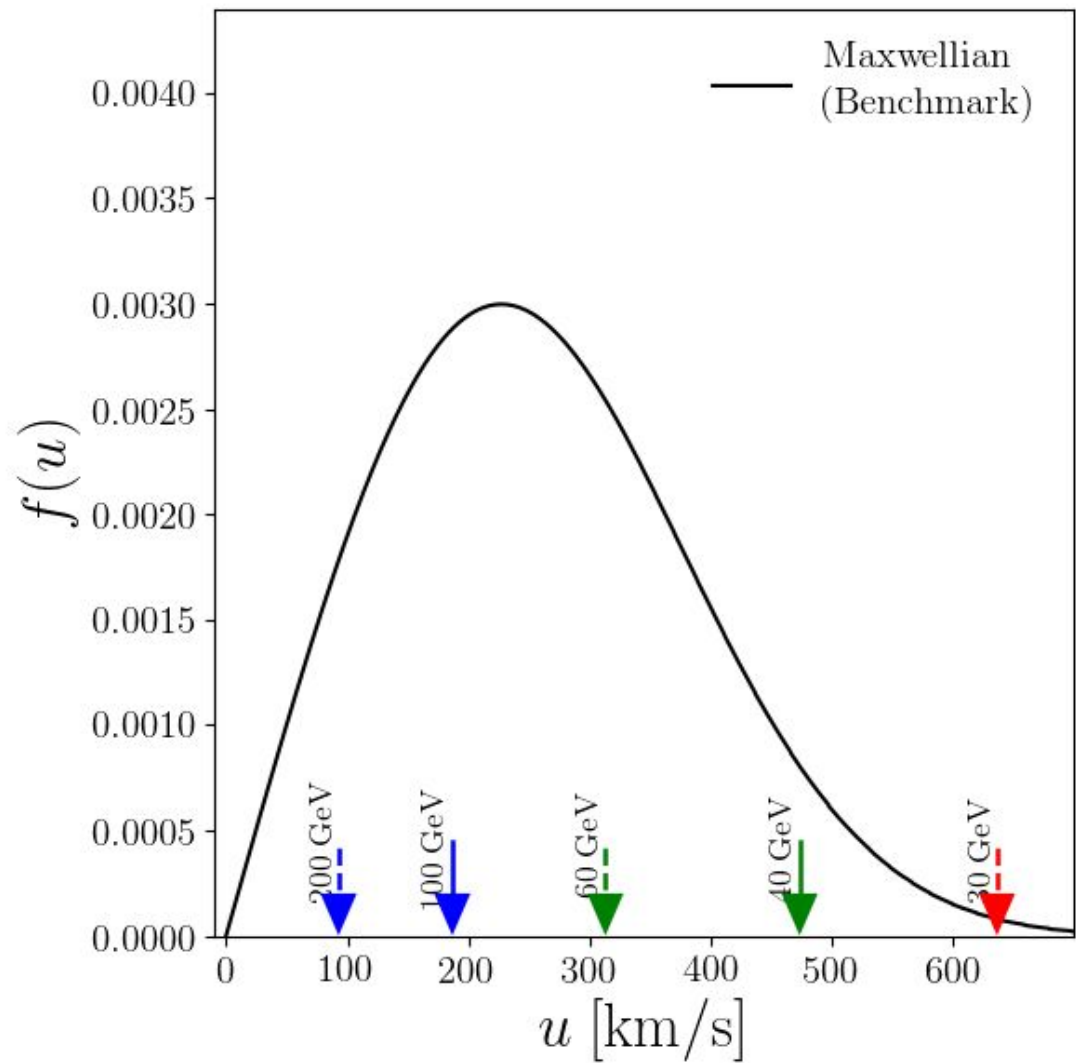
($V_{\text{sun}}=220$ km/s ; $\rho_{\text{sun}} \sim 0.4\text{-}0.3$ GeV/cm³; $v_{\text{esc}}=544$ km/s ; $f(v)$ =Maxwellian distribution)

- Direct extrapolation by fitting $f(v)$ from Cosmological "Milky-Way like" simulations
- Dynamical phase space prediction using MW macro features

e.g Eddington (Eddington, Iacox 2018), Action angle (Binney , Posti) etc.

1. SHM

Takes standard assumptions as they are and use them to generate the $f(v)$



Three main ways to approach $f(v)$

- Take the standard halo model (SHM)

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- Maxwellian exhibit problems with the **tail** and **hat** of the $f(v)$
 - Assumes an isothermal halo density profile $\rho \propto r^{-2}$
 - **Degenerations** in other values
- Direct extrapolation by fitting $f(v)$ from Cosmological "Milky-Way like" simulations
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e.g Eddington (Eddington 1916, Lacroix 2018), Action angle (Posti 2015) etc.

2. Cosmological simulations

To extrapolate data or use fits on $f(v)$ obtained in simulations of "MW-like galaxy"***

***: how MW-like can a simulation be?

Ingredients

Different fluids are modelled using different techniques.

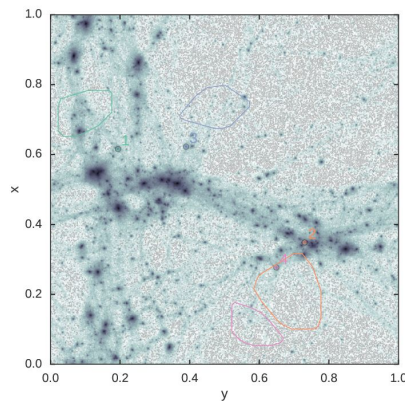
1. **Dark matter** as a collisionless fluid (Vlasov equation)
2. **Gas** as a compressible ideal gas (Euler equations)
3. **Stars** as a collisionless fluid (Vlasov equation)
4. Various chemical species as passive scalars and associated reactions

Possible extra ingredients:

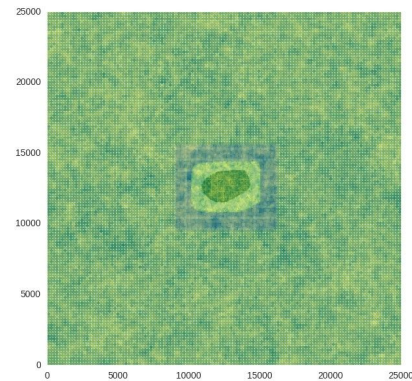
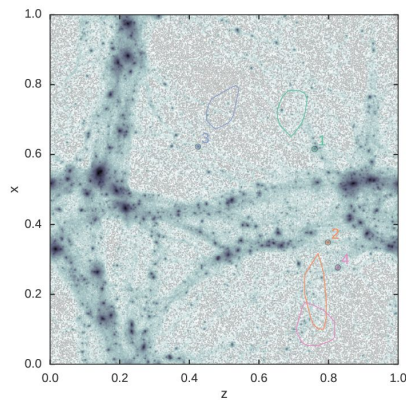
5. Metals and dust grains as passive scalars or as new fluids
6. Massive neutrinos as a quasi-relativistic fluid
7. Magnetic fields as a divergence free vector field
8. Supermassive black holes as individual accreting particles
9. Cosmic rays as an additional energy variables or as a new fluid

Zoom-in Simulations

1. detect one halo of interest in a cosmological simulation.
2. compute the Lagrangian volume in the low resolution IC
3. generate high-resolution IC by adding high frequency waves to the low resolution initial Gaussian random field
4. use the Lagrangian volume as a map to initialize high resolution particles.
5. do the high resolution simulation and check for contamination
6. eventually, compute a better initial Lagrangian volume and re-do the simulation



$z=0$



$z=100$

Star formation

Schmidt law for star formation:

$$\dot{\rho}_* = \epsilon_* \frac{\rho g}{t_{\text{ff}}} \text{ for } \rho > \rho_*$$

Option 1: constant efficiency (Krumholz & Tan (2007))

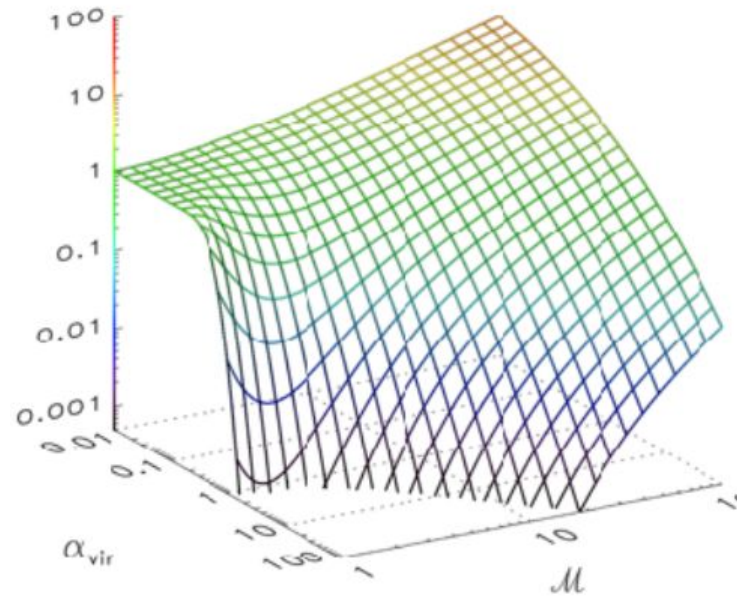
Option 2: calculated efficiency (Federrath & Klessen (2012))

$$p(s) = \frac{1}{\sqrt{2\pi\sigma_s^2}} \exp\left(-\frac{(s + \frac{1}{2}\sigma_s^2)^2}{2\sigma_s^2}\right)$$

Among some of the models we use: Krumholz & McKee (2005)

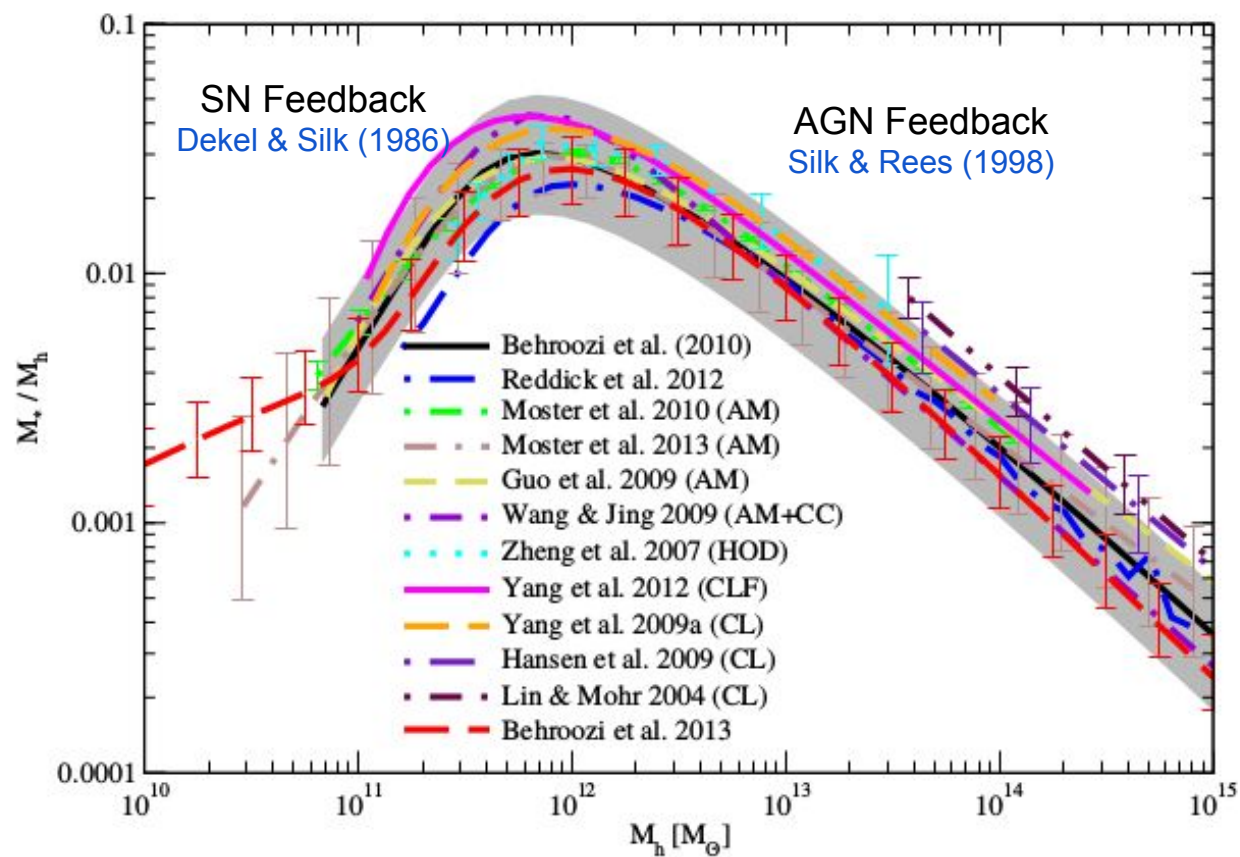
$$\sigma_s^2 = \ln(1 + b^2 \mathcal{M}^2) \quad \mathcal{M} = \frac{\sigma_T}{c_s}$$

$$\rho_{\text{crit}} \propto \alpha_{\text{vir}} \mathcal{M}^2 \quad \alpha_{\text{vir}} = \frac{\sigma_T^2}{G\rho_0\Delta^2}$$



$$\epsilon_{ff} = \exp(3/8\sigma_s^2) \left(1 + \text{erf} \left(\frac{\sigma_s^2 - s_{\text{crit}}}{\sqrt{2\sigma_s^2}} \right) \right)$$

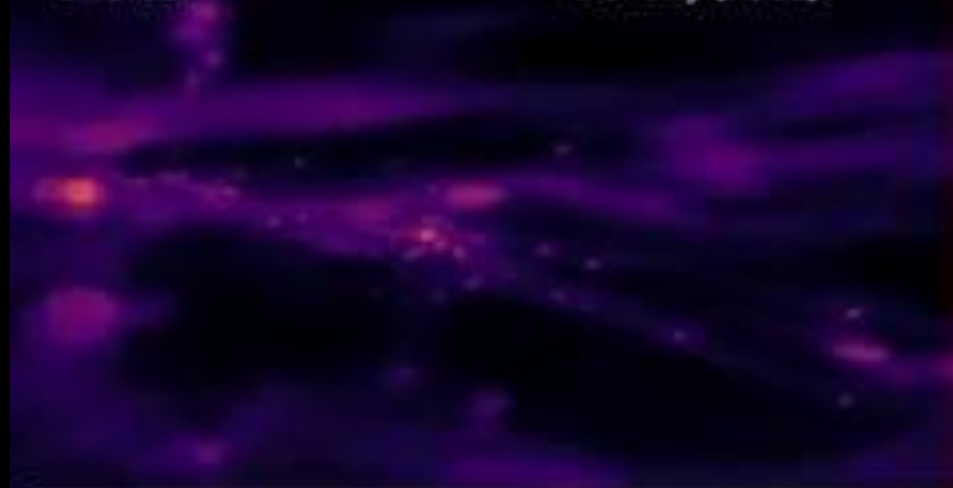
Feedback



Behroozi et al. (2013)

$z=1.89$

Density [MCC]



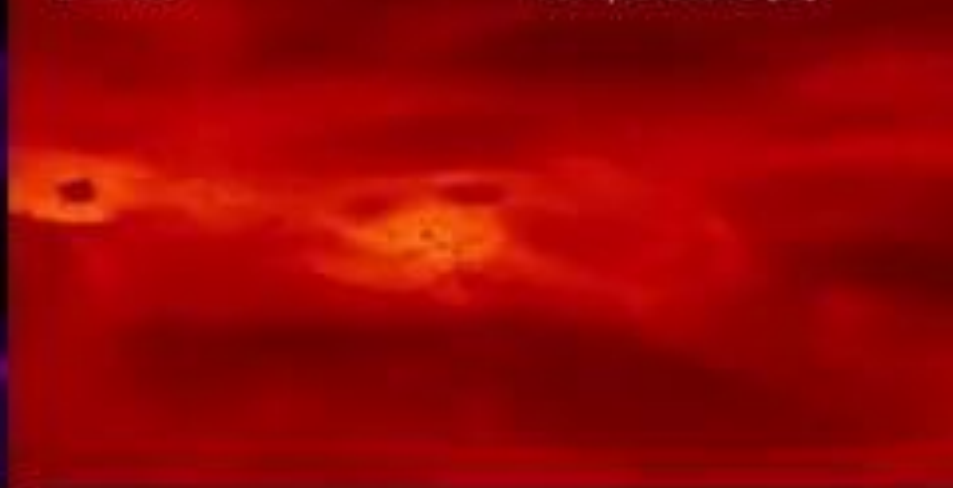
$z=1.89$

Stars



$z=1.89$

Temperature [K]



$z=1.89$

DM



luminosity

$z = 0.0$

ρ

σ

c_s

T

Schmidt law + Delayed Cooling

luminosity

$z = 0.0$

ρ

σ

c_s

T

Multi - ff KM + Delayed Cooling

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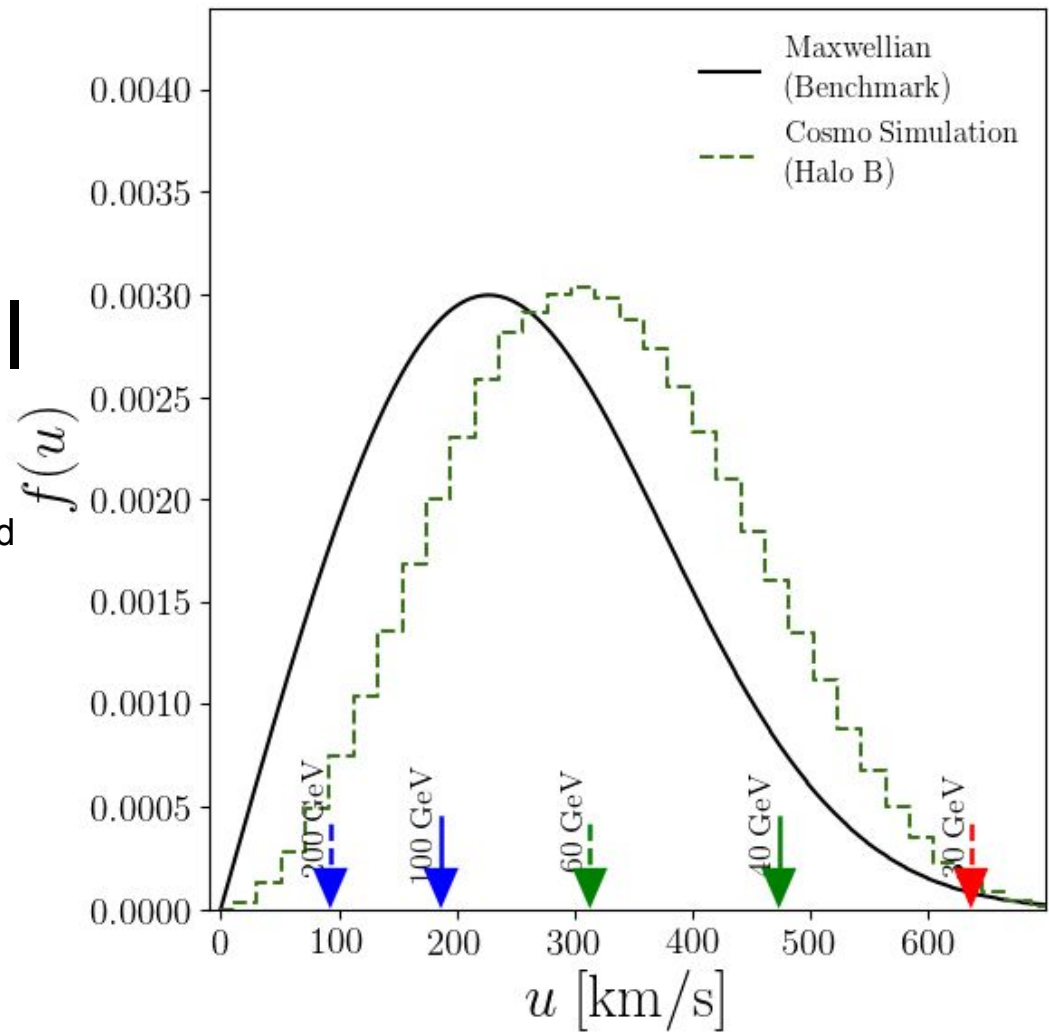
T

Multi - ff KM + Mechanical FB

17

2. Cosmological simulations

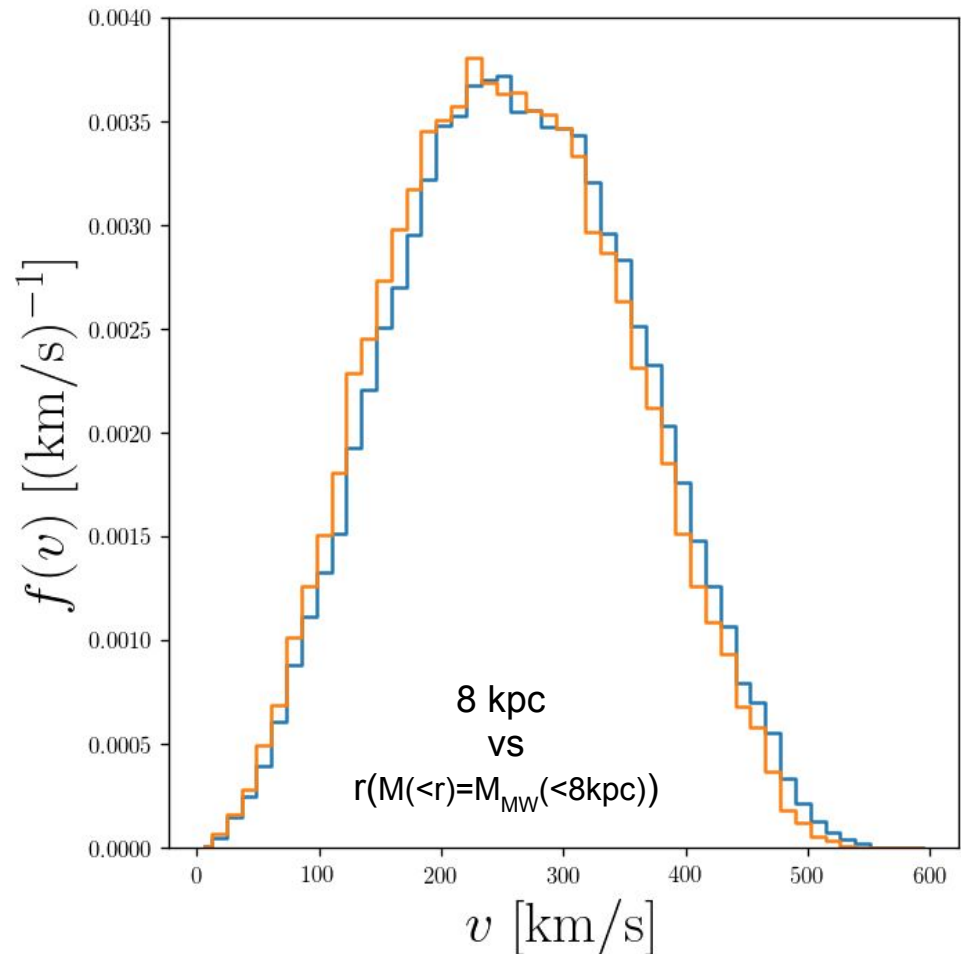
Extrapolate data or use fits on $f(v)$ obtained in simulations of "MW-like galaxy"**



** : how MW-like can a simulation be?

2. Cosmological simulations

Extrapolate data or use fits on $f(v)$ obtain in simulations of "MW-like galaxy"*** at 8 kpc**

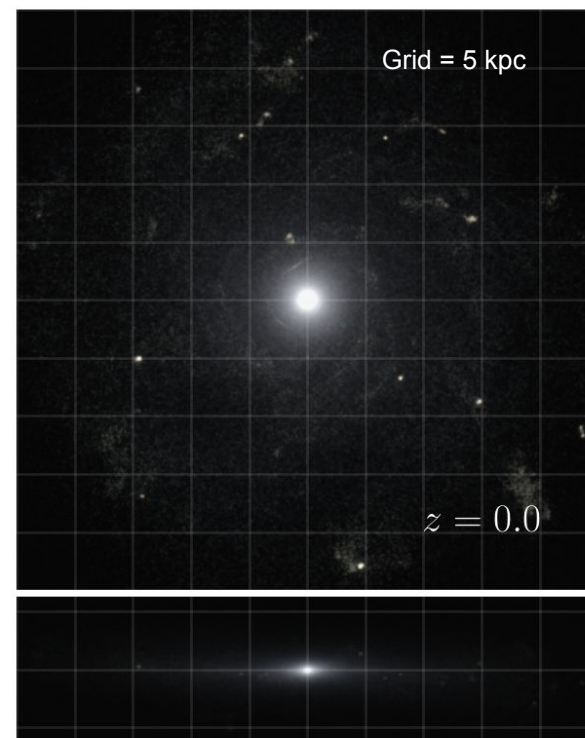
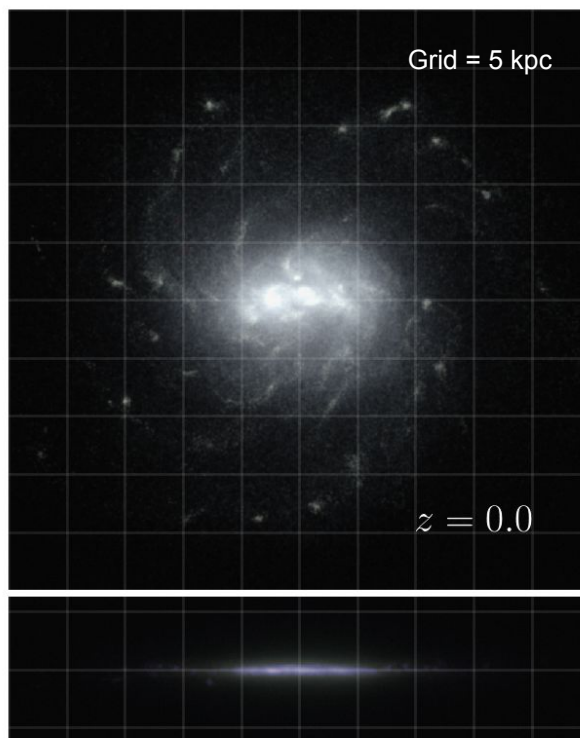


**: how MW-like can a simulation be?

***: what is the meaning of 8 kpc in your simulations with respect to 8kpc in the MW

We use 2 cosmological simulations of spiral galaxies in a MW size halo

- Simulated with AMR code RAMSES
- Similar baryonic physics implementation
 - Star formation
 - SN feedback
- **Ingredients:**
 - Dark Matter
 - Gas
 - Stars



- Usual comparisons with MW are done in their respective publications: RC, TF, SHMR, SFR.. **we present here extra checks**

Halo B

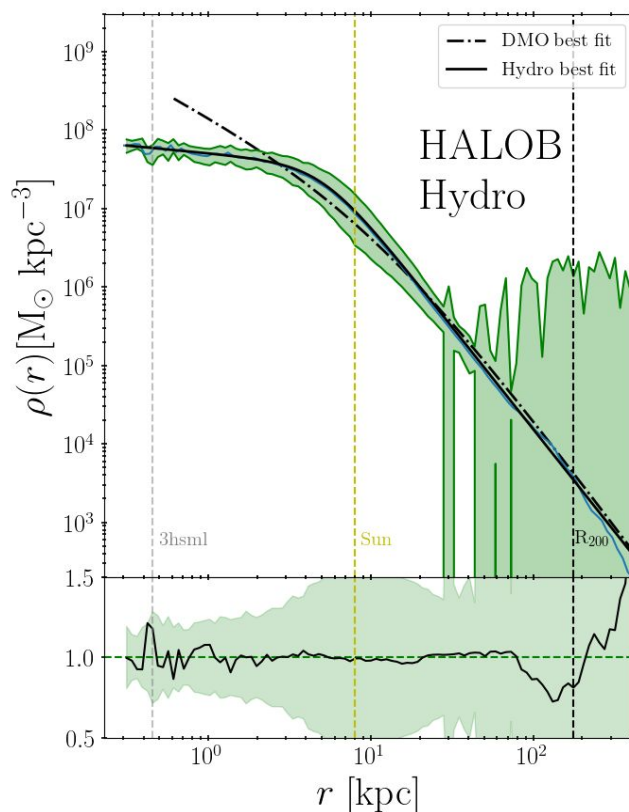
Boxsize = 20 Mpc
 $M_{\text{DM}} = 0.6 \times 10^{12} M_{\text{star}} = 7 \times 10^{10}$
 Resolution = 150 pc
 Mollitor et al [arXiv:1405.4318]

Mochima

Boxsize = 36 Mpc
 $M_{\text{DM}} = 0.9 \times 10^{12} M_{\text{star}} = 3 \times 10^{10}$
 Resolution = 35 pc
 Nunez-Castineyra et al. in prep

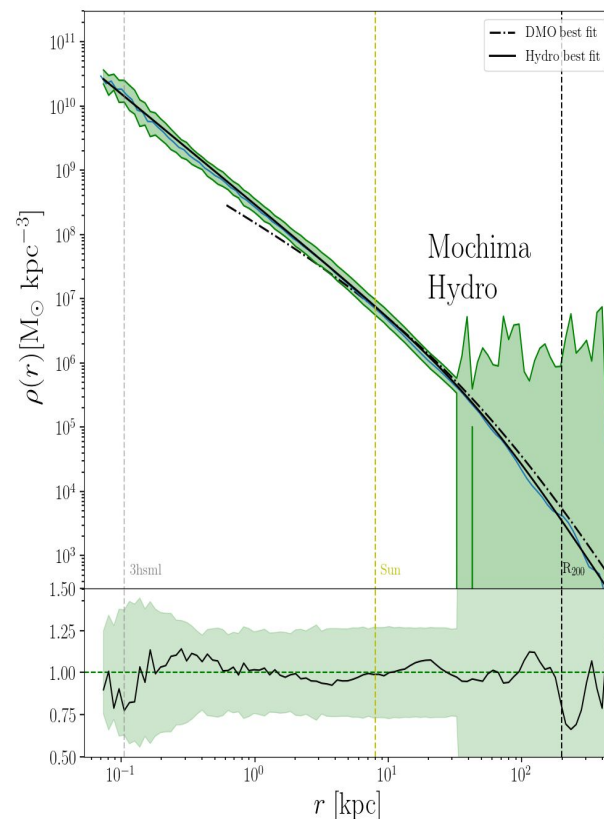
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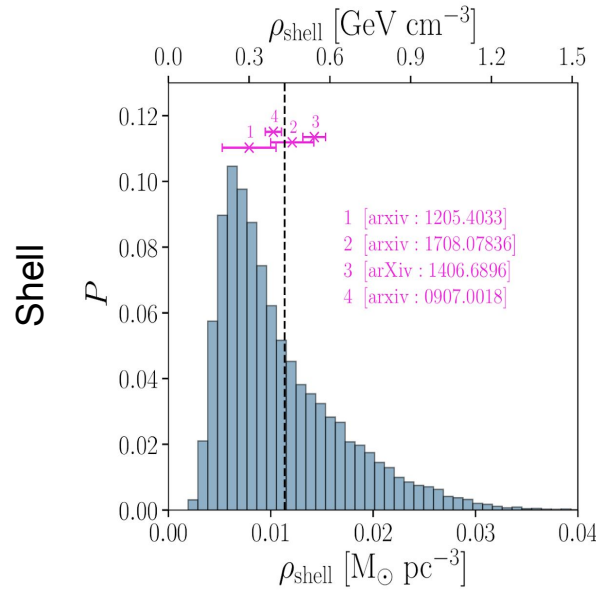
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Local DM density

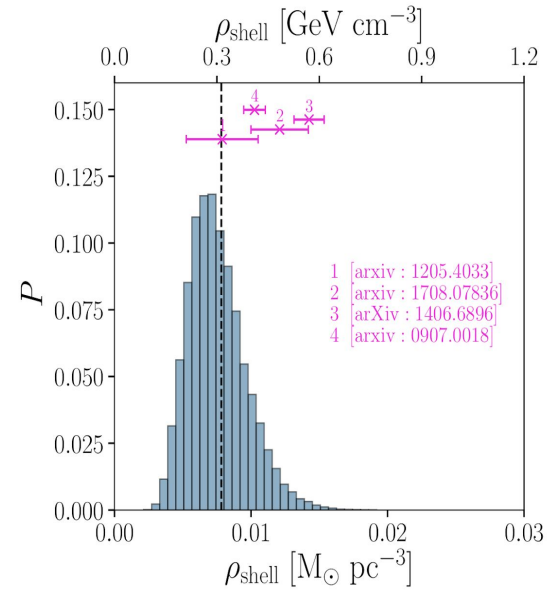
Check of the **local density value** in the **simulation** with **analytical predictions** and **observations** in **two volumetric selection**

- Ring
- Shell

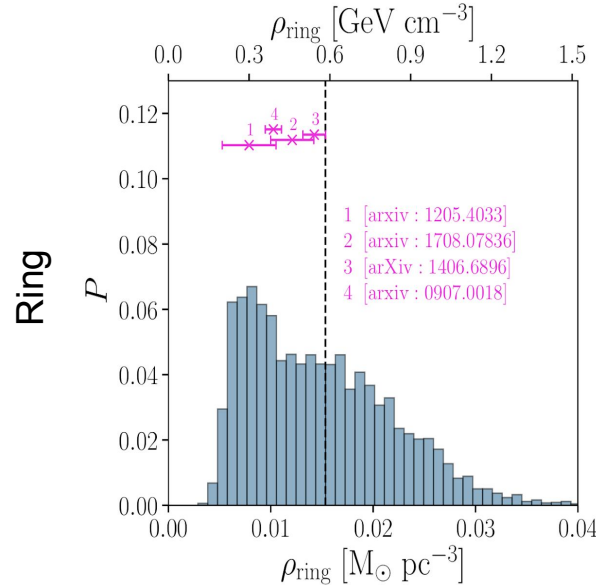
Centered at $r = 8$ kpc and a thickness of 2 kpc



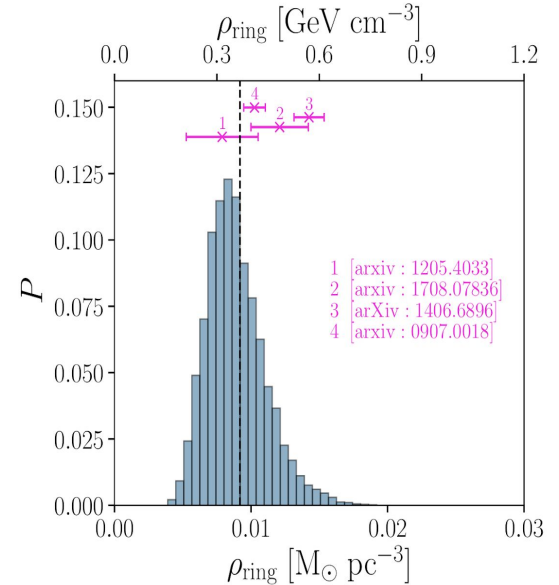
(a) Halo B shell



(b) Mochima shell



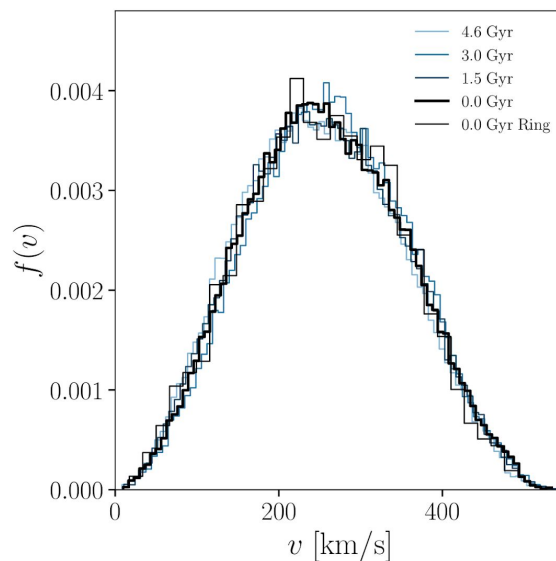
(c) Halo B ring



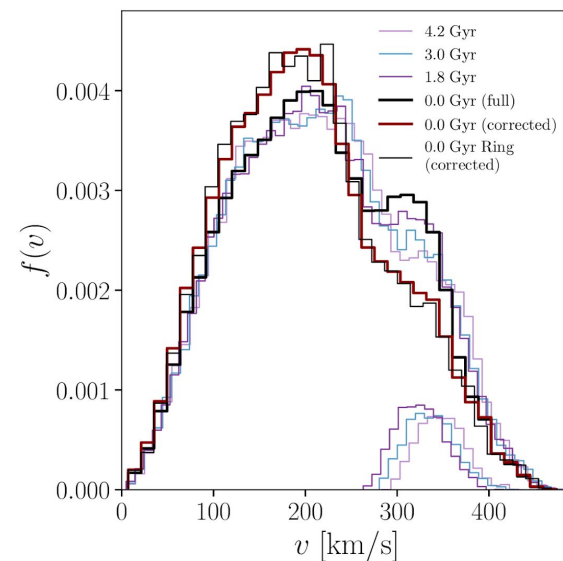
(d) Mochima ring

Check for equilibrium

Usually assumed for the MW



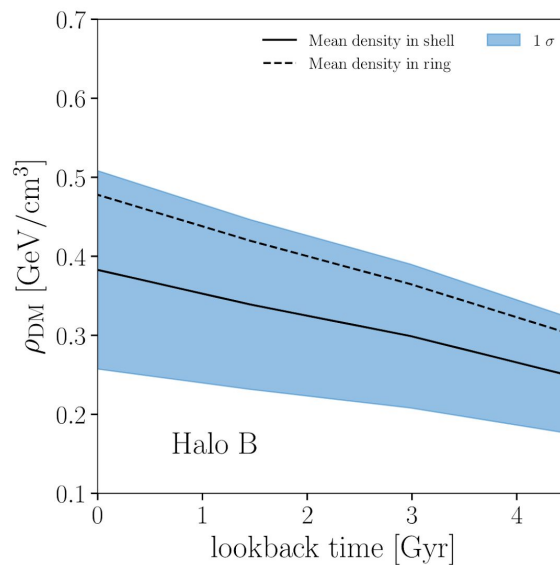
(a) Halo B shell.



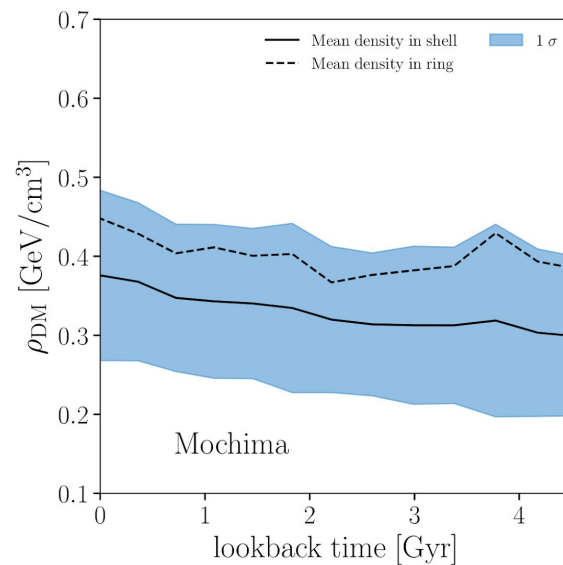
(b) Mochima shell

- Local density over time

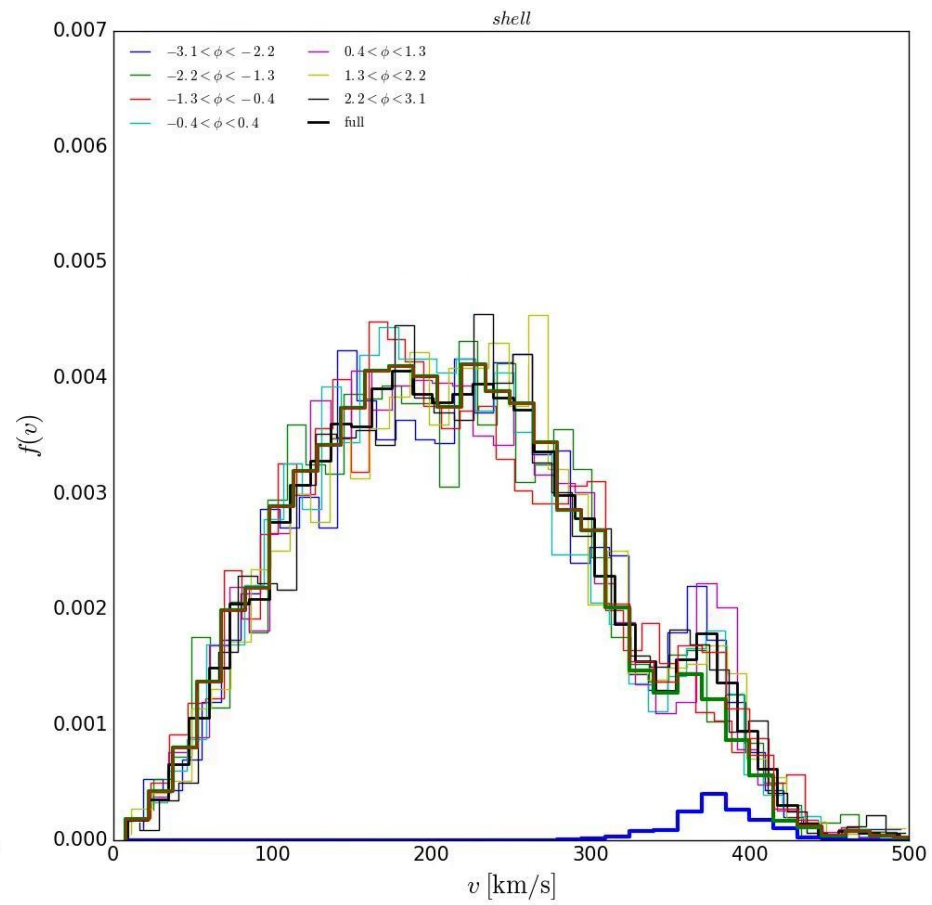
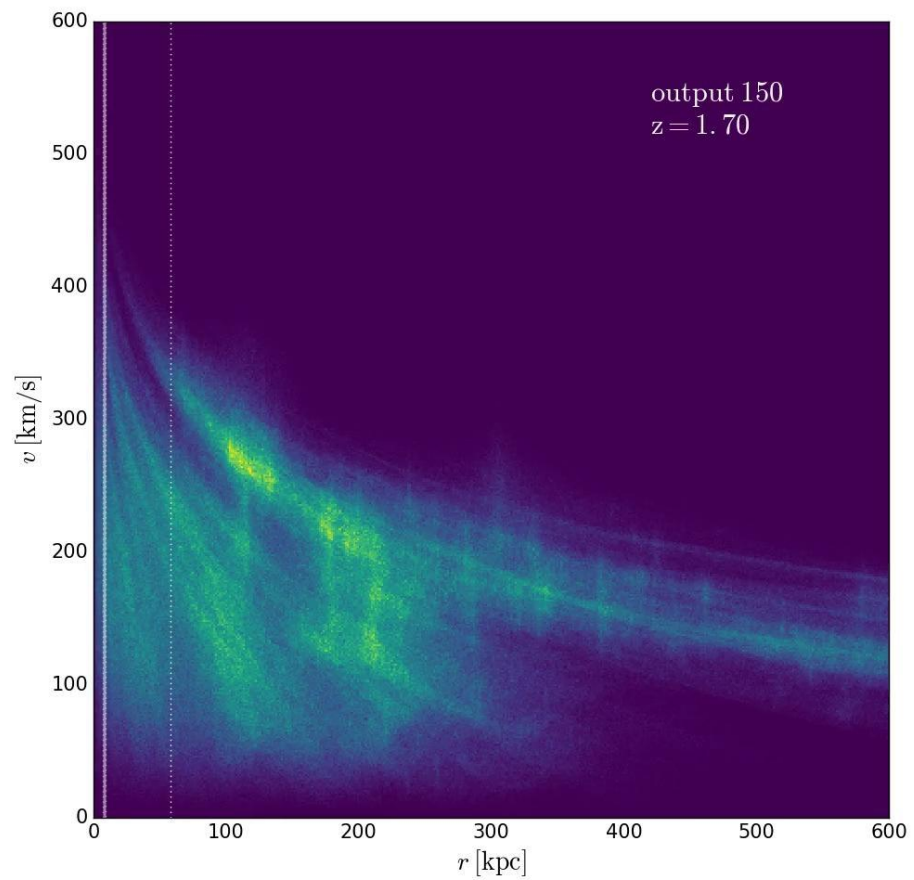
- $f(v)$ over time



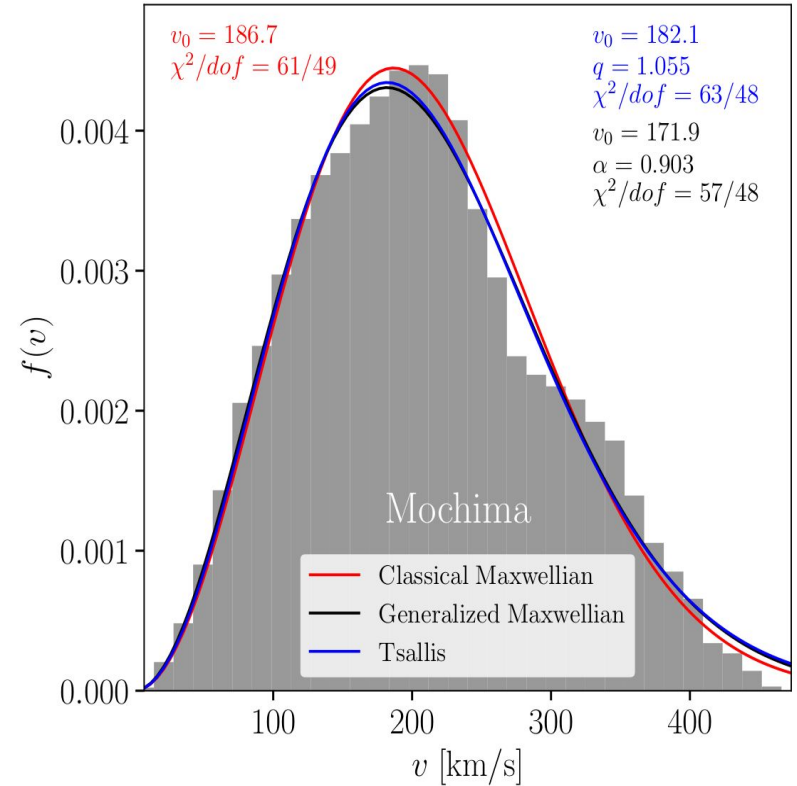
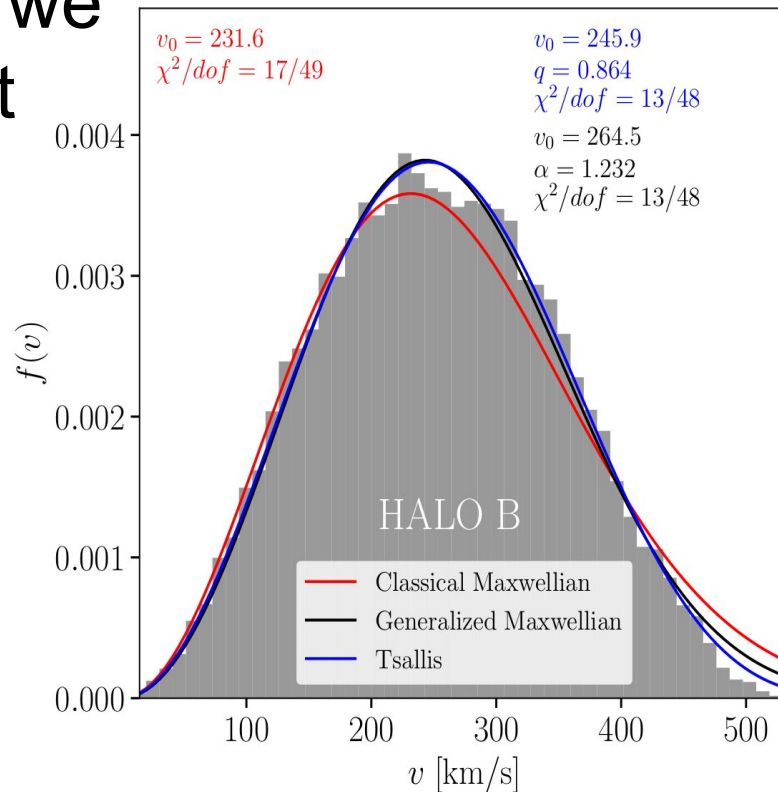
(c) Halo B



(d) Mochima



Then we
can fit



Take your favorite fitting formula and go ahead...

$$f(\vec{v}) = \frac{1}{N} e^{-(\vec{v}^2/v_0^2)}$$

Classical Maxwellian

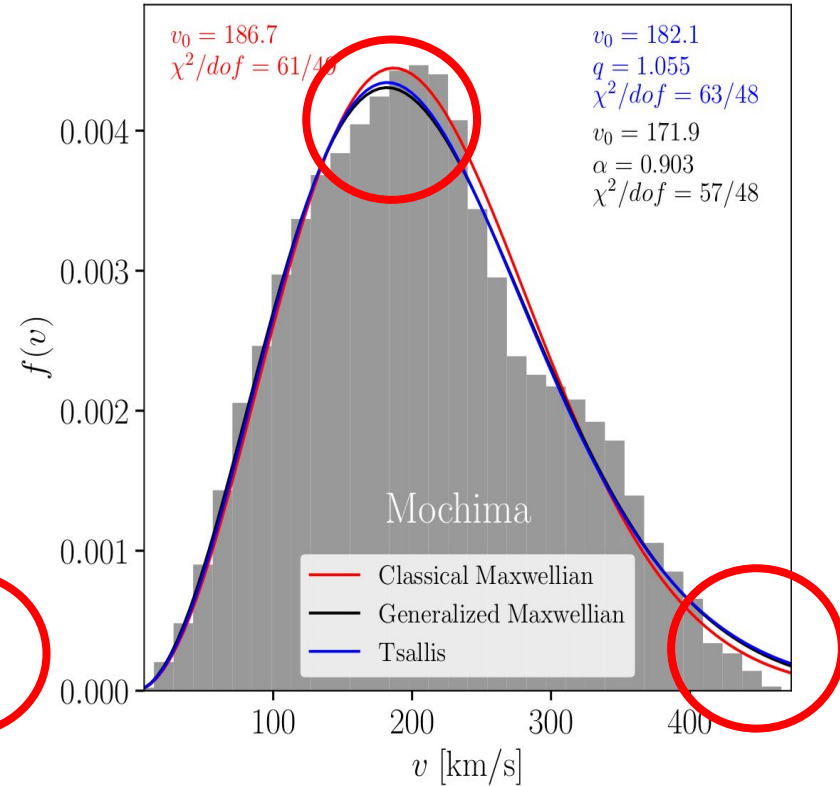
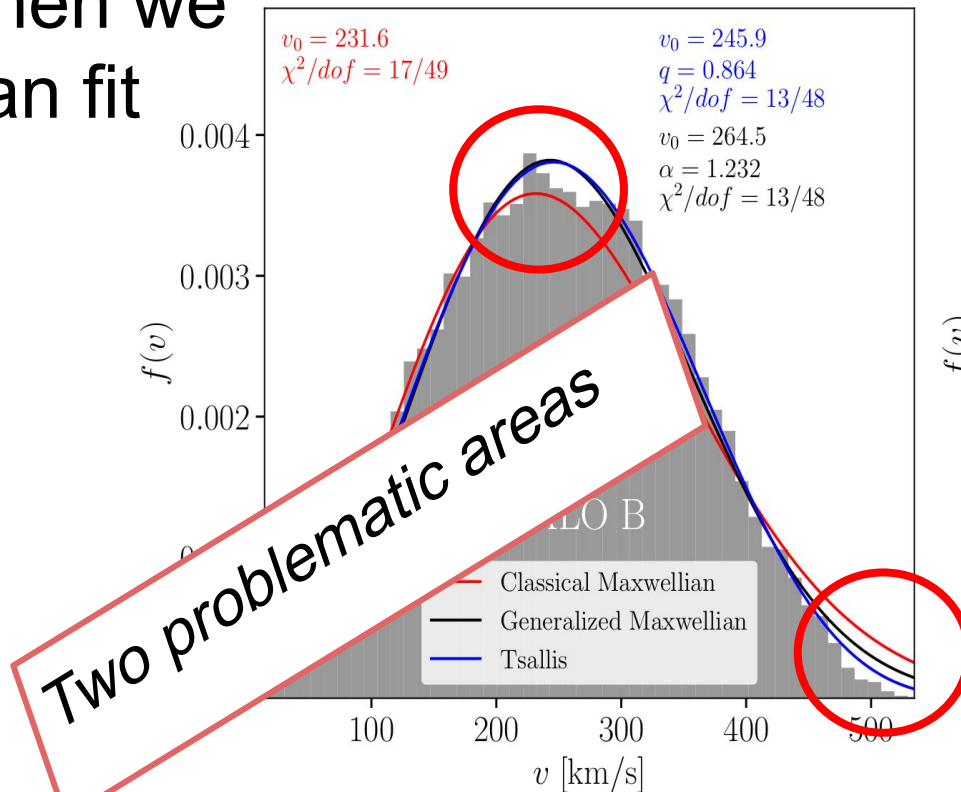
$$f(\vec{v}) = \frac{1}{N} e^{-(\vec{v}^2/v_0^2)^\alpha}$$

Generalized Maxwellian

$$f(\vec{v}) = \frac{1}{N} \left(1 - (1 - q) \frac{\vec{v}^2}{v_0^2} \right)^{q/(1-q)}$$

Tsallis

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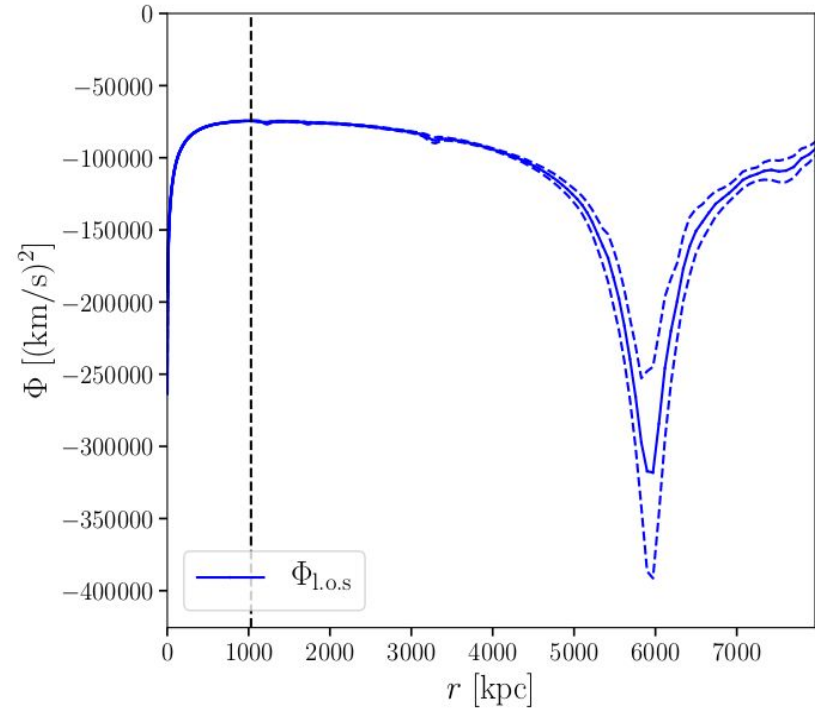
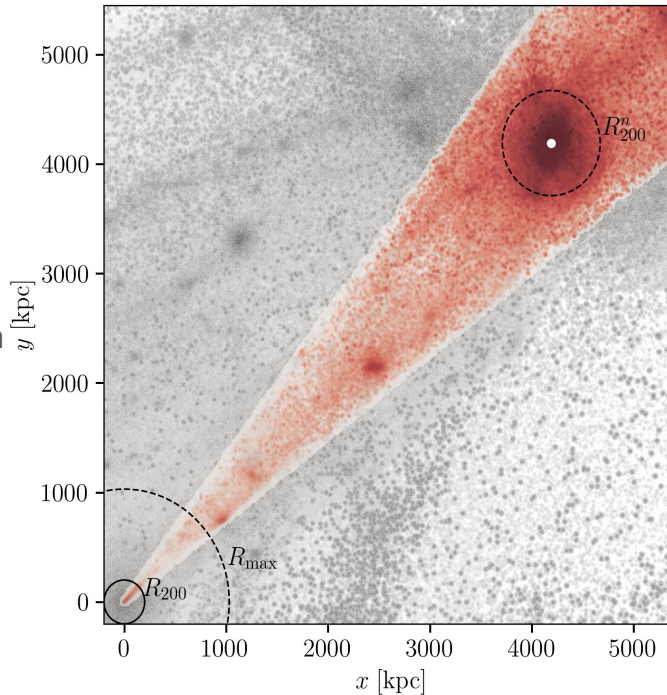
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Tsallis

Introducing v_{esc}

To find v_{esc} we
first find R_{max}

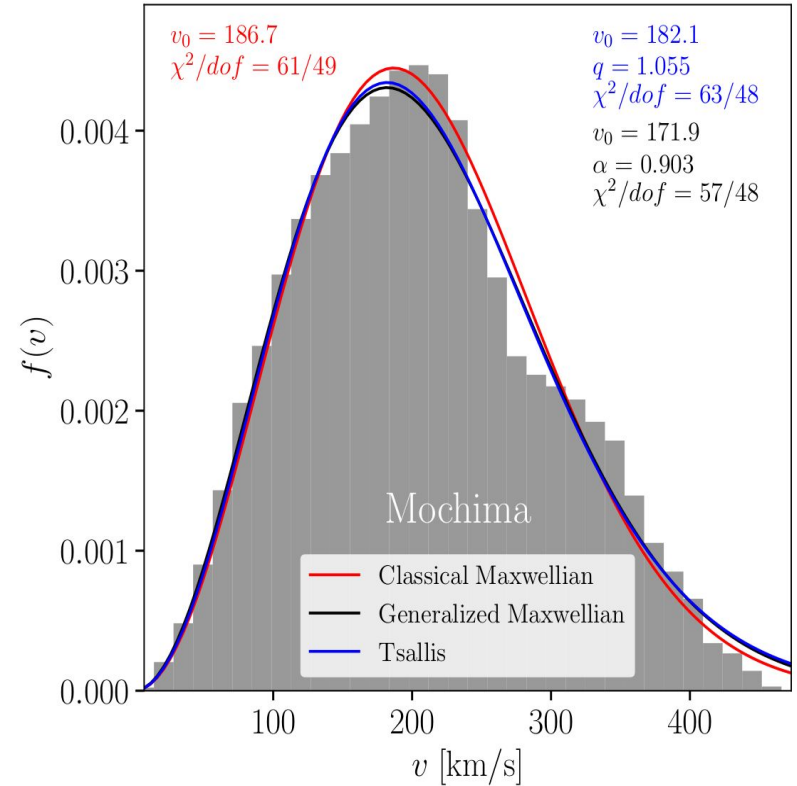
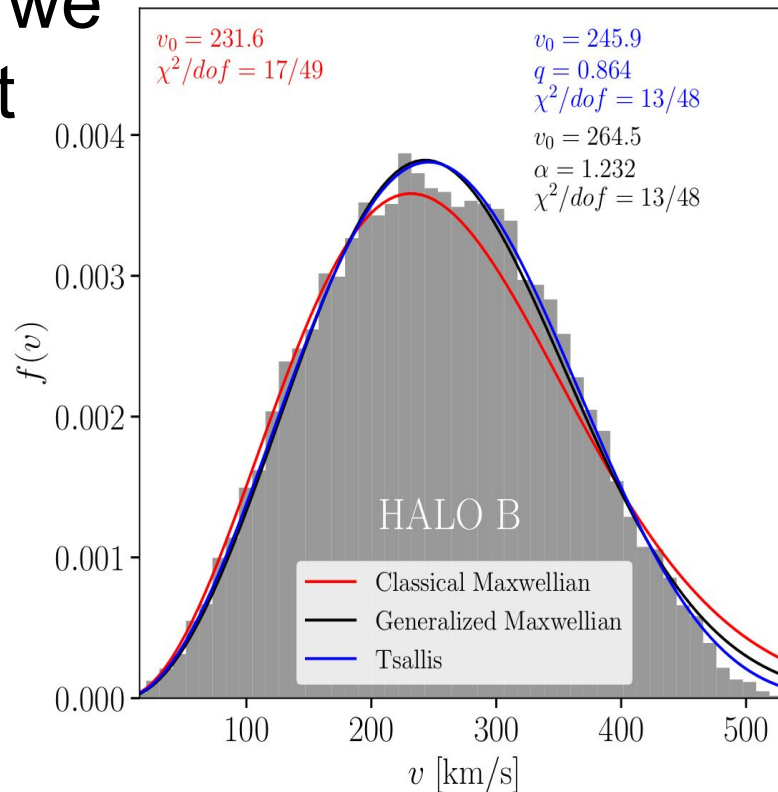
Radius at the point
where the potential in
the line between the
halo and the biggest
neighbour reach a
maximum



$$v_{\text{esc}} \equiv \sqrt{2\Psi(r)}$$

$$\Psi(r) = \Phi(R_{\text{max}}) - \Phi(r)$$

Then we
can fit



Take your favorite fitting formula and go ahead...

$$f(\vec{v}) = \frac{1}{N} e^{-(\vec{v}^2/v_0^2)}$$

Classical Maxwellian

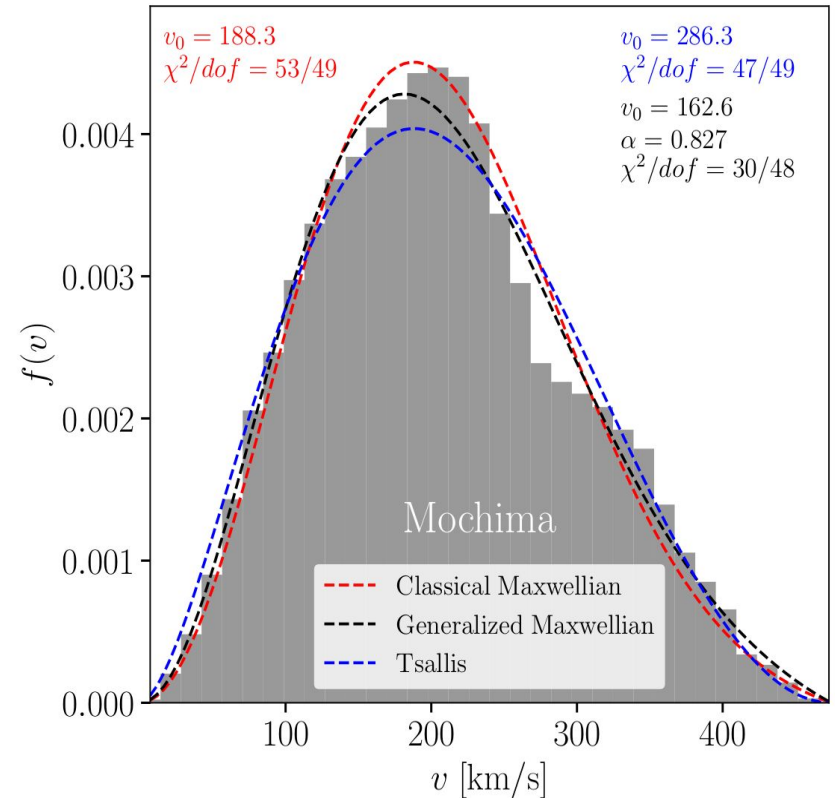
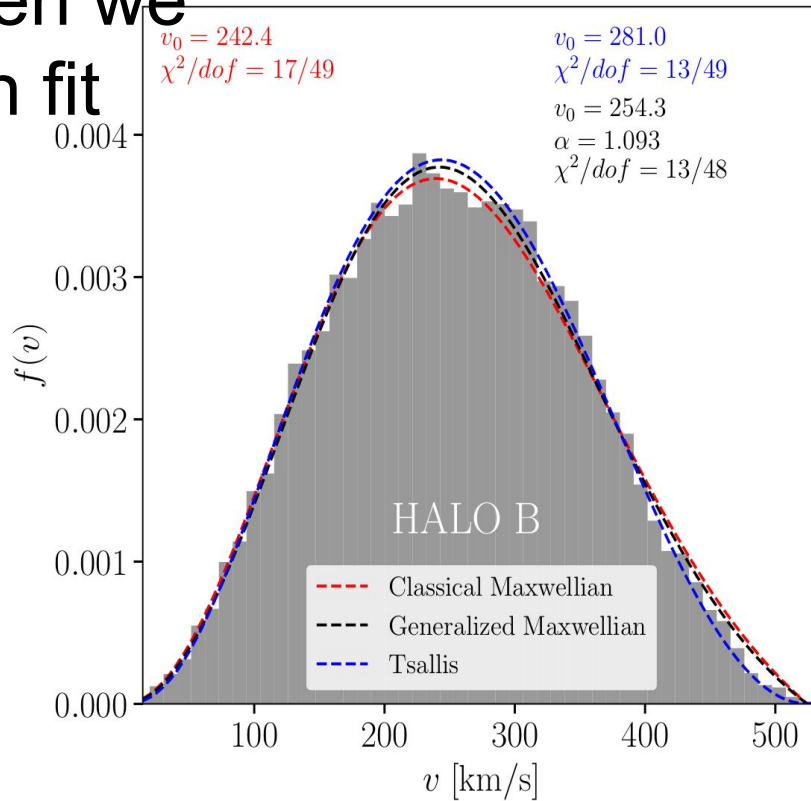
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Generalized Maxwellian

$$f(\vec{v}) = \frac{1}{N} \left(1 - (1 - q) \frac{\vec{v}^2}{v_0^2} \right)^{q/(1-q)}$$

Tsallis

Then we
can fit



Take your favorite fitting formula and go ahead... now including v_{esc}

$$f(\vec{v}) = \frac{1}{N} \left(e^{-(\vec{v}^2/v_0^2)} - e^{-(v_{\text{esc}}^2/v_0^2)} \right) \quad f(\vec{v}) = \frac{1}{N} \left(e^{-(\vec{v}^2/v_0^2)^\alpha} - e^{-(v_{\text{esc}}^2/v_0^2)^\alpha} \right) \quad f(\vec{v}) = \frac{1}{N} \left(1 - (1 - q) \frac{\vec{v}^2}{v_0^2} \right)^{q/(1-q)}$$

Classical Maxwellian

Generalized Maxwellian

Tsallis

$$q = 1 - (v_0^2/v_{\text{esc}}^2)$$

Three main ways to approach $f(v)$

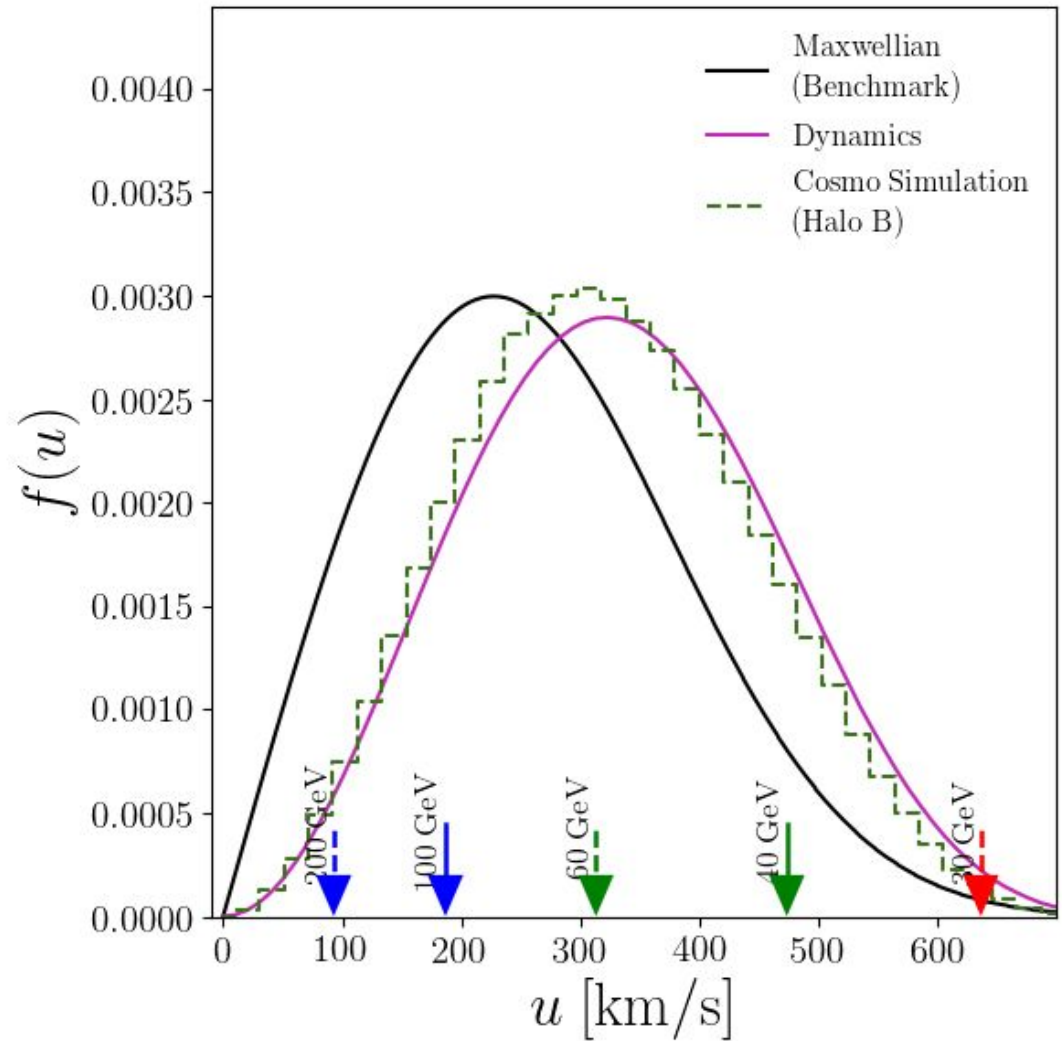
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- Assumes an isothermal halo density profile $\rho \propto r^{-2}$
- **Degenerations** in other values
- Direct extrapolation by fitting $f(v)$ from Cosmological "Milky-Way like" simulations
 - No warranty that of the "**Milky-way likeness**" of the simulation
 - The meaning of **8kpc** in the simulation
- Dynamical phase space prediction using MW macro features

e.g Eddington (Eddington 1916, Iacox 2018), Action angle (Posti 2015) etc.

2. Predictions from dynamics



Eddington inversion



Predictions from [the Eddington method](#) as studied by Lacroix et al. ([Binney - Tremaine](#)) of $f(v)$

vs

fully consistent objects build in a [Zoom-in Cosmological Simulation](#).

Density profile $\rightarrow$ Eddington inversion $\rightarrow f(\mathcal{E}) \rightarrow f(v)$

$$f(\vec{r}, \vec{v}) = f(\mathcal{E}, L)$$

$$\rho_{DM}(r) + \rho_{baryons}(r)$$

$$\mathcal{E} = \Psi(r) - \frac{v^2}{2}$$

$$\Psi = \Phi(r) - \Phi(r_{max})$$

$$\frac{d\rho}{d\Psi}$$

Assuming spherical symmetry and isotropy

$$f(\mathcal{E}) = \frac{1}{\sqrt{8\pi^2}} \left(\frac{1}{\sqrt{\mathcal{E}}} \left[\frac{d\rho}{d\Psi} \right]_{\Psi=0} + \int_0^{\mathcal{E}} \frac{d\Psi}{\sqrt{\mathcal{E} - \Psi}} \frac{d^3\rho}{d\Psi^3} \right)$$

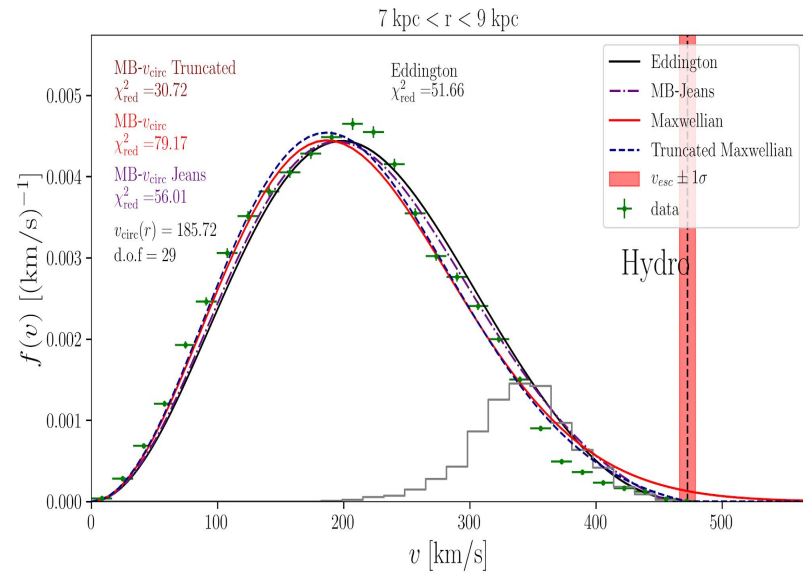
Eddington prediction vs Maxwellian approach

Two ways of building the mean of maxwellian $f(v)$

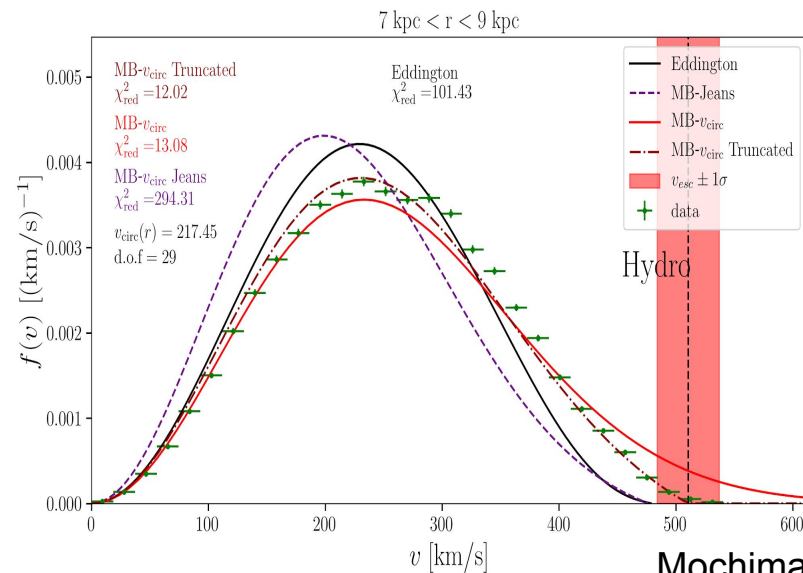
1) From the contained mass as

$$v_0 = v_c(r) = \sqrt{\frac{GM(r)}{r}}$$

2) By solving the **Jeans equation** for the velocity using the contain mass again.

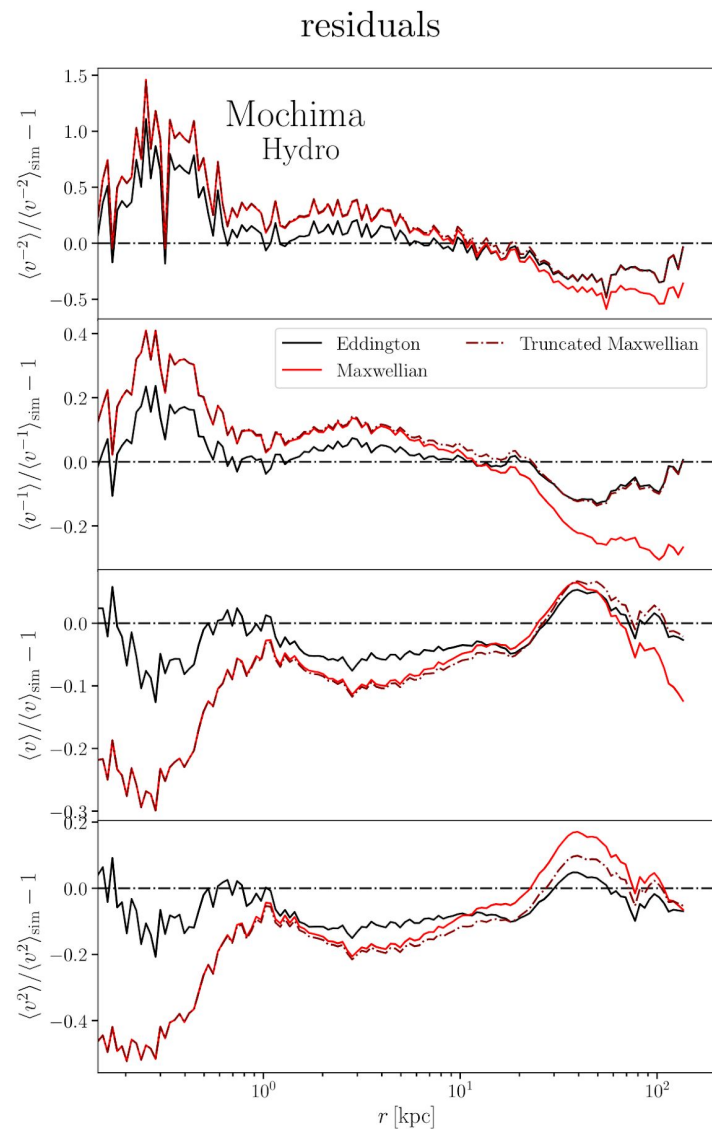
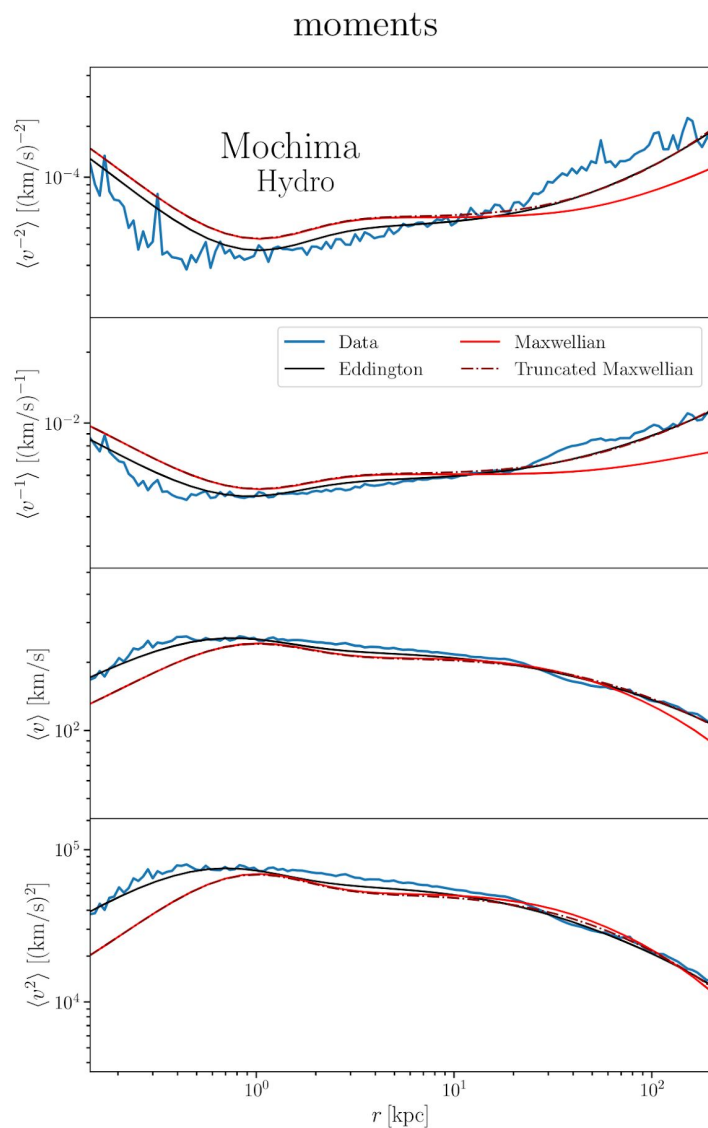


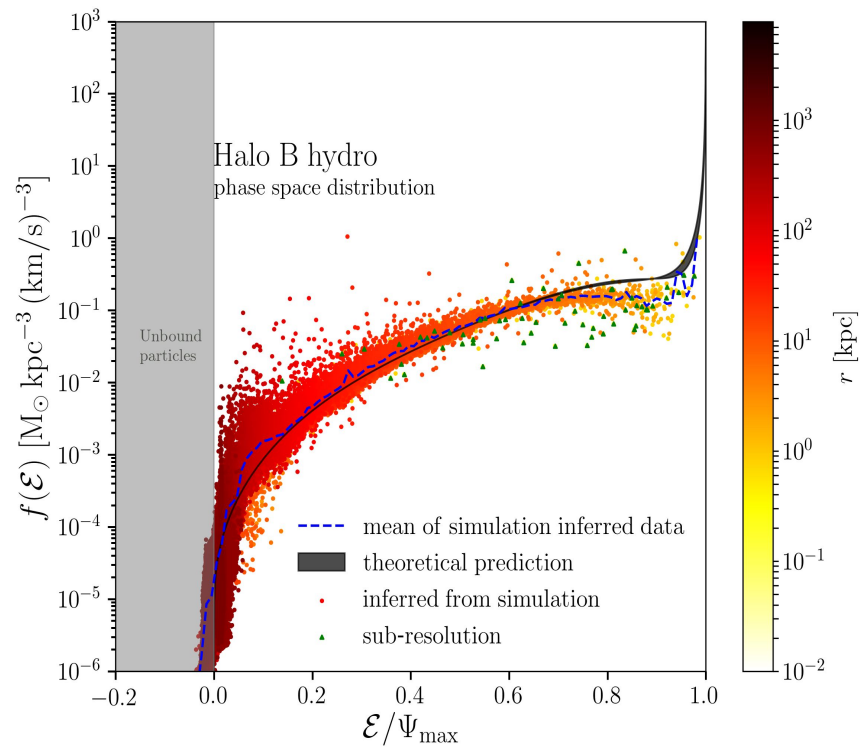
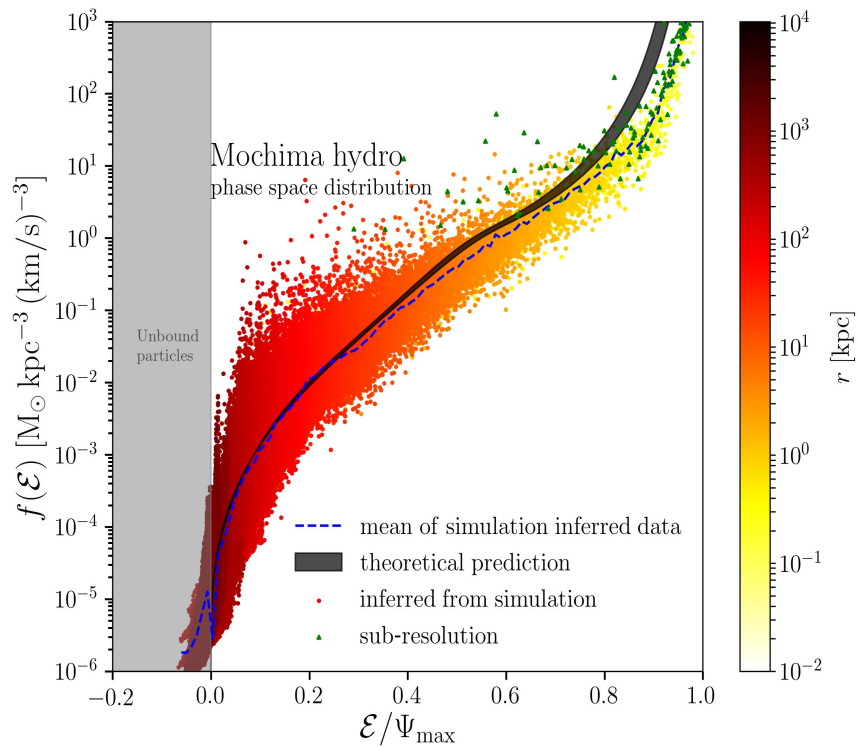
HALO B



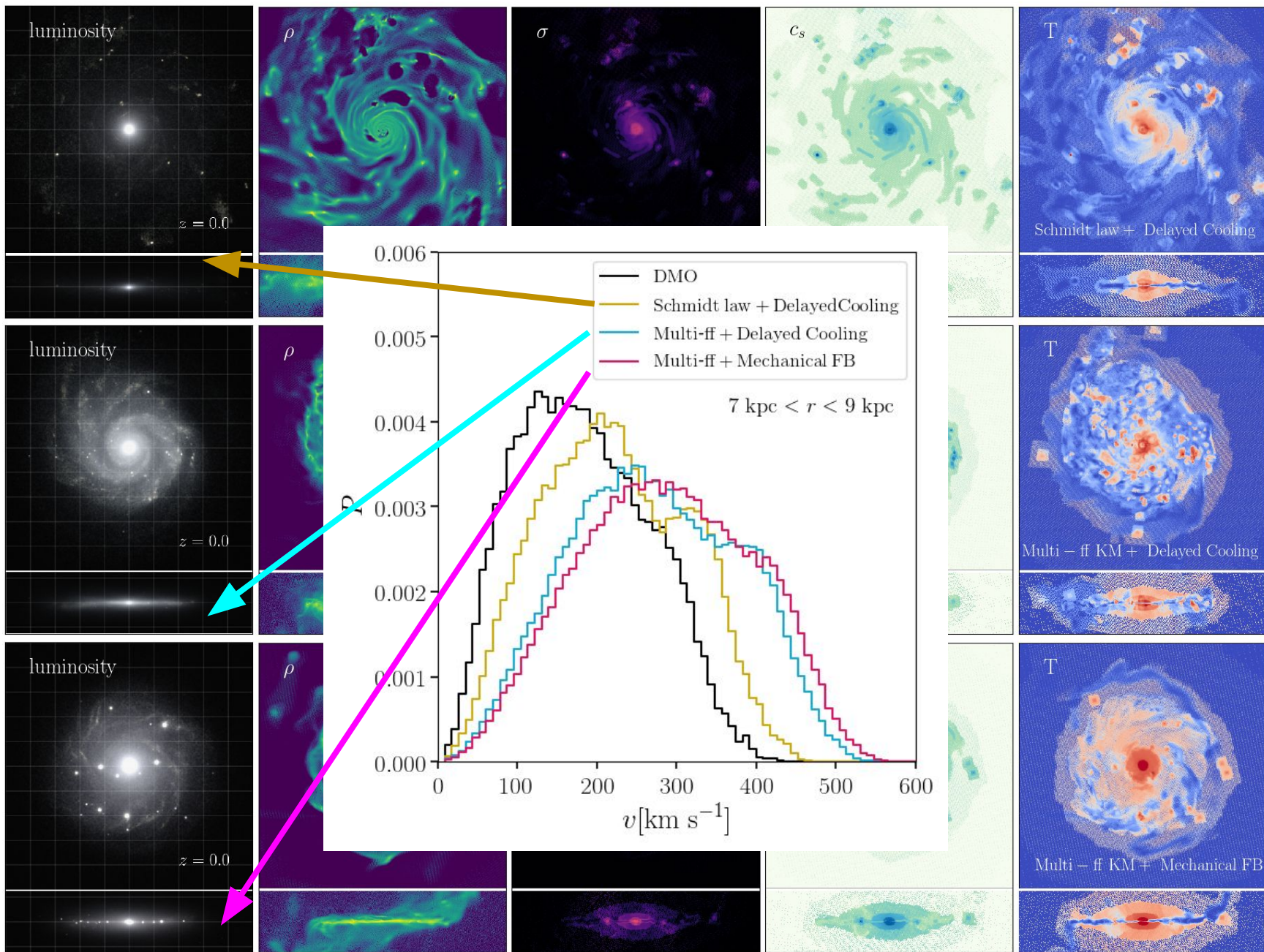
Mochima

Some more Eddington results

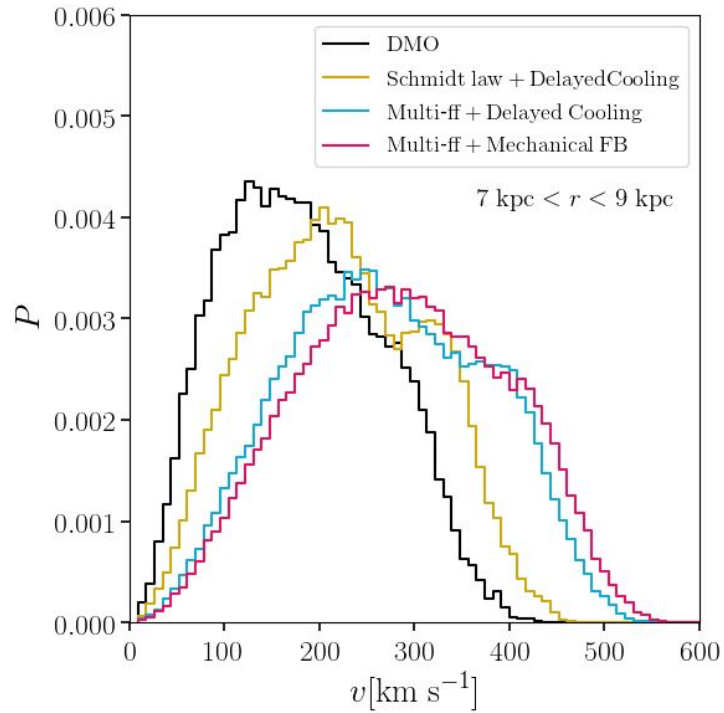




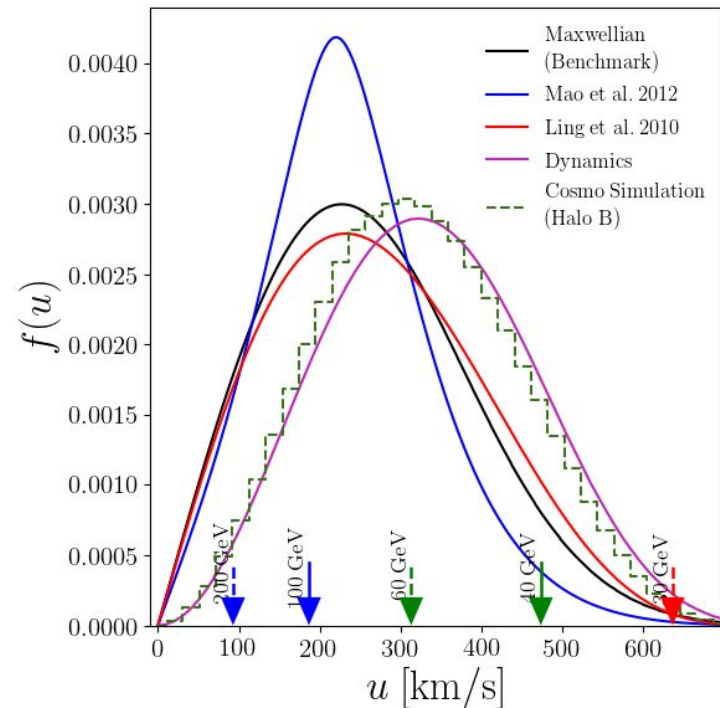
What about baryonic physics in the
simulation
(SF, Feedback)



Point of emphasis on the variability of the $f(v)$

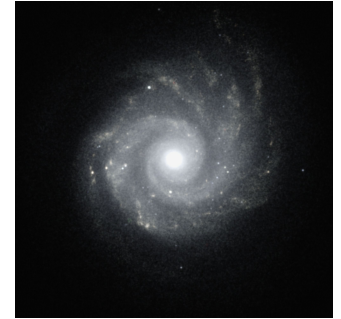
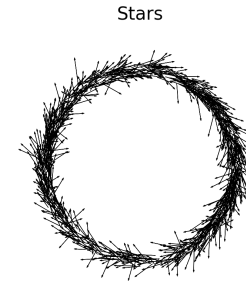
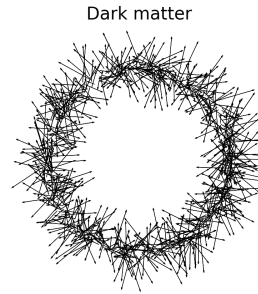


In simulations the implementation of baryonic physics will have an effect on the final distribution.
[Nunez-Castineyra et al. in prep](#)



The discussed cases + some extra from the literature

DM capture by the Sun



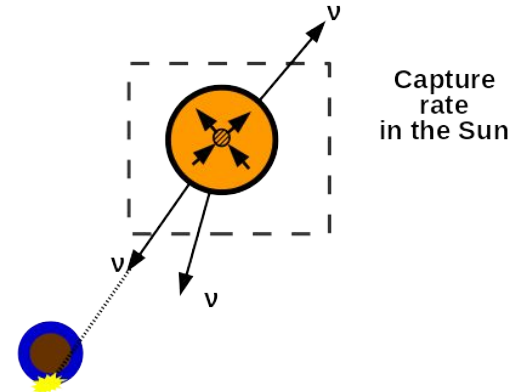
The number of captured WIMPs evolved as

$$\frac{dN_\chi}{dt} = C - 2\Gamma_A = C - C_A N_\chi^2$$

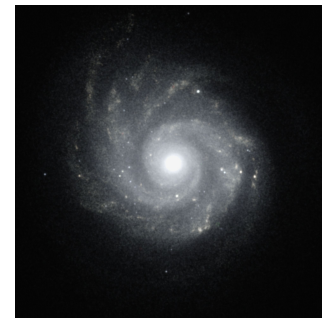
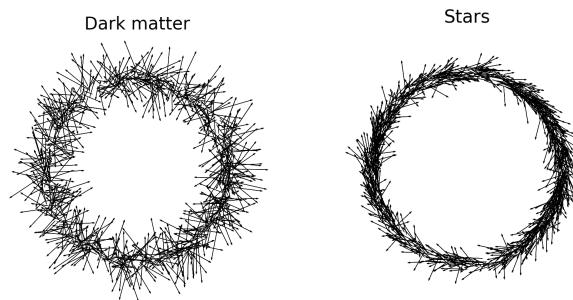
Once you solve it, it can be proved that

$$\Gamma_A = \frac{1}{2}C \tanh^2(t/\tau)$$

(A. Gould 1987)
(Jungman, Kamionkowski 1996)

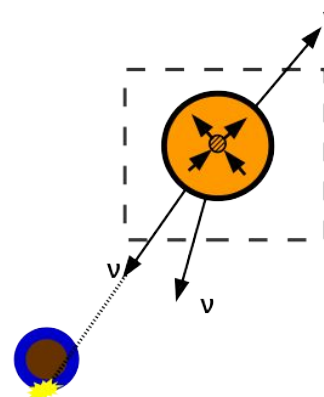


DM capture by the Sun



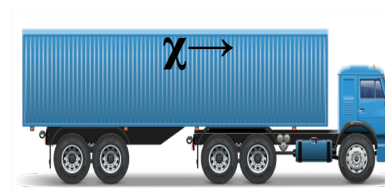
$$\frac{dC}{dV} = \left[\frac{\rho}{M_\chi} \int_0^{u_m} du \frac{f(u)}{u} \right] w \Omega(w)$$

Astrophysics and Particle Physics come together



Capture rate in the Sun

(A. Gould 1987)
(Garani & Palomares-Ruiz 2017)



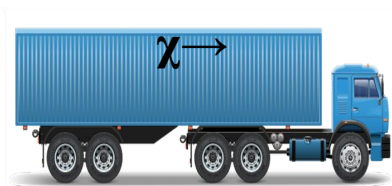
←(n/p)

DM capture by the Sun

$$\frac{dC}{dV} = \left[\frac{\rho}{M_\chi} \int_0^{u_m} du \frac{f(u)}{u} \right] w \Omega(w)$$

Astrophysics and Particle Physics come together

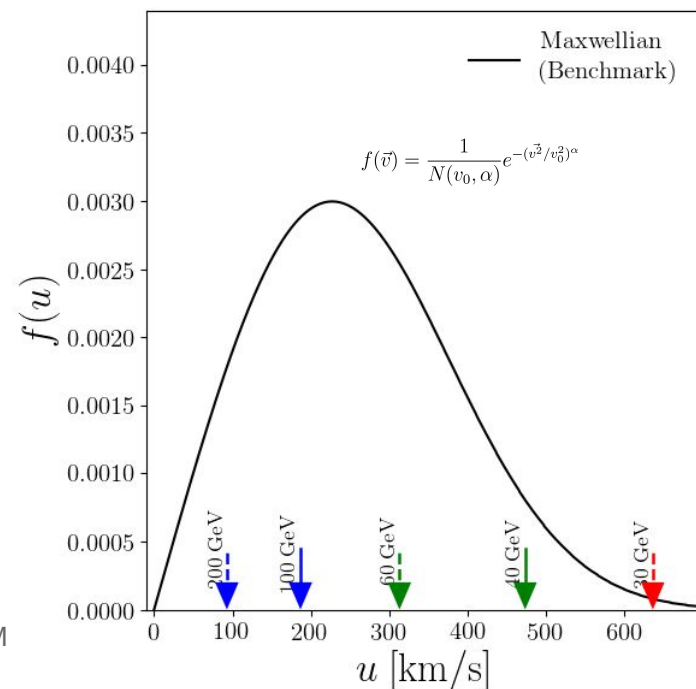
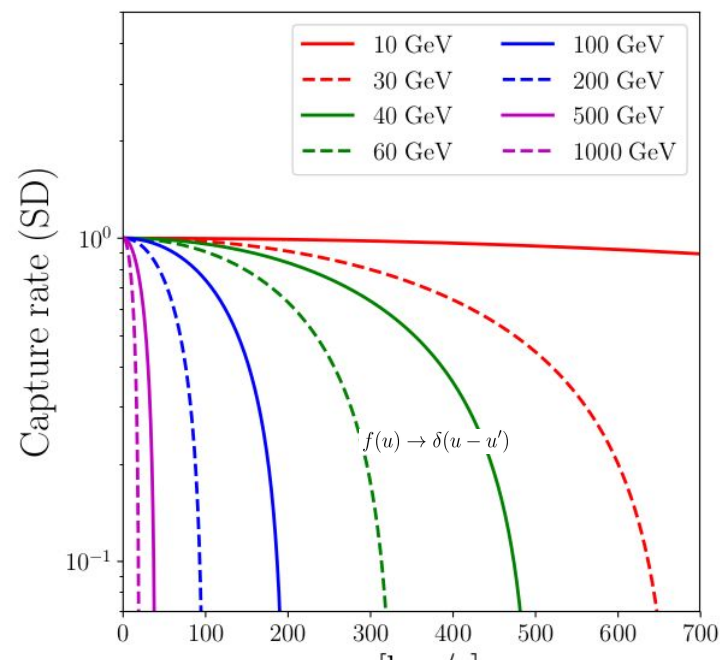
Capture in Sun: Low velocity part
Direct Detection: High velocity tail



←(n/p)



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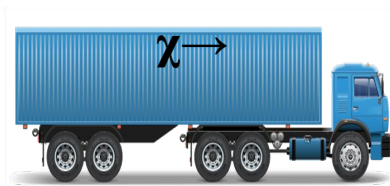


DM capture by the Sun

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Astrophysics and Particle Physics come together

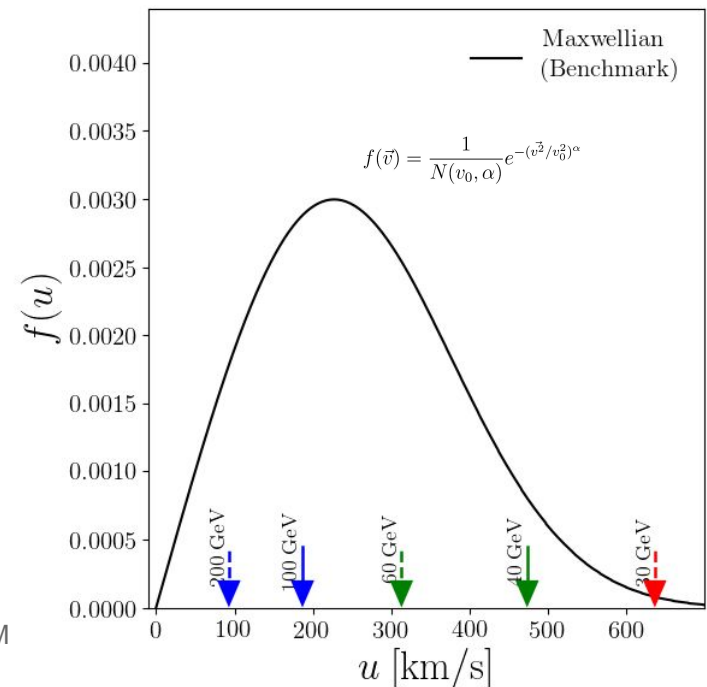
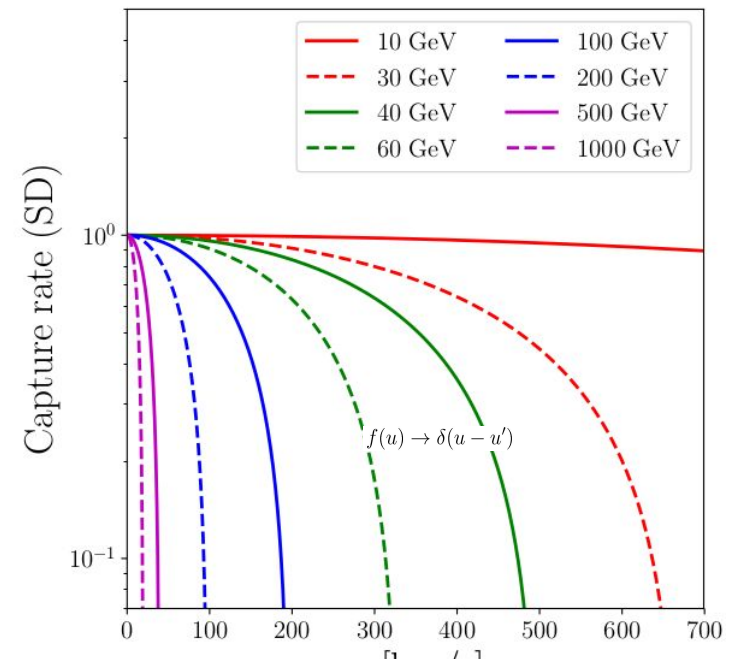
Capture in Sun: Low velocity part
Direct Detection: High velocity tail



←(n/p)



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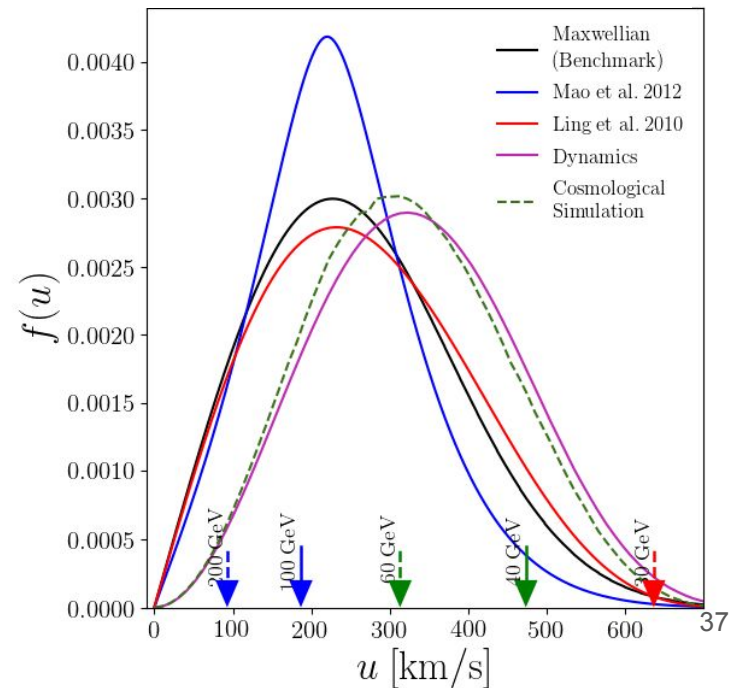
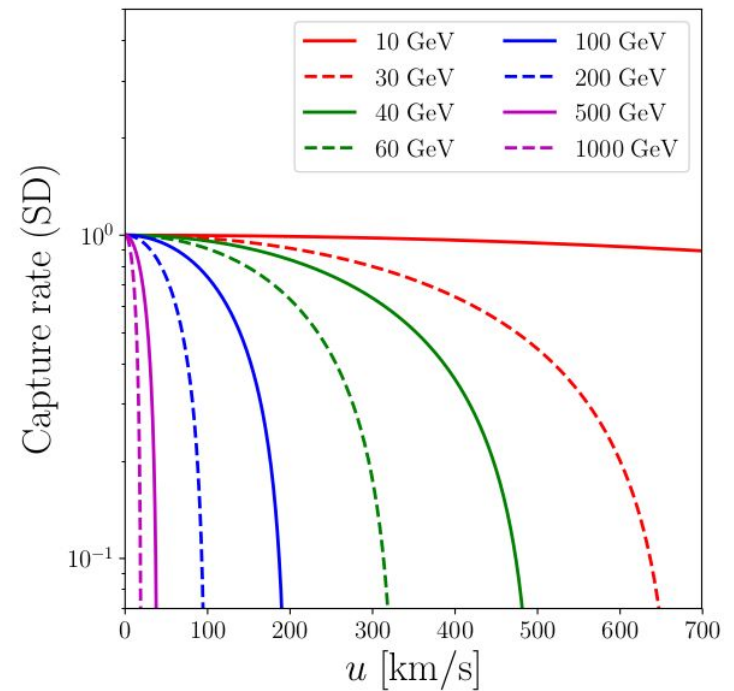
DM capture by the Sun

$$\frac{dC}{dV} = \left[\frac{\rho}{M_\chi} \int_0^{u_m} du \frac{f(u)}{u} \right] w \Omega(w)$$

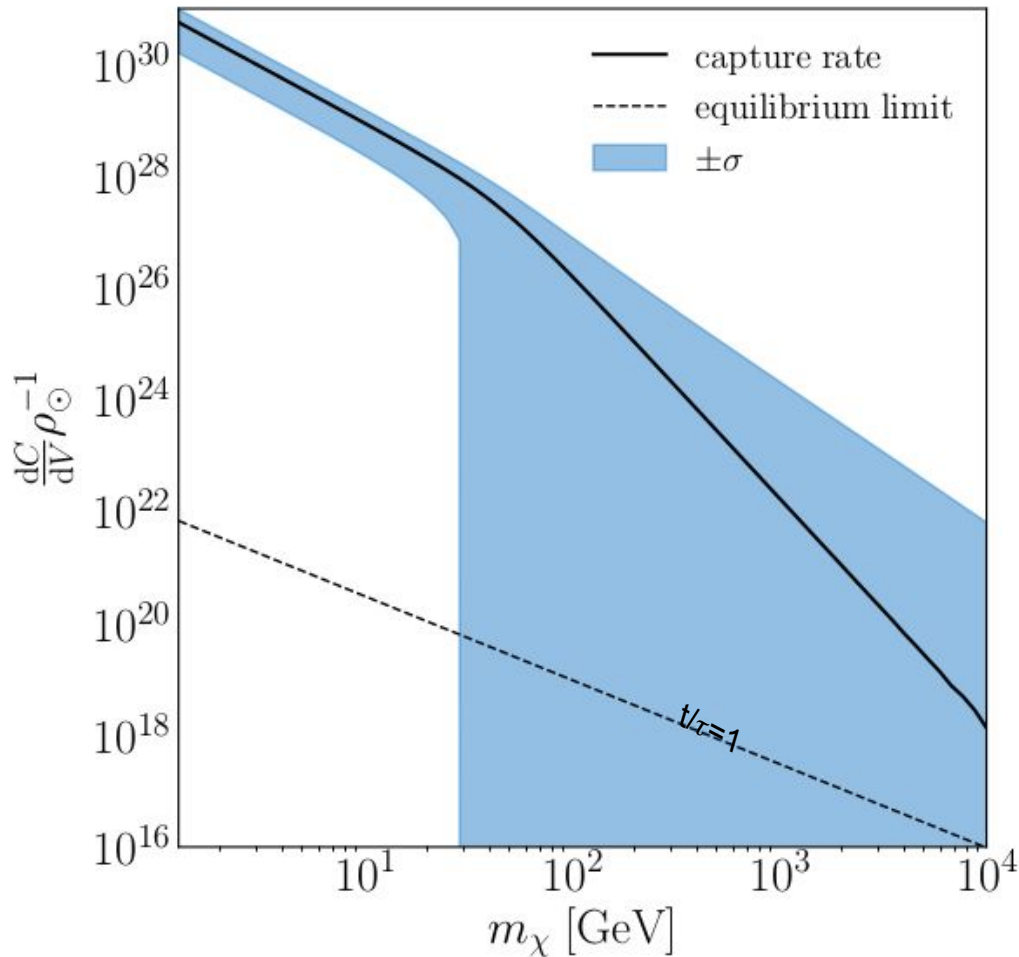
Astrophysics and Particle Physics come together

Astrophysical Uncertainties:

- Assumptions on the Sun's composition (Wikström & Edjo 2009)
- Assumptions on the galactic DM features (Choi 2014 , A. Green 2017)
- DM velocity distribution. (Ling 2010, Mao 2012)
- Intrinsic uncertainty of capture ([arxiv:1906.11674])



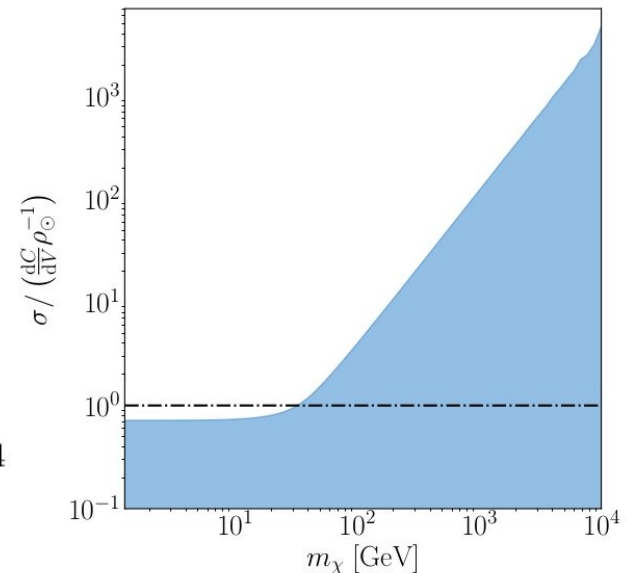
Intrinsic uncertainty



Nunez-Castineyra, Nezri & Bertin
[arxiv:1906.11674]

$$\frac{dC}{dV} = \left[\frac{\rho}{M_{\chi}} \int_0^{u_m} du \frac{f(u)}{u} \right] w \Omega(w)$$

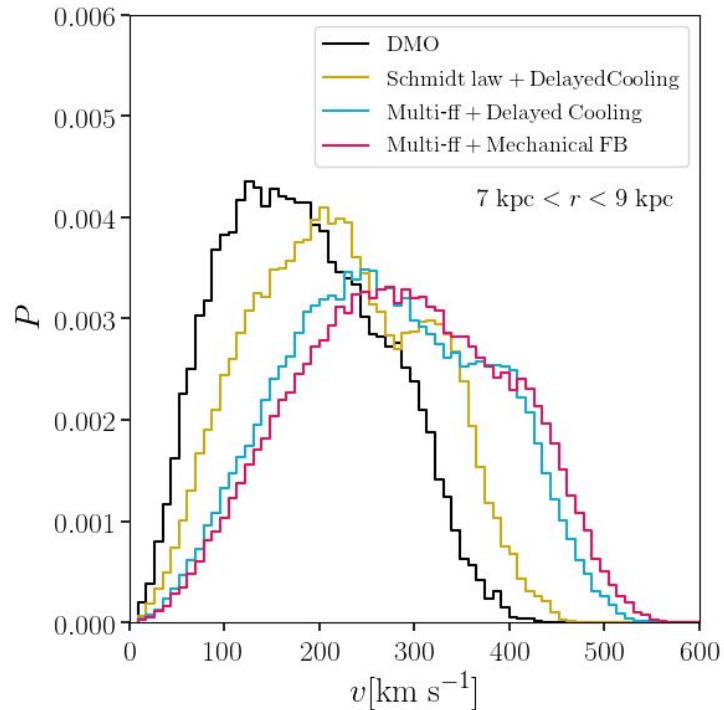
Astrophysics and Particle Physics come together



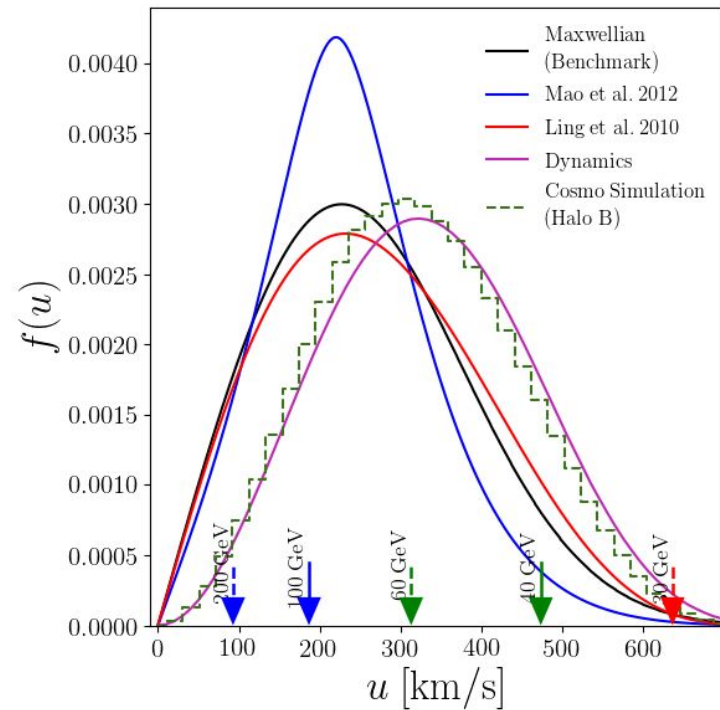
Results on Capture

- Departure from **capture/annihilation equilibrium** during periods of low capture is an unlikely scenario. **The Sun is** safely in **equilibrium** for WIMPs with m_χ . few TeV.
- The **peak/hat and the tail** of the simulations $f(v)$ are usually hard to fit. Adding the **escape velocity** in the fits improves the consistency with the tail of the distribution.
- The $f(v)$ s obtained with the **Eddington approach** bring additional information on the possible distributions that can be assumed. (have better agreement with simulations data than the standard Maxwellian VDF)
- **The merger history** of the halo could leave specific features in the $f(v)$ that **out of reach for usual functions**.
- The level of **variability** on the capture rate can reach up to **20%** depending on the assumptions $f(v)$
- The intrinsic errors, **the variance**, of the capture rate leads to **dramatic uncertainties**, especially for **$m_\chi > 30$ GeV**.

Point of emphasis on the variability of the $f(v)$



In simulations the implementation of baryonic physics will have an effect on the final distribution.
[Nunez-Castineyra et al. in prep](#)



The discussed cases + some extra from the literature

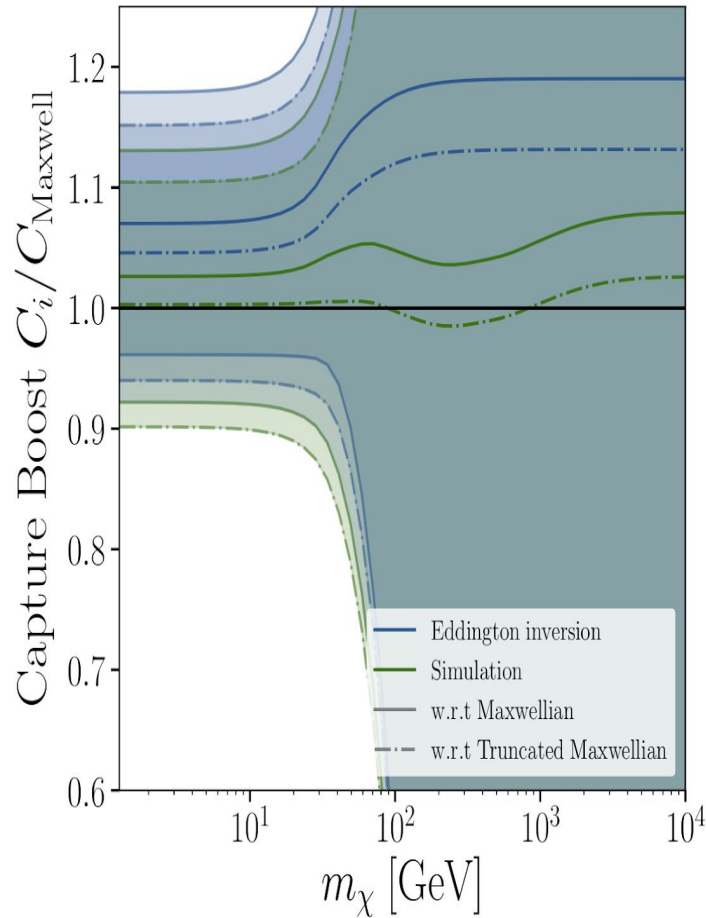
Three main ways to approach $f(v)$

- Take the standard halo model (SHM)
 - Maxwellian exhibit problems with the **tail** and **hat** of the $f(v)$
 - Assumes an isothermal halo density profile $\rho \propto r^{-2}$
 - **Degenerations** in other values
- Direct extrapolation by fitting $f(v)$ from Cosmological "Milky-Way like" simulations
 - No warranty that of the **"Milky-way likeness"** of the simulation
 - The meaning of **8kpc** in the simulation
- Dynamical phase space prediction using MW macro features
- e.g Eddington (Eddington 1916, Iacox 2018), Action angle (Posti 2015) etc.
 - Requires validation from simulations. Iacox et al in prep

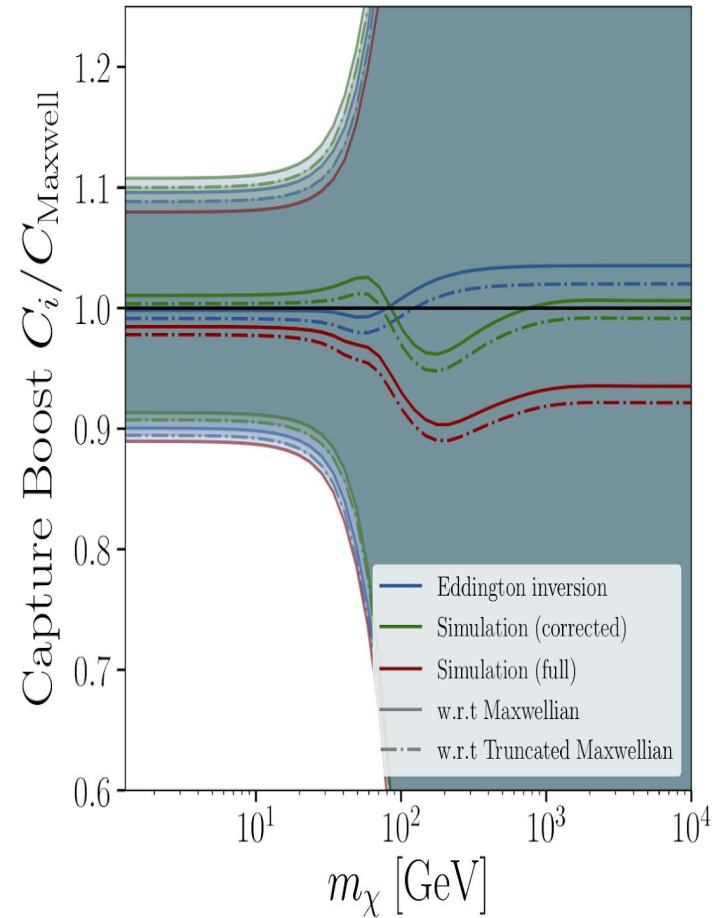
Perspectives: **GAIA era** will bring strong progresses + (directional) **direct detection** experiment and **neutrino telescopes**

Thank you

Capture boost

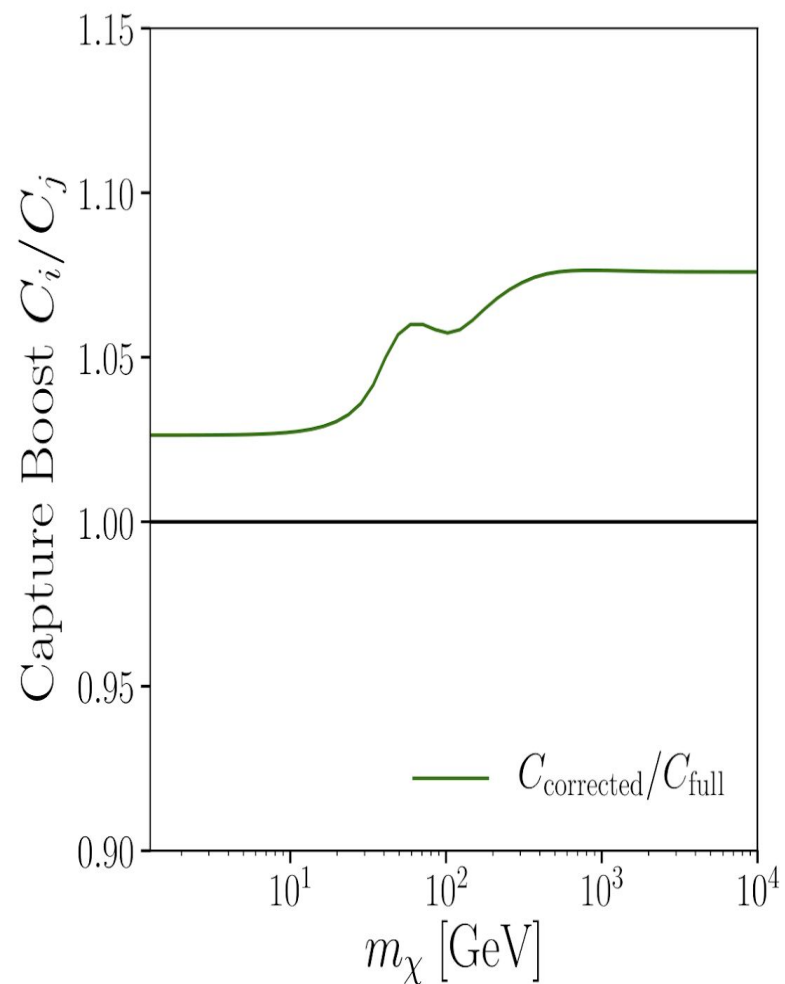


(a) Halo B shell

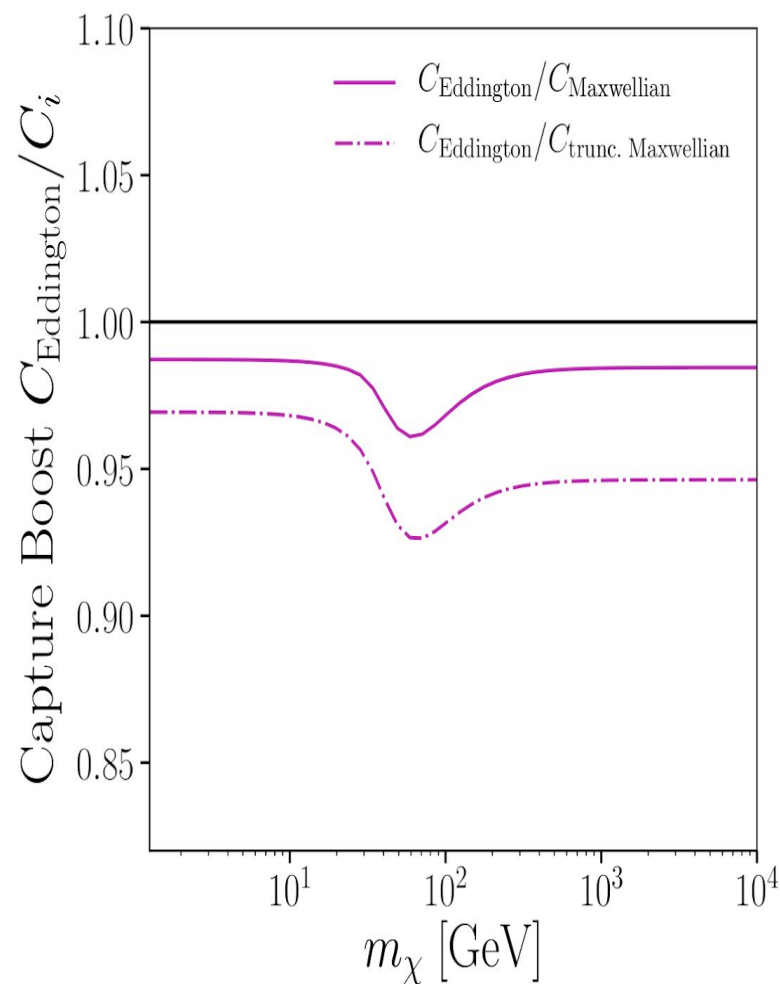


(b) Mochima shell

Capture boost



(c) Mochima



(d) Milky Way mass model