QUASINORMAL MODES OF BLACK HOLES IN MODIFIED AND QUANTUM GRAVITY

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Quasinormal modes of the Schwarzschild black hole

Moulin, A.B., Martineau, Universe 5 (2019) no.9, 202

$$g_{\mu\nu} = g^0_{\mu\nu} + \delta g_{\mu\nu} \qquad \delta R_{\mu\nu} = 0$$

– purely outgoing at infinity;
– purely ingoing at the event horizon.

$$\psi \propto e^{-i\omega t} = e^{-i(\omega_R + i\omega_I)t} = e^{\omega_I t}\cos(\omega_R t + \phi)$$

$$T = \frac{2\pi}{\omega_R} \qquad \tau = \frac{1}{\omega_I}$$

– $\omega_I < 0$: exponential damping (stable);
– $\omega_I > 0$: exponential growth (unstable).

$$\frac{1}{\sqrt{-g}}\partial_\mu(g^{\mu\nu}\sqrt{-g}\partial_\nu\psi) = 0$$

$$\begin{cases} s = 0 \text{ for scalar perturbations,} \\ s = 1 \text{ for electromagnetic perturbations,} \\ s = 2 \text{ for axial gravitational perturbations,} \end{cases}$$

and it is also worth noticing here that by symmetry we must have $\ell \geq 0$ for scalar perturbations, $\ell \geq 1$ for electromagnetic perturbations and $\ell \geq 2$ for gravitational perturbations.

$$V_\ell(r) = \left(1 - \frac{2M}{r}\right)\left[\frac{\ell(\ell+1)}{r^2} + \frac{2M(1-s^2)}{r^3}\right]$$

Review C. Chirenti, 1708.04476v2

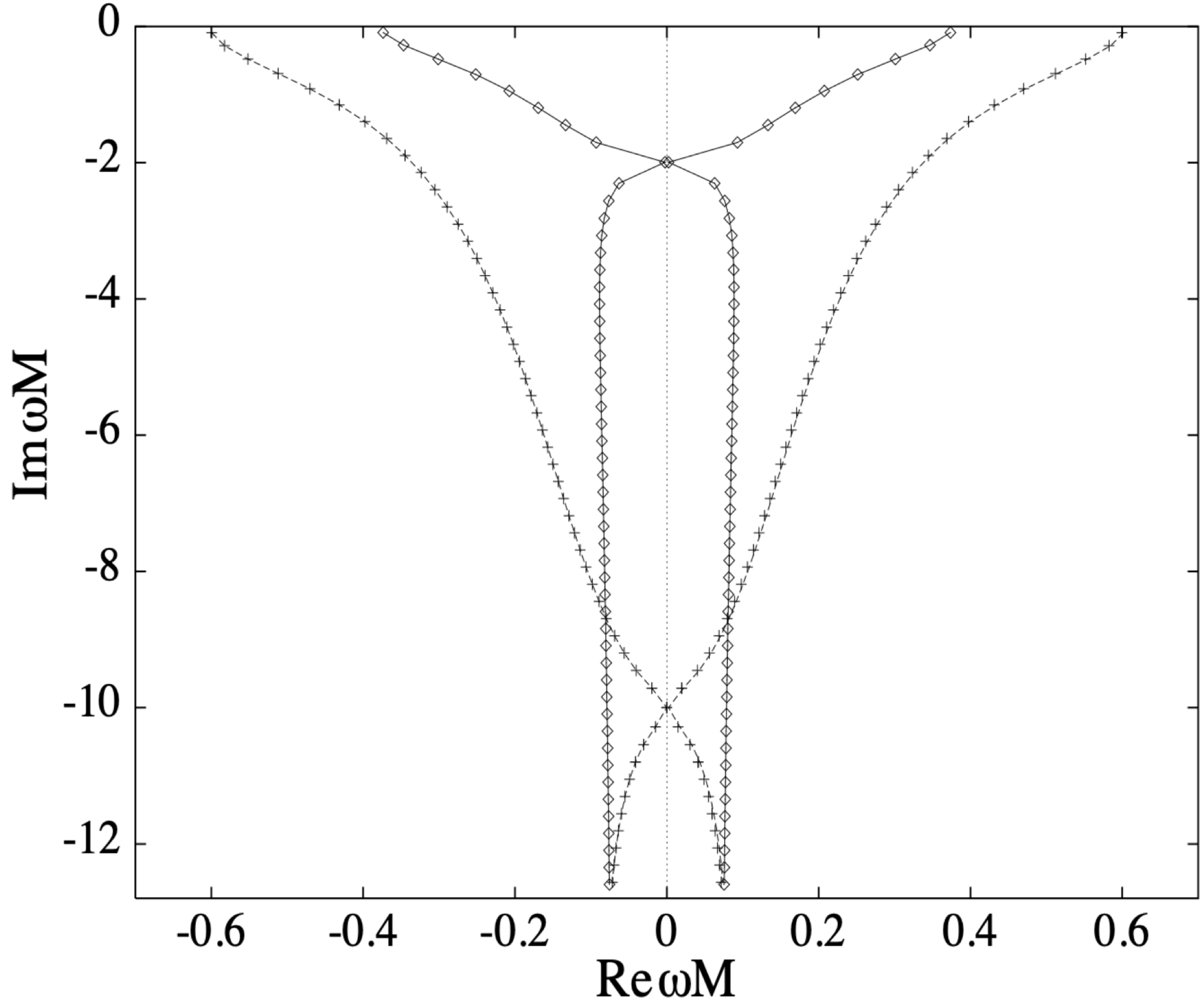
$$ds^2 = -e^{2\nu}dt^2 + e^{2\mu_2}dr^2 + e^{2\mu_3}d\theta^2 + \\ + e^{2\psi}(d\phi - \omega\, dt - q_2\, dr - q_3 d\theta)^2\,,$$

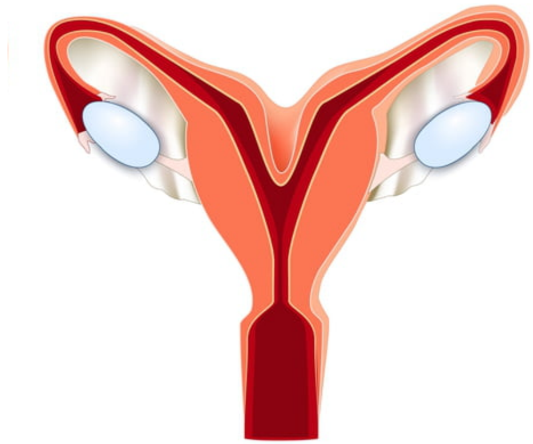
1. the perturbation that gives small values to the metric coefficients that were zero (ω, q_2, q_3): this perturbation induces frame dragging and imparts a rotation to the black hole; this is called the "axial" perturbation.
2. the perturbation that gives small increments to the already non-zero metric coefficients $(e^{2\nu}, e^{2\mu_2}, e^{2\mu_3}, e^{2\psi})$: this is called the "polar" perturbation.

$$V_\ell^{\text{axial}}(r) = \left(1 - \frac{2M}{r}\right)\left[\frac{\ell(\ell+1)}{r^2} - \frac{6M}{r^3}\right]$$

$$V_\ell^{\text{polar}}(r) = \frac{2}{r^3}\left(1 - \frac{2m}{r}\right) \times \\ \times \frac{9M^3 + 3c^2Mr^2 + c^2(1+c)r^3 + 9M^2cr}{(3M+cr)^2}$$

with $c = \ell(\ell+1)/2 - 1$.





Utérus bicorne

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Featured in Physics Editors' Suggestion

Testing the No-Hair Theorem with GW150914

Maximiliano Isi, Matthew Giesler, Will M. Farr, Mark A. Scheel, and Saul A. Teukolsky
Phys. Rev. Lett. **123**, 111102 – Published 12 September 2019

Cornell University, Ithaca, New York 14853, USA
(Dated: August 12, 2019)

We analyze gravitational-wave data from the first LIGO detection of a binary black-hole merger (GW150914) in search of the ringdown of the remnant black hole. Using observations beginning at the peak of the signal, we find evidence of the fundamental quasinormal mode and at least one overtone, both associated with the dominant angular mode ($\ell = m = 2$), with 3.6σ confidence. A ringdown model including overtones allows us to measure the final mass and spin magnitude of the remnant exclusively from postinspiral data, obtaining an estimate in agreement with the values inferred from the full signal. The mass and spin values we measure from the ringdown agree with those obtained using solely the fundamental mode at a later time, but have smaller uncertainties.

Beyond GR

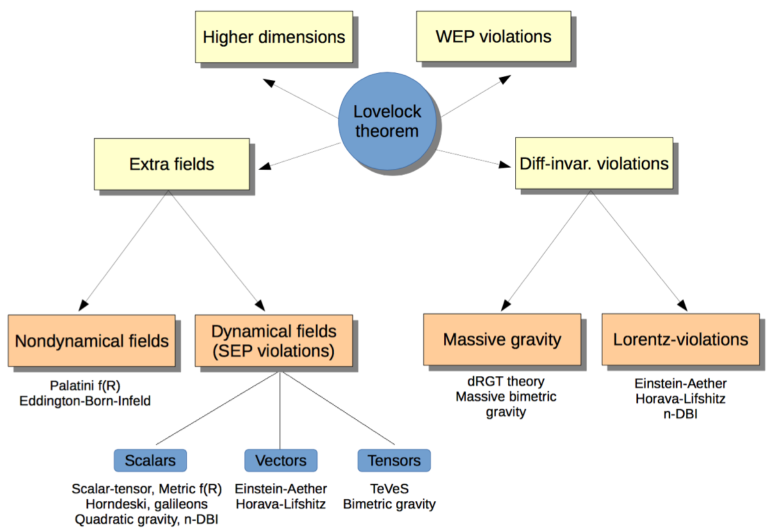
$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$

$$V(r) = f(r)\left(\frac{\lambda + 2(f(r)-1)}{r^2} - \frac{f'(r)}{r}\right)$$

$$ds^2 = A(r)dt^2 - B(r)^{-1}dr^2 - H(r)d\theta^2 - H(r)\sin^2\theta d\phi^2$$

$$\frac{d^2 Z}{dr^{*2}} + (\omega^2 - V(r))Z = 0$$

$$V(r) = \frac{1}{2H^2}\left(\frac{dH}{dr^*}\right)^2 + \frac{\mu^2 A}{H} - \frac{1}{\sqrt{H}}\frac{d^2}{dr^{*2}}\left(\sqrt{H}\right)$$



We use a 6th order WKB approximation scheme

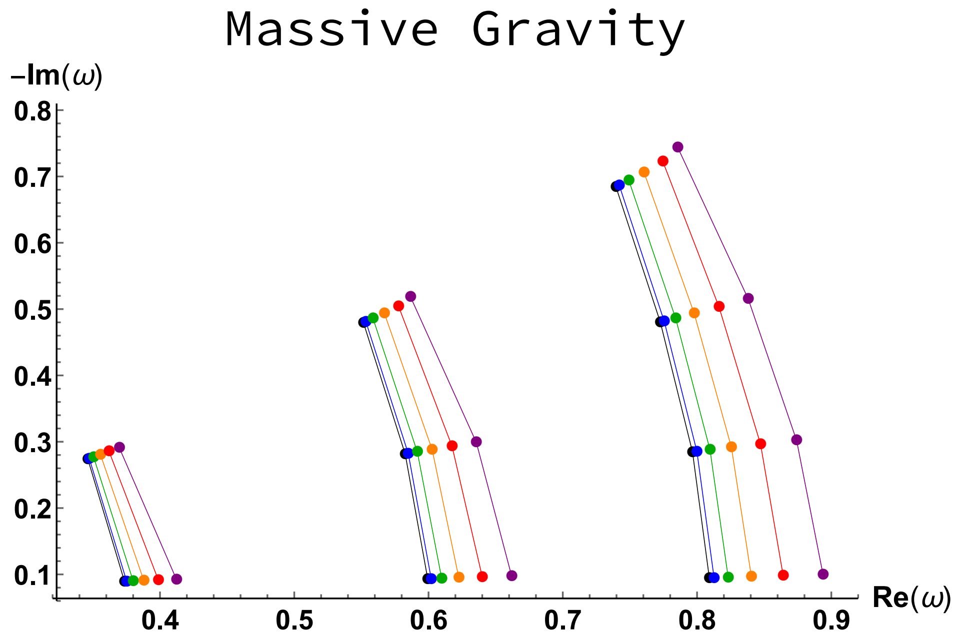
$$\omega^2 = V_0 - i\sqrt{-2V_0''}\left(\sum_{j=2}^{6}\Lambda_j + n + \frac{1}{2}\right)$$

1. Massive gravity

One of the first motivations for modern massive gravity – which can be seen as a generalization of GR – was the hope to account for the accelerated expansion of the Uni- verse by generating a kind of Yukawa-like potential for gravitation

$$S = \frac{1}{16\pi}\int \mathrm{d}^4x\sqrt{-g}\left(R + m^2\mathcal{U}(g,\phi^a)\right)$$

$$f(r) = 1 - \frac{2M}{r} + \frac{\Lambda r^2}{3} + \gamma r + \epsilon$$



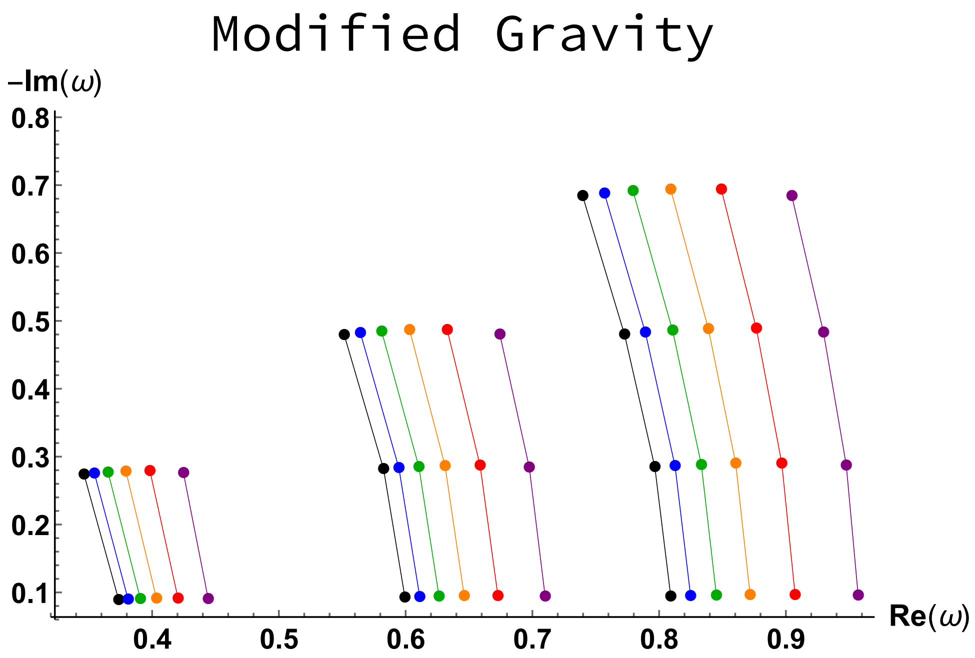
2. Modified STV gravity

MOG allows the gravitational constant, a vector field coupling, and the vector field mass to vary with space and time. We chose

- the case where the field equations for Bμν are non-linear, as the phenomenology is then richer,
- α < αc = 0.67 where there are two horizons and an appropriate potential behavior for the WKB approximation to hold.

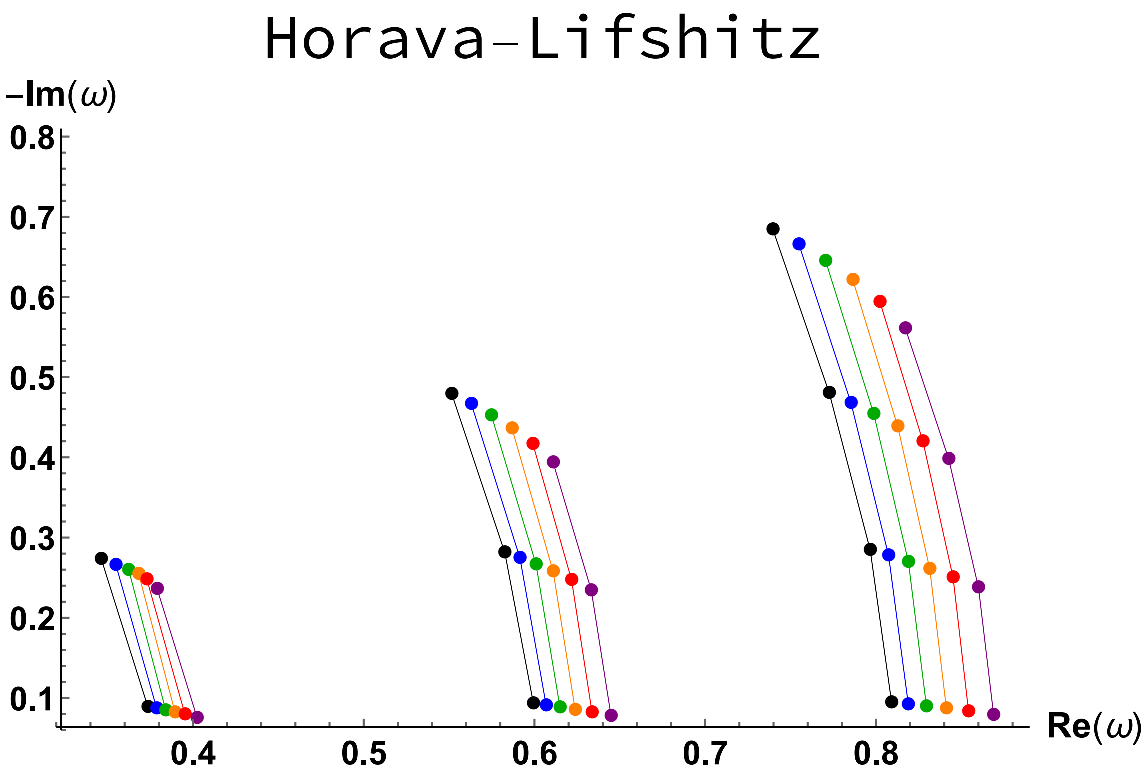
$$G = G_N(1+\alpha)$$

$$f(r) = 1 - \frac{2M}{r} + \frac{\alpha(1+\alpha)M^2}{r^2}$$



3. Hořava-Lifshitz gravity

Horava-Lifshitz gravity bets on the fundamental nature of the quantum theory instead of relying on GR principles. It is a renormalizable UV-complete gravitational theory which is not Lorentz invariant in 3 + 1 dimensions

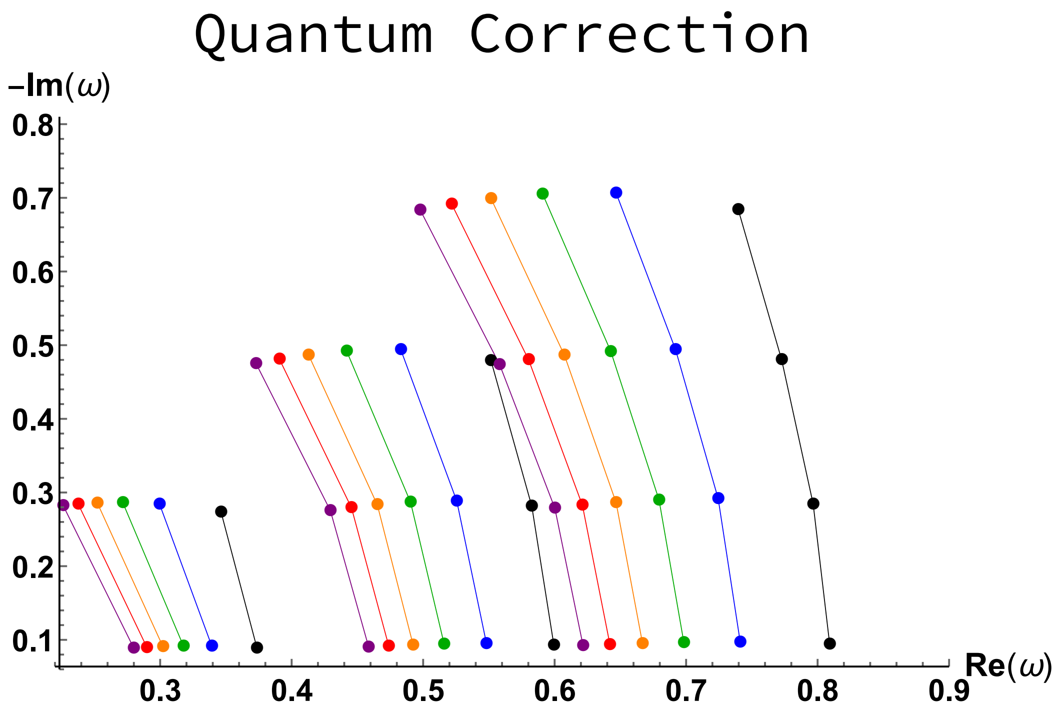


$$f(r) = \frac{2(r^2 - 2Mr + \beta)}{r^2 + 2\beta + \sqrt{r^4 + 8\beta M r}}$$

4. $\hbar$ correction

Quantum corrections to the Newtonian gravitational potential can be rigorously derived without having a full quantum theory of gravity at disposal.

$$f(r) = 1 - \frac{2M}{r} + \gamma \frac{2M}{r^3}.$$



5. LQG polymeric BH

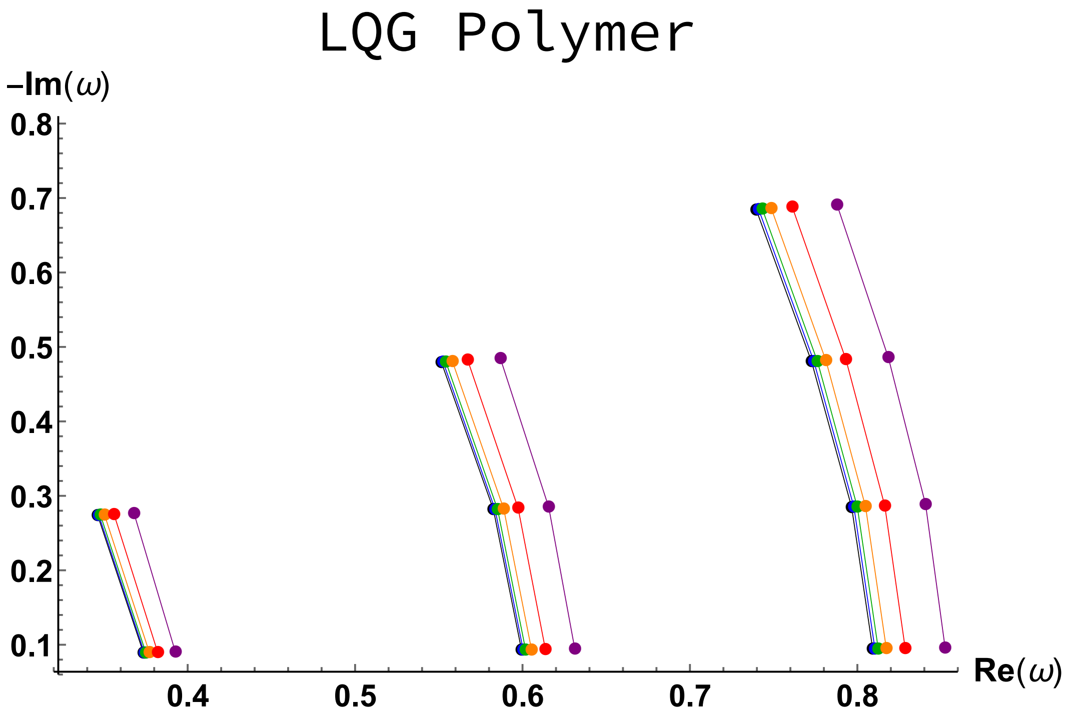
$$A(j) = 8\pi\gamma_{BI}\sqrt{j(j+1)}$$

$$ds^2 = -G(r)dt^2 + \frac{dr^2}{F(r)} + H(r)d\Omega^2 \ ,$$

$$G(r) = \frac{(r-r_+)(r-r_-)(r+r_*)^2}{r^4+a_o^2} \ ,$$

$$F(r) = \frac{(r-r_+)(r-r_-)r^4}{(r+r_*)^2(r^4+a_o^2)} \ ,$$

$$H(r) = r^2 + \frac{a_o^2}{r^2} \ ,$$



Distinguishing between those models with observations is challenging :

- there exist degeneracies, for given overtone and multipole numbers, between the models – when taking into account that the values of the parameters controlling the deformation are unknown.
- the intrinsic characteristics of the observed black holes are also unknown, which induces other degeneracies
- the study should be extended to Kerr black hole, which also adds some degeneracies in addition to the complexity

Some interesting trends can however be underlined. For all models, the effect of modifying the gravitational theory are more important for the real part than for the imaginary part of the complex frequency of the QNMs.
Some "trends" are specific to each studied. In addition, the sign of the frequency shift, and its dependance upon the overtone and multipole numbers is characteristic of a given extension of GR.

If features beyond GR were to be observed, the direction of the frequency shift in the complex plane would already allow to exclude models, as this article shows.

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Possible links between high order overtones and quantum gravity (Hod, Maggiore)

A toy model for cumulative quantum gravity

A.B., Martineau, Moulin, Martinon, Phys.Lett. B795 (2019) 346-350

$$l_R \sim \mathcal{R}^{-1/2} \qquad \mathcal{R}^2 := R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda}$$

Disregards cumulative effects

→ Let us assume a « quantumness »

$$q = l_P \, \mathcal{R} \, \tau. \qquad \tau = \sqrt{1 - \frac{2M}{r}}\, t,$$

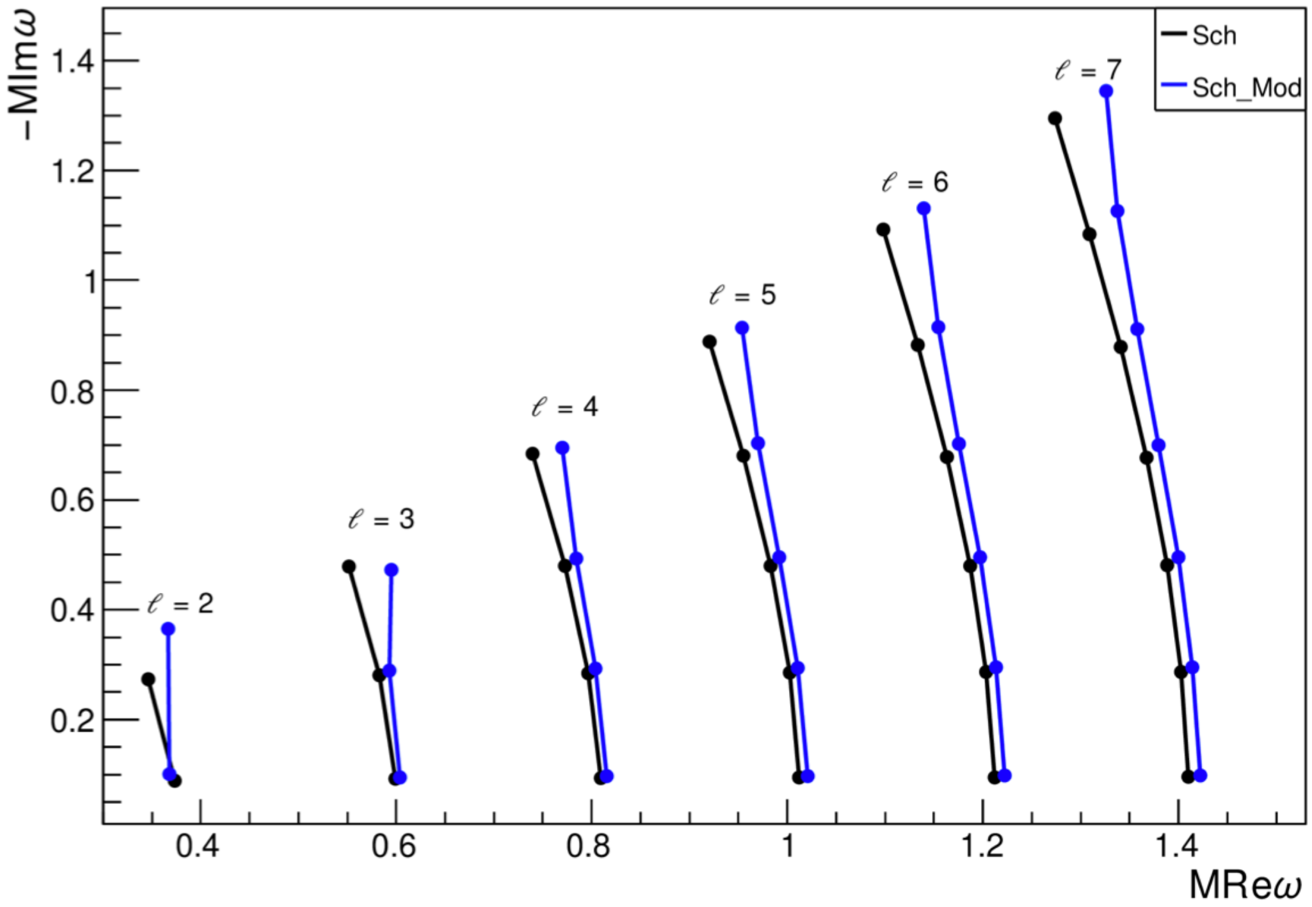
$$q(r) = \frac{M}{r^3} \left(1 - \frac{2M}{r}\right)^{\frac{1}{2}} t.$$

With a maximum at

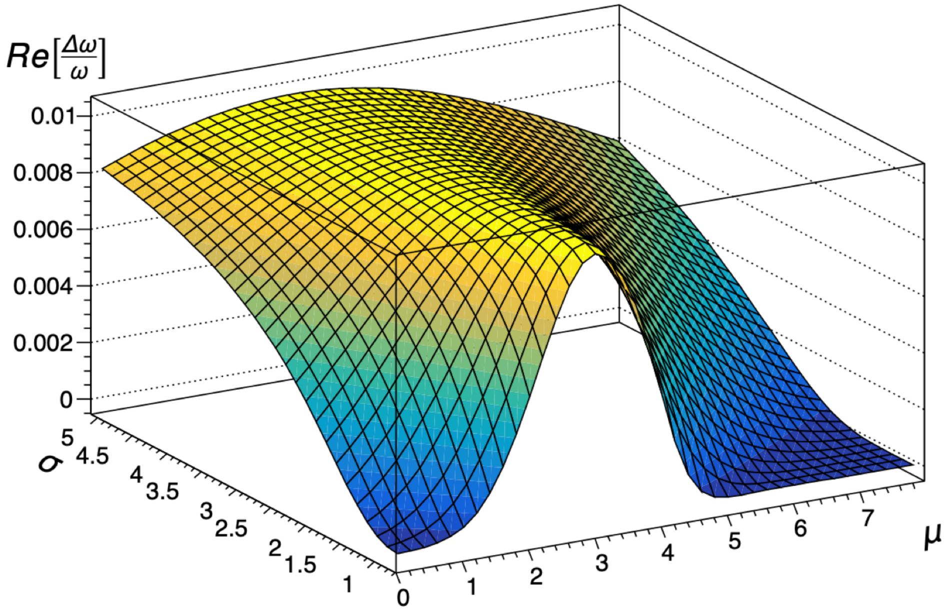
$$r = 2M\left(1 + \tfrac{1}{6}\right)$$

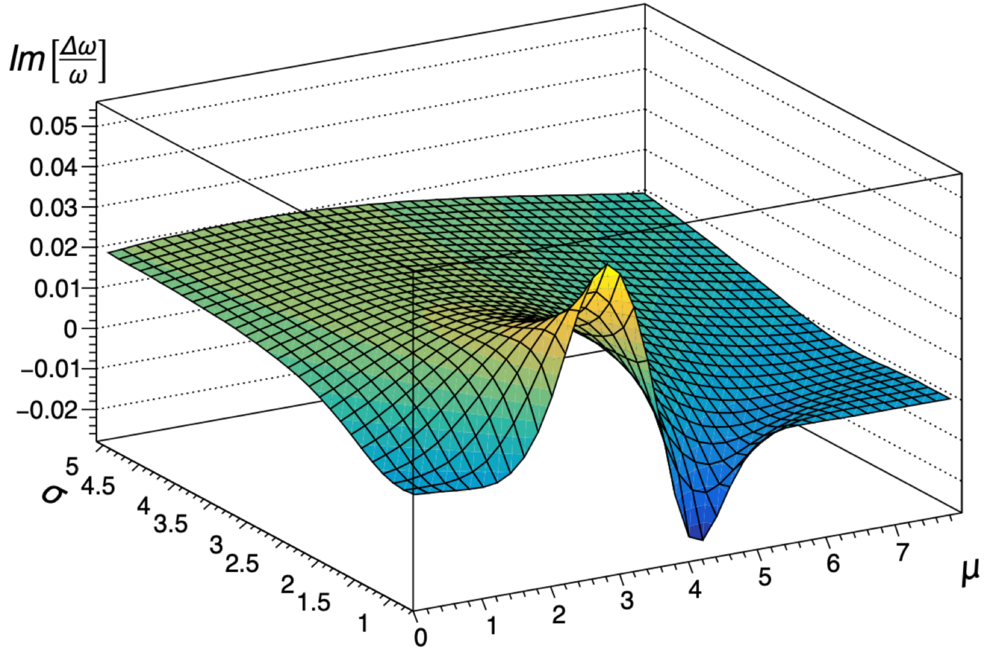
We assume in general

$$f(r) = \left(1 - \frac{2M}{r}\right)\left(1 + Ae^{-\frac{(r-\mu)^2}{2\sigma^2}}\right)^2$$



−MImω
1.4
1.2
1
0.8
0.6
0.4
0.2
0.4
0.6
0.8
1
1.2
1.4
MReω
Sch
Sch_Mod
ℓ = 2
ℓ = 3
ℓ = 4
ℓ = 5
ℓ = 6
ℓ = 7


$Re\left[\frac{\Delta\omega}{\omega}\right]$
0.01
0.008
0.006
0.004
0.002
0
q
5
4.5
4
3.5
3
2.5
2
1.5
1
0
1
2
3
4
5
6
7
μ


$Im\left[\frac{\Delta\omega}{\omega}\right]$
0.05
0.04
0.03
0.02
0.01
0
−0.01
−0.02
q
5
4.5
4
3.5
3
2.5
2
1.5
1
0
1
2
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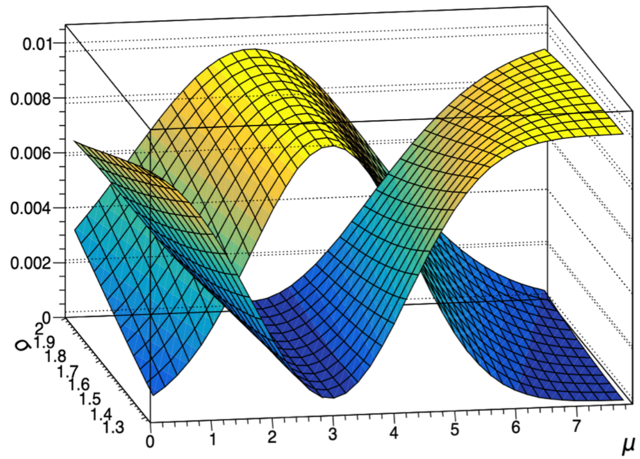
7
μ


0.01
0.008
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0.004
0.002
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q
2
1.9
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1.7
1.6
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1.4
1.3
0
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2
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4
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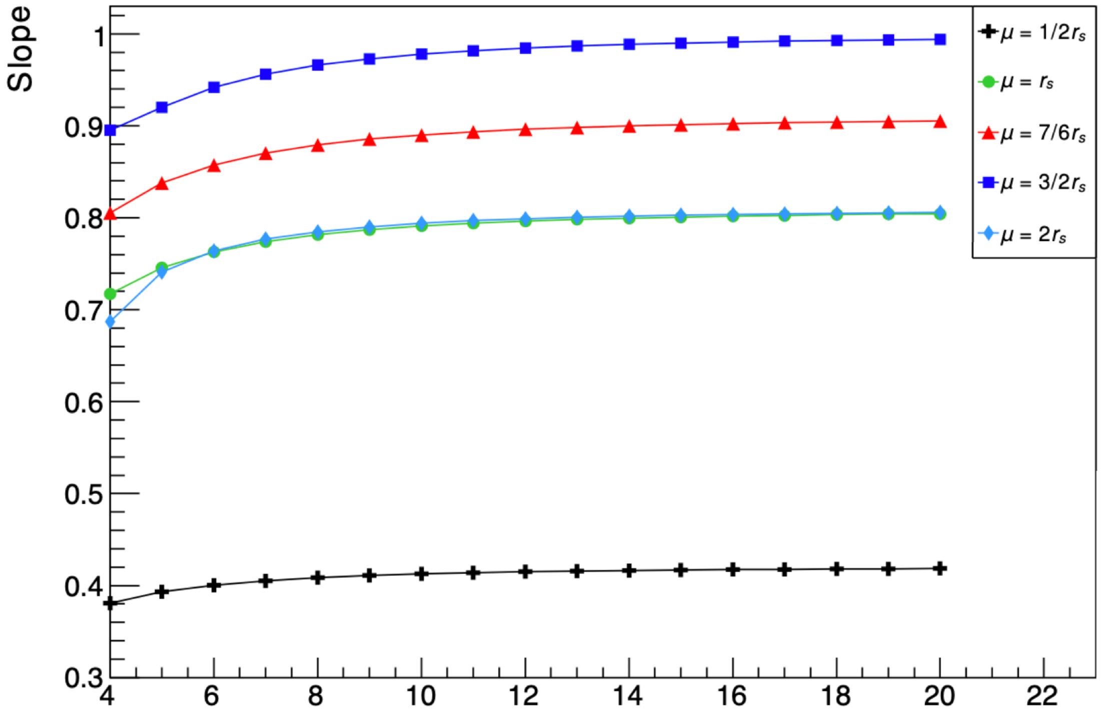
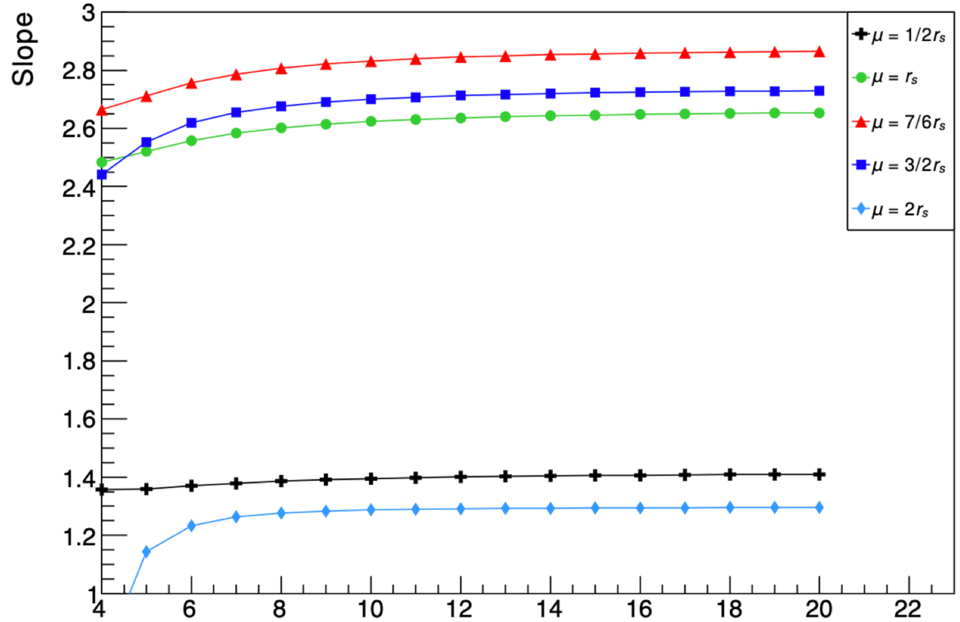
6
7
μ



Slope
$\mu = 1/2r_s$
$\mu = r_s$
$\mu = 7/6r_s$
$\mu = 3/2r_s$
$\mu = 2r_s$



Slope
$\mu = 1/2r_s$
$\mu = r_s$
$\mu = 7/6r_s$
$\mu = 3/2r_s$
$\mu = 2r_s$

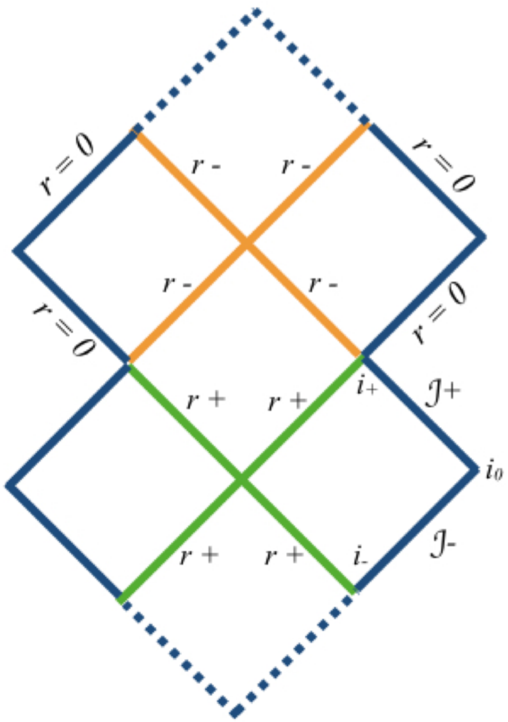
THE HAYWARD METRIC

$$F(r) = 1 - \frac{2m(r)}{r} \qquad m(r) = \frac{M\,r^3}{r^3 + 2ML^2}$$

$$L/M \sim 0.72 \qquad \left(\frac{\Delta\omega}{\omega}\right) \sim 5\%$$

Quantum fields in the background spacetime of a polymeric BH

Moulin, Martineau, Grain, A.B., Class.Quant.Grav. 36 (2019) no.12, 125003



$$\frac{dN}{dt} = \frac{1}{e^{\frac{\omega}{T_H}} \pm 1}\sigma(M, s, \omega)\frac{d^3k}{(2\pi)^3}$$

$$\sigma(M, s, \omega) = \sum_{l=0}^{\infty} \frac{(2j+1)\pi}{\omega^2}|A_{l,s}|^2$$

$$ds^2 = G(r)dt^2 - \frac{dr^2}{F(r)} - H(r)d\Omega^2 \ ,$$

$$G(r) = \frac{(r - r_+)(r - r_-)(r + r_x)^2}{r^4 + a_o^2} \ ,$$

$$F(r) = \frac{(r - r_+)(r - r_-)r^4}{(r + r_x)^2(r^4 + a_o^2)} \ ,$$

$$H(r) = r^2 + \frac{a_o^2}{r^2} \ ,$$

$$\frac{\sqrt{GF}}{H}\frac{\partial}{\partial r}\left(H\sqrt{GF}\frac{\partial R(r)}{\partial r}\right) + \left(\omega^2 - \frac{G}{H}l(l+1)\right)R(r) = 0,$$

$$\left(\frac{\partial^2}{\partial r^{*2}} + \omega^2 - V(r^*)\right)\Psi(r) = 0,$$

$$V(r) = \frac{G}{H}l(l+1) + \frac{1}{2}\sqrt{\frac{GF}{H}}\frac{\partial}{\partial r}\left(\sqrt{\frac{GF}{H}}\frac{\partial H}{\partial r}\right).$$

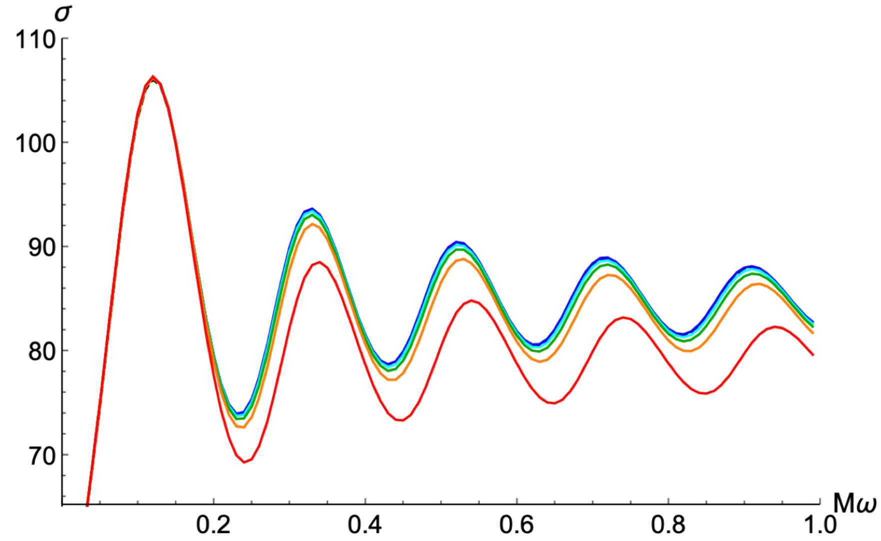


FIG. 2. Emission cross section for a scalar field with energy ω in the background spacetime of a LBH of mass M for different values of ϵ ($\epsilon = \gamma\delta$ measures the "quantumness" of spacetime). From bottom to top: $\epsilon = 10^{\{-0.3,-0.6,-0.8,-1,-3\}}$. The blue line, corresponding to $\epsilon = 10^{-3}$ is superposed with the cross section for a Schwarzschild BH.

$$
\begin{aligned}
(D+\epsilon-\rho)P^0+(\delta^*+\pi-\alpha)P^1 &= i\mu_*\overline{Q}^{1'}, \\
(\Delta+\mu-\gamma)P^1+(\delta+\beta-\tau)P^0 &= -i\mu_*\overline{Q}^{0'} \\
(D+\epsilon^*-\rho^*)\overline{Q}^{0'}+(\delta+\pi^*-\alpha^*)\overline{Q}^{1'} &= -i\mu_*P^1, \\
(\Delta+\mu_*-\gamma^*)\overline{Q}^{1'}+(\delta^*+\beta^*-\tau^*)\overline{Q}^{0'} &= i\mu_*P^0,
\end{aligned}
$$

with

$$
D=l^i\partial_i; \qquad \Delta=n^i\partial_i; \qquad \delta=m^i\partial_i; \qquad \delta^*=\overline{m}^i\partial_i.
$$

$$
\frac{1}{C_1}\frac{\partial^2 R_+}{\partial r^{*2}}-\frac{1}{2\sqrt{C_1}}\frac{\partial R_+}{\partial r^{*2}}+\left(\frac{\omega^2}{C1}+i\frac{\omega}{\sqrt{C1}}\right)R_+=0.
$$

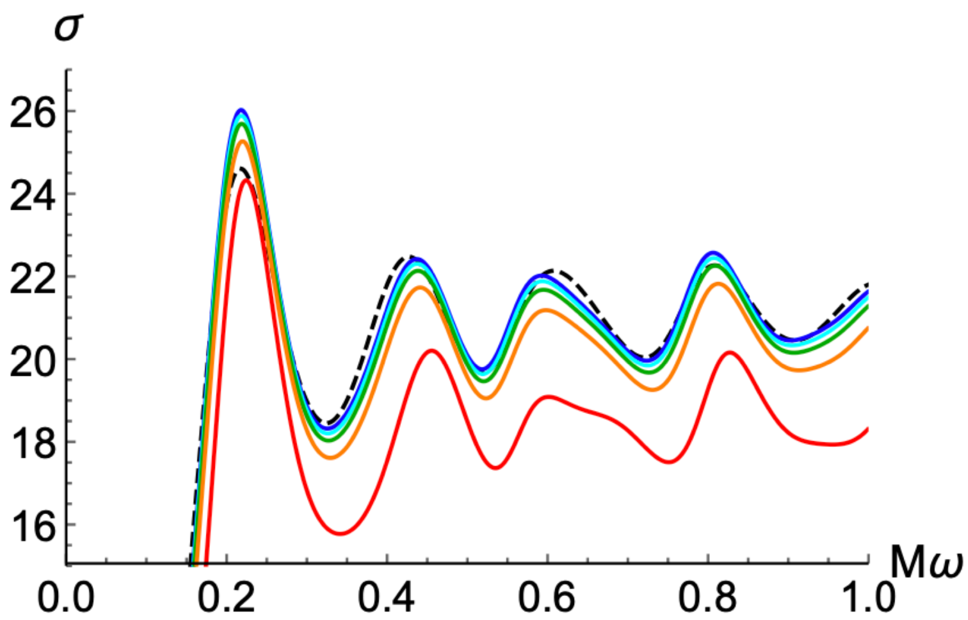


FIG. 3. Emission cross section for a fermionic field, with energy ω, in the background spacetime of a LBH of mass M. From bottom to top: $\epsilon=10^{\{-0.3,-0.6,-0.8,-1,-3\}}$. The dashed dark curve corresponds to the Schwarzschild cross section.

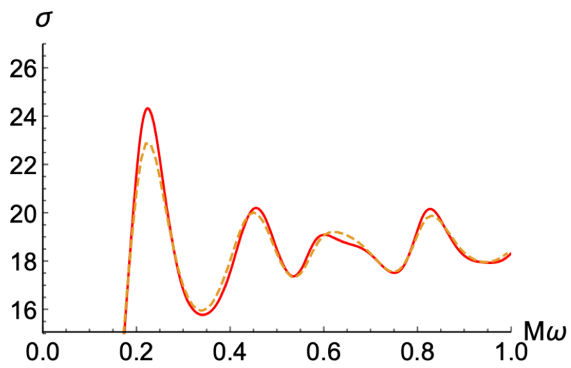
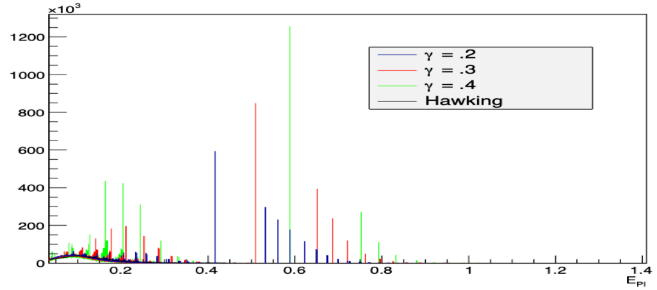
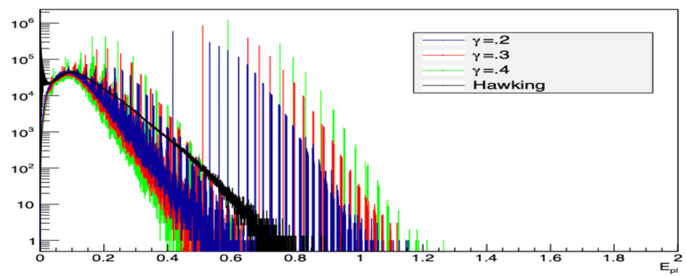


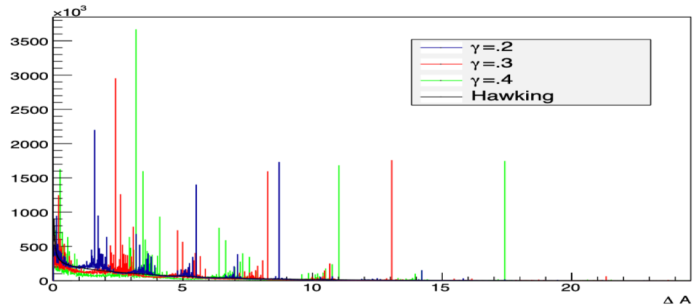
FIG. 4. Emission cross section for a fermionic field, with energy ω, in the background spacetime of a LBH of mass M, for $\epsilon = 10^{-0.3}$. The dashed curved corresponds to $a_0 = 0$ and the plain curve to the usual LQG value, $a_0 = A_{\min}/8\pi = \sqrt{3}\gamma/2$.

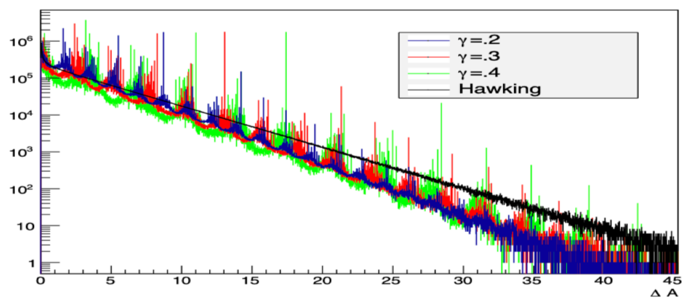
The effects are generically small but the trend is quite clear. Phenomenologically, large values of the polymerization parameter could be probed by a decreased cross section, together with a slight frequency shift for fermions. In addition, the non-vanishing minimum area leaves a specific footprint on the first peak.

A.B., Noui, Perez, Phys.Rev. D92 (2015) no.12, 124046





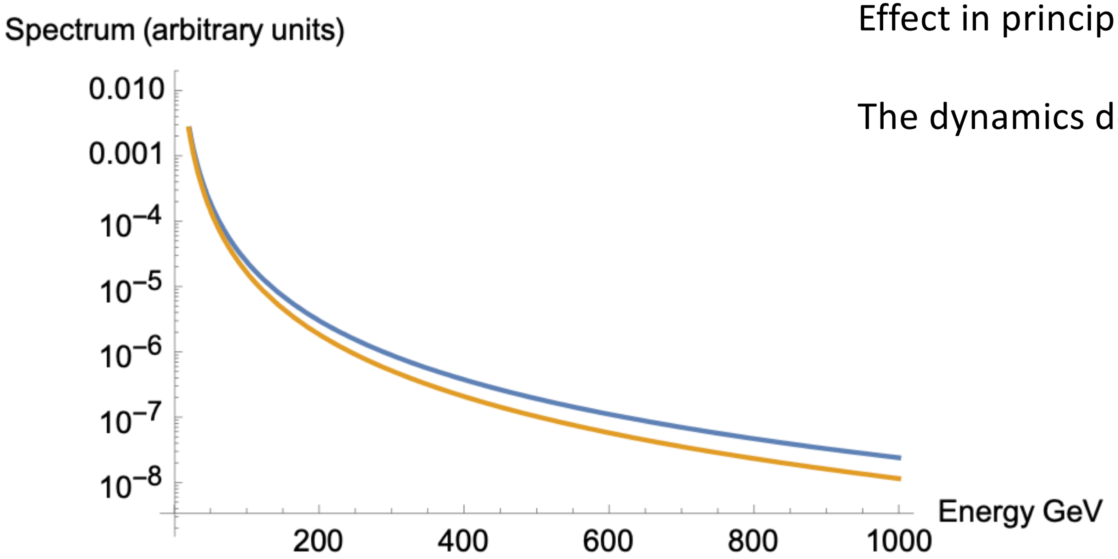




Evaporation from a local quantum gravity perspective

A.B., Phys. Rev. Lett. 117 (2016) 271301

$$A_j = 8\pi\gamma l_{Pl}^2 \sum_{n=1}^{N} \sqrt{j_n(j_n+1)},$$



Effect in principle detectable even at arbitrary high masses.

The dynamics does not wash out the effect.

Isopectrality

Moulin, A.B., arXiv:1906.09930

$$ds^2 = B(r)dt^2 - B(r)^{-1}dr^2 - r^2 d\Omega^2.$$

$$B(r) = (1+\alpha) - \frac{2M}{r} + \beta r$$

Other approaches : bouncing black holes

A.B., Moulin, Martineau, Phys.Rev. D97 (2018) no.6, 066019

$$\tau = -8m \ln v_o > \tau_q = 4k \frac{m^2}{l_p}$$