

Exact black hole solutions of 5D dilaton gravity and its holographic applications.

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based on works with

Irina Aref'eva (MI RAS, Moscow), Giuseppe Policastro (ENS, Paris)

[JHEP05\(2019\)117](#) and Vu H. Nguyen (BLTP JINR,VAST) [1906.12316](#), [TMPH](#)

Black holes and neutron stars in Modified Gravity

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MOTIVATION : HOLOGRAPHY

- is useful tool for theoretical insights into systems at strong coupling: ultrarelativistic heavy-ion collisions, cold atom systems, quantum simulators, “ultrafast” techniques in condensed matter physics, etc.
- allows to do calculations in real time, at non-zero baryonic density and finite temperature.
- reformulates the problem of quantum field theory into a dual classical gravitational problem in a space-time with an extra dimension.

Particularly, **4d** CFT \Leftrightarrow Gravity in **5d** AdS_5 .

$$S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} (R - \Lambda).$$

- AdS solution, • AdS-Schwarzschild BH, • Kerr-AdS BH.

AdS \Leftrightarrow $T = 0$ CFT

BH AdS \Leftrightarrow **thermal** CFT (non-spinning/spinning)

T of CFT is identified with the Hawking temperature of BH.

Example: QCD phases

- Chiral limit ($m_q = 0$): **UV region** - massless vector fields, **IR region** - massless pseudoscalars (pions)
- Lattice: QCD at high T has a quasi-conformal behaviour ($T_\mu^\mu = 0$)

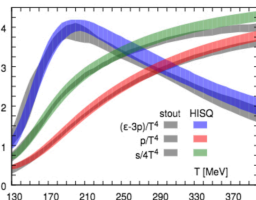


Figure: Bazavov et al, PRD 90 (2014) 094503

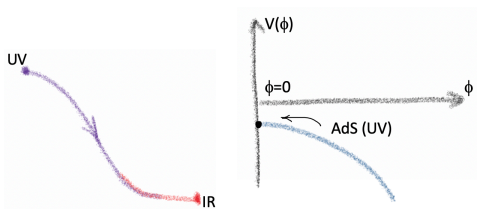
- high T QCD deconfined phase - Quark Gluon Plasma
- The viscosity-to-entropy ratio for QGP from **holography** $\frac{\eta}{s} = \frac{1}{4\pi}$.
Polcastro, Son, Starinets, Phys.Rev.Lett.87:081601,2001

Holographic picture for deviations from conformality

- CFT_4 has a description in terms of gravity in AdS_5 :
 $S = \int dx^5 du \sqrt{-g} (R - \Lambda).$
- An operator $\mathcal{O}(x)$ corresponds to a dynamical bulk field $\phi(x, u)$
- $\phi(x, 0)$ – a source for the \mathcal{O} in the CFT

$$S = \int dx^4 du \sqrt{-g} \left[R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right].$$

- $\phi(x, u) = \alpha u^{d-\Delta} + \dots \Leftrightarrow S = S_{CFT} + \int d^4x \alpha \mathcal{O}(x)$
- $\alpha = 0$ – undeformed CFT, bulk scalar – const., spacetime is AdS
- $\alpha \neq 0$ corresponds to relevant coupling for the CFT; deform. AdS



The models

- General analysis: U. Gürsoy, E. Kiritsis, L. Mazzanti, F. Nitti, Holography and Thermodynamics of 5D Dilaton-gravity, JHEP 0905:033,2009
- Improved holographic QCD Gursoy,Kiritsis' 07, Gubser'08

For asymptotically AdS UV $\lambda \rightarrow 0$ $V(\lambda) = V_0 + v_1\lambda + v_2\lambda^2 + \dots$

For confinement in the IR $\lambda \rightarrow \infty$ $V(\lambda) \sim \lambda^Q(\log \lambda)^P$

- Perturbative analysis near extrema of the potential Gürsoy et al.'17,Kiritsis et al' 16'17'18'19

- Single exponent potential $V = V_0(1 - X^2)e^{-\frac{8}{3}X\phi}$, $X < 0$ Gürsoy, Järvinen, Policastro'16
- Two exponent potential $V = C_1e^{2k_1\phi} + C_2e^{2k_2\phi}$, $C_1 < 0, C_2 > 0, k > 0$ Aref'eva, AG, Policastro'19

5D Dilaton Gravity with exponential potential

$$S = \frac{1}{2\kappa^2} \int d^4x \int du \sqrt{-g} \left(R - \frac{4}{3} (\partial\phi)^2 + V(\phi) \right) - \frac{1}{\kappa^2} \int_{\partial} d^4x \sqrt{-\gamma},$$

- $V = V_0(1 - X^2)e^{-\frac{8}{3}X\phi}$, $X < 0$ Chamblin, Reall, *Nucl.Phys.B562(1999)*; Charmousis, *Class.Quant.Grav.* 19 (2002)

$$\begin{aligned} ds_{CR}^2 &= e^{2A(u)} \left(-f(u)dt^2 + \delta_{ij}dx^i dx^j \right) + \frac{du^2}{f(u)} \\ e^A &= e^{A_0} \lambda^{\frac{1}{3X}}, \quad f = 1 - b\lambda^{-\frac{4(1-X^2)}{3X}}, \quad \lambda = e^\phi = \left(a - 4X^2 \frac{u}{\ell} \right)^{\frac{3}{4X}}. \end{aligned}$$

Gürsoy, Järvinen, Policastro'15; QNMs: Betzios, Gürsoy, Järvinen, Policastro'17'18

- $V = C_1 e^{2k_1\phi} + C_2 e^{2k_2\phi}$; $C_1 < 0, C_2 > 0$.
new holographic backgrounds Aref'eva, AG, Policastro'19

5D Dilaton Gravity with exponential potential

The action reads

$$S = \frac{1}{2\kappa^2} \int d^4x \int du \sqrt{-g} \left(R - \frac{4}{3}(\partial\phi)^2 + V(\phi) \right) - \frac{1}{\kappa^2} \int_{\partial} d^4x \sqrt{-\gamma},$$

$V(\phi) = C_1 e^{2k_1\phi} + C_2 e^{2k_2\phi}$, C_i , k_i , $i = 1, 2$ are some constants.

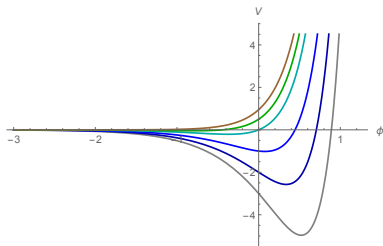


Figure: The behaviour of the potential $V(\phi)$ for $C_1 < 0$, $C_2 > 0$.

The ansatz for the metric and the dilaton

$$ds^2 = -e^{2A(u)} dt^2 + e^{2B(u)} \sum_{i=1}^3 dy_i^2 + e^{2C(u)} du^2, \quad \phi = \phi(u).$$

The gauge $C = A + 3B$.

The sigma-model

$$x^1 = A, \quad x^2 = B, \quad x^3 = \phi, \quad x = C.$$

$$L = \frac{1}{2} G_{MN} \dot{x}^M \dot{x}^N - V, \quad V = -\frac{1}{2} \sum_{s=1}^2 C_s e^{2(x^1 + 3x^2 + k_s x^3)}, \quad \cdot \equiv \frac{d}{du}.$$

$$(G_{MN}) = \begin{pmatrix} 0 & -3 & 0 \\ -3 & -6 & 0 \\ 0 & 0 & \frac{4}{3} \end{pmatrix}, \quad M, N = 1, 2, 3.$$

(G_{MN}) – minisuperspace metric on the target space \mathcal{M}

$$L = \frac{1}{2} \langle \dot{x}, \dot{x} \rangle + \frac{C_1}{2} e^{\langle \textcolor{red}{V}, x \rangle} + \frac{C_2}{2} e^{\langle \textcolor{red}{W}, x \rangle}.$$

V – time-like, W – spacelike vectors on \mathcal{M} (the basis is (e_1, e_2, e_3))

$$\langle V, V \rangle = 3 \left(k_1^2 - \frac{16}{9} \right), \quad \langle W, W \rangle = 3 \left(k_2^2 - \frac{16}{9} \right), \quad \langle V, \textcolor{red}{W} \rangle = 3 \left(k_1 k_2 - \frac{16}{9} \right).$$

LET $\langle V, \textcolor{red}{W} \rangle = 0 \Leftrightarrow k_1 k_2 = \frac{16}{9}, \quad k_1 = k, \quad k_2 = \frac{16}{9k}, \quad 0 < k < 4/3.$

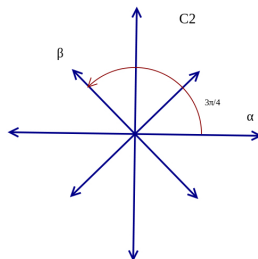
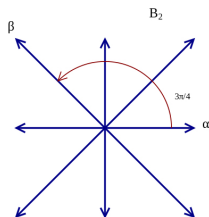
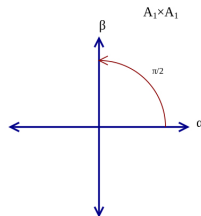
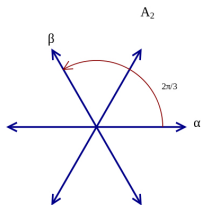
The new basis

$$e'_1 = \frac{V}{\|V\|}, \quad e'_2 = \frac{W}{\|W\|}, \quad \langle e'_i, e'_j \rangle = \eta_{ij}, \quad (\eta_{ij}) = \text{diag}(-1, 1, 1).$$

$$X^i = \eta_{ii} \langle e'_i, x \rangle, \quad x^i = \sum_{j=1}^3 S_j^i X^j, \quad e'_j = \sum_{i=1}^3 S_j^i e_i.$$

S_j^i – components of general Lorentz transformations.

Root systems



The $A_1 \times A_1$ -mechanical model

Let V and W vectors to root vectors of $su(2) \oplus su(2)$ Lie algebra

$$L = \frac{1}{2} \sum_{i,j=1}^3 \eta_{ij} \dot{X}^i \dot{X}^j + \frac{C_1}{2} e^{\eta_{11} |\langle V, V \rangle|^{1/2} X^1} + \frac{C_2}{2} e^{\eta_{22} |\langle W, W \rangle|^{1/2} X^2},$$

$$E_0 = \frac{1}{2} \sum_{i,j=1}^3 \eta_{ij} \dot{X}^i \dot{X}^j - \frac{C_1}{2} e^{\eta_{11} |\langle V, V \rangle|^{1/2} X^1} - \frac{C_2}{2} e^{\eta_{22} |\langle W, W \rangle|^{1/2} X^2}.$$

Liouville equations for $sl(2)$ -Toda chains ($sl(2) \cong su(2)$)

$$\begin{aligned} \ddot{X}^s &= -\sqrt{|\langle R_s, R_s \rangle|} C_s e^{\eta_{ss} |\langle R_s, R_s \rangle|^{1/2} X^s}, \quad s = 1, 2, \\ \ddot{X}^3 &= 0, \quad \text{with} \quad \langle R_1, R_1 \rangle = \langle V, V \rangle, \quad \langle R_2, R_2 \rangle = \langle W, W \rangle. \end{aligned}$$

Gavrilov, Ivashchuk, Melnikov'9407019

Lü, Pope, 9607027, 9604058

Lü, Yang, 1307.2305

The solution to the $A_1 \times A_1$ - mechanical model

The solution reads

$$\begin{aligned} X^1 &= |\langle V, V \rangle|^{-1/2} \ln(F_1^2(u - u_{01})), \\ X^2 &= -|\langle W, W \rangle|^{-1/2} \ln(F_2^2(u - u_{02})), \\ X^3 &= p^3 u + q^3, \end{aligned}$$

with

$$F_s(u - u_{0s}) = \begin{cases} \sqrt{|\frac{C_s}{2E_s}|} \sinh \left[\sqrt{\frac{|E_s \langle R_s, R_s \rangle|}{2}} (u - u_{0s}) \right], & \eta_{ss} C_s > 0, \eta_{ss} E_s > 0, \\ \sqrt{|\frac{C_s}{2E_s}|} \sin \left[\sqrt{\frac{|E_s \langle R_s, R_s \rangle|}{2}} (u - u_{0s}) \right], & \eta_{ss} C_s > 0, \eta_{ss} E_s < 0, \\ \sqrt{\frac{|\langle R_s, R_s \rangle C_s|}{2}} (u - u_{0s}), & \eta_{ss} C_s > 0, E_s = 0, \\ \sqrt{|\frac{C_s}{2E_s}|} \cosh \left[\sqrt{\frac{|E_s \langle R_s, R_s \rangle|}{2}} (u - u_{0s}) \right], & \eta_{ss} C_s < 0, \eta_{ss} E_s > 0, \end{cases}$$

$u_{0s}, E_s, E_s, p^3, q^3$ are constants of integration.

$$S_1^i = \frac{V^i}{|\langle V, V \rangle|^{1/2}}, \quad S_2^i = \frac{W^i}{\langle W, W \rangle^{1/2}}, \quad \alpha^i = S_3^i p^3, \quad \beta^i = S_3^i q^3$$

The general solution

$$ds^2 = F_1^{\frac{8}{9k^2-16}} F_2^{\frac{9k^2}{2(16-9k^2)}} \left(-e^{2\alpha^1 u} dt^2 + e^{-\frac{2}{3}\alpha^1 u} d\vec{y}^2 \right) + F_1^{\frac{32}{9k^2-16}} F_2^{\frac{18k^2}{16-9k^2}} du^2$$

$$\phi = -\frac{9k}{9k^2-16} \log F_1 + \frac{9k}{9k^2-16} \log F_2$$

with F_1 and F_2 given by

$$F_1 = \sqrt{\left| \frac{C_1}{2E_1} \right|} \sinh(\mu_1 u), \quad \mu_1 = \sqrt{\left| \frac{3E_1}{2} \left(k^2 - \frac{16}{9} \right) \right|},$$

$$F_2 = \sqrt{\left| \frac{C_2}{2E_2} \right|} \sinh(\mu_2 (u - u_0)), \quad \mu_2 = \sqrt{\left| \frac{3E_2}{2} \left(\left(\frac{16}{9} \right)^2 \frac{1}{k^2} - \frac{16}{9} \right) \right|},$$

where $0 < k < 4/3$ and u is positive and $u > u_0$.

Moreover, one has the constraint

$$E_1 + E_2 + \frac{2}{3}(\alpha^1)^2 = 0, \quad E_1 < 0, \quad E_2 > 0.$$

Constraints

$$E_1 + E_2 + \frac{2(\alpha^1)^2}{3} = 0.$$

- 1 $\alpha^1 = 0$ 4d Poincaré invariant solutions, $|E_1| = |E_2|$
- 2 $\alpha^1 \neq 0$ **no** Poincaré invariance $|E_1| \neq |E_2|$, candidates for **black hole**
 - Conditions from the $V(\phi)$: $C_1 < 0, C_2 > 0, 0 < k < 4/3$.
 - Constants of integration $u_0 > 0$

$$u > u_0$$

$$u_{01} = 0$$

- Possible horizon: $u \rightarrow +\infty$.

Black hole solution

■ $u = +\infty$ is the horizon

■ $\mu_2 = \mu_1 = -\frac{4}{3}\alpha^1$

$$ds^2 = \mathcal{C} \mathcal{X} \left(-e^{-2\mu u} dt^2 + d\vec{y}^2 \right) + \mathcal{C}^4 \mathcal{X}^4 e^{-2\mu u} du^2,$$

$$\mathcal{X} = (1 - e^{-2\mu u})^{-\frac{8}{16-9k^2}} (1 - e^{-2\mu(u-u_0)})^{\frac{9k^2}{2(16-9k^2)}},$$

$$\mathcal{C} \equiv 2^{\frac{16}{(16-9k^2)}} (3\mu)^{\frac{1}{2}} |C_1|^{\frac{8}{2(9k^2-16)}} \left(\frac{C_2}{k} e^{-2\mu u_0} \right)^{\frac{9k^2}{4(16-9k^2)}} (16-9k^2)^{-\frac{1}{4}}.$$

$$\phi = \frac{9k}{9k^2-16} \log \left[\sqrt{\left| \frac{E_1 C_2}{E_2 C_1} \right|} \frac{\sinh(\mu(u-u_0))}{\sinh(\mu u)} \right], \quad \lim_{u \rightarrow +\infty} \phi \rightarrow const$$

Hawking temperature:

$$T = \frac{2}{3\pi} \frac{|\alpha^1|}{\mathcal{C}^{3/2}} = \frac{1}{2\pi} \frac{\mu}{\mathcal{C}^{3/2}}.$$

Null geodesics imply

$$ds^2 = 0,$$

$$t - t_0 = \int_{u_0}^u d\bar{u} \mathcal{C}^{3/2} \left(1 + \dots\right) \xrightarrow[u \rightarrow \infty]{} \infty.$$

Both the scalar curvature and Kretschmann scalar tend to zero with $\mu_1 = \mu_2$ and $u \rightarrow +\infty$.

Near $u_0 = 0$, the solutions turns to have the asymptotics as the Chamblin-Reall solution governed by the single exponential potential

$$ds^2 \sim z^{\frac{8}{9k^2-4}} \left(-dt^2 + d\bar{y}^2 + dz^2\right), \quad z \sim \frac{16-9k^2}{9k^2-4} u^{\frac{4-9k^2}{16-9k^2}}.$$

with the dilaton

$$\lim_{\phi_{u \rightarrow \epsilon}} = -\frac{9k}{16-9k^2} \log \left[\frac{4}{3k} \sqrt{\frac{C_2}{|C_1|}} \frac{\sinh(-\mu u_0)}{\mu \epsilon} \right].$$

5d AdS-Schwarzschild solution, $u_0 = 0$

$$ds^2 = \mathcal{C} (1 - e^{-2\mu u})^{-\frac{1}{2}} (-e^{-2\mu u} dt^2 + d\vec{y}^2) + \mathcal{C}^4 (1 - e^{-2\mu u})^{-2} e^{-2\mu u} du^2,$$

$$\mu = -\frac{4}{3}\alpha^1, \quad \mathcal{C} = (2\sqrt{2})^{1/2} \left(\frac{C_1}{E_1} \right)^{\frac{4}{9k^2-16}} \left(\frac{C_2}{E_2} \right)^{\frac{9k^2}{4(16-9k^2)}}.$$

The dilaton

$$\phi = \frac{9k}{2(16-9k^2)} \log \left| \frac{C_1 E_2}{C_2 E_1} \right|.$$

The curvature

$$R = -\frac{5\mu^2}{\mathcal{C}^4}.$$

$$ds^2 = \frac{1}{z^2} \left(-f(z) dt^2 + d\vec{y}^2 + \frac{dz^2}{f(z)} \right),$$

$$z = z_h (1 - e^{-2\mu u})^{\frac{1}{4}}, \quad \mathcal{C} = z_h^{-2}, \quad f = 1 - \left(\frac{z}{z_h} \right)^4.$$

Free energy through black hole thermodynamics

$$d\mathcal{F} = -s dT, \quad \text{with the black brane entropy density} \quad s = \frac{V_3}{4} \mathcal{C}^{\frac{3}{2}}.$$

$$sT = \frac{V_3}{2\pi} \mu, \quad \mathcal{F} = - \int s dT = - \frac{V_3}{2\pi} \int_0^\mu \frac{\mu'}{T} \frac{dT}{d\mu'} d\mu'.$$

The temperature

$$T = \frac{2}{3\pi Q^{3/2}} \left| \frac{3}{4} \mu \right|^{1/4} e^{\frac{27k^2}{4(16-9k^2)} u_0} \mu,$$

$$T = \frac{\sqrt{2}}{3^{3/4} \pi Q^{3/2}} \mu^{1/4} e^{-\frac{27k^2}{4(16-9k^2)} \operatorname{arcsinh}(\frac{\mu}{\Lambda})}, \quad \text{with} \quad \Lambda = \frac{\mu}{\sinh(-\mu u_0)}.$$

$$\mathcal{F} = -\frac{V_3}{8\pi} \left(\mu - \frac{27k^2}{16-9k^2} (\sqrt{\Lambda^2 + \mu^2} - \Lambda) \right),$$

AdS case: $u_0 \rightarrow 0$, $\Lambda \rightarrow 0$ the free energy $\mathcal{F} = -\frac{V_3}{8\pi} \mu$.

Free energy through the holography

- F.E. of the black hole = the renormalized on-shell action $I_{bulk}^{ren} + I_{GH}^{ren}$
- It's convenient to come to the so-called domain wall coordinates

$$ds^2 = e^{2\mathcal{A}} \left(-f(w)dt^2 + d\vec{x}^2 \right) + \frac{dw^2}{f(w)}.$$

For our black hole solution we have

$$\mathcal{A} = \frac{1}{2} \log \mathcal{C} + \frac{1}{2} \log \mathcal{X}(w),$$

with the coordinate transformation

$$dw = \mathcal{C}^2 \mathcal{X}(u)^2 e^{\frac{8}{3}\alpha^1 u} du$$

$$\mathcal{F} \sim -(I_{bulk}^{ren} + I_{GH}^{ren})$$

For the bulk Lagrangian and the corresponding action we have

- $\sqrt{g} \left(R - \frac{4}{3}(\partial\phi)^2 - V(\phi) \right) = \frac{2}{3}e^{4A}V$, since $R = \frac{5}{3} + \frac{4}{3}(\partial\phi)^2$.
- $V = -(12\mathcal{A}'^2 + 3\mathcal{A}'')f - 3\mathcal{A}'f$
- $\mathcal{L}_{bulk} = -2\frac{d}{dw} \left(e^{4A}\mathcal{A}'f \right)$
- The regularized Einstein action reads $I_E^\epsilon = 2V_3\beta e^{4A(\epsilon)}\mathcal{A}'(\epsilon)f(\epsilon)$

For the GH term we have

- The extrinsic curvature reads $K = \frac{1}{2}h^{ab}n^w\partial_w h_{ab} = \frac{\sqrt{f}}{2} \left(8\mathcal{A}' + \frac{f'}{f} \right)$,
where $w = \epsilon$, $n^w = \sqrt{f}$, $n^i = 0$.
- $I_{GH}^\epsilon = V_3\beta e^{4A(\epsilon)}(8\mathcal{A}'(\epsilon)f(\epsilon) + f'(\epsilon))$,

$$\frac{I_{reg}}{\beta V_3} = -e^{4A}(6\mathcal{A}'f + f')|_{w=\epsilon}.$$

$$\frac{I_{reg}^{on-shell}}{\beta V_3} = - \left(6\mathcal{A}'(u) + \frac{f'(u)}{f(u)} \right) \Big|_{u=\epsilon}, \quad \text{with} \quad dw = e^{4A}f du.$$

Free energy through the holographic on-shell action

The expansion of \mathcal{A} near $u \sim 0$ $\mathcal{A} \sim -\frac{4}{16-9k^2} \log u + \mathcal{A}_0 + \mathcal{A}_1 u + \dots$,
with

$$\mathcal{A}_0 = \frac{1}{2} \log \mathcal{C} - \frac{4}{16-9k^2} \log(2\mu) + \frac{9k^2}{4(16-9k^2)} \log(1 - e^{2\mu u_0}),$$

$$\mathcal{A}_1 = \frac{4\mu}{16-9k^2} + \frac{9k^2}{2(16-9k^2)} \frac{\mu}{e^{-2\mu u_0} - 1}.$$

$$\frac{I_{reg}}{\beta V_3} = \frac{1}{16-9k^2} \left(\frac{24}{\epsilon} + \mu \left(8 - 18k^2 - \frac{27k^2}{e^{-2\mu u_0} - 1} \right) \right).$$

The counterterm ([Papadimitriou, JHEP 08\(2011\) 119](#))

$$I_{ct} = -\frac{8\gamma}{3} \int d^4x \sqrt{h} e^{k\phi}.$$

The asymptotics of ϕ is given by $\phi \sim \frac{9k}{16-9k^2} \log u + \phi_0 + \phi_1 u + \dots$

$$\begin{aligned}\phi_0 &= -\frac{9k}{16-9k^2} \log \left(\frac{4}{3k} \sqrt{\frac{C_2}{C_1}} \frac{\sinh(-\mu u_0)}{\mu} \right), \\ \phi_1 &= -\frac{9k}{16-9k^2} \mu \coth(-\mu u_0).\end{aligned}$$

$$\mathcal{L}_{ct} = -\frac{24}{16-9k^2} \left(\frac{1}{\epsilon} + 4\mathcal{A}_1 + k\phi_1 \right) (1 - \mu\epsilon) = -\frac{24}{16-9k^2} \frac{1}{\epsilon} + o(\epsilon).$$

The renormalized action is then

$$\frac{I_{ren}}{\beta V_3} = \frac{I_{reg} + I_{ct}}{\beta V_3} = \frac{1}{2} \left(\mu - \frac{27k^2}{16-9k^2} \sqrt{\Lambda^2 + \mu^2} \right), \quad \Lambda = \frac{\mu}{\sinh(-\mu u_0)}.$$

$$\mathcal{F} \sim -\frac{1}{2} \left(\mu - \frac{27k^2}{16-9k^2} (\sqrt{\Lambda^2 + \mu^2} - \Lambda) \right).$$

Free energy

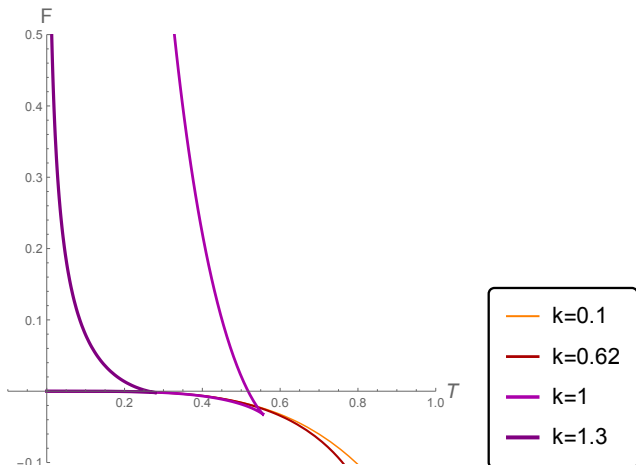


Figure: The dependence of the free energy F on the temperature T for the different shapes of the potential (different k , $C_1 = -2$, $C_2 = 2$).

The holographic Wilson loops

The expectation value of the holographic WL can be defined through the Nambu-Goto action \mathcal{S}_{NG}

$$\langle W(\mathcal{C}) \rangle \sim e^{-\mathcal{S}_{NG}}$$

Maldacena'98

The expectation value of the WL of size $T \times \ell$ is related with $q\bar{q}$ -potential

$$\langle W \rangle \sim e^{-V_{q\bar{q}}(\ell)T}$$

The potential of the quark antiquark interaction as

$$V_{q\bar{q}} = \frac{1}{T} \mathcal{S}_{NG}$$

The Nambu-Goto action is defined as

$$\mathcal{S}_{NG} = -\frac{1}{2\pi\alpha'} \int d^2\sigma \sqrt{-\det h}, \quad h_{\alpha\beta} = e^{\frac{4}{3}\phi} G_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu,$$

$G_{\mu\nu}$ is the background metric, the world-sheet coordinates σ^α , $\alpha = 0, 1$, and the embedding functions $X^\mu = X^\mu(\sigma^\alpha)$

The holographic Wilson loops

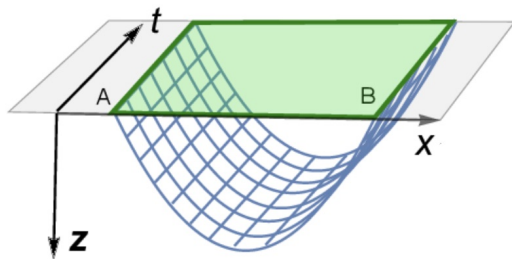


Figure: The time-like rectangular Wilson loop as a minimal surface

Holographic Wilson loops

AG&Vu Nguyen'19 TMPh

We choose the following gauge

$$\sigma^0 = t, \quad \sigma^1 = x_1, \quad u = u(x_1)$$

The Nambu-Goto action in the string frame

$$\frac{\ell}{2} = \int du \frac{ce^{3A}}{\sqrt{e^{4A+\frac{8}{3}\phi} - c^2}}$$

and for the Nambu-Goto action we have the following relation

$$\frac{\mathcal{S}_{NG}}{2} = \frac{T}{2\pi\alpha'} \int du \frac{e^{7A+\frac{8}{3}\phi}}{\sqrt{e^{4A+\frac{8}{3}\phi} - c^2}}.$$

Let us define the so-called effective potential with $u' = 0$ as

$$V_{eff} = e^{2A+\frac{4}{3}\phi} = F_1^{\frac{4(2-3k)}{9k^2-16}} F_2^{\frac{3k(3k-8)}{2(16-9k^2)}}.$$

Holographic WL for $T \neq 0$

The effective potential

$$V_{eff} = \mathcal{C} e^{-\mu u} \left(\frac{4e^{-\mu u_0}}{3k} \sqrt{\frac{C_2}{|C_1|}} \right)^{\frac{12k}{9k^2-16}} (1-e^{-2\mu(u-u_0)})^{\frac{3k(8-3k)}{2(9k^2-16)}} (1-e^{-2\mu u})^{\frac{4(2-3k)}{9k^2-16}}.$$

The distance between quarks and the Nambu-Goto action can be represented in terms of V_{eff} as

$$\frac{\ell}{2} = \int_0^{u_*} du \frac{e^{-2\phi} e^{\frac{\mu}{2}u} V_{eff} \sqrt{V_{eff}}}{\sqrt{\frac{V_{eff}^2(u)}{V_{eff}^2(u_*)} - 1}}$$

and

$$\mathcal{S}_{NG} = \frac{T}{\pi\alpha'} \int_0^{u_*} du \frac{e^{-2\phi} e^{\frac{\mu}{2}u} V_{eff}^3 \sqrt{V_{eff}}}{\sqrt{V_{eff}^2(u) - V_{eff}^2(u_*)}},$$

correspondingly.

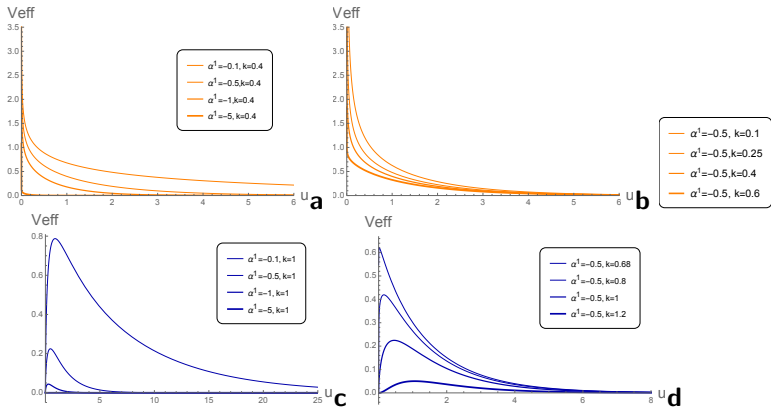


Figure: V_{eff} as a function of u for the holographic RG flows at finite temperature: **a),c)** we fix k varying $\alpha^1 = -\frac{3}{4}\mu$, **b),d)** we fix $\alpha^1 = -\frac{3}{4}\mu$ varying k .

Thank you for attention!