Einstein-scalar-Gauss-Bonnet Theories: Black Holes, Wormholes and Particle-like Solutions

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Outline

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Introduction

Einstein's General Theory of Relativity based on his field equations

$$\boxed{R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}}$$

may determine, upon choosing the form of the energy-momentum tensor $T_{\mu\nu}$, the form of spacetime around:

- an ordinary massive body such as the Sun
- a black hole
- a wormhole

GR was experimentally verified at Solar System scales but black holes and wormholes were considered as merely mathematical curiosities

Black Holes in General Relativity

General Relativity admits three families of Black-Hole solutions:

Schwarzschild (1916) | Reissner-Nordstrom (1921)

Kerr(-Newman) (1963)

According to the "no-hair" theorems of GR (Birkhoff; Israel; Carter; Price; Hartle; Teitelboim; Bekenstein), a BH may be characterized only by its mass M, electromagnetic charge Q and angular-momentum J.

A BH has no colour, baryon and lepton number, or scalar charges...

That's very restrictive... But, Einstein's theory of General Relativity, although a beautiful theory, is not considered as the final theory of Gravity...

Generalised Theories of Gravity

A generalised theory of gravity could have the form

$$S = \int d^4x \sqrt{-g} \left[f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma}, \Phi_i) + \mathcal{L}_X(\Phi_i) \right]$$

Such a theory arises in the context of: string effective theory at low energies, Lovelock effective theory in four dimensions, scalar-tensor (Horndeski, Galileon or DHOST) theories...

Are there, then, many novel black-hole solutions beyond the limits of GR? Are the old GR solutions not valid any more?

Today, we know that black holes are legitimate astrophysical objects (of the Kerr family) that populate our universe and may give valuable information about the strong(er)-gravity regime and thus for the validity of any modified gravity

Scalar No-Hair Theorems

- The old 'No-Hair Theorem' (Bekenstein, 1972; Teitelboim, 1972) excluded novel BH solutions in minimally-coupled scalar-tensor theories:

  "There are no static black-hole solutions with scalar hair"

- This was evaded for black-hole solutions with a Skyrme field (Luckock & Moss, 1986; Droz et al, 1994) or conformally-coupled scalar field (Bekenstein, 1974)
- The "novel No-Hair Theorem" (Bekenstein, 1995) applied in non-minimally-coupled scalar fields, and was extended to general scalar-tensor theories (Sotiriou & Faraoni, 2012; Hui & Nicolis, 2013)
- These were again evaded in the case of dilatonic BHs (Kanti et al, 1996), coloured BHs (Torii et al, 1997; Kanti et al, 1997), rotating BHs (Kleihaus et al, 2011; Pani et al, 2011) and the shift-symmetric Galileon BHs (Babichev & Charmousis, 2014; Sotiriou & Zhou, 2014)

The Einstein-Scalar-Gauss-Bonnet Theory

We considered the following generalised theory of gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi + f(\phi) \, R^2_{GB} \right],$$

with $f(\phi)$ a coupling function between a scalar field ϕ and the GB term

$$R^2_{GB} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$$

This theory has a number of attractive points:

- It contains a quadratic gravitational term (next important term in strong-curvature regimes) but leads to field equations with up to 2nd-order derivatives, and with no Ostrogradski instabilities
- It is a very "rich" theory: it leads to cosmological singularity-free solutions (Antoniadis, Rizos & Tamvakis, 1994; P.K., Rizos and Tamvakis, 1998), wormholes (Kanti, Kleihaus & Kunz, 2011) and BHs with scalar hair: for $f(\phi) \sim e^\phi$, we get the Dilatonic BHs (P.K., Mavromatos et al, 1996; 1998) and for $f(\phi) \sim \phi$, the shift-symmetric static black holes (Sotiriou & Zhou, 2014)

Novel Einstein-Scalar-GB Black-Hole Solutions

• The Basic Question: For what forms of the coupling function $f(\phi)$ can one get a static, spherically-symmetric black-hole solution?

Keeping therefore the form of $f(\phi)$ arbitrary, we assume the line-element

$$ds^2 = -e^{A(r)}dt^2 + e^{B(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

while the equations of motion read

$$\nabla^2\phi + \dot{f}(\phi)R^2_{GB} = 0\,, \qquad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu}$$

where

$$T_{\mu\nu} = -\frac{1}{4}g_{\mu\nu}(\partial\phi)^2 + \frac{1}{2}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}(g_{\rho\mu}g_{\lambda\nu} + g_{\lambda\mu}g_{\rho\nu})\eta^{\kappa\lambda\alpha\beta}\tilde{R}^{\rho\gamma}{}_{\alpha\beta}\nabla_\gamma\partial_\kappa f\,,$$

$$\Downarrow$$

$$\boxed{A'' = \frac{P}{S}\,, \qquad \phi'' = \frac{Q}{S}\,, \qquad P, Q, S = g(r, \phi, \phi', A')}$$

Novel Einstein-Scalar-GB Black-Hole Solutions

For the existence of a regular black-hole horizon, we demand that

$$e^{A(r)} \to 0, \quad e^{-B(r)} \to 0, \quad \phi(r) \to \phi_h$$

Demanding that ϕ'' is also finite at the horizon r_h, we find the constraint

$$\phi'_h = \frac{r_h}{4\dot{f}_h}\left(-1 \pm \sqrt{1 - \frac{96\dot{f}_h^2}{r_h^4}}\right), \qquad \dot{f}_h^2 < \frac{r_h^4}{96}$$

Using the above, the field equations give the complete solution near the horizon

$$e^A = a_1(r - r_h) + \dots, \qquad e^{-B} = b_1(r - r_h) + \dots,$$

$$\phi = \phi_h + \phi'_h(r - r_h) + \phi''_h(r - r_h)^2 + \dots$$

Asymptotically-flat solutions

At large distances from the horizon, we assume a power series expansion in $1/r$, and by substituting in the equations of motion, we find

$$e^A = 1 - \frac{2M}{r} + \frac{MD^2}{12r^3} + \frac{24MD\dot{f} + M^2D^2}{6r^4} + \dots$$

$$e^B = 1 + \frac{2M}{r} + \frac{16M^2 - D^2}{4r^2} + \frac{32M^3 - 5MD^2}{4r^3} + \mathcal{O}\left(\frac{1}{r^4}\right) + \dots$$

$$\phi = \phi_\infty + \frac{D}{r} + \frac{MD}{r^2} + \frac{32M^2D - D^3}{24r^3} + \frac{12M^3D - 24M^2\dot{f} - MD^3}{6r^4} + \dots$$

It is in order $\mathcal{O}(1/r^4)$ that the explicit form of the coupling function $f(\phi)$ first makes its appearance

Thus, a general coupling function f does not interfere with the existence of an asymptotically-flat limit for the spacetime

Asymptotically-flat solutions

Can we smoothly connect these two asymptotic solutions? Bekenstein's *Novel No-Hair* theorem (1995) said no, because:

- "at radial infinity: T_r^r is positive and decreasing"

Indeed, even in the presence of the GB term: $T_r^r \simeq \phi'^2/4 \simeq D^2/4r^4 + \ldots$

- "near the BH horizon: T_r^r is negative and increasing"

If true, the smooth connection of the two demands an extremum - this is *excluded* by the positivity of energy in ordinary scalar-tensor theories

However, in the Einstein-scalar-Gauss-Bonnet theories with general f, the second clause is not true. Instead, we find that

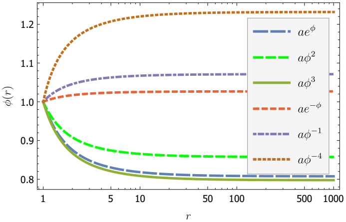
$$sign(T_r^r)_h = -sign(\dot{f}_h \phi'_h) = sign(1 \mp \sqrt{1 - 96\dot{f}^2/r_h^4}) > 0$$

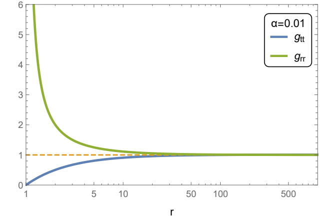
The regularity of the horizon automatically guarantees the positivity of T_r^r

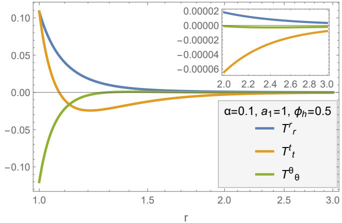
Asymptotically-flat solutions

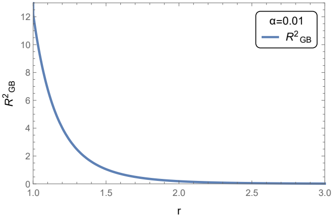
Choosing $f(\phi)$ and then (ϕ_h, ϕ'_h), we found numerous BH solutions:

(Antoniou, Bakopoulos & P.K., PRL 2018, PRD 2018)

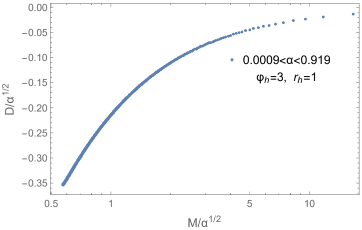


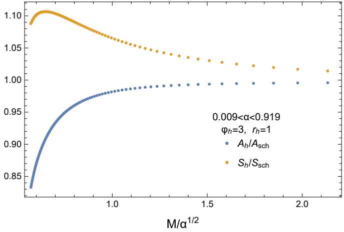






Asymptotically-flat solutions





- The scalar charge D is a "secondary" conserved quantity
- In the limit of large mass, all GB black holes reduce to the Schwarzschild solution
- The entropy of the GB black holes may exceed that of the Schwarzschild solution (shown that of $f(\phi) \sim 1/\phi$)
- All GB black holes are smaller than the corresponding Schwarzschild solution and have a minimum mass

Novel Einstein-Scalar-GB Black-Hole Solutions

There is a huge literature on different types of solutions with a non-trivial scalar field such as black holes and neutron stars...

Spontaneous Scalarisation: a GR solution (with $\phi = const.$) in a scalar-tensor theory of gravity is destabilised, and a novel solution with a non-trivial scalar field emerges (Damour & Esposito-Farese, 1993) if

$$\left.\frac{df(\phi)}{d\phi}\right|_{\phi=\phi_0} = 0\,, \qquad \frac{d^2 f(\phi)}{d\phi^2} > 0 \qquad (\Rightarrow m^2_{eff} < 0 \text{ for } \delta\phi)$$

These hold for $f \sim 1 - e^{-\phi^2}$ (Doneva & Yazadjiev, 2018), $f = a\phi^2$ (Silva et al, 2018) and $f \sim \phi^2 + a\phi^4$ (Minamitsuji & Ikeda, 2018) when $\phi_0 = 0$

For other coupling functions such as $f = \lambda e^{\phi}$ or $f = a\phi$, the above conditions do not hold – for these, the process of **natural scalarisation** takes place: solutions with non-trivial scalar field emerge having the Schwarzschild solution as their large mass limit

The Einstein-Scalar-Gauss-Bonnet Theory with Λ

We extend the previous theory by adding a cosmological constant

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} - \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi + f(\phi) \, R^2_{GB} - \Lambda \right],$$

In this case, the field equations remain unchanged apart from the shift

$$T_{\mu\nu} \to T_{\mu\nu} - \Lambda \, g_{\mu\nu}$$

Despite the minimal change, the situation differs from the previous one:

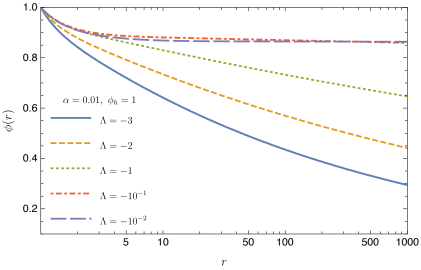
- The spacetime is not asymptotically-flat but is expected to reduce to a (Anti)-de Sitter background
- A regular black-hole horizon emerges provided that

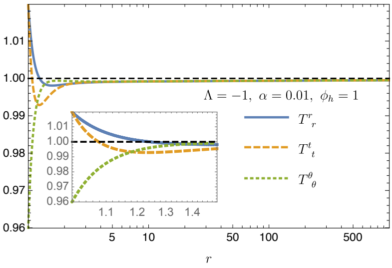
$$\phi'_h = \frac{16\Lambda r_h \dot{f}^2 \, (\Lambda r_h^2 - 3) + \Lambda r_h^5 - r_h^3 \mp \sqrt{R}}{4\dot{f}[r_h^2 - \Lambda(r_h^4 - \dot{f}^2)]}$$

Asymptotically Anti-de Sitter solutions

For $\Lambda < 0$, we find complete BH, asymptotically Anti-de Sitter solutions:

(Bakopoulos, Antoniou & P.K., PRD 2019)





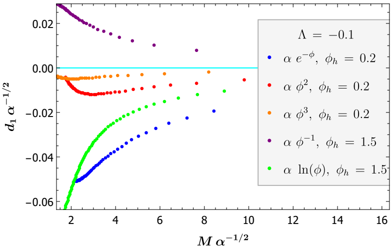
We found similar solutions for $f(\phi) = e^{\pm\phi}, \phi^{\pm 2n}, \phi^{\pm(2n+1)}, \ln\phi, \ldots$

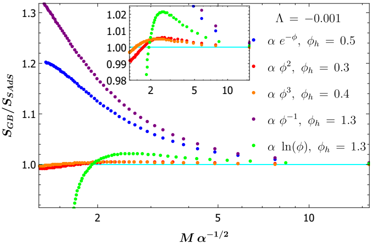
BH solutions with an Anti-de Sitter asymptotic behaviour emerge as easily as the ones with Minkowski asymptotic behaviour

Asymptotically Anti-de Sitter solutions

At large distances, the scalar field behaves as:

$$\phi(r) = \phi_\infty + d_1 \ln r + \frac{d_2}{r^2} + \ldots$$

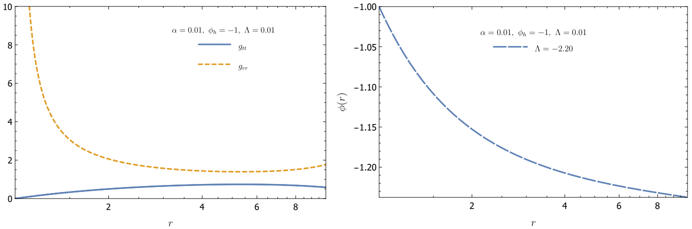




As M increases, d_1 tends to zero and the entropy S_h adopts the Schwarzschild value $A_h/4$

Asymptotically de Sitter solutions

For $\Lambda > 0$, though, no complete BH, asymptotic de Sitter solution was found...



However, employing the alternative metric ansatz

$$ds^2 = -e^{-2\delta(r)}N(r)\,dt^2 + N^{-1}(r)\,dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2),$$

solutions with an asymptotic de Sitter behaviour were found (Brihaye, Herdeiro and Radu, 2019; Bakopoulos & PK, work in progress, see also Charmousis, Crisostomi, Gregory and Stergioulas, 2019)

Pure scalar-GB solutions?

In the asymptotically-flat case, the quadratic GB term is negligible at large distances but is very important near the horizon. Is there a class of BH solutions that may be attributed almost entirely to the scalar-GB combination?

If we ignore the Ricci term in the field eqns and assume that $e^A \to 0$, as $r \to r_*$, we get a family of solutions with a spacetime singularity but a finite energy-momentum tensor

If, on the other hand, we assume that $e^B \to \infty$, as $r \to r_*$, we get

$$e^{-B} = 2\ln(r/r_*)\,, \qquad \phi \to \phi_*\,, \qquad e^A \to const.$$

This family of solutions has no spacetime singularities, a regular field and a finite energy-momentum tensor

In both cases, no hairy black-hole solutions emerge – their existence relies on the synergy between the Ricci and GB terms

Pure scalar-GB solutions?

The Ricci term stands for ordinary gravity and provides the attractive force that creates the positive curvature around the black hole

The scalar-GB combination, however, creates a repulsive effect since

$$p_r = -\rho = \frac{2b_1}{r_h^2}\,|(\phi' \dot{f})_h| > 0$$

All GB black holes have a mimimum horizon radius defined by

$$|\dot{f}_h| \leq r_{h,min}^2/4\sqrt{6}$$

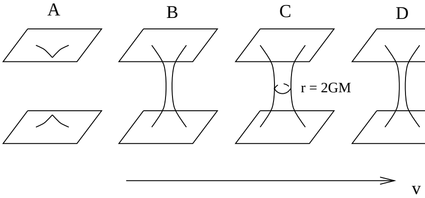
Solutions with $r_h < r_{h,min}$ would have a larger curvature of spacetime, a stronger GB term and a repulsive force large enough to destroy the horizon

The phase space of solutions of the pure scalar-GB theory contain interesting cosmological solutions (inflationary or singularity-free)
– perhaps particle-like ones or wormholes?

(Kanti, Gannouji & Dadhich, 2015; Antoniadis, Rizos & Tamvakis, 1994)

Wormholes in General Relativity

Wormholes are well motivated objects since, in GR, they hide inside the horizon of a black hole:



The region inside the horizon of a Schwarzschild BH, $r < r_h = 2M$, is dynamical and a throat appears in place of the singularity as time goes by

But the throat closes so quickly that not even a light signal can pass through (Einstein-Rosen, 1935; Wheeler, 1955)

The Reissner-Nordstrom and Kerr geometries have generically internal tunnels - but the internal Cauchy horizons are unstable

Wormholes in General Relativity

Looking for a traversable wormhole, Morris & Thorne (1988) disconnected the wormhole from the black hole. Using an ansatz of the form

$$ds^2 = -e^{2\Phi(r)}\,dt^2 + \left(1 - \frac{b(r)}{r}\right)^{-1} dr^2 + r^2\,(d\theta^2 + \sin^2\theta\, d\varphi^2),$$

they demanded

- an asymptotically-flat regime: $\Phi \to 0$ and $b/r \to 0$, for $r \to \infty$
- the absence of a horizon or singularity: $\Phi(r)$ everywhere regular
- the presence of a throat at $r_{min} = b_0$ where $b(r_{min}) = b_0$

The above demand in turn an energy-momentum tensor

$$T_{tt} = \rho\,, \qquad T_{rr} = -\tau\,, \qquad T_{\theta\theta} = T_{\varphi\varphi} = p$$

satisfying $\tau \geq \rho \;\Rightarrow$ violation of energy conditions $\Rightarrow$ Exotic Matter

Wormholes in EsGB Theory

An additional family of solutions was found in the EdGB theory, i.e the EsGB theory with $f(\phi) = \alpha e^{\phi}$ (P.K., Mavromatos et al, 1996) where

$$e^A \simeq a_0 + ..., \quad e^{-B} = b_1(r - r_0) + ... , \quad \phi \simeq \phi_0 + \phi_1\sqrt{r - r_0} + ...$$

All components of $T_{\mu\nu}$ and scalar invariants were finite

Under a redefinition $l^2 = r^2 - r_0^2$, the line-element becomes:

(P.K., Kleihaus & Kunz, PRL 2011, PRD 2012)

$$ds^2 = -e^{A(l)}dt^2 + e^{B(l)}dl^2 + (l^2 + r_0^2)\,(d\theta^2 + \sin^2\theta d\varphi^2)$$

where now

$$e^A = a_0 + a_1 l + ... , \quad e^{B(r)} = b_0 + b_1 l + ...$$

$$\phi(l) = \phi_0 + \phi_1 l + ...$$

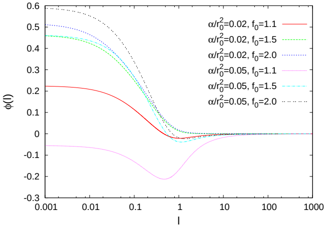
The above describes a wormhole with the throat at $r = r_0$ or $l = 0$

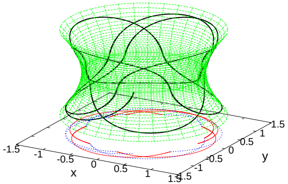
Wormholes in EsGB Theory

At large distances, we obtain

$$e^A \simeq 1 - \frac{2M}{l} + \ldots, \qquad e^B = 1 + \frac{2M}{l} + \ldots\,, \qquad \phi \simeq \phi_\infty + \frac{D}{l} + \ldots\,,$$

where M and D are the mass and scalar charge of the wormhole





Regular symmetric solutions arise if a perfect fluid (non-exotic!) and a gravitational source term are introduced at the throat

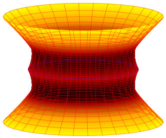
$$S = \int d^3x\sqrt{h}\,(\lambda_1 + \lambda_0 e^{\phi}\tilde{R})$$

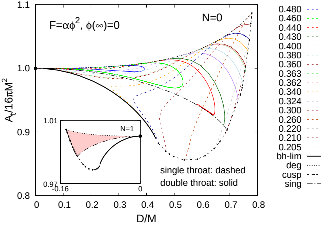
Wormholes in EsGB Theory

In the context of the EsGB theory with an arbitrary $f(\phi)$, we searched for wormholes by employing a new set of coordinates:

$$ds^2 = -e^{A(\eta)}dt^2 + e^{\Gamma(\eta)}\left[d\eta^2 + (\eta^2+\eta_0^2)\,(d\theta^2+\sin^2\theta d\varphi^2)\right]$$

We found regular wormhole solutions for every form of $f(\phi)$ with an asymptotically-flat behaviour and with a single or double throat





All EsGB WHs are bounded by the corresponding BHs and are free of any exotic matter

(Antoniou, Bakopoulos, P.K., Kleihaus, Kunz, 1904.13091 [hep-th])

Particle-like Solutions in EsGB Theory

We finally looked for solutions with a regular spacetime, with no singularities, no horizons and no throats

$$ds^2 = -e^{A(r)}dt^2 + e^{B(r)}\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right]$$

At large distances, the asymptotic behaviour is the same as for black holes and wormholes. At small distances, for e.g. $f(\phi) = \alpha\phi^2$, we find

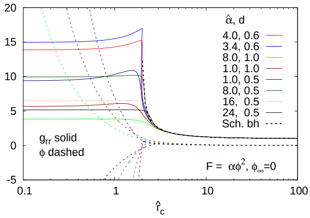
$$A(r) = A_0 + A_2 r^2 + A_3 r^3 + \dots\,, \quad \phi = -\frac{c_0}{r} + \phi_0 + \phi_1 r + \phi_2 r^2 + \phi_3 r^3 + \dots$$

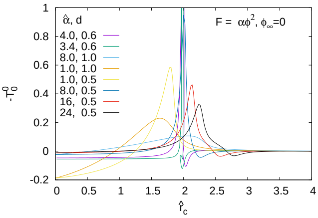
All scalar invariants are finite everywhere, as are also all components of $T_{\mu\nu}$ despite the singularity in ϕ: (Kleihaus, Kunz & P.K., 1910.02121)

$$\rho(0) = -\frac{3}{32\alpha}\,, \qquad p(0) = \frac{2}{32\alpha}$$

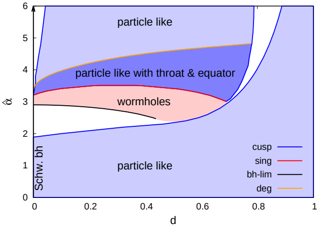
(in agreement with Brihaye, Hartmann and Urrestilla, 2019)

Particle-like Solutions in EsGB Theory





The energy-momentum tensor has a regular, shell-like behaviour and vanishes at a very small radius qualifying these solutions as ultra compact objects



Conclusions

- The Generalised Theories of Gravity may be the way forward in gravitational physics
- The Einstein-scalar-Gauss-Bonnet theory is a very promising type of a quadratic theory that admits a variety of solutions
- Regulal Black Holes with scalar hair, that have an asymptotically-flat or (Anti)-de Sitter behaviour, emerge for any form of the coupling function
- Wormholes with no need for exotic matter and particle-like solutions also emerge, again for every form of $f(\phi)$
- The phase space of the solutions of the pure scalar-GB theory needs to be further investigated, however, this does not seem to contain black holes

Bounds on the EsGB Theory

Up to now, GR has proven to be compatible with every observable and every process in our universe – modifications due to extra fields or terms should be small

Observations may impose bounds on the parameters of the generalised theory - here, we focus on the GB coupling α defined through the relation $f(\phi) = \alpha \tilde{f}(\phi)$ with $[\alpha] = L^2$

• **Bounds from Observed Black Holes:** The EsGB theory predicts a minimum mass for the BHs through the constraint (P.K., Mavromatos et al, 1996)

$$\dot{f}^2 \leq \frac{r_h^4}{96} \Rightarrow \sqrt{\alpha} \leq 1.2 \left(\frac{M}{M_\odot}\right) \times 10^5\,\mathrm{cm}$$

for $f(\phi) = \alpha e^{\phi}$. For $M \simeq 5 M_\odot$, then $\sqrt{\alpha} \leq 6.2 \times 10^5\,\mathrm{cm}$

Bounds on the EsGB Theory

• **Bounds from Solar System:** Using the Shapiro time delay and the uncertainty in the Mercury's orbit, the GB coupling α must obey the constraints (Amendola, Charmousis & Davis, 2007)

$$\sqrt{\alpha} \leq 1.3 \times 10^{12}\,\mathrm{cm} \qquad \mathrm{and} \qquad \sqrt{\alpha} \leq 1.9 \times 10^{13}\,\mathrm{cm}$$

• **Bounds from Quasi-Normal Modes:** Differ by the GR prediction by a few %, thus a bound may be set (Blazquez-Salcedo et al, 2016)

$$\sqrt{\alpha} \leq 10 \left(\frac{50}{\rho}\right)^{1/4} \left(\frac{M}{M_\odot}\right) \times 10^5\,\mathrm{cm}$$

For LVC detections, the signal-to-noise ratio is $\rho \simeq 10$, leading to $\sqrt{\alpha} \leq 7.5 \times 10^5\,\mathrm{cm}$. For the Einstein Telescope, $\rho \simeq 100$, leading to $\sqrt{\alpha} \leq 4.2 \times 10^5\,\mathrm{cm}$.

Bounds on the EsGB Theory

• **Bounds from the Speed of GWs:** The almost simultaneous detection of GWs and EM radiation from the BNS GW170817 (Abbott et al, 2017) imposed the constraint $|c_g/c - 1| \leq \times 10^{-16}$

In the case of localised gravitational solutions, we obtain the result (Kobayashi, Motohashi and Suyama, 2012)

$$c_g/c - 1 = \frac{8D\dot{f}(\phi_\infty)}{r^3}$$

which, even if $\dot{f}(\phi_\infty) \neq 0$ and $D \neq 0$, decays very fast.

• **Bounds from Scalar Dipole Radiation:** From the phase evolution of the gravitational waveform, due to the emission of scalar dipole radiation, as found in GW151226 and GW170608, it was found (Nair et al, 2019)

$$\sqrt{\alpha} \leq 5.4 \times 10^5 \,\mathrm{cm}$$

Bounds on the EsGB Theory

- **Bounds from Binary Orbital Decay:** From the observed period decay rate of the BH low-mass x-ray binary A0620-00, which should be increased by the emission of scalar dipole radiation, it is found (Yagi, 2012)

$$\sqrt{\alpha} \leq 1.9 \times 10^5\,\mathrm{cm}$$

Novel Einstein-Scalar-GB Black-Hole Solutions

- **Old No-Hair Theorem:** it uses the scalar equation

$$\int d^4x\,\sqrt{-g}\,f(\phi)\left[\nabla^2\phi + \dot{f}(\phi)\,R^2_{GB}\right] = 0$$

Integrating by parts, we obtain

$$\int d^4x\,\sqrt{-g}\,\dot{f}(\phi)\left[\partial_\mu\phi\,\partial^\mu\phi - f(\phi)\,R^2_{GB}\right] = 0$$

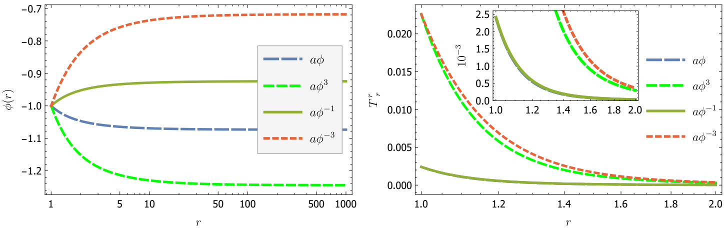
Since, $\partial_\mu\phi\,\partial^\mu\phi > 0$, the above holds only for $f(\phi)\,R^2_{GB} > 0$. Silva et al, with a slightly different manipulation, found instead that $\ddot{f}\,R^2_{GB} > 0$.

Should we combine both constraints? How *many constraints* are there? How can we impose this *integral* constraint before we determine the solutions (the novel no-hair theorem is a *local* one)?

If the old no-hair theorem is trustable, BH's with $f(\phi) < 0$ do not exist...

Novel Einstein-Scalar-GB Black-Hole Solutions

Surprise! They do....!



Solutions that *violate* the old no-hair theorem but *respect* the novel no-hair theorem do arise for several $f(\phi)$'s.

But the sign of $f(\phi)$ affects directly the entropy of the BH solution

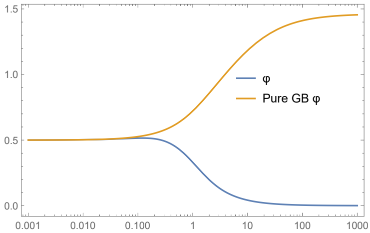
$$S = \frac{A_h}{4} + 4\pi f(\phi_h) > 0$$

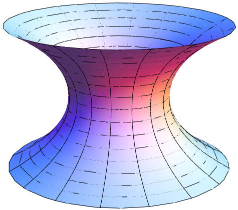
If, for the same M, BHs arise with both positive and negative $f(\phi)$, these will have different entropies with $S_+ > S_-$

Synergy between Ricci and GB terms

A repulsive force is also necessary for the throat of a wormhole to remain open. In addition, no limit exists that restricts the magnitude of the GB term

Then, as in the cosmological case, regular pure GB wormhole solutions do emerge that in fact do not violate the energy conditions!





(A. Bakopoulos, P.K. and N. Pappas, in preparation)