Echoes from Horizonless Objects

Kostas Kokkotas & **Sebastian Völkel**
Theoretical Astrophysics, University of Tübingen

Classes of Compact Objects

Conventional

- Black – Holes
- Neutron stars

Un-conventional or Exotic

- Constant Density Stars
  - & anisotropic stars

- **The simplest model of an ultra compact object is the incompressible fluid star.**
- The star is assumed to be of uniform density, which is an extreme assumption for the equation of state.
- Besides the nice property that the TOV equations can be solved analytically, these stars can be **as compact as the Buchdahl limit of M/R = 4/9**.
- This makes them a **suitable choice for analytic studies that represent an extremely stiff equation of state.**

$$m(r) = \begin{cases} \frac{4}{3}\pi\rho_0 r^3, & \text{for } r \leq R, \\ \frac{4}{3}\pi\rho_0 R^3, & \text{for } R < r, \end{cases}$$

$$P(r) = \rho_0 \frac{\left(1 - 2Mr^2/R^3\right)^{1/2} - \left(1 - 2M/R\right)^{1/2}}{3\left(1 - 2M/R\right)^{1/2} - \left(1 - 2Mr^2/R^3\right)^{1/2}}, \quad \text{for } r \leq R.$$

$$g_{00} = \frac{1}{4}\left[3\left(1 - \frac{2M}{R}\right)^{1/2} - \left(1 - \frac{2Mr^2}{R^3}\right)^{1/2}\right]^2,$$

$$g_{11} = \left(1 - \frac{2Mr^2}{R^3}\right)^{-1}.$$

Classes of Compact Objects

Conventional

- Black – Holes
- Neutron stars

Un-conventional or Exotic

- Constant Density Stars
- Wormholes

- Exotic objects, which are commonly studied not only in GR, but also in alternative theories of gravity.
- Here we consider the Damour & Solodukhin, which mimics the Schwarzschild exterior arbitrarily well.
- Besides the mass M, it has only one additional parameter λ , which scales the deviation to the Schwarzschild black hole.
- Even for small values of λ , there is no event horizon, but a throat connecting two isometric, asymptotically flat regions. It is located close to where the event horizon would have been.

$$\mathrm{d}s^2 = -\left(g(r)+\lambda^2\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{g(r)} + r^2\left(\mathrm{d}\theta^2 + \sin^2(\theta)\mathrm{d}\phi^2\right)$$

Classes of Compact Objects

Conventional

- Black – Holes
- Neutron stars

Un-conventional or Exotic

- Constant Density Stars
- Wormholes
- Gravastars
- …

- Gravitational vacuum condensate stars (**gravastars**) have been proposed as alternatives to BHs.
- Can be constructed to be as compact as Schwarzschild black holes and are therefore an **interesting candidate to mimic black holes.**
- The spherically symmetric model can be made up of different layers, but always features a de Sitter condensate, with $P = -\rho$, as core.
- The gravastar proposal features thermodynamic and quantum properties, which circumvent some of the problems related to classical BHs, such as the information loss problem.
- Depending on the chosen equation of state for the shell, stable solutions were reported.
- In order to circumvent the classical Buchdahl limit, gravastars have anisotropic pressure and are therefore quite different from normal stars

Classes of Compact Objects

Conventional

- Black – Holes
- Neutron stars

Un-conventional or Exotic

- Constant Density Stars
- Wormholes
- Gravastars
- Quantum gravity inspired models (fuzzballs, firewalls, **area quantization**, …)

- The BH horizon consists of quantized patches

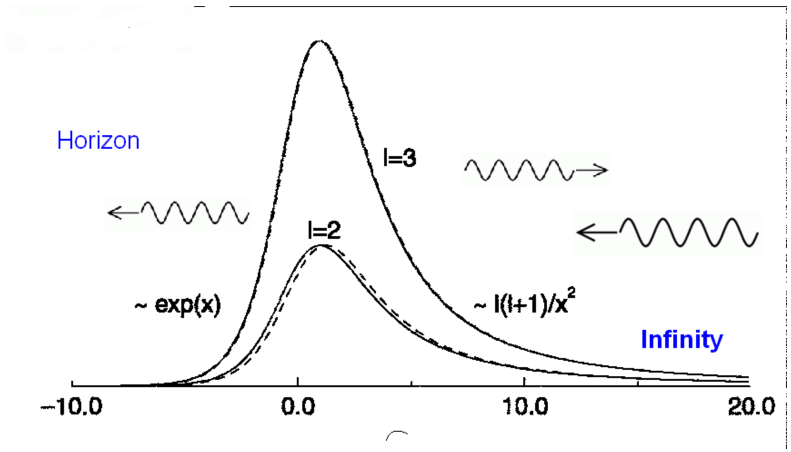
$$A_n = \alpha \ell_p^2 n = 4\pi \left(2M\right)^2$$

- By inverting, one finds a discrete spectrum for possible BH masses.
- Thus one should expect that not every incoming particle can be absorbed, but rather gives some non-zero reflectivity.
- This can be connected to possible GW effects (*)

(*)
- Foit-Kleban "*Testing quantum BHs with GWs*" CQG 2019
- Cardoso, Foit, Kleban "*GW echoes from BH area quantization*" JHEP 2019
- Coates, Völkel, Kokkotas "*Spectral Lines of Quantized, Spinning BHs and their Astrophysical Relevance*" PRL 2019

QNMs of Black Holes

$$\frac{d^2}{dr^{*2}}\Psi(r) + \left(\omega_n^2 - V(r)\right)\Psi(r) = 0, \qquad V(r) = \left(1 - \frac{2M}{r}\right)\left[\frac{l(l+1)}{r^2} - \frac{6M}{r^3}\right]$$ Black-Holes



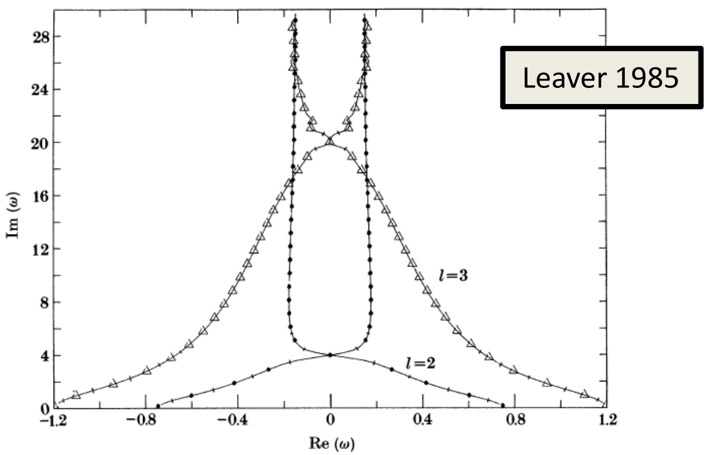


FIGURE 1. First 60 Schwarzschild quasi-normal frequencies for $l = 2$ and $l = 3$. The odd-order frequencies are prominently marked; a few-even order frequencies are indicated as short bars perpendicular to the curves connecting the points.

WKB method

$$\boxed{\omega^2 = \left[V_0 - i(-2V_0'')^{1/2}\Lambda(n)\right] - i\left(n+\frac{1}{2}\right)(-2V_0'')^{1/2}\left[1+\alpha^{-1}\Omega(n)\right]}$$

Schutz, Will 1984
Iyer, Will 1985,86
KK, Schutz 1988

$$\begin{aligned}\Omega(n) &= \frac{\alpha}{2Q_0''}\left[\frac{5}{6912}\left(\frac{Q_0^{(3)}}{Q_0''}\right)^4(77+188\alpha^2) - \frac{1}{384}\left(\frac{Q_0^{(3)2}Q_0^{(4)}}{Q_0''^3}\right)(51+100\alpha^2)\right.\\ &+ \frac{1}{2304}\left(\frac{Q_0^{(4)}}{Q_0''}\right)^2(67+68\alpha^2) + \frac{1}{288}\left(\frac{Q_0^{(3)}Q_0^{(5)}}{Q_0''^2}\right)(19+28\alpha^2)\\ &- \frac{1}{288}\left(\frac{Q_0^{(6)}}{Q_0''}\right)(5+4\alpha^2) \quad \text{where} \quad \alpha = n+\frac{1}{2} \qquad (112)\end{aligned}$$

Bohr-Sommerfeld Formula for BH QNMs

KK 1992
Dunham 1932

The classical form of BS rule is:

$$\int_{x_A}^{x_B} Q(x)^{1/2} dx = \left(n + \frac{1}{2}\right)\pi \tag{113}$$

where x_A, x_B are the roots of $Q(x) = 0$ (turning points).

If one assumes:

$$Q \equiv \omega^2 - V \sim Q_0 + \frac{1}{2} Q_0''(x - x_0)^2, \quad Q_0'' \equiv d^2Q/dx^2 . \tag{114}$$

Then the Schutz-Will formula can be derived trivially & analytically

$$\frac{Q_0}{\sqrt{2Q_0''}} = i\left(n + \frac{1}{2}\right), \Rightarrow \quad \omega^2 = V_0 - i\left(n + \frac{1}{2}\right)\sqrt{2V_0''} \quad n = 0, 1, 2, \ldots \tag{115}$$

The formal proof assumes the following form of the Bohr-Sommerfeld formula:

$$\oint_C \sum_{k=0}^{\infty} S_{2k}(z) dz \approx 2i\left(n + \frac{1}{2}\right)\pi \tag{116}$$

Equations for Stellar Oscillations

Polar oscillations

$$-\frac{1}{c^2}\frac{\partial^2 S}{\partial^2 t}+\frac{\partial^2 S}{\partial^2 r_*}+L_1(S,F,\ell)=0,$$

$$-\frac{1}{c^2}\frac{\partial^2 F}{\partial^2 t}+\frac{\partial^2 F}{\partial^2 r_*}+L_2(S,F,H,\ell)=0,$$

$$-\frac{1}{(c_s)^2}\frac{\partial^2 H}{\partial^2 t}+\frac{\partial^2 H}{\partial^2 r_*}+L_3(H,H',S,S',F,F',\ell)=0,$$

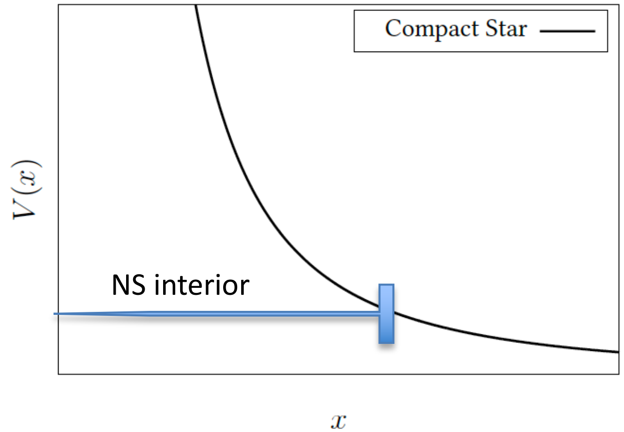
Thorne 1967

...

Allen, Andersson, KK, Schutz 1998

Axial Oscillations

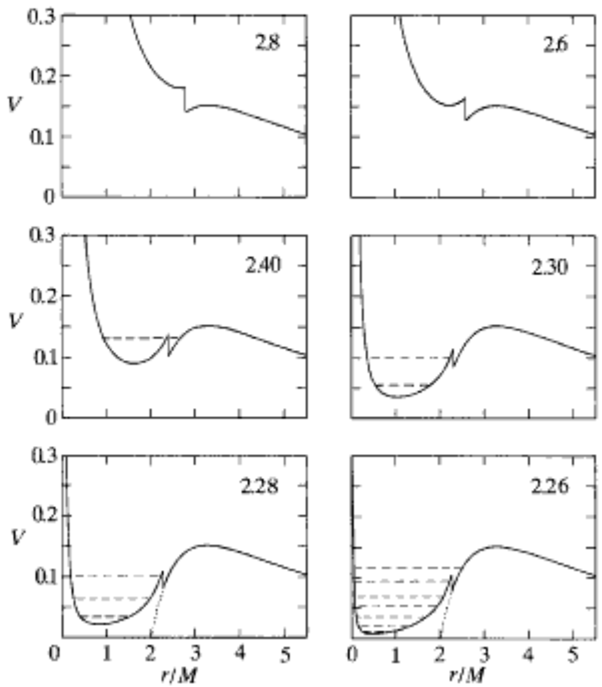
$$-\frac{1}{c^2}\frac{\partial^2 X}{\partial^2 t}+\frac{\partial^2 X}{\partial^2 r_*}+\frac{e^v}{r^3}\left[\ell(\ell+1)r+\boxed{r^3(\rho-p)}-6M\right]=0$$



90s: An interesting observation by Chandrasekhar-Ferrari

1991: A reconsideration of the axial modes

$$X_{,r,r} - \frac{2}{ry}\left(1 - 2\epsilon r^2 \frac{y_1}{3y_1 - y}\right) X_{,r} - \frac{(l-1)(l+2)}{r^2 y} X + \frac{4\sigma^2}{y(3y_1 - y)^2} X = 0,$$



CF 1991

Table 1. (a) $R/M = 2.26$ and (b) $R/M = 2.28$.

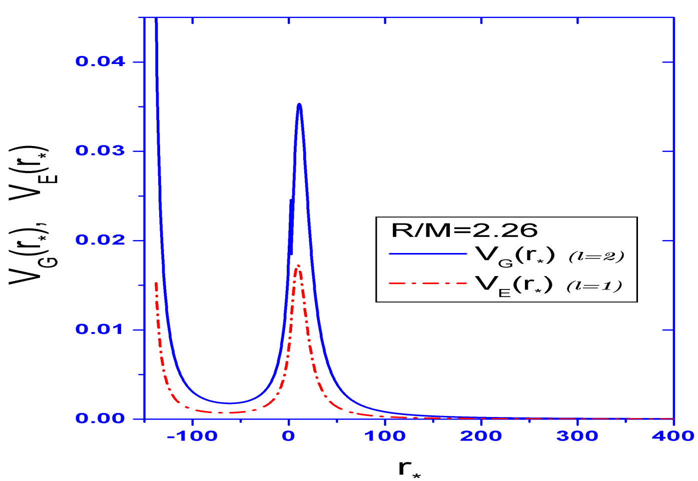
(a) $\Re(\omega_i)$	(a) $\Im(\omega_i)$	(b) $\Re(\omega_i)$	(b) $\Im(\omega_i)$
0.2139	$.2430\times10^{-8}$	0.3690	$.1231\times10^{-5}$
0.2910	$.7747\times10^{-7}$	0.5008	$.5294\times10^{-4}$
0.3680	$.1073\times10^{-5}$	0.6281	$.9202\times10^{-3}$
0.4446	$.9519\times10^{-4}$	0.7482	$.7241\times10^{-2}$
0.5206	$.6339\times10^{-4}$	0.8667	.0243
0.5956	$.3381\times10^{-3}$	0.9911	.0455
0.6690	$.1432\times10^{-2}$	1.1221	.0641
0.7409	$.4508\times10^{-2}$	1.2604	.0794
0.8128	.0101	1.4057	.0922
0.8859	.0172	1.5561	.1023
0.9604	.0245	1.7091	.1097
1.0361	.0310	1.8629	.1151
1.1131	.0367	2.0168	.1194
1.1915	.0416	2.1707	.1232
1.2712	.0458	2.3246	.1267

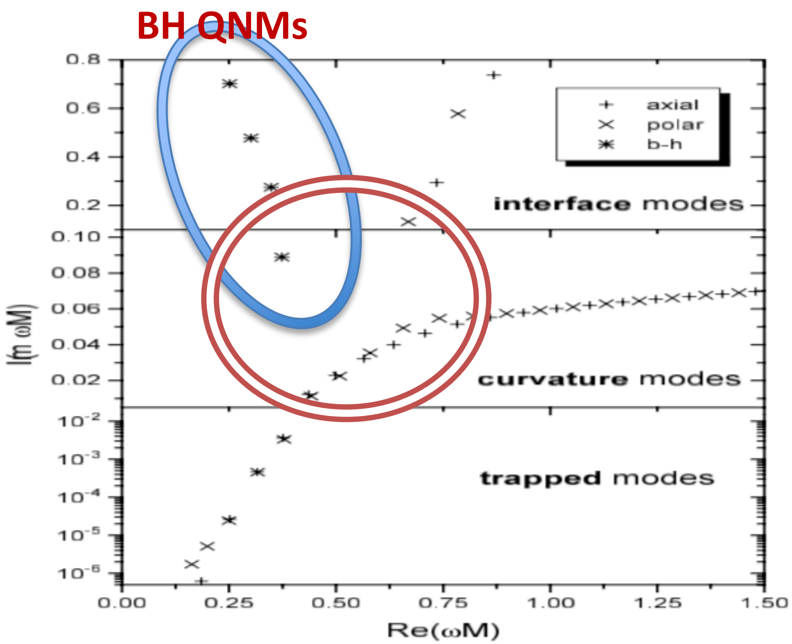
Kokkotas 1994

The AXIAL QNM Spectrum

$$\frac{d^2}{dr^{*2}}\Psi(r) + \left(\omega_n^2 - V(r)\right)\Psi(r) = 0,$$

$$V(r) = \frac{e^{2\nu}}{r^3}\left[l(l+1)r + \boxed{r^3(\rho - p(r))} - 6M(r)\right]$$

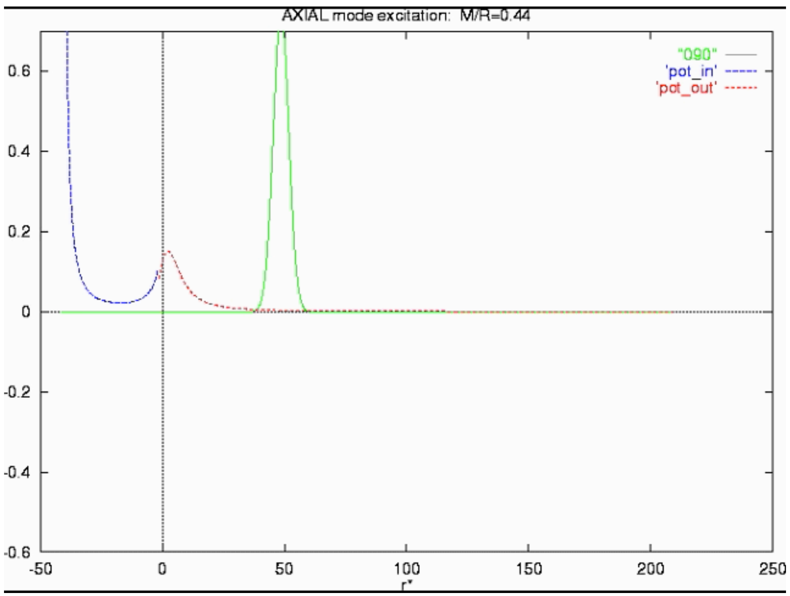


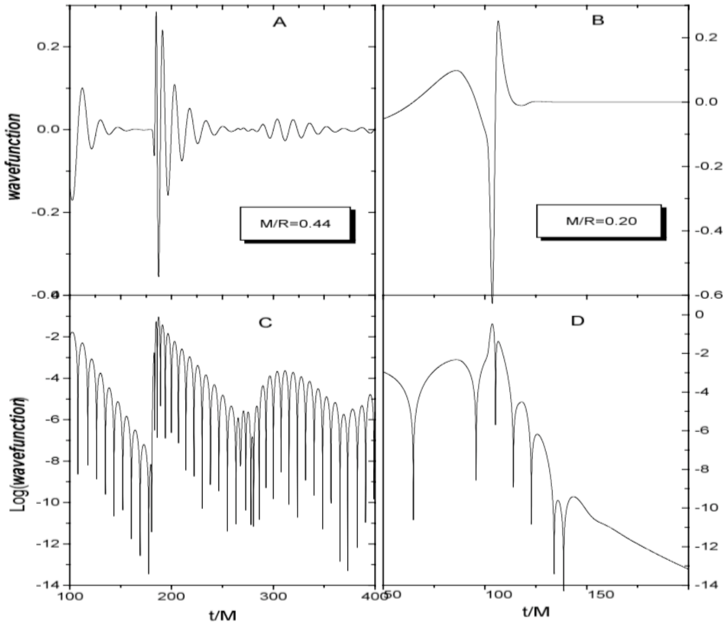


KK 1994, ...

Stellar POLAR and AXIAL spacetime associated spectra similar but not identical as for BHs

An interesting “toy” problem





KK: Les Houches 1995

Axial spectra & "Echoes"

$$\frac{\mathrm{d}^2}{\mathrm{d}r^{*2}}\Psi(r) + \left(\omega_n^2 - V(r)\right)\Psi(r) = 0,$$

$$V(r) = \frac{e^{2\nu}}{r^3}\left[l(l+1)r + \boxed{r^3(\rho - p(r))} - 6M(r)\right]$$

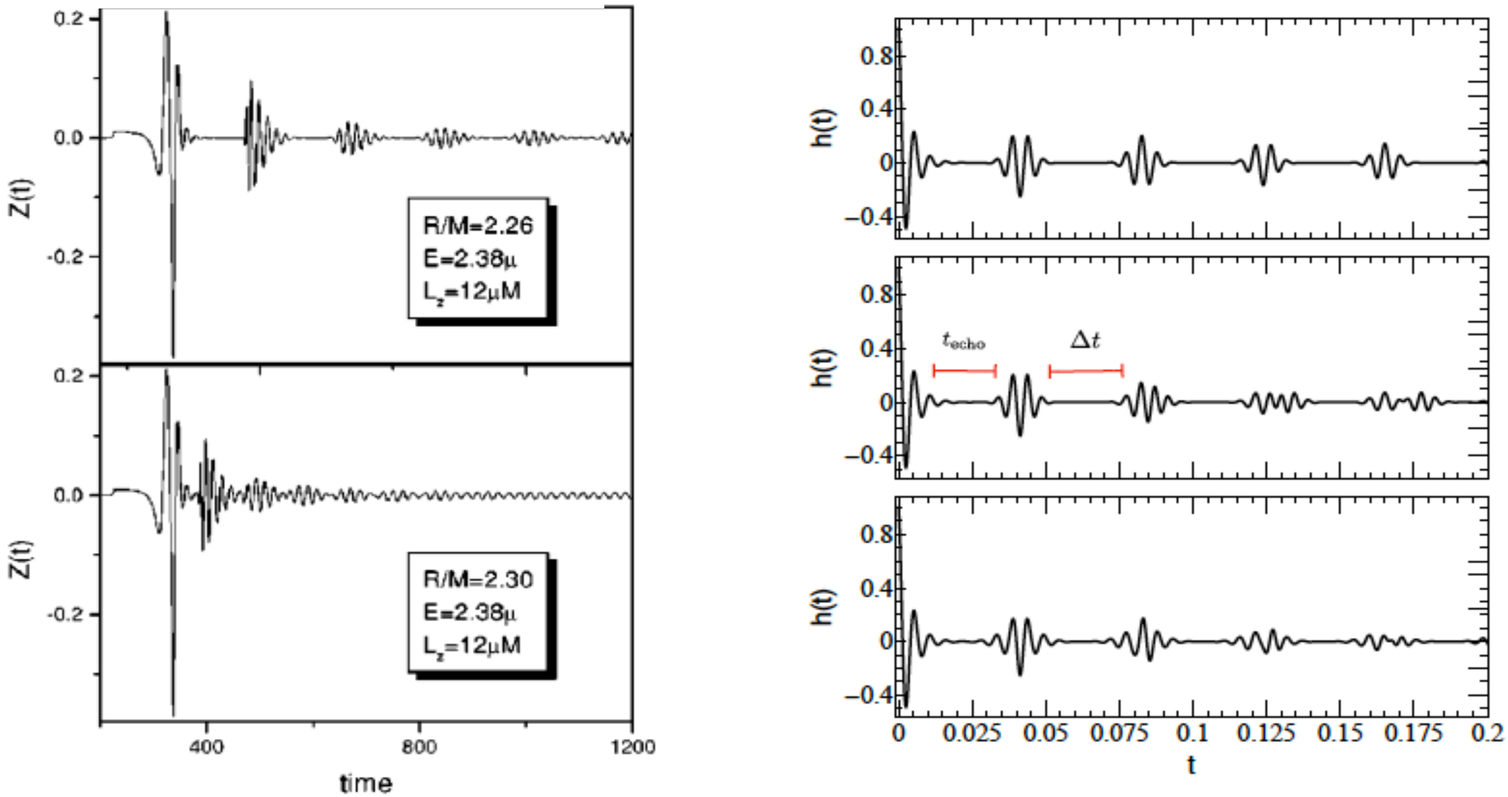
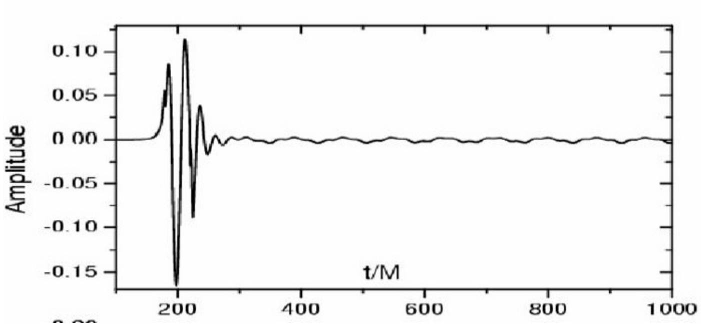


Figure 2: Left: Axial perturbations ultra compact stars, Ferrari & Kokkotas (2000). Right: Phenomenological template for parameter estimation: Maselli, Völkel & Kokkotas (2018).

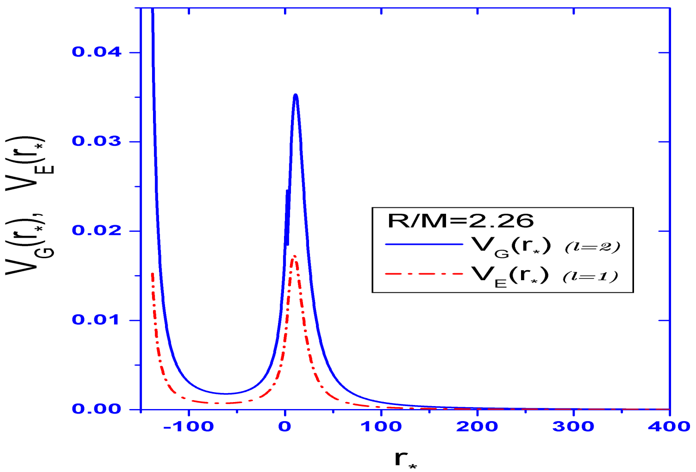
"W-mode" or "Ergoregion" Instability or "Superadiant Instability"

- Perturbations of the spacetime, similar to the QNMs of BHs (KK+Schutz 1986-1992)
- Frequencies (typical) 5-12kHz (for normal NS)
- Damping times (typical) ≥0.1ms
- For very compact stars they become exciting! (Chandrasekhar+Ferrari 1991)



- The creation of an ergosphere signals the onset of an instability (Friedman 1978, Comins+Schutz 1978)
- The dragging is so strong that any timelike backwards moving trajectory gets dragged forward

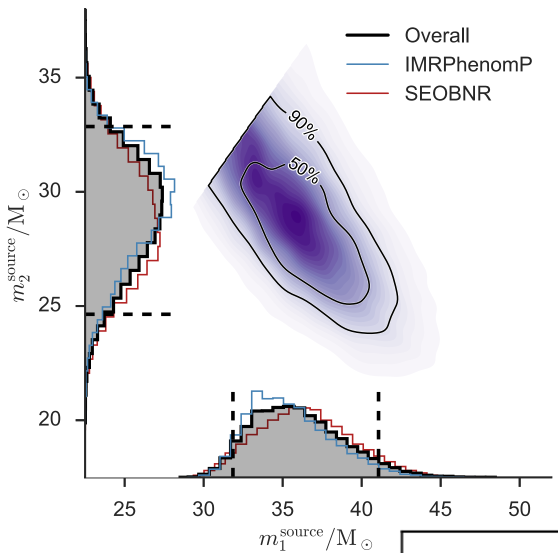
- Growth time of the order of tenths of secs
- It sets in quite early for very compact NS *($R/M \sim 2.26$, $\Omega \sim 0.19\Omega_{Kepler}$)*

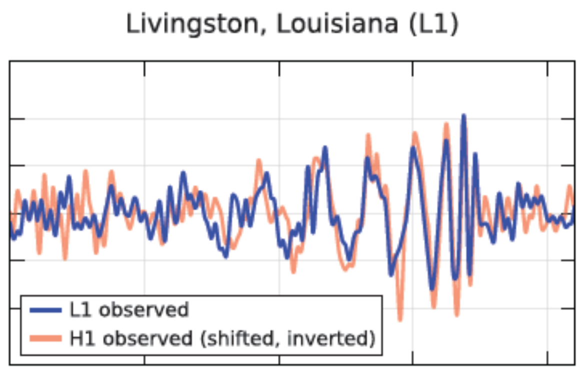


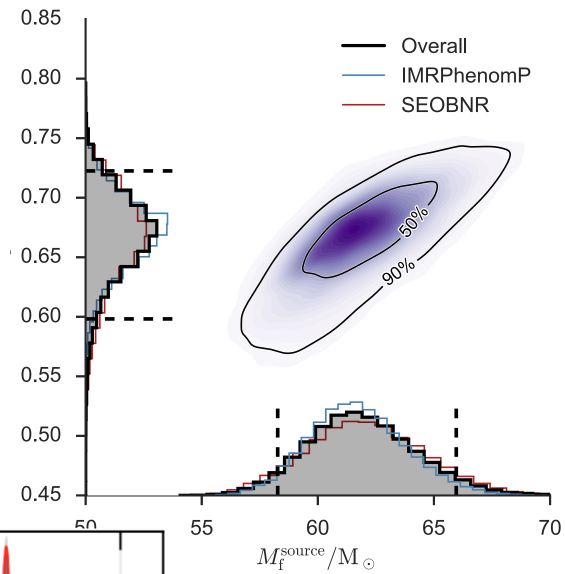
- No direct way for viscosity to suppress the instability (!)
- Nonlinear saturation (?)

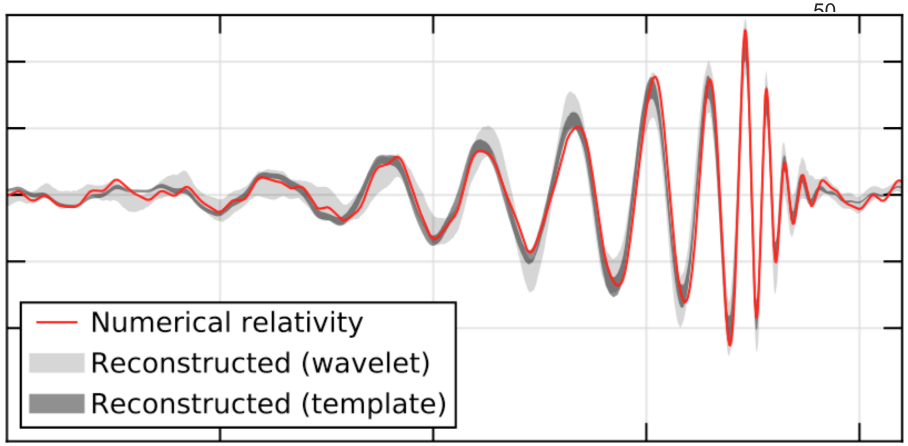
Kokkotas-Ruoff-Andersson 2004

Finding the parameters of GW150914









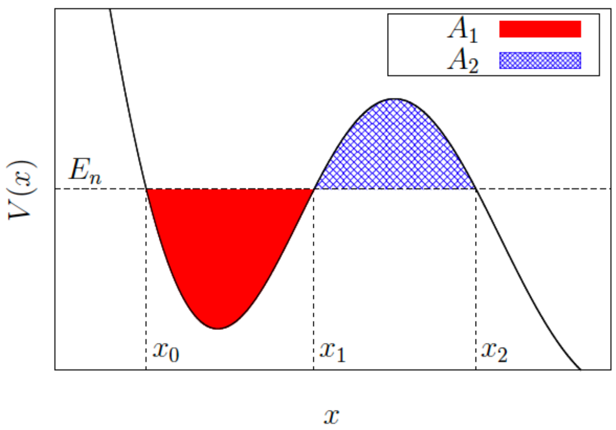
Phys. Rev. Lett. 116, 241102 (2016)

Semi-Analytic Sudy of the Axial QNMs

Völkel-Kokkotas 2017

$$\frac{d^2}{dr^{*2}}\Psi(r) + \left(\omega_n^2 - V(r)\right)\Psi(r) = 0, \qquad V(r) = \frac{e^{2\nu}}{r^3}\left[l(l+1)r + \boxed{r^3(\rho - p(r))} - 6M(r)\right]$$

Using WKB theory it is possible to include higher order corrections and to generalize the Bohr-Sommerfeld rule to other type of potentials.



2 turning points

$$\int_{x_0}^{x_1}\sqrt{E_n - V(x)}\,dx = \pi\left(n + \frac{1}{2}\right) + \frac{1}{24}\frac{\partial^2}{\partial E_n^2}\int_{x_0}^{x_1}\left(\frac{dV(x)}{dx}\right)^2\frac{dx}{\sqrt{E_n - V(x)}}$$

3 turning points

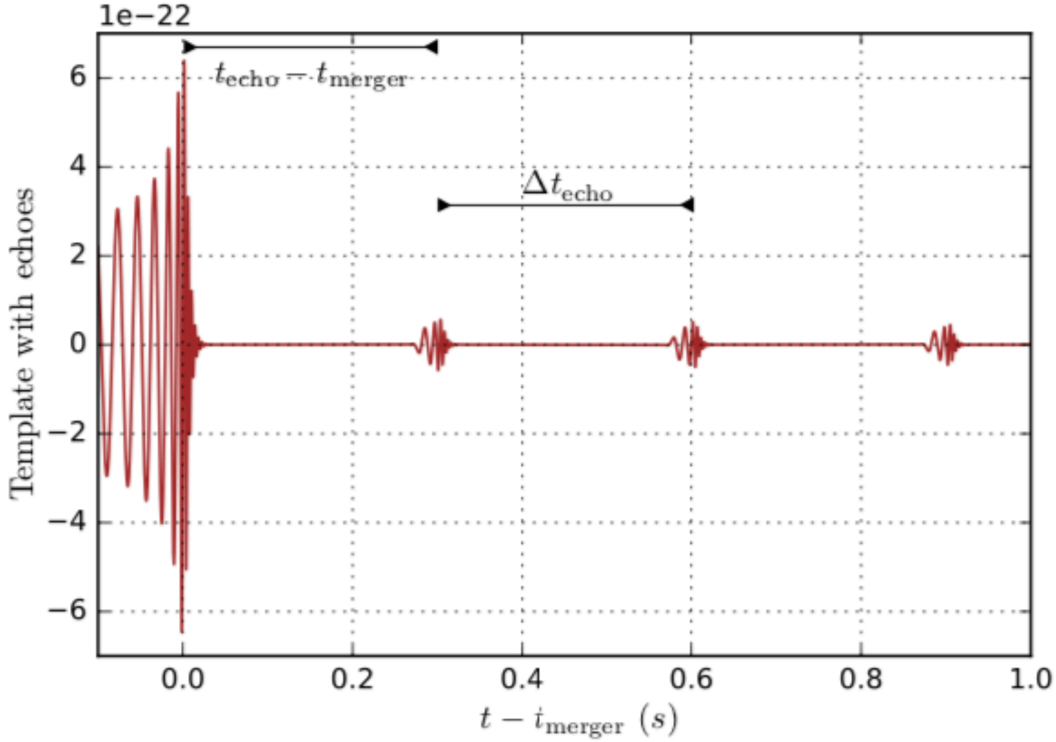
$$\int_{x_0}^{x_1}\sqrt{E_n - V(x)}\,dx = \pi\left(n + \frac{1}{2}\right) - \frac{i}{4}\exp\left(2i\int_{x_1}^{x_2}\sqrt{E_n - V(x)}\,dx\right)$$

Damping Time

$$E_{1n} = -\frac{1}{2}\exp\left(2i\int_{x_1}^{x_2}\sqrt{E_{0n} - V(x)}\,dx\right)\left(\int_{x_0}^{x_1}\frac{1}{\sqrt{E_{0n} - V(x)}}\,dx\right)^{-1}$$

	BS fitting		BS low		BS high		Numerical	
n	$\mathrm{Re}(\omega_n)$	$\lvert\mathrm{Im}(\omega_n)\rvert$	$\mathrm{Re}(\omega_n)$	$\lvert\mathrm{Im}(\omega_n)\rvert$	$\mathrm{Re}(\omega_n)$	$\lvert\mathrm{Im}(\omega_n)\rvert$	$\mathrm{Re}(\omega_n)$	$\lvert\mathrm{Im}(\omega_n)\rvert$
0	0.1060	2.88×10^{-9}	0.1068	1.01×10^{-9}	0.1091	1.29×10^{-9}	0.1090	1.24×10^{-9}
1	0.1530	9.54×10^{-8}	0.1462	3.16×10^{-8}	0.1485	3.73×10^{-8}	0.1484	3.95×10^{-8}
2	0.1970	1.14×10^{-6}	0.1856	4.13×10^{-7}	0.1879	4.72×10^{-7}	0.1876	5.47×10^{-7}
3	0.2377	7.92×10^{-6}	0.2251	3.47×10^{-6}	0.2274	3.88×10^{-6}	0.2267	4.85×10^{-6}
4	0.2756	3.97×10^{-5}	0.2646	2.28×10^{-5}	0.2668	2.52×10^{-5}	0.2654	3.23×10^{-5}
5	0.3114	1.60×10^{-4}	0.3040	1.40×10^{-4}	0.3063	1.52×10^{-4}	0.3036	1.72×10^{-4}
6	0.3453	5.54×10^{-4}	0.3411	5.51×10^{-4}	0.3441	6.14×10^{-4}	0.3410	7.30×10^{-4}
7	0.3778	1.71×10^{-3}	0.3784	2.00×10^{-3}	0.3752	1.81×10^{-3}	0.3777	2.30×10^{-3}

“Echoes” from the “Abyss”



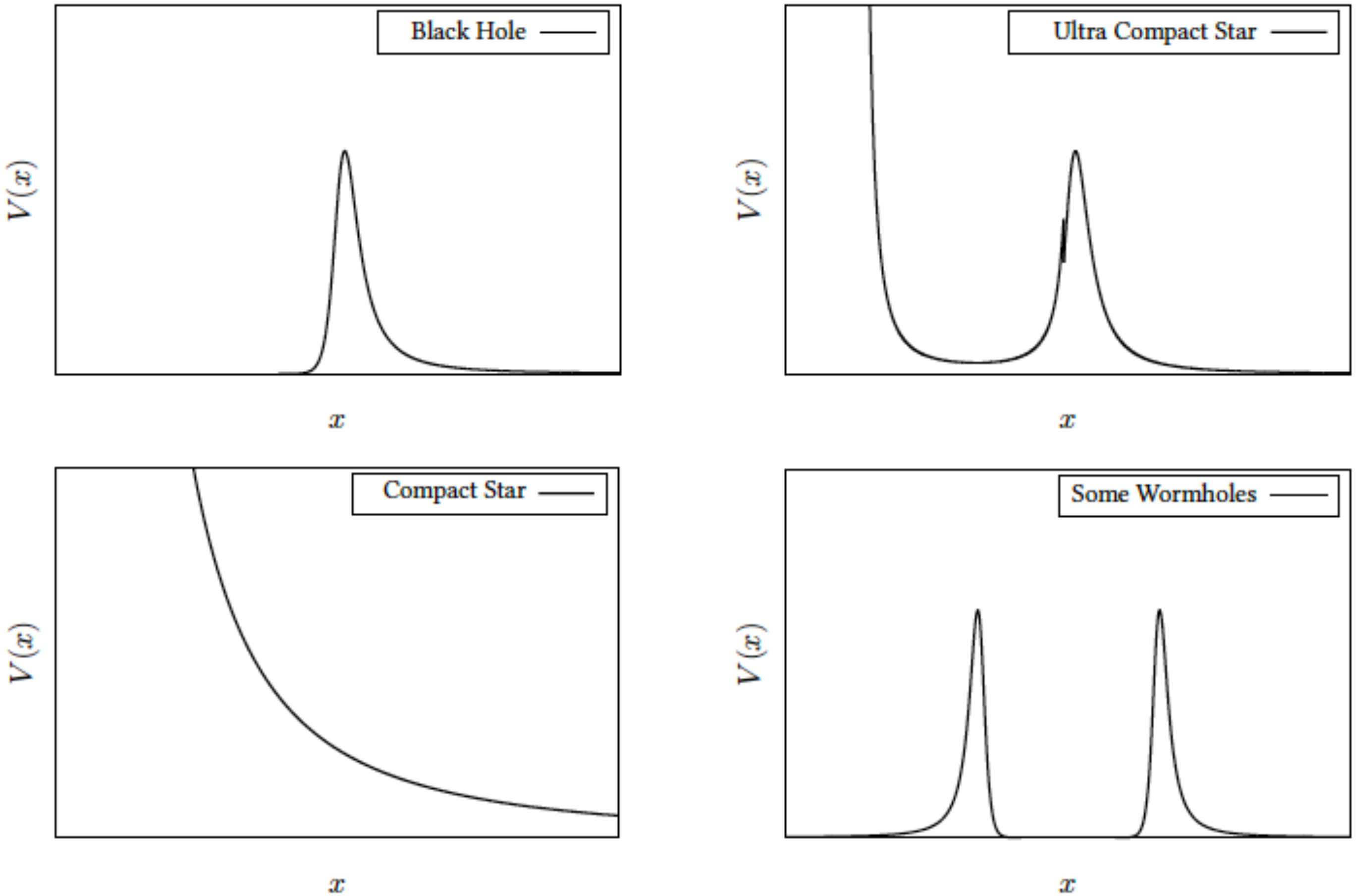
Abedi, Dykaar, Afshordi PRD (2017)

Searched for **observational signatures of echoes** in the GW data released by advanced LIGO following the 3 black hole merger events GW150914, GW151226, and LVT151012.

Different types of axial perturbation potentials

$$\frac{\mathrm{d}^2}{\mathrm{d}r^{*2}}\Psi(r) + \left(\omega_n^2 - V(r)\right)\Psi(r) = 0,$$

$$V(r) = \frac{e^{2\nu}}{r^3}\left[l(l+1)r + \boxed{r^3(\rho - p(r))} - 6M(r)\right]$$



Inversion Method for Different types of potentials

Wheeler 1976

$$\frac{\mathrm{d}^2}{\mathrm{d}r^{*2}}\Psi(r) + \left(\omega_n^2 - V(r)\right)\Psi(r) = 0,$$

$$V(r) = \frac{e^{2\nu}}{r^3}\left[l(l+1)r + \boxed{r^3(\rho - p(r))} - 6M(r)\right]$$

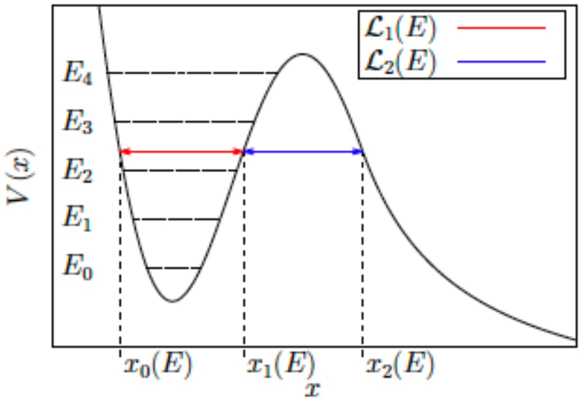


Figure 3: Völkel & Kokkotas (2017)

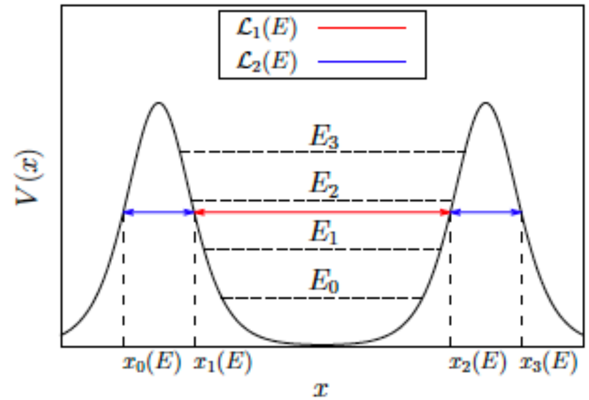


Figure 4: Völkel & Kokkotas, (2018)

$$\mathcal{L}_1(E) = x_1 - x_0 = 2\frac{\partial}{\partial E}\int_{E_{\min}}^{E}\frac{n(E')+1/2}{\sqrt{E-E'}}\mathbf{d}E'$$

$$\mathcal{L}_2(E) = x_2 - x_1 = -\frac{1}{\pi}\int_{E}^{E_{\max}}\frac{(\mathbf{d}T(E')/\mathbf{d}E')}{T(E')\sqrt{E'-E}}\mathbf{d}E'$$

-- Wheeler (1976); Chadan & Sabatier (1989); Lazenby & Griffiths (1980);
-- Bonatsos, Daskaloyannis & Kokkotas 1992-4; Gandhi and Efthimiou (2006)
-- Völkel and Kokkotas (2017,2); Völkel (2018,1); Völkel & Kokkotas (2018,2)

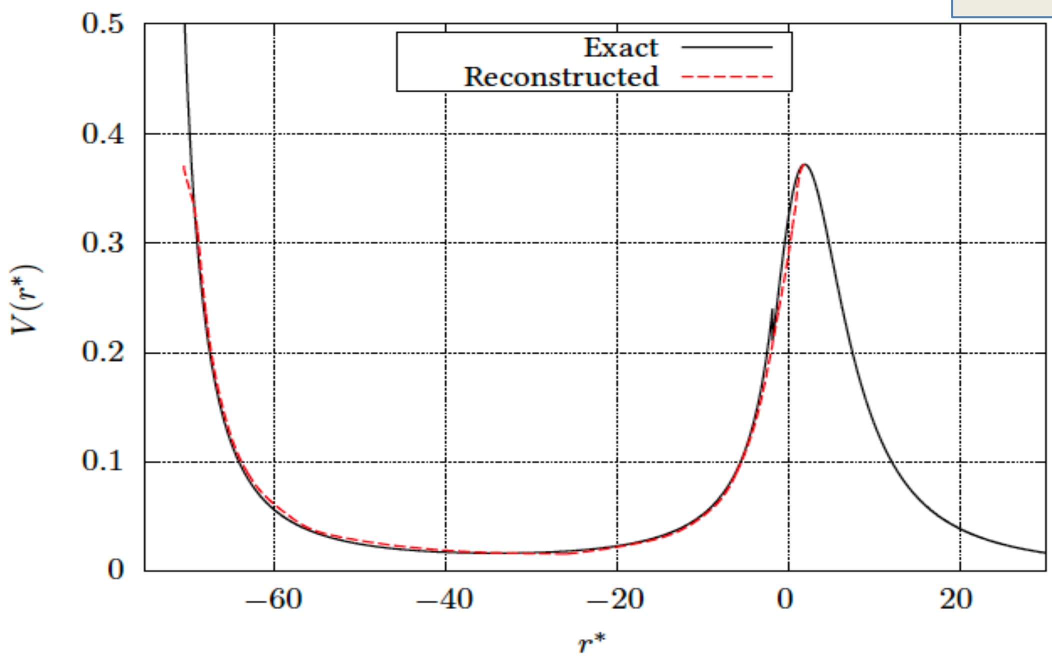
Inversion Method for stellar potentials

$$\frac{d^2}{dr^{*2}}\Psi(r) + \left(\omega_n^2 - V(r)\right)\Psi(r) = 0,$$

$$V(r) = \frac{e^{2\nu}}{r^3}\left[l(l+1)r + \boxed{r^3(\rho - p(r))} - 6M(r)\right]$$

Example for ultra compact constant density star $C \approx 0.44$

Similar for gravastars



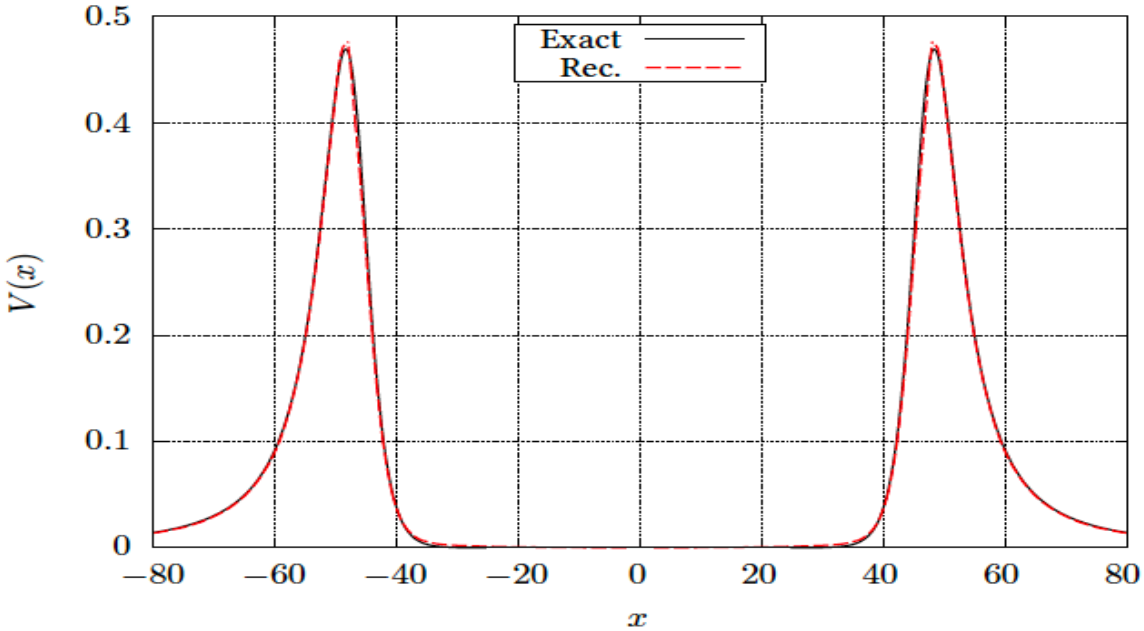
Reconstructed axial perturbation potential, constant density star, $l = 3$, taken from Völkel and Kokkotas (2017,2).

Inversion Method for DS wormhole

$$\frac{d^2}{dr^{*2}}\Psi(r) + \left(\omega_n^2 - V(r)\right)\Psi(r) = 0,$$

$$V(r) = \frac{e^{2\nu}}{r^3}\left[l(l+1)r + \boxed{r^3(\rho - p(r))} - 6M(r)\right]$$

Example for Damour-Solodukhin wormhole $C \approx 0.5$

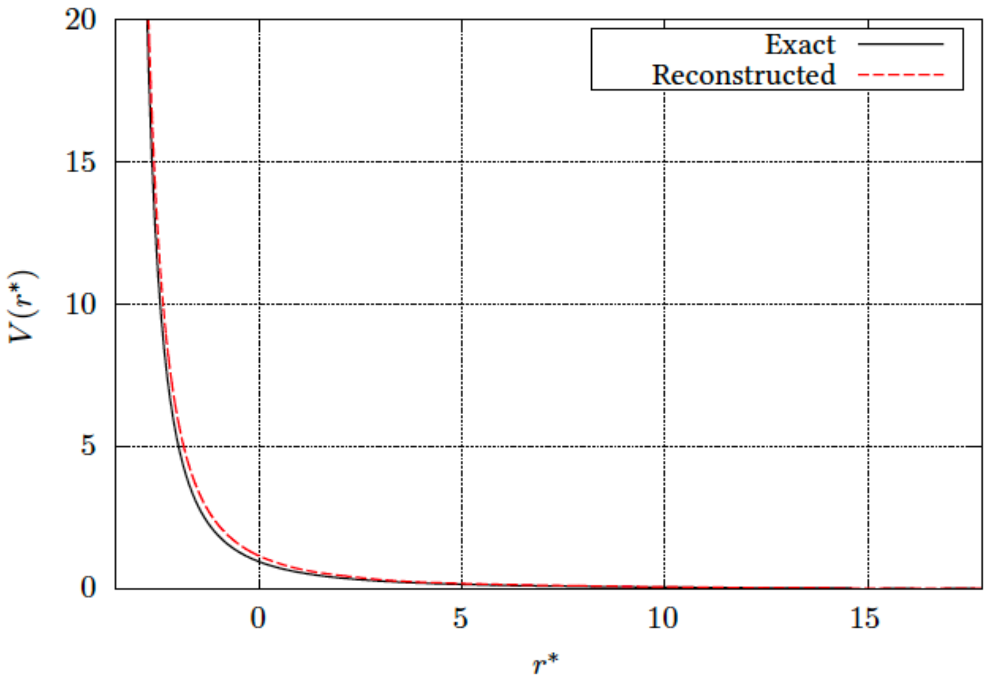


Reconstructed scalar perturbation potential, Damour-Solodukhin wormhole, $l = 3$, taken from Völkel and Kokkotas (2018,2).

Inversion Method for normal NS

$$\frac{\mathrm{d}^2}{\mathrm{d}r^{*2}}\Psi(r) + \left(\omega_n^2 - V(r)\right)\Psi(r) = 0, \qquad V(r) = \frac{e^{2\nu}}{r^3}\left[l(l+1)r + \boxed{r^3(\rho - p(r))} - 6M(r)\right]$$

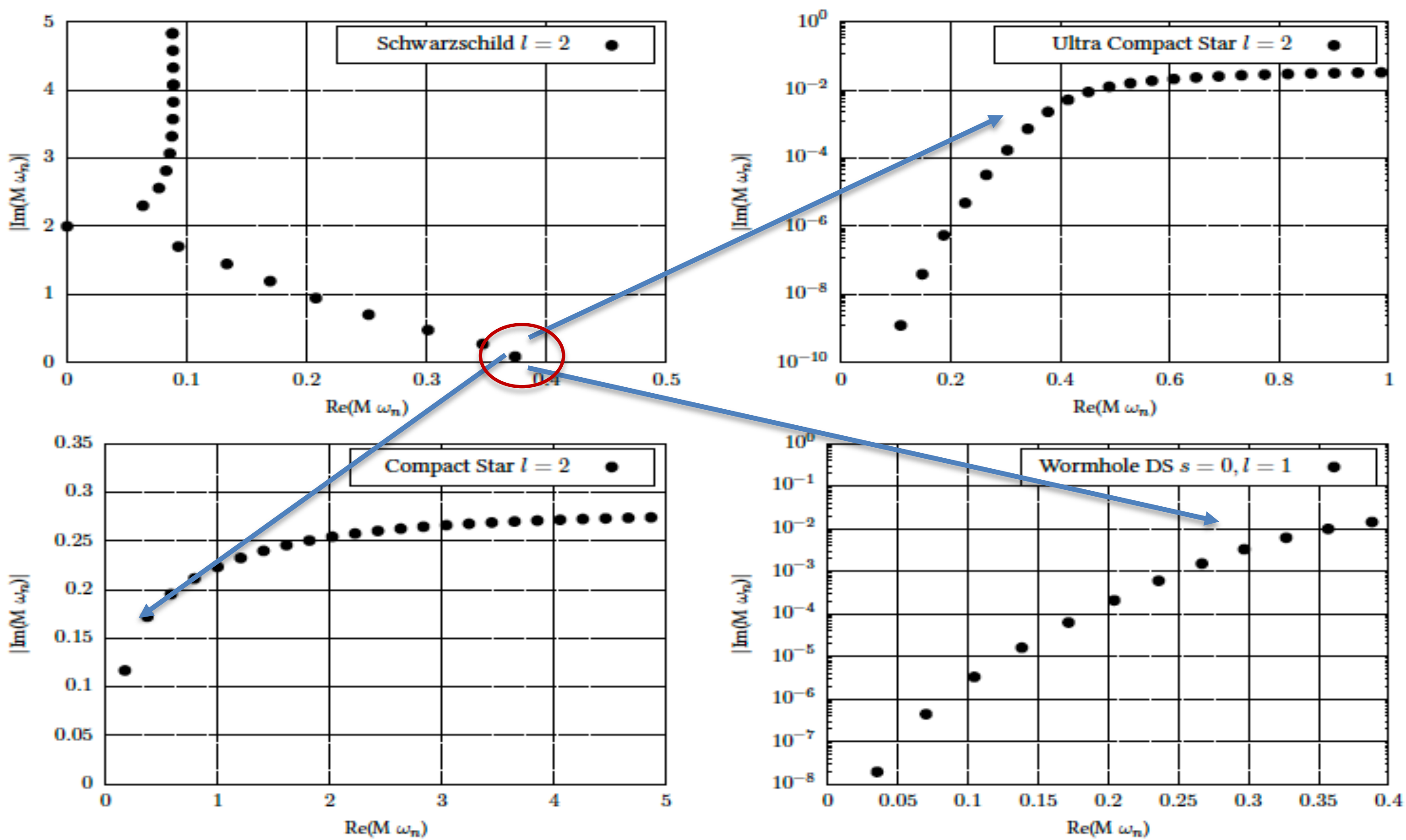
Example for neutron star polytrope $C \approx 0.15$



Reconstructed axial perturbation potential, neutron star polytrope, $l = 3$, Völkel and Kokkotas (2019).

Different types of axial spectra

$$\frac{d^2}{dr^{*2}}\Psi(r) + \left(\omega_n^2 - V(r)\right)\Psi(r) = 0, \qquad V(r) = \frac{e^{2\nu}}{r^3}\left[l(l+1)r + \boxed{r^3(\rho - p(r))} - 6M(r)\right]$$



Inversion Problem for Hawking Radiation

Völkel, Konoplya, KK PRD 2019

Assuming Hawking radiation can be described by

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \sum_l N_l \left|\mathcal{A}_l\right|^2 \frac{\omega}{\exp\left(\omega/T_{\mathrm{H}}\right) - 1} \frac{\mathrm{d}\omega}{2\pi},$$

T_H Hawking temperature, $\mathcal{A}_l$ greybody factors, N_l multiplicities [4]

$$\begin{aligned} \Psi &= e^{-i\omega r_*} + Re^{i\omega r_*}, \quad & r_* \to +\infty, \\ \Psi &= Te^{-i\omega r_*}, & r_* \to -\infty, \end{aligned}$$

reflection R and transmission T

$$\left|\mathcal{A}_\ell\right|^2 = 1 - \left|R_\ell\right|^2 = \left|T_\ell\right|^2.$$

[4] Details in Kanti, Kodama, Konoplya, Pappas, and Zhdenko (2009)

Inversion Problem for Hawking Radiation

Völkel, Konoplya, KK PRD 2019

Analytic approximation given by Gamow formula

$$T(E) = \exp\left(2i\int_{x_0}^{x_1}\sqrt{E - V(x)}\mathrm{d}x\right), \qquad (5)$$

E energy, $V(x)$ potential barrier, and x_0 and x_1 classical turning points[5].

Can be inverted to find width of potential barrier[6]

$$\mathcal{L}(E) \equiv x_1 - x_0 = \frac{1}{\pi}\int_{E}^{E_{\mathrm{max}}} \frac{(\mathrm{d}T(E')/\mathrm{d}E')}{T(E')\sqrt{E' - E}}\mathrm{d}E', \qquad (6)$$

But, how do we get the individual greybody factors?

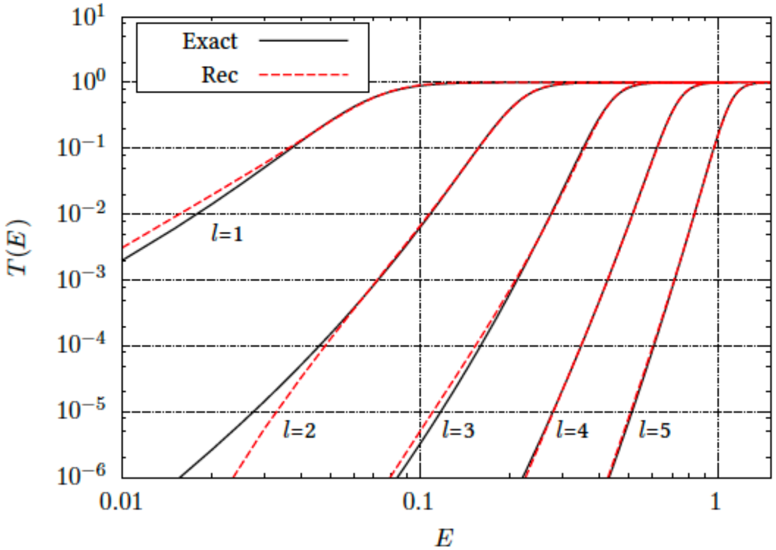
---

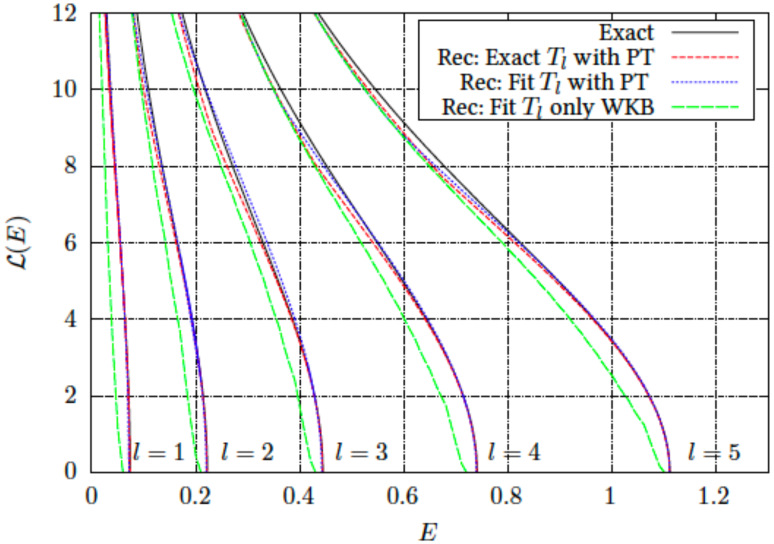
[5] Defined by $E = V(x)$

[6] Cole & Good PRA (1978)

Inversion Problem for Hawking Radiation

Völkel, Konoplya, KK PRD 2019





Reconstruction of the **Schwarzschild transmissions** $T_l(E)$ from Hawking spectrum fitting

Reconstruction of the **Schwarzschild potential barrier** widths $\mathbf{L_l(E)}$ from given transmissions $T_l(E)$.

Disco Balls in the Sky? (i)

Foit & Kleban 2019, Cardoso, Foit & Kleban 2019

BH area quantization could be a mechanism to produce echoes as well as characteristic absorption lines in GW observations of merging BHs

Area of quantized BH

$$A = \alpha \ell_p^2 n = 4\pi r_s^2 = 4\pi \left(\frac{2GM}{c^2}\right)^2$$

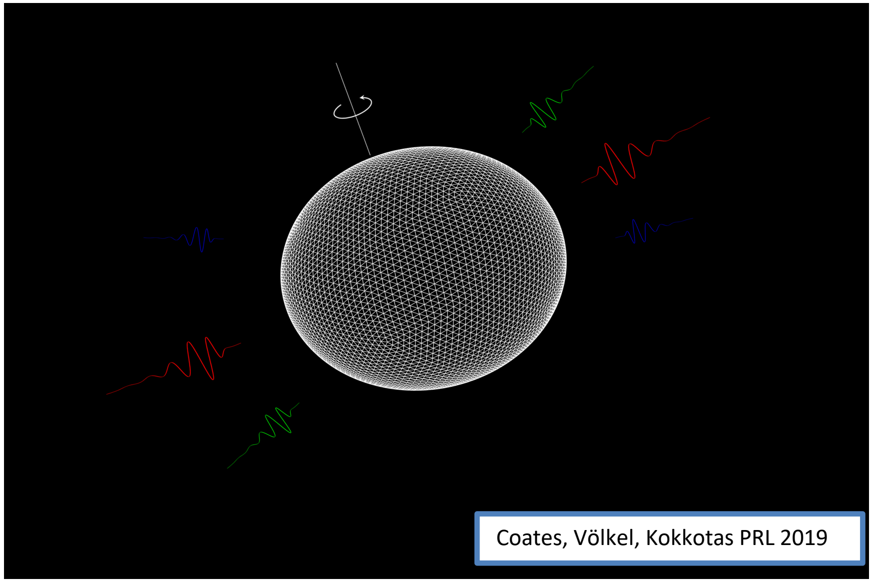
$$\boxed{1 < \alpha < 30}$$

The spectrum of emission or absorption of radiation by quantum BHs occurs in a series of evenly-spaced lines. For Schwarszchild

$$\Delta A = \alpha \frac{\hbar G}{c^3} \Delta N = 32\pi \frac{G^2}{c^4} M \Delta M$$

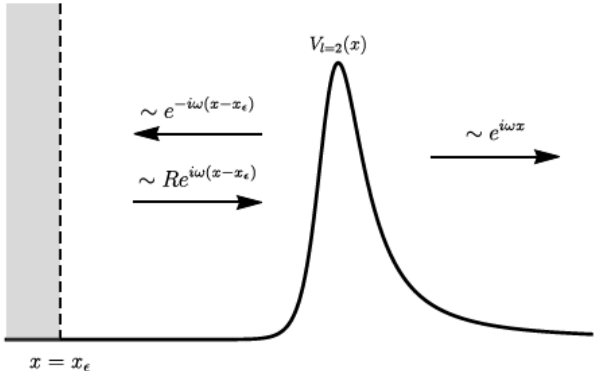
therefore the frequency absorbed or emitted by a BH is quantized (n = |ΔN| is the change in the area quantum)

$$\omega_n = \left|\frac{\Delta M}{\hbar}\right| = \frac{n\alpha}{32\pi} \frac{c}{MG} = \frac{n\alpha}{16\pi} \frac{c}{r_s}$$



Coates, Völkel, Kokkotas PRL 2019

Radiation that falls towards the horizon will be reflected if it has the "wrong" frequency, while radiation with the frequencies ω_n will be absorbed.



Disco Balls in the Sky? (ii)

Coates, Völkel, Kokkotas PRL 2019

Bekenstein & Mukhanov: the ratio between **width** and **spacing** of non-spinning BH states approaches a small constant

Mass of the n[th] state

$$M_n = \sqrt{\frac{\alpha n c \hbar}{16\pi G}}$$

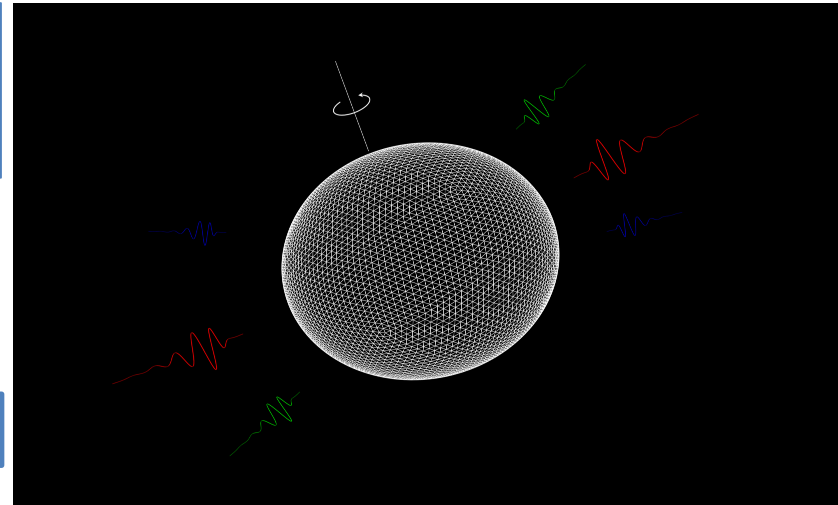
The transition from the n to n-δn has energy

$$(M_n - M_{n-\delta n})\, c^2 = \hbar \omega_{n,\delta n}.$$

Matching the power output by the spontaneous decay to the power output of Hawking radiation one can then find an estimate for the energy emitted in Hawking radiation

$$P_H = \frac{\langle E_{\text{emitted}} \rangle}{\tau}$$ Or $$P_H = \langle \omega \rangle \Gamma.$$

Γ is the **energy width** of the state



Coates, Völkel, Kokkotas PRL 2019

Energy width of the state

$$\begin{aligned}\tilde{\Gamma} =& 9\frac{\sqrt{n}+\sqrt{n-1}}{320 n}\sqrt{\frac{c^5 \hbar \pi}{G\alpha^3}}\\ =& \frac{9}{160}\sqrt{\frac{\pi \hbar c^5}{G\alpha^3 n}}\left(1-\frac{1}{4n}\right) + \mathscr{O}(n^{-5/2})\end{aligned}$$

This shows that this estimate of the **width goes to zero for large black holes** (large n)

Disco Balls in the Sky? (iii)

Coates, Völkel, Kokkotas PRL 2019

The most important astrophysical BHs, from an observational point of view, are expected to have non-negligible spin **(a~0.7)**

Thus along with the area the spin should be also quantized

$$J = \hbar j$$

The transition frequency from
n → n -δn and **j → j - δj**

$$\omega = \frac{c^2}{\hbar}\left[M(n,j) - M(n-\delta n, j-\delta j)\right]$$

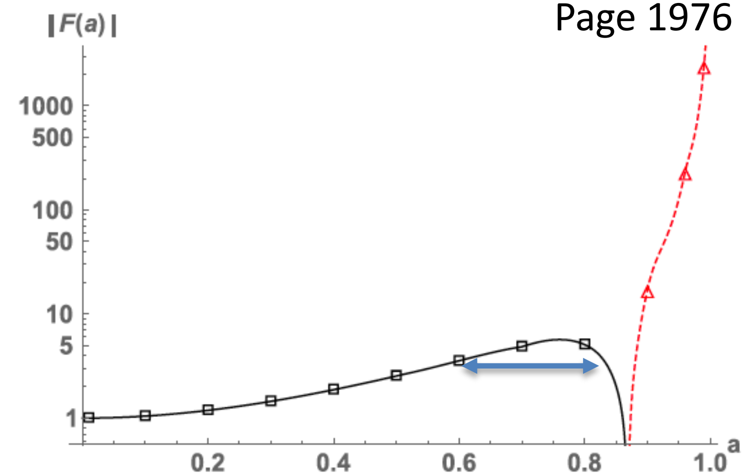
The energy width of the state will be:

$$\Gamma = -\frac{\hbar \dot{n}}{\langle \delta n \rangle} = -\frac{\hbar \dot{n}}{r \langle \delta n_0 \rangle}$$

$\dot{n}$ is the rate of change of the area quantum number

and $\quad j = 0 \text{ for } r = 1$

Page 1976



The influence of the spin

$$\frac{\Gamma \omega_0}{\Gamma_0 \omega_{\mathrm{ref}}} = \frac{F(a)}{r},$$

This means that, for the lowest value of α~4ln(2) the natural linewidth can cause neighboring states to completely overlap or that the states of astrophysicaly interesting BHs are likely to overlap

i.e. NO REFLECTION!!

Conclusions

- “ECHOES” are beautiful and exciting toy problems
- Not yet convincing Astrophysical scenario for their excitation
- If observed there will be dramatic consequences for both Astrophysics and Theoretical Physics
- The machinery to interpret them it is still in its infancy (mainly axial & non-rotating perturbations)
- Sooner or later the observations will provide the final answer

THANK YOU