

# Modified gravity in DHOST theories

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Astroparticules  
et Cosmologie

# Introduction

- So far, **GR** seems compatible with all observations.
- Several motivations for exploring **modified gravity**
  - Quantum gravity effects
  - Understand cosmological acceleration (or possibly dark matter)
  - Explore alternative gravitational theories
  - **Testing gravity**
- Many models of **dark energy or modified gravity**
- Here: **Generalized framework** for scalar-tensor theories, allowing for **2nd order derivatives** in their Lagrangian

# Higher order scalar-tensor theories

- Traditional scalar-tensor theories :  $\mathcal{L}(\nabla_\lambda \phi, \phi)$

$$S = \int d^4x \sqrt{-g} \left[ F(\phi)^{(4)}R - Z(\phi)\partial_\mu\phi\partial^\mu\phi - U(\phi) \right] + S_m[\psi_m; g_{\mu\nu}]$$

- **Generalized theories** with second order derivatives

$$\mathcal{L}(\nabla_\mu \nabla_\nu \phi, \nabla_\lambda \phi, \phi)$$

- In general, they contain an **extra degree of freedom**, expected to lead to **Ostrogradsky instabilities**

$$L(\ddot{q}, \dot{q}, q)$$

- But possible to avoid for special Lagrangians

# Degenerate theories

EOM higher order but **no extra DOF** if the Lagrangian is “degenerate”.

$$L = \frac{1}{2}a\ddot{\phi}^2 + b\ddot{\phi}\dot{q} + \frac{1}{2}c\dot{q}^2 + \frac{1}{2}\dot{\phi}^2 - V(\phi, q) \quad \begin{cases} ac - b^2 \neq 0 & : 3 \text{ dof} \\ ac - b^2 = 0 & : 2 \text{ dof} \end{cases}$$

$$L_{\text{deg}} = \frac{1}{2}c \left( \dot{q} + \frac{b}{c}\ddot{\phi} \right)^2 + \frac{1}{2}\dot{\phi}^2 - V(\phi, q) \quad [x \equiv q + \frac{b}{c}\dot{\phi} \quad \text{and} \quad \phi]$$

DL & K. Noui ‘1510 ‘1512



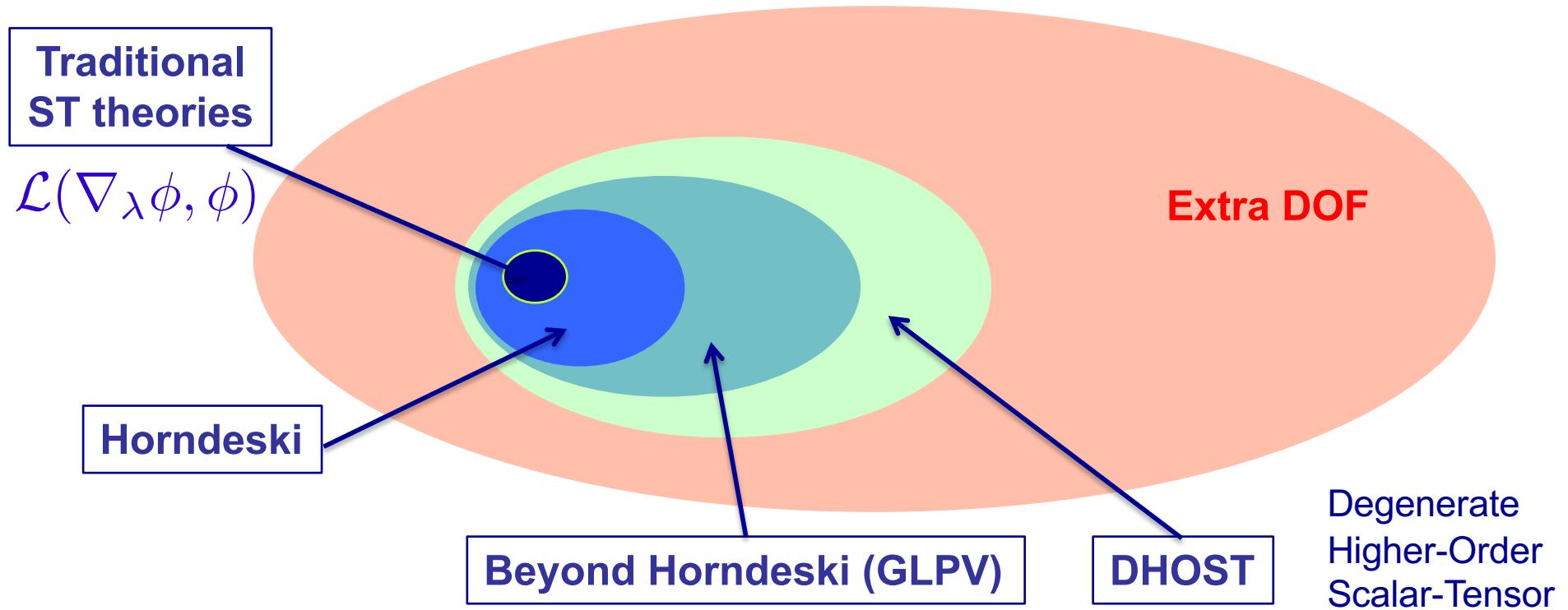
**DHOST theories**  
(Degenerate Higher-Order Scalar-Tensor)

$$\phi(t) \longrightarrow \phi(x^\mu)$$

$$q(t) \longrightarrow g_{\mu\nu}$$

# Higher order scalar-tensor theories

- Generalized theories:  $\mathcal{L}(\nabla_\mu \nabla_\nu \phi, \nabla_\lambda \phi, \phi)$
- DHOST: most general family of covariant scalar-tensor theories with a **single scalar DOF**



# DHOST theories

- Action [DL & Noui '15 ; Ben Achour, Crisostomi, Koyama, DL, Noui & Tasinato '16]

$$S[g, \phi] = \int d^4x \sqrt{-g} \left[ P(X, \phi) + Q(X, \phi) \square \phi + f_2(X, \phi) {}^{(4)}R + \sum_{I=1}^5 a_I(X, \phi) L_I^{(2)} + f_3(X, \phi) G_{\mu\nu} \phi^{\mu\nu} + \sum_{I=1}^{10} b_I(X, \phi) L_I^{(3)} \right]$$

with  $X \equiv \nabla_\mu \phi \nabla^\mu \phi$   
 $\phi_{\mu\nu} \equiv \nabla_\nu \nabla_\mu \phi$

with the 5 **quadratic** Lagrangians

$$L_1^{(2)} = \phi_{\mu\nu} \phi^{\mu\nu}, \quad L_2^{(2)} = (\square \phi)^2, \quad L_3^{(2)} = (\square \phi) \phi^\mu \phi_{\mu\nu} \phi^\nu$$

$$L_4^{(2)} = \phi^\mu \phi_{\mu\rho} \phi^{\rho\nu} \phi_\nu, \quad L_5^{(2)} = (\phi^\mu \phi_{\mu\nu} \phi^\nu)^2$$

and the 10 **cubic** ones

$$L_1^{(3)} = (\square \phi)^3, \quad L_2^{(3)} = (\square \phi) \phi_{\mu\nu} \phi^{\mu\nu}, \quad \dots, \quad L_{10}^{(3)} = (\phi_\mu \phi^{\mu\nu} \phi_\nu)^3$$

# DHOST theories

- The functions  $f_2(X, \phi)$ ,  $a_I(X, \phi)$ ,  $f_3(X, \phi)$  and  $b_I(X, \phi)$  must satisfy **degeneracy conditions**.

 DHOST theories

- Subsets of DHOST theories
  - Beyond Horndeski or GLPV [ Gleyzes, DL, Piazza & Vernizzi ]

$$f_2 = G_4, \quad a_1 = -a_2 = 2G_{4X} + XF_4, \quad a_3 = -a_4 = 2F_4$$

$$f_3 = G_5, \quad 3b_1 = -b_2 = \frac{3}{2}b_3 = G_{5X} + 3XF_5, \quad -2b_4 = b_5 = 2b_6 = -b_7 = 6F_5$$

- Horndeski theories:  $F_4 = 0, F_5 = 0$

# Disformal transformations

- Transformations of the metric [Bekenstein '93]

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(X, \phi) g_{\mu\nu} + D(X, \phi) \partial_\mu \phi \partial_\nu \phi$$

- Starting from an action  $\tilde{S}[\phi, \tilde{g}_{\mu\nu}]$ , one can define the new action

$$S[\phi, g_{\mu\nu}] \equiv \tilde{S}[\phi, \tilde{g}_{\mu\nu} = C g_{\mu\nu} + D \phi_\mu \phi_\nu]$$

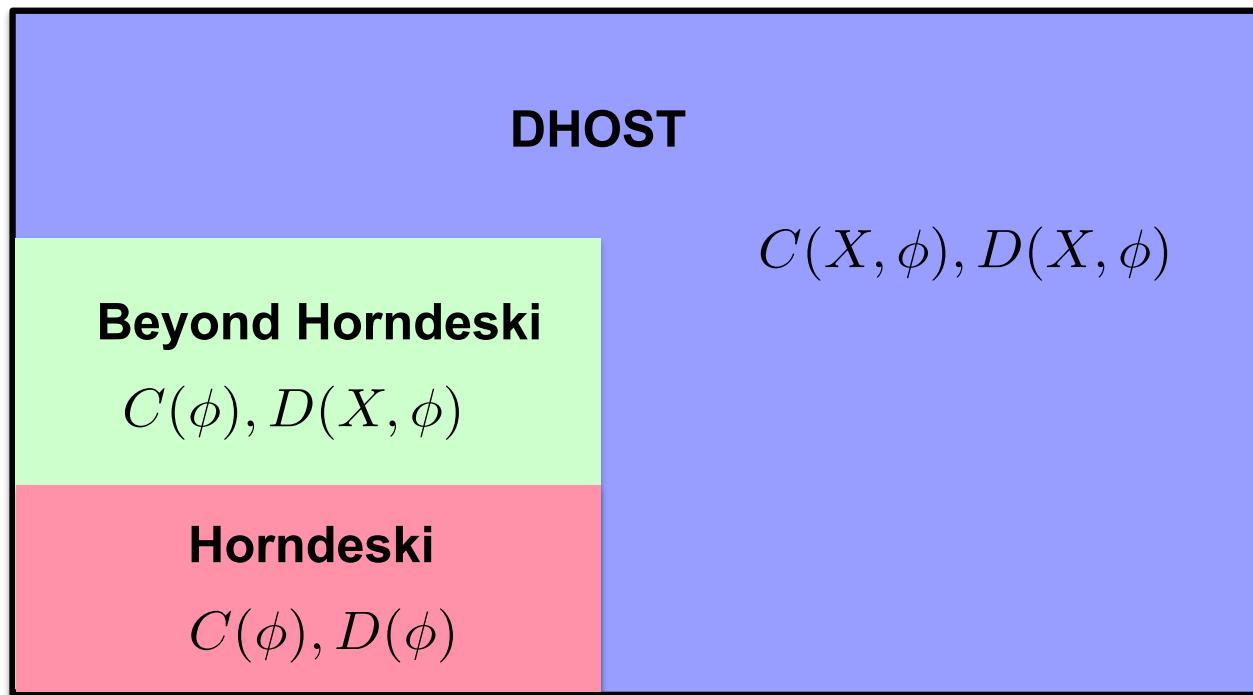
- Disformal transformation of quadratic DHOST theories ?

$$\tilde{S} = \int d^4x \sqrt{-\tilde{g}} \left[ \tilde{f}_2 {}^{(4)}\tilde{R} + \sum_I \tilde{a}_I \tilde{L}_I^{(2)} \right]$$

The structure of DHOST theories is preserved and all classes are stable.

# Disformal transformations

- Stability under  $g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = C g_{\mu\nu} + D \partial_\mu \phi \partial_\nu \phi$



- When **matter** is included (with minimal coupling), two disformally related theories are **physically inequivalent** !

# DHOST in cosmology

- Cosmological background:  $a(t)$ ,  $\phi(t)$
- **Quadratic action** (ADM form in the uniform  $\phi$  gauge)

$$S_{\text{quad}} = \int d^3x dt a^3 \frac{M^2}{2} \left\{ \delta K_{ij} \delta K^{ij} - \left( 1 + \frac{2}{3} \alpha_L \right) \delta K^2 + (1 + \alpha_T) \left( R \frac{\delta \sqrt{h}}{a^3} + \delta_2 R \right) \right. \\ \left. + H^2 \alpha_K \delta N^2 + 4H \alpha_B \delta K \delta N + (1 + \alpha_H) R \delta N + 4\beta_1 \delta K \delta \dot{N} + \beta_2 \delta \dot{N}^2 + \frac{\beta_3}{a^2} (\partial_i \delta N)^2 \right\}$$

- Horndeski:  $\alpha_K$ ,  $\alpha_B$ ,  $\alpha_T$ ,  $\alpha_M$
- Beyond Horndeski:  $\alpha_H$  DHOST:  $\beta_1$

- The parameters  $\alpha_L, \beta_1, \beta_2, \beta_3$  are related by

$$\mathcal{C}_I : \alpha_L = 0, \beta_2 = -6\beta_1^2, \beta_3 = -2\beta_1 [2(1 + \alpha_H) + \beta_1(1 + \alpha_T)]$$

$$\mathcal{C}_{II} : \beta_I = \mathcal{F}_I(\alpha_L, \alpha_T, \alpha_H)$$

# Scalar degree of freedom

- Scalar perturbations:  $\delta N$ ,  $N_i \equiv \partial_i \psi$ ,  $h_{ij} = a^2(t) e^{2\zeta} \delta_{ij}$
- Quadratic action for the **physical degree of freedom**:

$$S^{(2)} = \frac{1}{2} \int dx^3 dt a^3 \left[ \mathcal{K}_t \dot{\zeta}^2 + \mathcal{K}_s \frac{(\partial_i \zeta)^2}{a^2} \right]$$

with e.g. for beyond Horndeski theories

$$\mathcal{K}_t \equiv \frac{\alpha_K + 6\alpha_B^2}{(1 + \alpha_B)^2}, \quad \mathcal{K}_s \equiv 2M^2 \left\{ 1 + \alpha_T - \frac{1 + \alpha_H}{1 + \alpha_B} \left( 1 + \alpha_M - \frac{\dot{H}}{H^2} \right) - \frac{1}{H} \frac{d}{dt} \left( \frac{1 + \alpha_H}{1 + \alpha_B} \right) \right\}$$

- Stability (neither ghost nor gradient instability)

$$\mathcal{K}_t > 0 \quad c_s^2 \equiv -\frac{\mathcal{K}_s}{\mathcal{K}_t} > 0$$

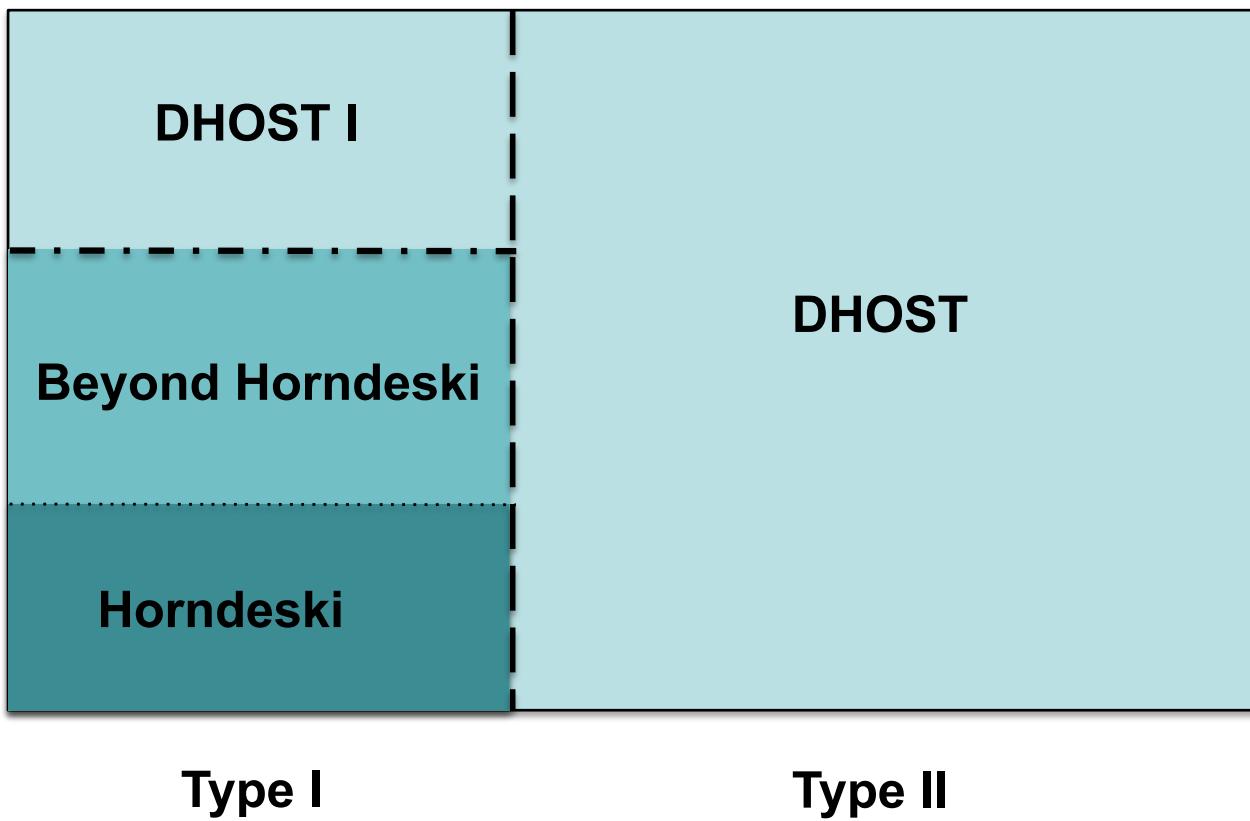
# Tensor degrees of freedom

- Quadratic action for the **tensor modes**:

$$S_{\gamma}^{(2)} = \frac{1}{2} \int dt d^3x a^3 \left[ \frac{M^2}{4} \dot{\gamma}_{ij}^2 - \frac{M^2}{4} (1 + \alpha_T) \frac{(\partial_k \gamma_{ij})^2}{a^2} \right]$$

- Stability:  $M^2 > 0$  and  $c_T^2 \equiv 1 + \alpha_T > 0$
- DHOST theories in category  $\mathcal{C}_{\text{II}}$ : gradient instability either in the scalar or the tensor sector

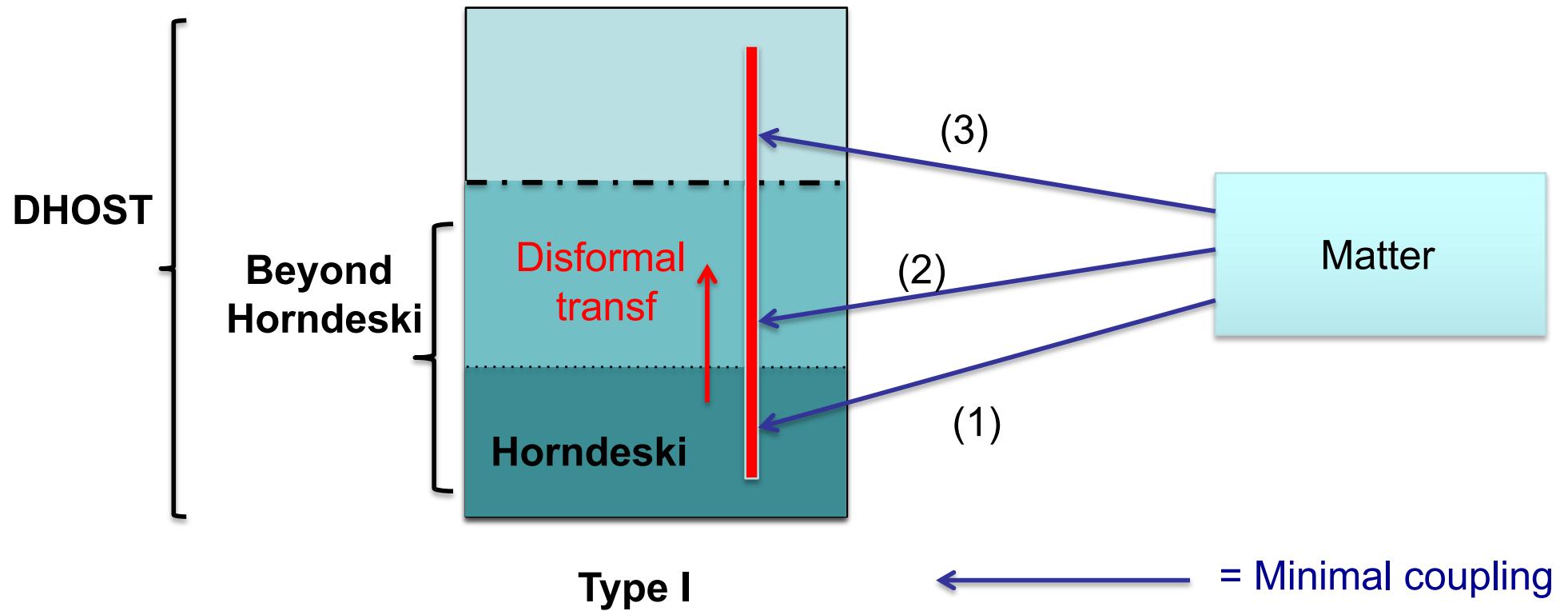
# DHOST theories



(Gradient instability either in  
the scalar or the tensor sector !)

# Coupling matter to gravity

- Disformal transformations:  $\tilde{g}_{\mu\nu} = C g_{\mu\nu} + D \partial_\mu \phi \partial_\nu \phi$



Theories (1), (2) & (3) are distinct because of the coupling to matter.

# GW170817 constraints

- Constraint on the speed of GW:  $|c_{\text{gw}} - c|/c \lesssim 10^{-15}$
- Assuming  $c_{\text{gw}} = c$  exactly (in any background) implies

1. Quadratic terms: with  $a_1 = 0$

$$L_{\text{ADM}} = (f - Xa_1)K_{ij}K^{ij} - f^{(3)}R$$

2. No cubic term [ e.g.  $K_j^i K_l^j K_i^l$  ,  $KK_j^i K_i^j$  ]

- Only quadratic DHOST theories (of type I) with  $a_1 = 0$

# DHOST theories with $c_g = c$

- Taking into account the degeneracy conditions,

$$a_1 = a_2 = 0 ,$$

**free functions:**  $f(X, \phi)$ ,  $a_3(X, \phi)$

$$a_4 = \frac{1}{8f_2} [48f_{2X}^2 - 8(f_2 - Xf_{2X})a_3 - X^2a_3^2] ,$$

$$a_5 = \frac{1}{2f_2} (4f_{2X} + Xa_3) a_3$$

- **Total Lagrangian: 4 free functions**

$$\begin{aligned} L_{c_g=1}^{\text{DHOST}} &= f_2(X, \phi) {}^{(4)}R + P(X, \phi) + Q(X, \phi) \square\phi \\ &\quad + a_3(X, \phi) \phi^\mu \phi^\nu \phi_{\mu\nu} \square\phi + a_4(X, \phi) \phi^\mu \phi_{\mu\nu} \phi_\lambda \phi^{\lambda\nu} \\ &\quad + a_5(X, \phi) (\phi_\mu \phi^{\mu\nu} \phi_\nu)^2 \end{aligned}$$

Horndeski:  $G_2(X, \phi)$ ,  $G_3(X, \phi)$ ,  $G_4(\phi)$

# Gravitational decay

- GW decay into dark energy

Creminelli, Lewandowski,  
Tambalo & Vernizzi '1809

vertex:  $\frac{1}{\Lambda_3^3} \ddot{\gamma}_{ij} \partial_i \pi \partial_j \pi$

$$\Lambda_3 \sim (M_P H_0^2)^{1/3}$$

$$\Gamma_{h \rightarrow \pi\pi} \sim (\alpha_H + 2\beta_1)^2 \frac{\omega_{\text{gw}}^7}{\Lambda_3^6}$$

- DHOST theories with  $a_3 = 0$  avoid this problem

$$L_{c_g=1, \text{no decay}}^{\text{DHOST}} = P(X, \phi) + Q(X, \phi) \square \phi + f(X, \phi) {}^{(4)}R + 6 \frac{f_X^2}{f} \phi^\mu \phi_{\mu\nu} \phi^{\nu\rho} \phi_\rho$$

# Gravitation in DHOST with $c_g = c$

DL, Saito, Yamauchi & Noui '1711 [see also Crisostomi & Koyama '1711  
and Dima & Vernizzi '1712]

- Quasi-static approximation on scales  $r \ll H^{-1}$

$$ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Psi)\delta_{ij}dx^i dx^j$$

$$\phi = \phi_c(t) + \chi(r)$$

- Equations of motion for  $\chi, \Phi$  and  $\Psi$ 
  - Scalar equation
  - Metric equations
- Matter source: spherical body with density  $\rho(r)$

# Gravitation in DHOST with $c_g = c$

DL, Saito, Yamauchi & Noui '1711 [see also Crisostomi & Koyama '1711  
and Dima & Vernizzi '1712]

- Gravitational laws

$$\frac{d\Phi}{dr} = \frac{G_N \mathcal{M}(r)}{r^2} + \Xi_1 G_N \mathcal{M}''(r),$$

$$\frac{d\Psi}{dr} = \frac{G_N \mathcal{M}(r)}{r^2} + \Xi_2 \frac{G_N \mathcal{M}'(r)}{r} + \Xi_3 G_N \mathcal{M}''(r)$$

$$\mathcal{M}(r) \equiv 4\pi \int_0^r \bar{r}^2 \rho(\bar{r}) d\bar{r}$$

with  $(8\pi G_N)^{-1} \equiv 2f(1 + \Xi_0)$

where the coefficients  $\Xi_I$  are given in terms of  $f, f_X, a_3$  and  $\dot{\phi}_c$

- Breaking of the Vainshtein screening **inside matter !**

In particular, sound speed is modified Babichev & Lehebel '1810

# Gravitation in DHOST with $c_g = c$

- The four coefficients  $\Xi_I$  depend on only 2 parameters

$$\Xi_0 = -\alpha_H - 3\beta_1, \quad \Xi_1 = -\frac{(\alpha_H + \beta_1)^2}{2(\alpha_H + 2\beta_1)},$$
$$\Xi_2 = \alpha_H, \quad \Xi_3 = -\frac{\beta_1(\alpha_H + \beta_1)}{2(\alpha_H + 2\beta_1)}.$$

- Constraints on the coefficients

$$\Xi_0 = \frac{G_{\text{gw}}}{G_N} - 1 \quad |\Xi_0| < 10^{-2}$$

Beltran Jimenez, Piazza & Velten 1507

$$\text{Stars: } -\frac{1}{6} < \Xi_1 \lesssim 7 \times 10^{-3} \quad [\text{Saito, Yamauchi, Mizuno, Gleyzes \& DL '15}]$$

[Sakstein 15]

- Special case:  $\alpha_H + 2\beta_1 = 0$

# Gravitation in DHOST with $a_1 = a_3 = 0$

[ Hirano, Kobayashi & Yamauchi; Crisostomi, Lewandowski & Vernizzi '1903 ]

- Inside matter ( $\beta_1 > 0$ )

$$\frac{d\Phi}{dr} = G_{\text{gw}}(1 + \varepsilon_{\Phi}^{\text{in}}) \frac{\mathcal{M}(r)}{r^2}, \quad \varepsilon_{\text{in}}^{\Phi} = \frac{\beta_1(2 - \beta_1)}{(1 - \beta_1)^2}$$

$$\frac{d\Psi}{dr} = G_{\text{gw}}(1 + \varepsilon_{\Psi}^{\text{in}}) \frac{\mathcal{M}(r)}{r^2}, \quad \varepsilon_{\text{in}}^{\Psi} = -\frac{\beta_1^2}{(1 - \beta_1)^2}$$

- Outside matter

$$\varepsilon_{\text{out}}^{\Phi} = \varepsilon_{\text{in}}^{\Phi} - \frac{\beta_1 v}{2\xi(1 - \beta_1)^2}, \quad \varepsilon_{\text{out}}^{\Psi} = \varepsilon_{\text{in}}^{\Psi} + \frac{\beta_1 v}{2\xi(1 - \beta_1)^2}.$$

$$v \equiv \alpha_B - \alpha_M(1 - \beta_1) + \beta_1, \quad \xi \equiv 2\alpha_B - \alpha_M(1 - \beta_1) + 2\beta_1 - 2\frac{\dot{\beta}_1}{H}$$

- Vainshtein screening broken **outside** !

Constraint :  $\beta_1 \lesssim 10^{-5}$  (or  $\beta_1 \lesssim 10^{-2}$  if  $v = 2\xi$ )

# **Compact objects in DHOST theories**

# Neutron stars in beyond Horndeski

Babichev, Koyama, DL, Saito & Sakstein '16

- Model

$$S = \int d^4x \sqrt{-g} \left[ M_P^2 \left( \frac{R}{2} - \Lambda \right) - k_2 X + f_4 L_4^{\text{bH}} \right]$$

with

$$L_4^{\text{bH}} = -X \left[ (\square\phi)^2 - (\phi_{\mu\nu})^2 \right] + 2\phi^\mu\phi^\nu [\phi_{\mu\nu}\square\phi - \phi_{\mu\sigma}\phi_\nu^\sigma]$$

- Cosmological solution: **de Sitter** with  $\dot{\phi} = v_0 \neq 0$ ,  $H \neq 0$

$$ds^2 = -(1 - H^2 r^2) dt^2 + \frac{dr^2}{1 - H^2 r^2} + r^2 d\Omega_2^2$$

$$\phi(r, t) = v_0 t + \frac{v_0}{2H} \ln(1 - H^2 r^2)$$

# Neutron stars in beyond Horndeski

Babichev, Koyama, DL, Saito & Sakstein '16

- Spherical symmetric solutions

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2d\Omega_2^2$$

with  $\nu(r) = \nu_{\text{cosmo}} + \delta\nu(r)$ ,  $\lambda(r) = \lambda_{\text{cosmo}} + \delta\lambda(r)$

$$\phi(t, r) = \phi_{\text{cosmo}}(t, r) + \delta\phi(r)$$

- **External solution:** Schwarzschild-de Sitter

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega_2^2, \quad f \equiv 1 - \frac{2G_N M}{r} - H^2 r^2$$

$$\phi(t, r) = v_0 \left[ t - \int dr \frac{\sqrt{1-f}}{f} \right] \quad G_N \equiv \frac{3G}{5\sigma^2 - 2}$$

# Neutron stars in beyond Horndeski

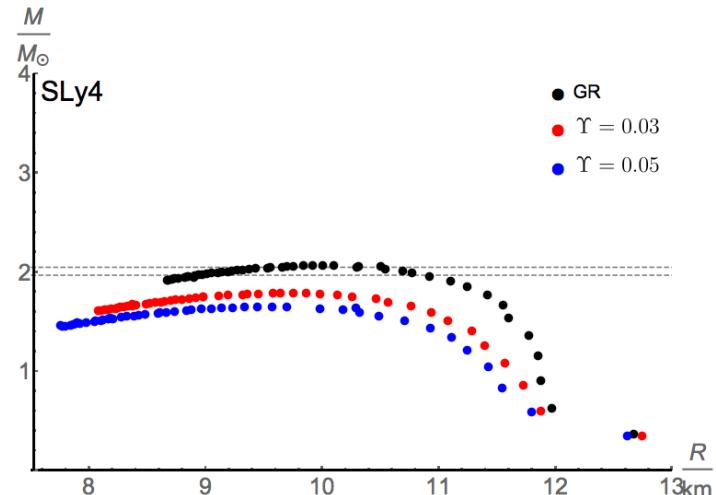
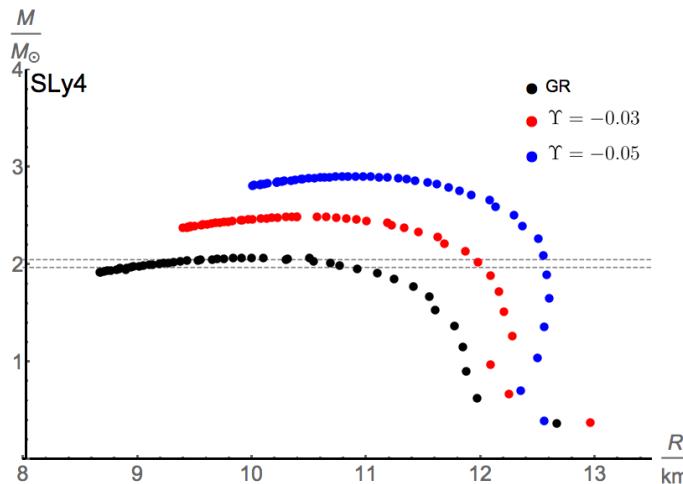
Babichev, Koyama, DL, Saito & Sakstein '16

- Internal solution

System analog to TOV equations

$$\Upsilon = 4\Xi_1 = -2\alpha_H$$

- Mass-radius relations



- Other equations of state
- Slowly rotating stars

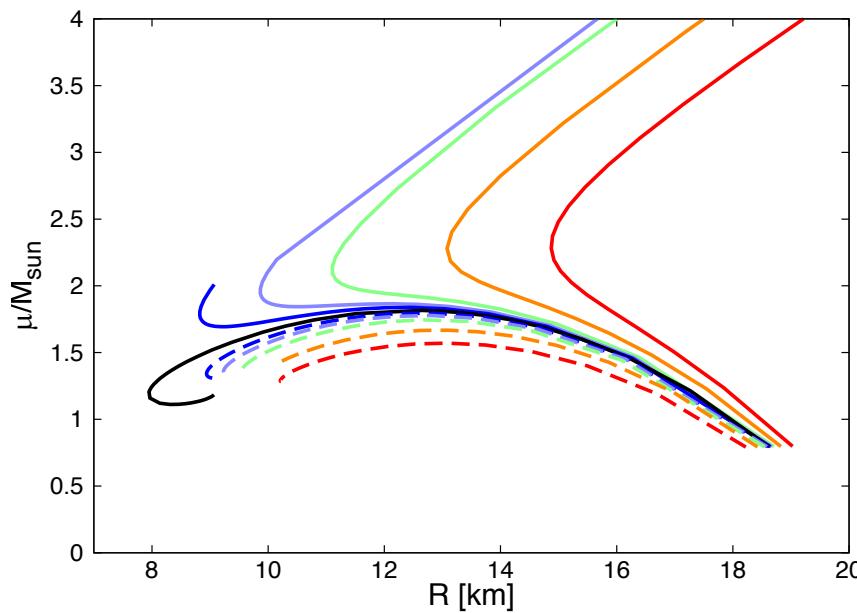
Sakstein, Babichev, Koyama, DL & Saito '16

# Neutron stars in DHOST

Kobayashi & Hiramatsu '1803

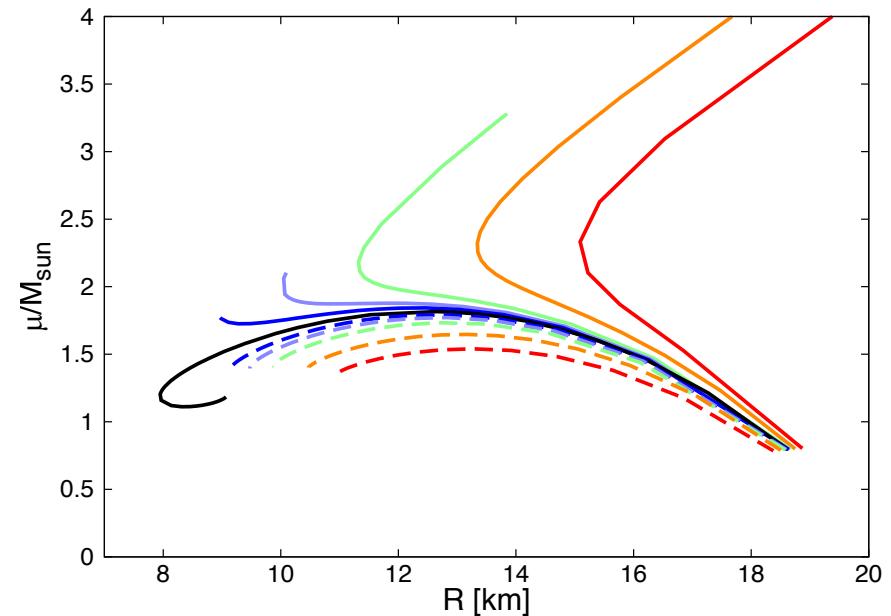
- Model with  $c_g = c$

$$f = \frac{M_p^2}{2} + \alpha X^2, \quad a_1 = 0, \quad a_3 = -8\alpha - \beta$$



Varying

$$\bar{\alpha} = \alpha v^4 / M_p^2$$



$$\bar{\beta} = \beta v^4 / M_p^2$$

# Black holes in DHOST

- Spherically symmetric solutions
  - Various exact solutions
  - Scalar field:  $\phi(r)$  or  $\phi = qt + \psi(r)$
- Linear perturbations:
  - Odd-parity & even-parity modes
- Rotating solutions
  - Solution: Kerr-de Sitter geometry  
*Charmousis, Crisostomi, Gregory & Stergioulas '1903*
  - Perturbations: see talk by M. Crisostomi tomorrow

Ask

E. Babichev

C. Charmousis

G. Esposito-Farese

A. Lehébel

# Conclusions

- **DHOST theories:** most general framework for scalar-tensor theories propagating a single scalar dof.
- Allowed space of models is strongly reduced **if one imposes  $c_{gw} = c$  and the no-decay constraint**, but this depends on the regime of interest.
- For cosmological perturbations, DHOST theories predict deviations from the standard model, which can be tested in future observations of large scale structures.
- One also finds interesting deviations from GR in the astrophysical context (e.g. neutron stars & black holes).