

# Exotic Compact Objects with Soft Hair

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*Black Holes and Neutron Stars in Modified Gravity - Meudon*

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work with\*

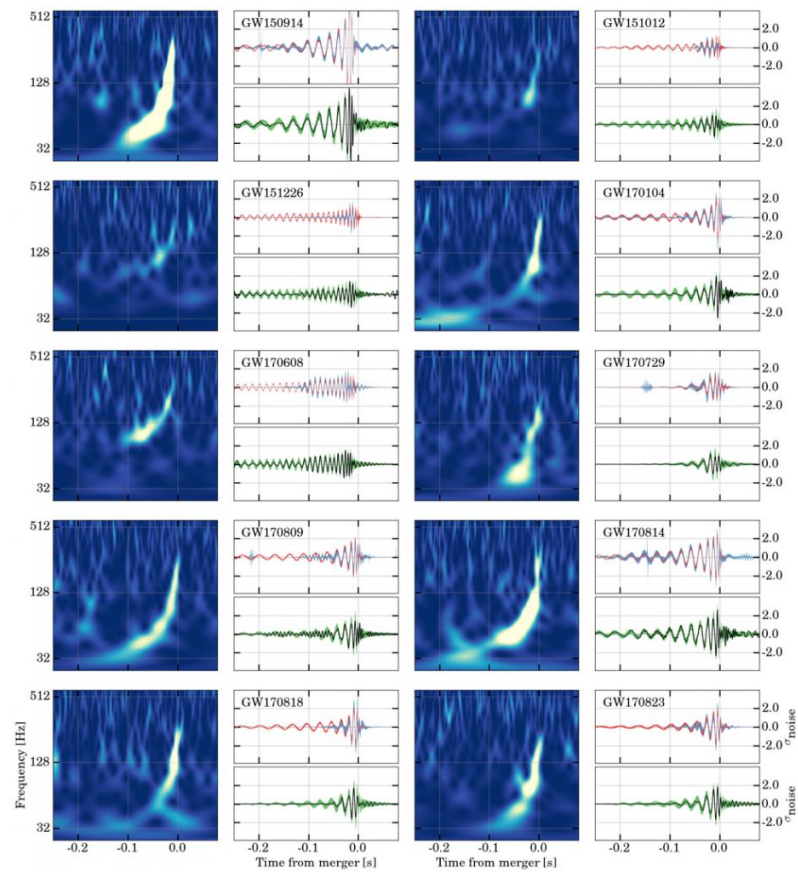
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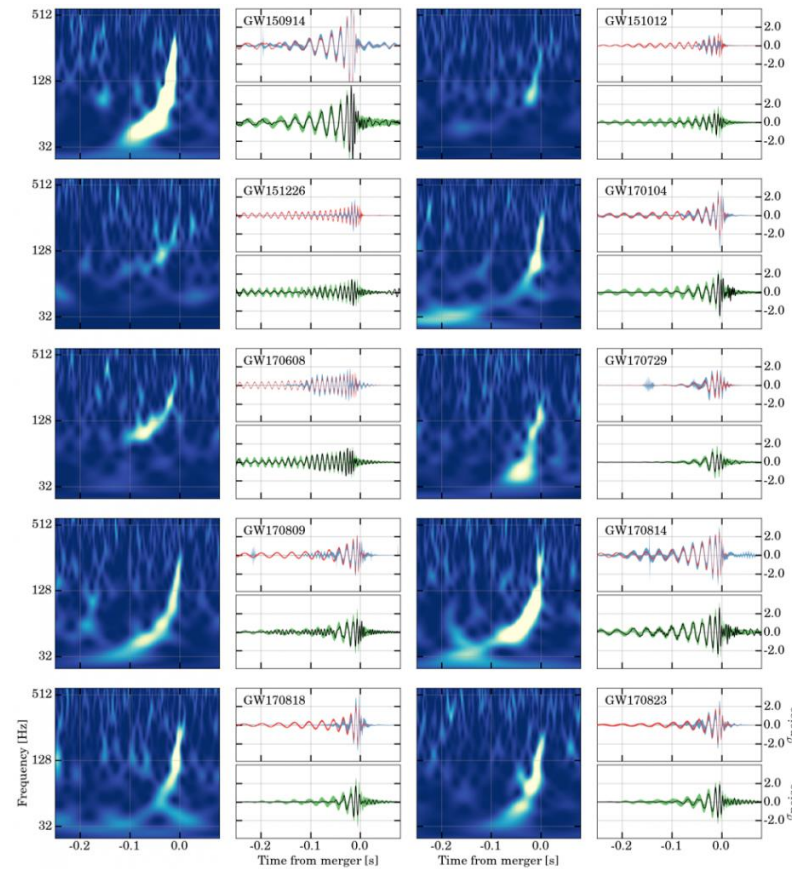
*\*not EPR but RPE*



01-02

BH-BH

01-02



Are they really black holes?  
Or very good impostors?

Impostors (aka mimickers)  
must be extremely compact

Exist over a wide mass range?

Stellar only?

All stellar range, or only low?

Supermassive too?

Black holes are scale-free

Very difficult to have scale-free material  
objects

Impostors may replace BHs entirely, or  
be a preferred alternative in certain  
phenomena/mass ranges

Horizonless impostors may play a role in  
BH info problem

(eg fuzzballs)



Impostors require drastically new physics

## **Exotic Compact Objects**

### **ECOs**

They may be unlikely, but

- foil to test BH paradigm
- discovery would be a bombshell

ECOs vs BHs

**BHs**

Extreme simplicity

# Uniqueness

Their physics is determined by just  
two numbers

$M, J$

# Kerr multipoles

*Geroch, Hansen*

mass:  $\mathcal{M}_\ell$

current:  $\mathcal{S}_\ell$

$$\mathcal{M}_0 = M$$

$$\mathcal{S}_1 = J$$

$$\mathcal{M}_\ell^{BH} + i\mathcal{S}_\ell^{BH} = M^{\ell+1} \left( i \frac{J}{M^2} \right)^\ell$$

dimensionless

$$\bar{\mathcal{M}}_\ell^{BH} + i\bar{\mathcal{S}}_\ell^{BH} = \left(i\frac{J}{M^2}\right)^\ell$$

eg

$$\bar{\mathcal{M}}_2^{BH} = -\left(\frac{J}{M^2}\right)^2 : \text{mass quadrupole}$$

$$\bar{\mathcal{S}}_3^{BH} = -\left(\frac{J}{M^2}\right)^3 : \text{current octupole}$$

$$\mathcal{M}_\ell^{BH} + i\mathcal{S}_\ell^{BH} = M^{\ell+1} \left( i \frac{J}{M^2} \right)^\ell$$

Any independent measure of **three**  
multipoles is a null test of Kerr

eg  $M, J$  and quadrupole  $\mathcal{M}_2$

**Quasinormal ringdown**

Two qnm's

**Tidal deformation**

Zero for Schwarzschild/slow Kerr

**Non-tidal multipoles?**



**ECOs**

Huge model dependence

Stationary compact objects:  
multipoles depend on specific physics model

Can we find a general parametrization?

What kind of physics  
—eg new scales of physics—  
is involved if a non-Kerr multipole is measured?

# ECOs in **Unmodified gravity**

A general, ECO-model-independent analysis

Agnostic about what goes on below the ECO  
surface

Assume vacuum GR rules the exterior of the  
ECO

*Unmodified gravity*

Exotism is *all in the matter*, none in the gravity

Vacuum GR rules exterior

⇒ matter-model-independent characterization  
of deviations from Kerr

Different ECO models  $\Rightarrow$  different boundary conditions at the surface of the ECO

Different boundary conds  $\Leftrightarrow$  multipoles

$$\begin{aligned}\mathcal{M}_\ell^{\text{ECO}} &= \mathcal{M}_\ell^{\text{BH}} + \delta\mathcal{M}_\ell, \\ \mathcal{S}_\ell^{\text{ECO}} &= \mathcal{S}_\ell^{\text{BH}} + \delta\mathcal{S}_\ell,\end{aligned}$$

# Soft ECOs

Curvature at the surface is comparable to  
curvature of black hole

Smooth limit to black hole

No scale of new physics is distinguishable from  
exterior



# Hard ECOs

Curvature at the surface is much larger than  
curvature of black hole

Singular limit to black hole

Imply new scale of physics

High-energy effects drastically modify near-  
surface geometry

Note:

Softness of ECO refers to soft scales noticeable from the exterior

Interior physics may still require very high energy physics (eg large pressures) in order to support object near the BH limit

This talk

## Soft ECOs

Perturbative analysis of Schwarzschild solution  
(slow rotation)

Stationary, axisymmetric

No equatorial symmetry assumed

Perturbations of Schwarzschild in  
vacuum GR  $R_{\mu\nu} = 0$

Multipolar decomposition

# Boundary conditions

Asymp flatness

+

ECO surface: ECOphysics

# ECOphysics

Boundary conds specified for each  $\ell \geq 2$  by

$$\mathcal{M}_\ell \text{ and } \mathcal{S}_\ell$$

General & physical parametrization of soft hair  
(after hair conditioning)

Multipoles couple at higher perturbation orders

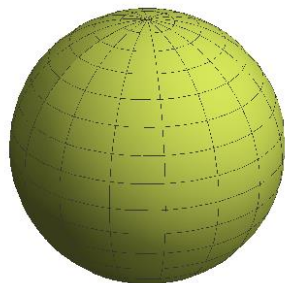
Simplest are sourced by lowest multipoles

Spin-induced

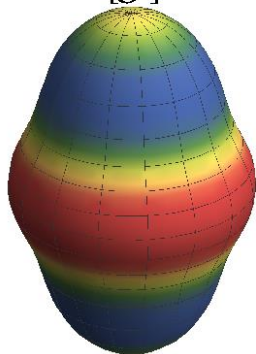
Mass-quadrupole-induced

We've generated a variety of them

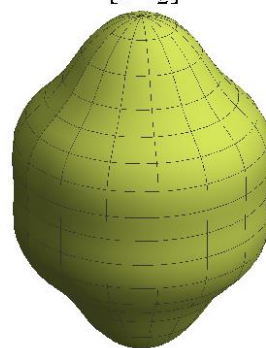
Schwarzschild



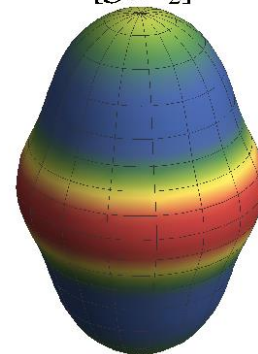
$[\mathcal{I}]$



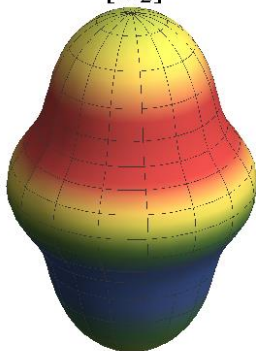
$[\mathcal{M}_2]$



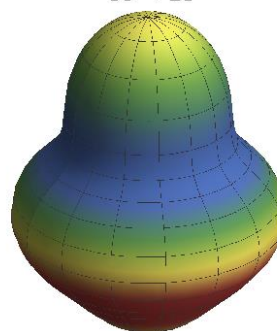
$[\mathcal{IM}_2]$



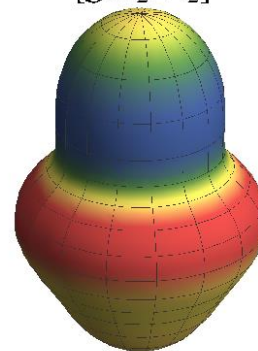
$[\mathcal{S}_2]$



$[\mathcal{IS}_2]$



$[\mathcal{IS}_2\mathcal{M}_2]$



color:  $g_{t\varphi}$



# Hair conditioner for ECOs

Softness of black hole limit

How large can soft hair be?

Depends on compactness and  
multipole type

# Compactness

$$\delta = \frac{r_0}{2M} - 1$$

$r_0$  = circumferential radius (axial symmetry)

# Bounds on multipoles from soft curvature

Induce a mass quadrupole  $\bar{\mathcal{M}}_2$  from spin

Kretschmann  $\mathcal{K} \sim \frac{1}{M^4} (\bar{\mathcal{M}}_2 - \bar{\mathcal{M}}_2^{BH}) \log \delta$

Hair conditioning

$$\Rightarrow \bar{\mathcal{M}}_2 - \bar{\mathcal{M}}_2^{BH} \sim \frac{1}{\log \delta}$$

# Bounds on multipoles from soft curvature

Non-spin-induced  $\bar{\mathcal{S}}_2$

$$\mathcal{K} \sim \frac{1}{M^4} \left( \frac{\bar{\mathcal{S}}_2}{\delta} \right)^2$$

Hair conditioning

$$\Rightarrow \bar{\mathcal{S}}_2 \sim \delta$$


Requiring regular BH limit (soft hair) implies


Non-spin-induced moments  
are  $\propto \delta$

Spin-induced moments  
are larger  $\propto \frac{1}{\log \delta}$

# *“Soft-hair theorem”* for ECOs

The closer a soft ECO is to a black hole,  
the less hairy it must be,  
with

$$\delta\mathcal{M}_\ell \rightarrow a_\ell \frac{\chi^\ell}{\log \delta} \mathcal{M}^{\ell+1}, \quad \delta\mathcal{S}_\ell \rightarrow c_\ell \frac{\chi^\ell}{\log \delta} \mathcal{M}^{\ell+1} \quad \text{spin-induced}$$


$$\delta\mathcal{M}_\ell \rightarrow b_\ell \mathcal{M}^{\ell+1} \delta, \quad \delta\mathcal{S}_\ell \rightarrow d_\ell \mathcal{M}^{\ell+1} \delta \quad \text{non-spin-induced}$$


Constraining soft ECOs with  
observations?  
(preliminary)

Spin-induced moments can be larger  
for given compactness  
 $\Rightarrow$  easier to constrain



EMRIs in LISA can constrain

$$\frac{\delta \mathcal{M}_2}{M^3} < 10^{-4}$$

for central supermassive object

Applied to soft ECOs:

$$\frac{\delta \mathcal{M}_2}{\mathcal{M}^3} \sim \begin{cases} |\log \delta|^{-1} \sim 10^{-2} \left| \log \left( \frac{10^6 M_\odot}{\ell_P} \right) \right|^{-1} \\ \delta^{-1} \sim 10^{-4} \left( \frac{L}{10^6 M_\odot} \right) \end{cases}$$

⇒ spin-induced multipolar deviations  
from Kerr can be constrained for  
objects with Planck-scale non-  
compactness

# Caveats

Vacuum GR exterior  
but valid if fall-off is fast

# Caveats

Perturbatively small multipoles

Not always valid: eg spin of boson star is  
quantized

Compactness not connected to BH limit

# Caveats

## Stability?

Depends on internal composition of object  
(or boundary conditions for time-  
dependent perturbations)

Must be assessed case-by-case

# Hard ECOs?

Stationary axisymmetric sector of vacuum GR is  
integrable

Exact solutions can be systematically  
constructed

# Hard ECOs?

Hard ECOs can be readily studied with low multipoles

No smooth limit to BH

# Hard ECOs?

May characterize hard scale vs compactness

Hard-hair theorem?



Thank you