

Hairy rotating black holes in cubic Galileon theory

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MEUDON
19/11/19

Outline

- 1 Context
- 2 The cubic Galileon
- 3 Approach
- 4 Static, spherically symmetric solutions
- 5 Rotating solutions

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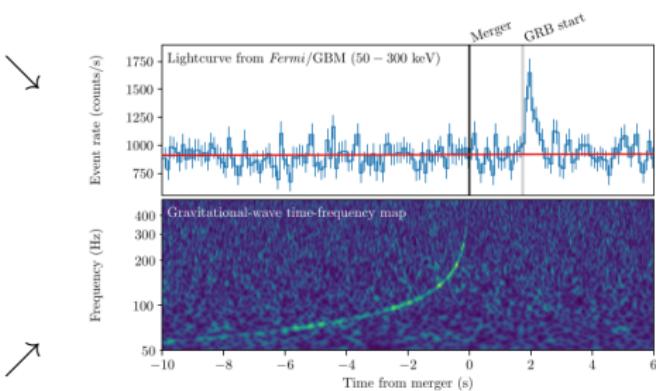
Strong-field tests: coalescing compact objects



FERMI



LIGO/VIRGO



GRB170817A & GW170817¹

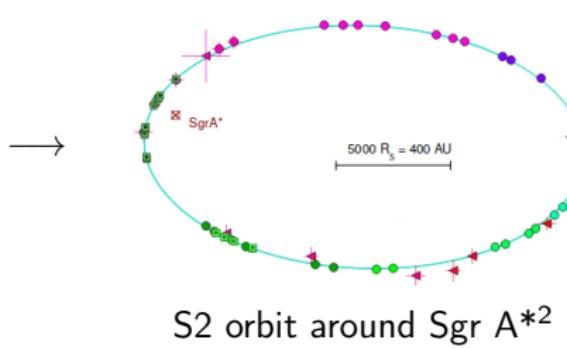
↳ constraint on c_{GW}

¹LIGO/Virgo collaboration, *ApJ* 848, 2 (2017)

Strong-field tests: high precision astrometry



GRAVITY

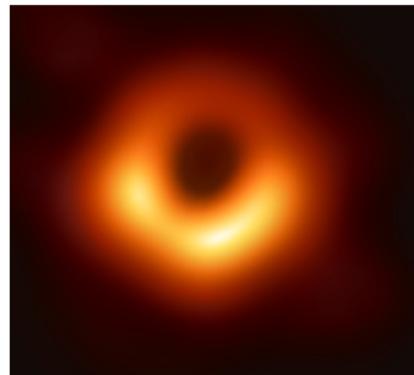


↳ constraints on timelike orbits

Strong-field tests: black hole shadows



EVENT HORIZON
TELESCOPE

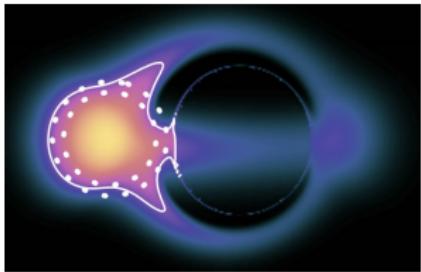


"Picture" of M87*³

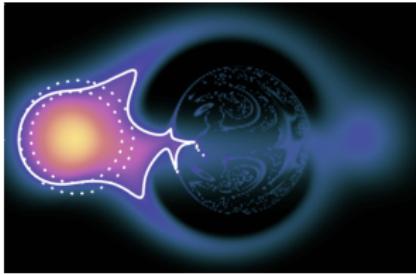
↳ constraints on null geodesics

³EHT collaboration, *APJL* 875, 1 (2019)

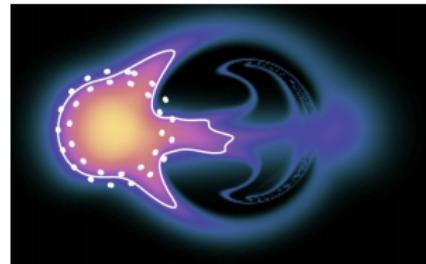
Predictions of GR



*Kerr black hole*⁴



*"Kerr black hole
with scalar hair"*⁴



*Boson star*⁵

⁴Vincent, Gourgoulhon, Herdeiro & Radu, *PRD* 94, 084045 (2016)

⁵Vincent, Meliani, Grandclément, Gourgoulhon & Straub, *CQG* 33, 105015 (2016)

Modified gravity

- Yet GR is not perfect: dark matter, dark energy, unification
 - ↳ modified theories: more “gravitational” fields, more dimensions...
 - ↳ scalar-tensor: cubic Galileon \subset Horndeski \subset DHOST

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The DGP model (2000)

- 5D-spacetime, matter restricted to hypersurface \mathcal{H} :

$$S_{DGP} = M^3 \int R^{(5)} \sqrt{|g^{(5)}|} d^5x + M_P^2 \int_{\mathcal{H}} [R + \mathcal{L}_m] \sqrt{|g|} d^4x$$

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- Reintroduced the Vainshtein mechanism

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- Reintroduced the Vainshtein mechanism
- “Soft” massive gravity on \mathcal{H} : continuum of gravitons
- Effective Lagrangian on \mathcal{H} + decoupling limit
 - ↳ scalar term $(\partial\phi)^2 \square\phi$

Cubic Galileon in vacuum

$$S_{CG} [g, \phi] = \int \left[\zeta (R - 2\Lambda) - \eta (\partial\phi)^2 + \gamma (\partial\phi)^2 \square\phi \right] \sqrt{|g|} d^4x$$

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↳ conserved current $J^\mu[g, \phi]$

↳ scalar equation: $\nabla_\mu J^\mu = 0$

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- Metric equations: $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}^{(\phi)}$

- Compatible with GW170817 & GRB170817A: $c_{GW} = c$
- Studied in context of:
 - black hole accretion
 - self-acceleration
 - self-tuning
 - tension with cosmological data⁶

⁶Leloup, Ruhlmann-Kleider, Neveu & de Mattia, *JCAP* 2019, 05 (2019)

No-hair theorems

Fields dressing a black hole metric are trivial for:

- 1970,1972: asympt. flat stationary BH of GR other than known^{7,8}
- 1971: vacuum stationary BH of Brans-Dicke⁹
- 2014: asympt. flat vacuum stationary BH of a large class of ST¹⁰

⁷Chase, *Commun. Math. Phys.* 19, 4 (1970)

⁸Bekenstein, *PRD* D 5, 2403 (1972)

⁹Hawking, *Commun. Math. Phys.* 25, 167 (1972)

¹⁰T.P. Sotiriou and V. Faraoni, *PRL* 108, 081103 (2012)

No-scalar-hair theorem for shift-symmetric Galileons

- Asymptotically flat, static, spherically symmetric BH + $\phi(r)$

↳ ϕ is trivial¹¹

¹¹L. Hui and A. Nicolis, *PRL* 110, 241104 (2013)

¹²Babichev, Charmousis, Lehébel & Moskalets, *JCAP*09, 011 (2016)

No-scalar-hair theorem for shift-symmetric Galileons

- Asymptotically flat, static, spherically symmetric BH + $\phi(r)$

↳ ϕ is trivial¹¹

- Introduce a linear time dependence¹²: $\phi = qt + \Psi(r)$

→ preserves spacetime symmetries

→ yields BH different from GR ones

↳ suggests a way to hairy rotating BH

¹¹L. Hui and A. Nicolis, *PRL* 110, 241104 (2013)

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Rotating black hole

- Scalar ansatz:

$$\phi(t, r, \theta) = qt + \Psi(r, \theta)$$

Rotating black hole

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- Circular black hole:

$$ds^2 = -N^2 dt^2 + A^2(dr^2 + r^2 d\theta^2) + B^2 r^2 \sin^2 \theta(d\varphi - \omega dt)^2$$

→ Adapted to rotating BH: Kerr, dilatonic Einstein-Gauss-Bonnet

→ Relevant for rotating stars with no meridional flow

Equations and boundary conditions

- Inject circular metric and scalar ansatz into the field equations:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}^{(\phi)}$$
$$\nabla_\mu J^\mu = 0$$

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- Set boundary conditions imposing:

- rotating event horizon ($N|_{\mathcal{H}} = 0, \omega|_{\mathcal{H}} = \Omega_{\mathcal{H}}$)

- flat asymptotics ($N_\infty = A_\infty = B_\infty = 1, \omega_\infty = 0$)

- Truncate decomposition onto basis functions (C++ library KADATH¹³):

$$\text{e.g. } A(r, \theta) = \sum_{i=0}^{N_r} \sum_{j=0}^{N_\theta} \tilde{A}_{ij} T_i(r) \cos(2j\theta)$$

→ PDE's transform into a nonlinear *algebraic* system $S(\tilde{X}) = 0$

¹³Grandclément, J. Comput. Phys. 229, 3334 (2010)

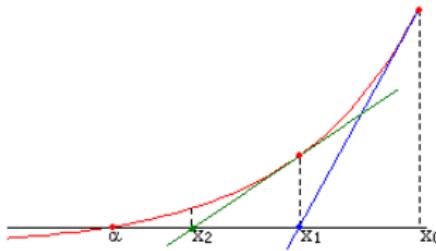
Numerical treatment

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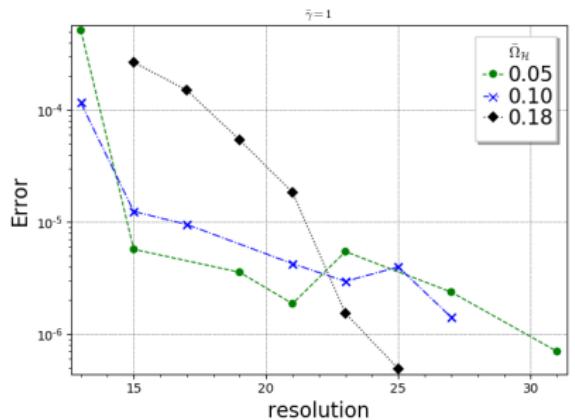
→ PDE's transform into a nonlinear *algebraic* system $S(\tilde{X}) = 0$

- Newton-Raphson iterations: $S(\tilde{X}^{(n)}) + dS_{\tilde{X}^{(n)}}(\tilde{X}^{(n+1)} - \tilde{X}^{(n)}) = 0$

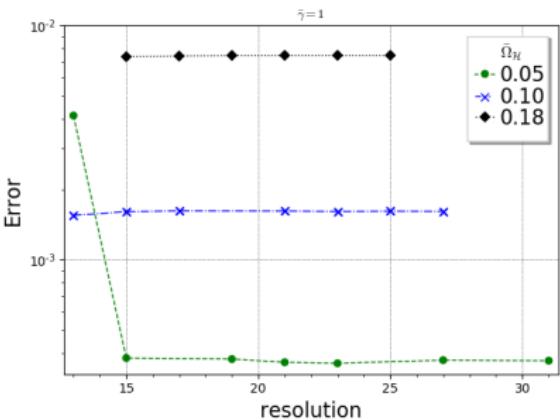


¹³Grandclément, J. Comput. Phys. 229, 3334 (2010)

Validation/violation of circularity assumption



Validation of equation ($r\theta$)
(fast decrease with resolution)



Violation of equation (tr)
(error independent of resolution)

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- No (dimensionless) angular velocity: $\bar{\Omega}_{\mathcal{H}} = r_{\mathcal{H}} \Omega_{\mathcal{H}} = 0$

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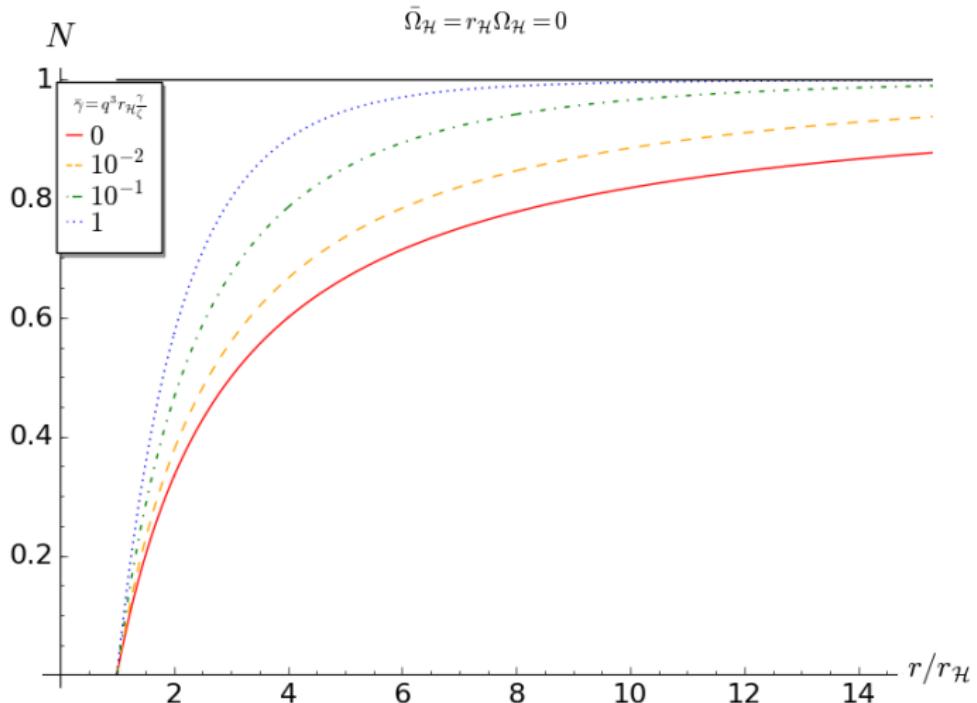
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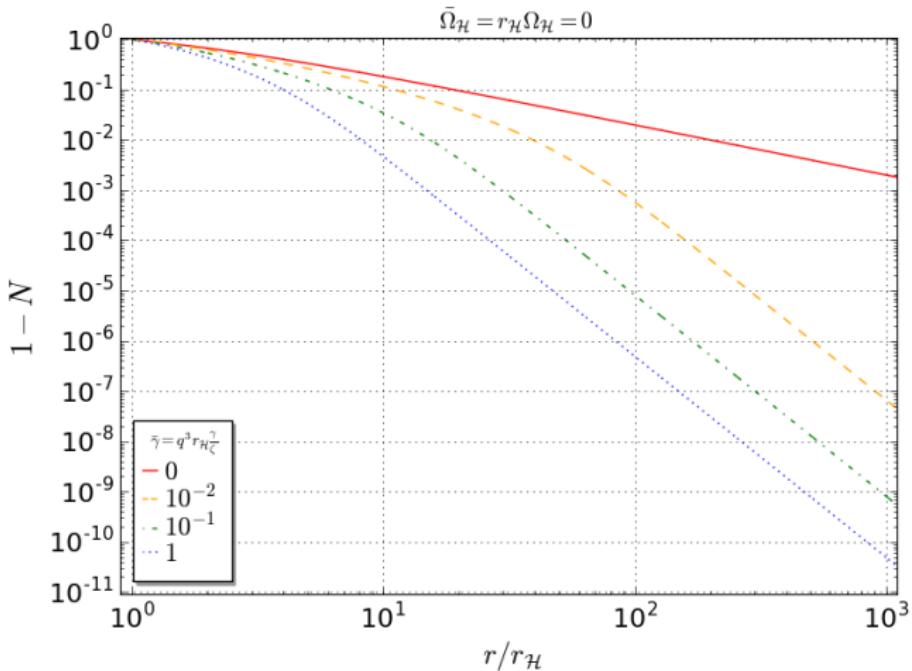
- Asymptotic flatness $\Rightarrow \eta = \Lambda = 0$:

$$S_{CG} [g, \phi] = \int \left[\zeta R + \gamma (\partial \phi)^2 \square \phi \right] \sqrt{|g|} d^4x$$

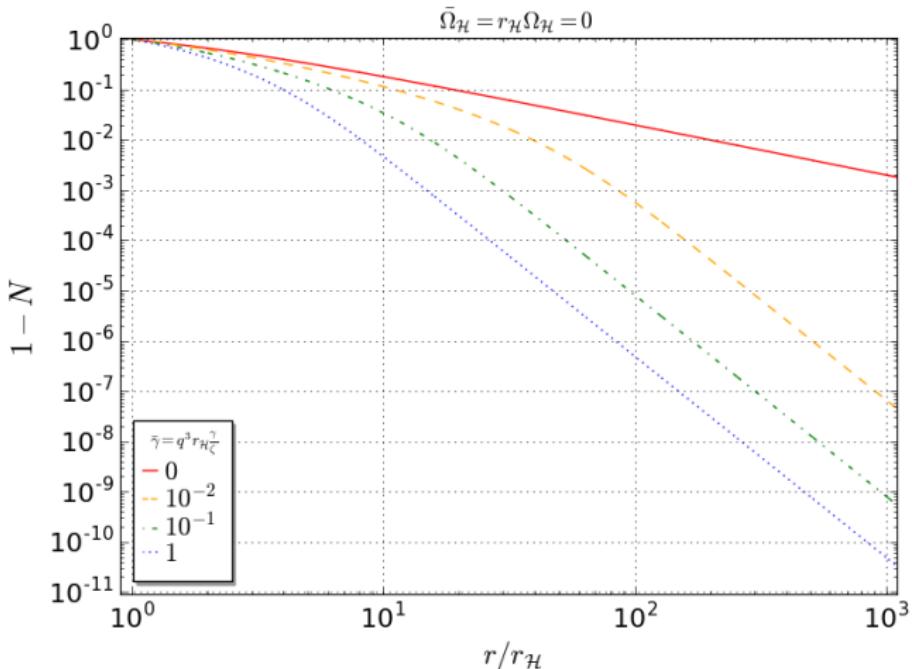
Radial profile of N



Asymptotic behaviour of N

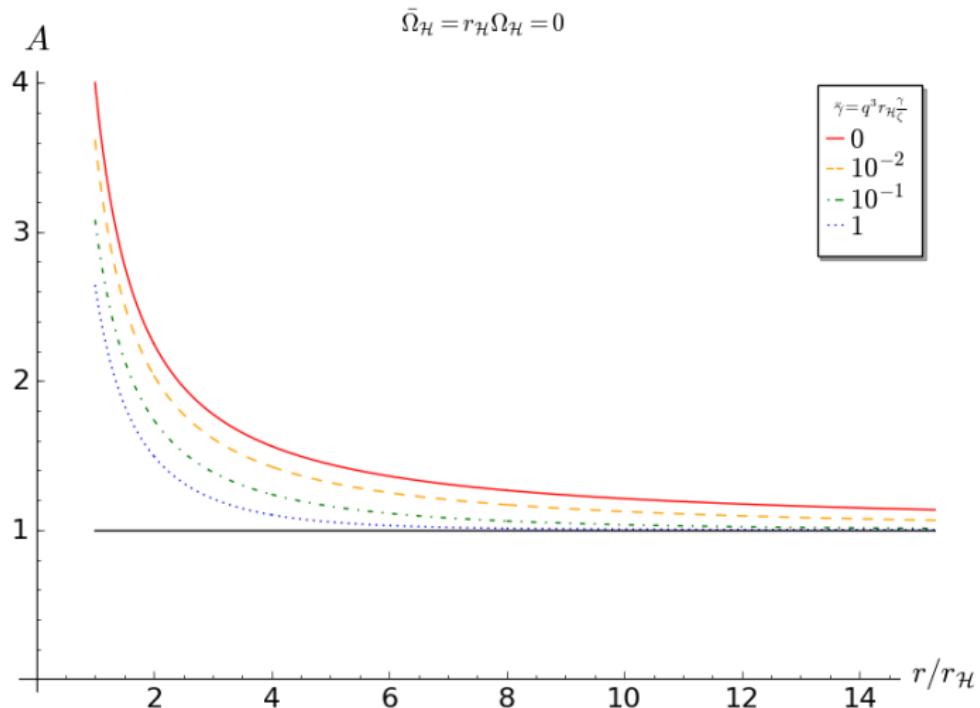


Asymptotic behaviour of N

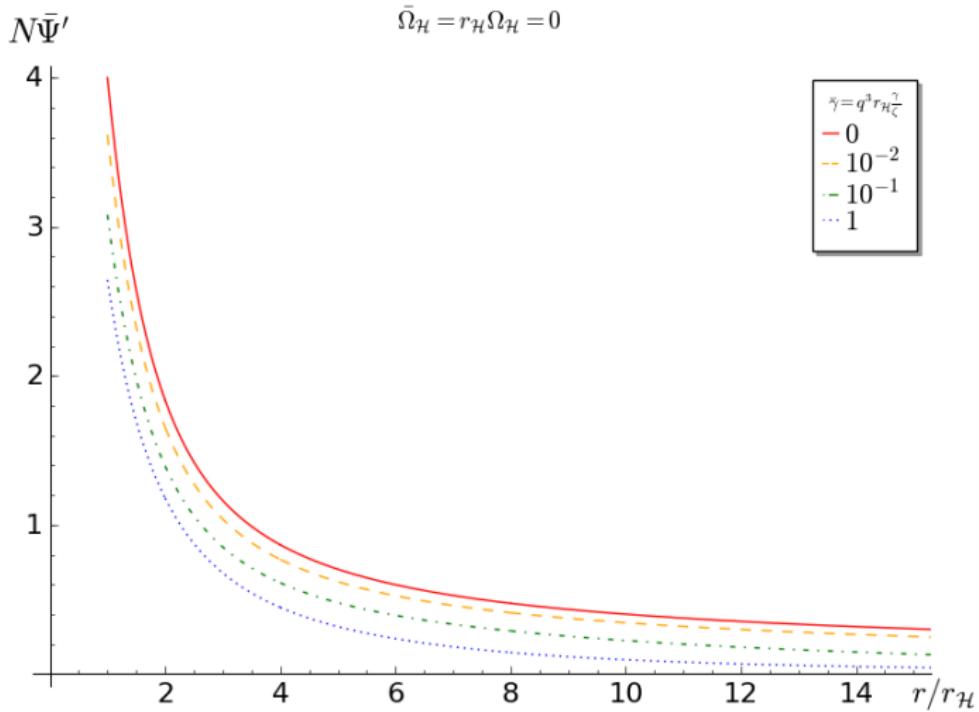


$$\longrightarrow N(r) \sim 1 - \frac{\alpha}{r^4} \text{ instead of } N(r) \sim 1 - \frac{M}{r}$$

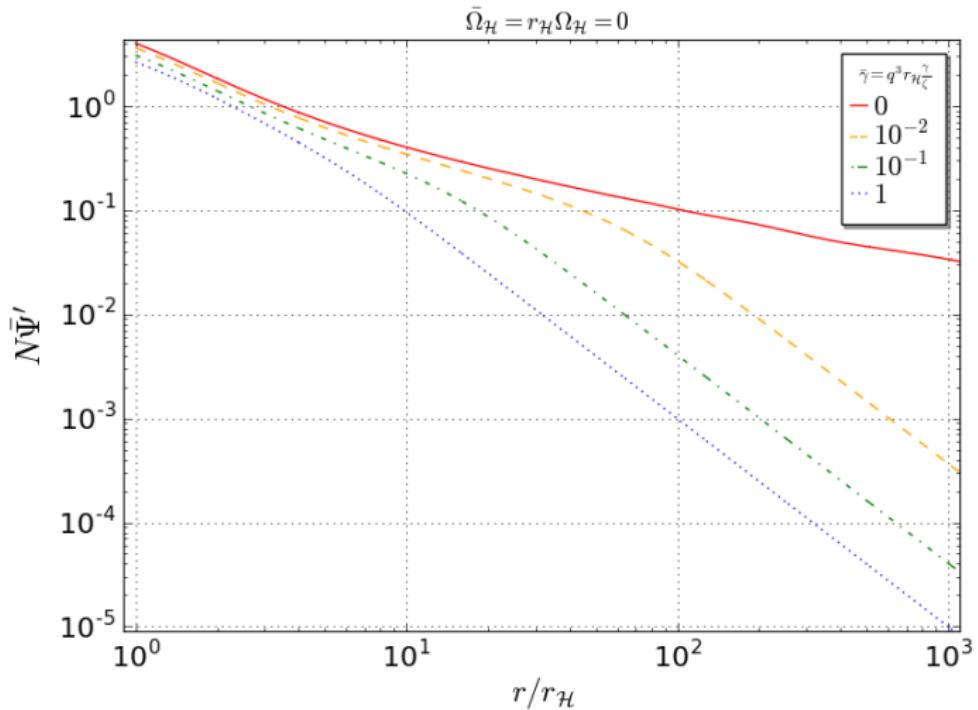
Radial profile of A



Radial profile of $N\bar{\Psi}'$



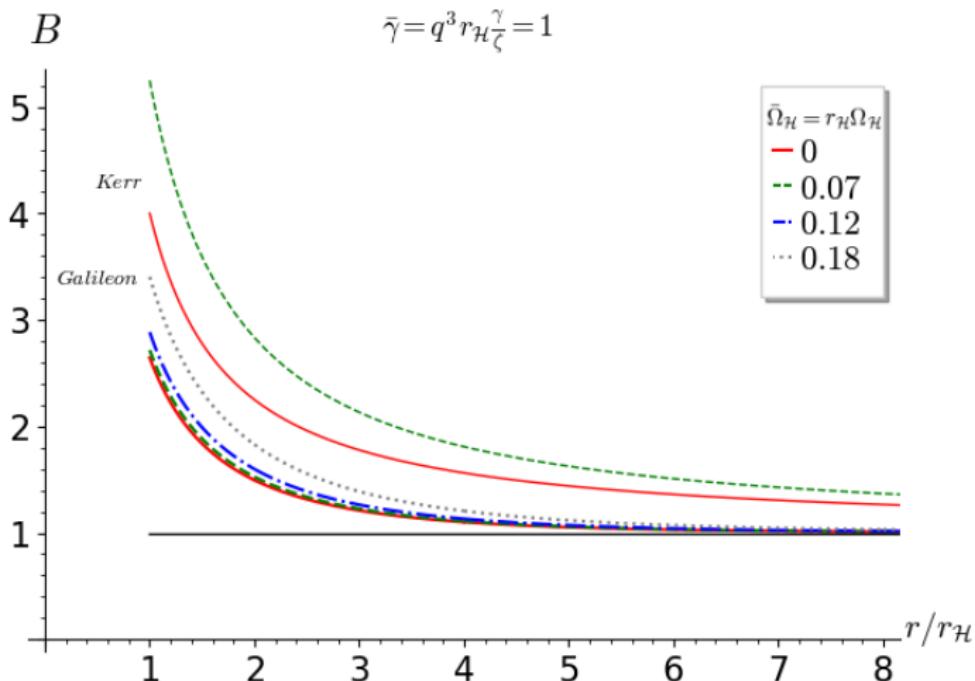
Radial profile of $Z = N\bar{\Psi}'$ ($\log - \log$)



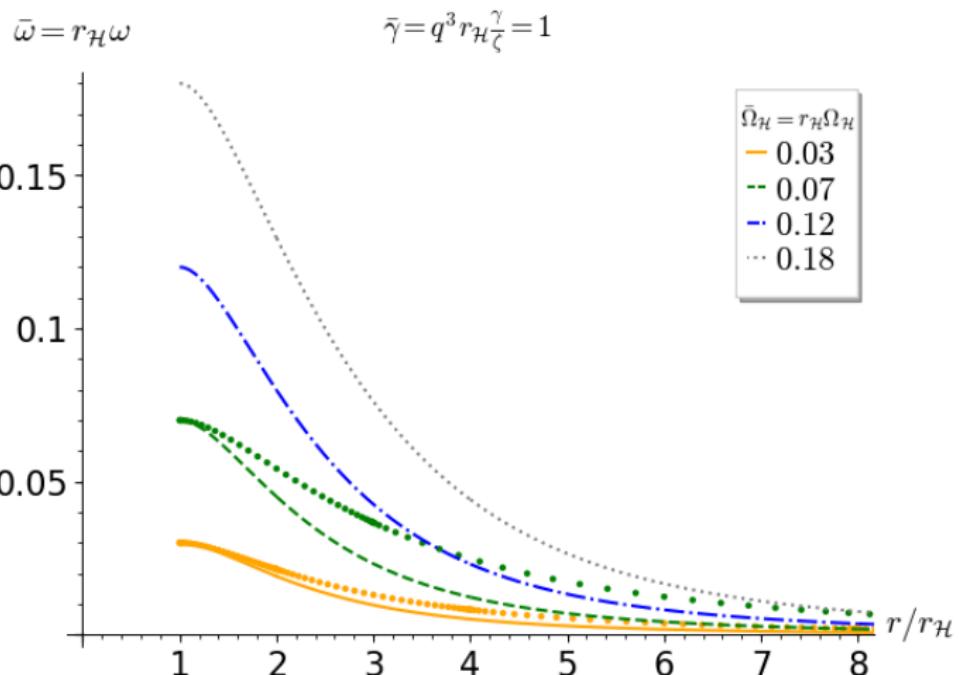
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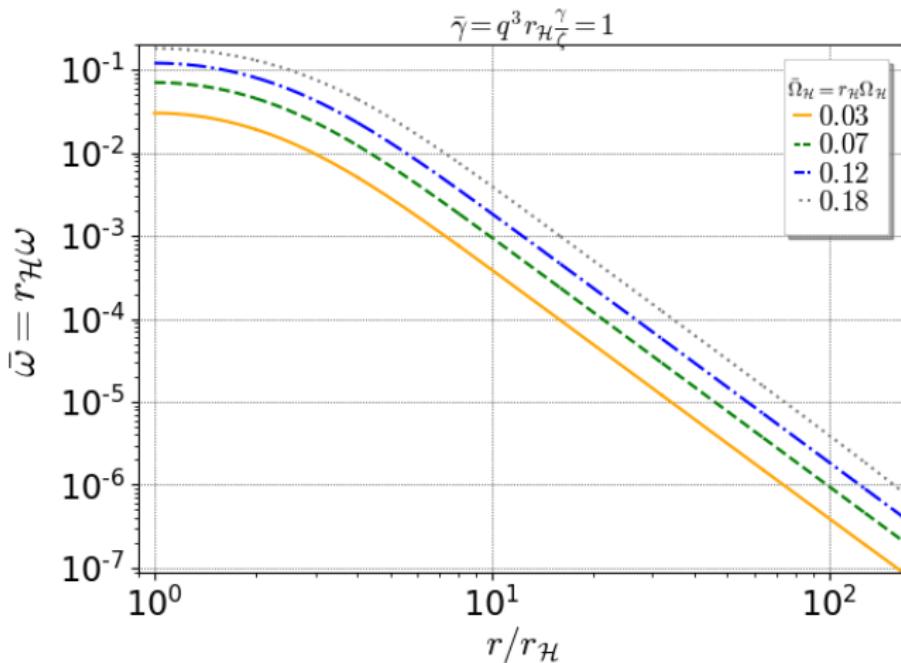
Radial profile of B



Radial profile of $\bar{\omega} = r_{\mathcal{H}}\omega$

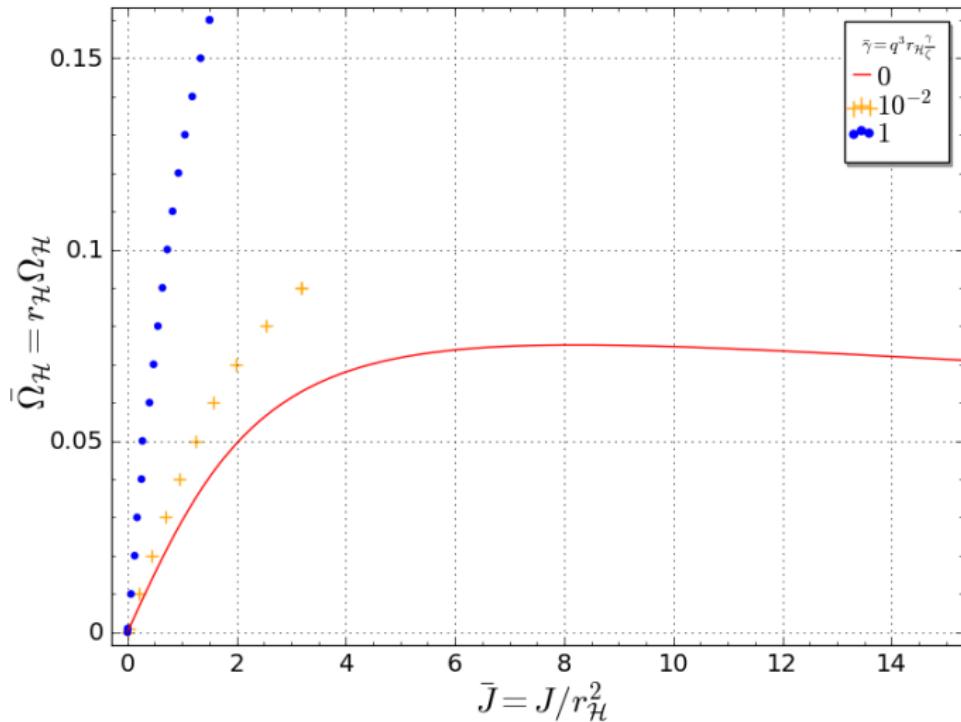


Asymptotic behaviour of $\bar{\omega} = r_{\mathcal{H}} \omega$



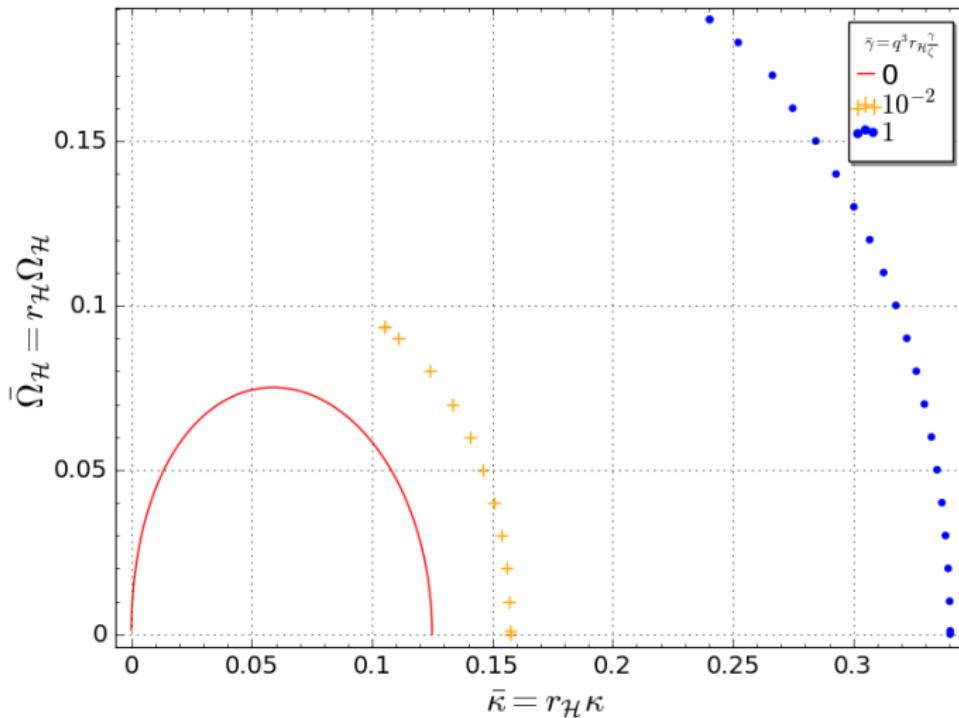
$$\longrightarrow \omega \sim \frac{2J}{r^3}$$

Angular momentum

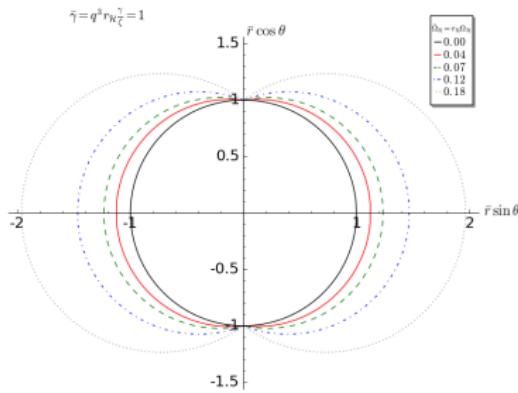


Surface gravity

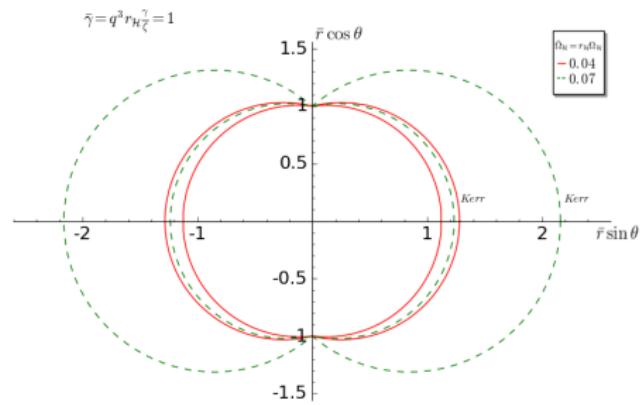
- Zeroth law holds, $\bar{\kappa} = \frac{1}{A} \partial_r N|_{\mathcal{H}}$



Ergoregion



Dependence on $\bar{\Omega}_H$

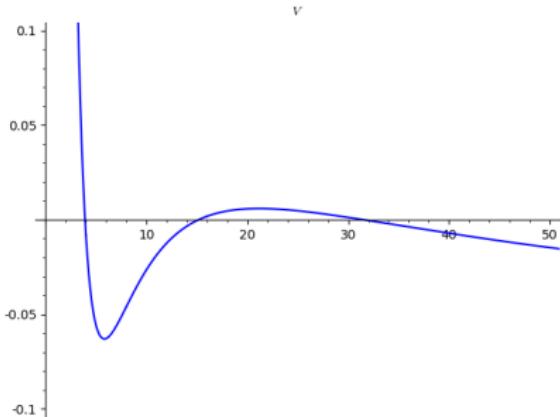


Comparison with Kerr

Next steps

- Studying the effective potential of geodesics:

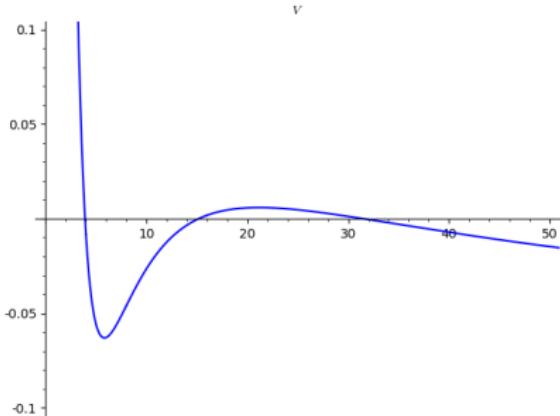
$$\left(\frac{dr}{d\tau} \right)^2 = \frac{1}{A^2} \left[\frac{1}{N^2} (\epsilon - \omega I)^2 - \frac{I^2}{B^2 r^2} - 1 \right]$$



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- Studying the effective potential of geodesics:

$$\left(\frac{dr}{d\tau}\right)^2 = \frac{1}{A^2} \left[\frac{1}{N^2} (\epsilon - \omega l)^2 - \frac{l^2}{B^2 r^2} - 1 \right]$$



- Non-flat asymptotics

Summary

- The cubic Galileon emerges from the DGP model
- It is consistent with $c_{GW} = c$
- Allowing a linear time-dependence for ϕ yields hairy solutions
- Hairy rotating black holes feature non negligible deviations from Kerr
- Orbits and shadow to come