Regularization, renormalization or why the standard model can be quite natural?

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following work done with

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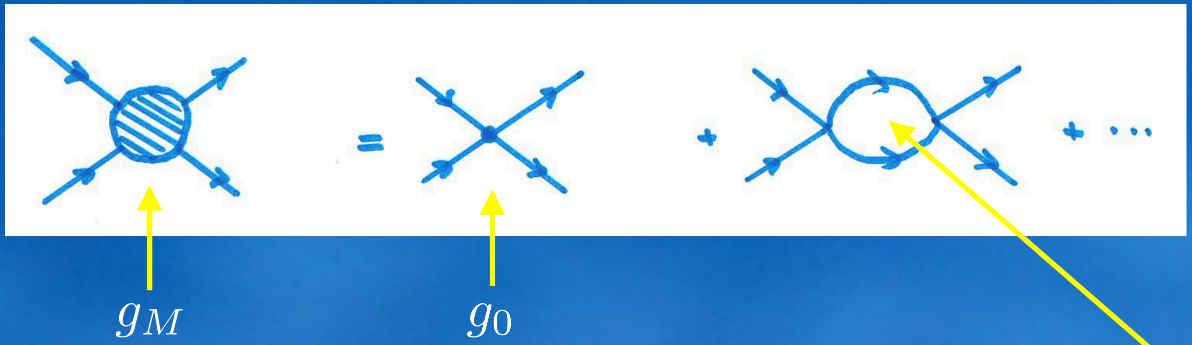
PLAN

- ❑ **Motivations**
  - ➢ **origin of divergences in quantum field theory**
- ❑ **Arbitrary scales and « universal » coupling constants**
  - ➢ **energy scales and scaling variables**
  - ➢ **« universal » coupling constants**
  - ➢ **« renormalized » coupling constants**
- ❑ **Fine-tuning and hierarchy of the standard model and beyond**
- ❑ **Landau pole and triviality of the scalar model**
- ❑ **Some immediate consequences**
- ❑ **Conclusions**

Motivations

❑ Anti-pasti

➢ **New Physics: one should clearly disentangle physical scales from spurious non-physical ones**



Test of the UV/IR properties of the theory

➢ **New scales appearing beyond tree level**

- ➥ **regularization procedures**
- ➥ **renormalization schemes**

➢ **Not trivial at all : need guiding principles**

- ➥ **mathematical coherence of the objects we manipulate**
- ➥ **constant link with physical observables**

➢ Mathematical origin of divergences

➥ quantum fields are distributions (N. Bogoliubov 1952)

➥ one should be able to define the product of two distributions at the same point (L. Schwartz 1951)

$$\mathcal{L} \sim \overline{q}(x)\ \phi(x)\ q(x)$$

➢ Two different types of approaches in order to solve the problem

➥ « à-posteriori » procedures

➥ « à-priori » procedures

❑ « A-posteriori » regularization procedures

➢ Cut-off Λ_C imposed for the calculation of each amplitude

poor's men regularization prior to 1972

$$\mathcal{O}_C \sim \int_0^{\Lambda_C} d^4k\ \mathcal{O}(m_0, g_0; k)$$

➢ The Lagrangian we start from is still not well-defined from a mathematical point of view

➢ Violation of gauge and Lorentz invariances

« A-priori » regularization procedures

- **Two possible ways to start from a well-defined Lagrangian**
  - **avoid the problem : dimensional regularization (DR)**
    - $D = 4 - \epsilon$
    - $\mathcal{A}_\epsilon = \int d^{4-\epsilon} \mathcal{L}_\epsilon(\psi, \partial_\mu \psi; m_0, g_0)$

('t Hooft *et al.* 1972)

  - **treat the problem : finite field theories**

(H. Epstein, V. Glaser 1973)
(G. Scharf 1995)
(J.M. Gracia-Bondia 2003)

$$\phi(x) \to \varphi(x) = \int d^4y \; \phi(y) \; \rho(|x-y|)$$

    - theory of distributions with well defined properties for the test function ρ
    - Ex. Taylor-Lagrange regularization procedure (TLRS)

F. Tkachov, hep-th/9911236

(P. Grangé, E. Werner 2011)

- Two very different behaviors
  - each elementary amplitude is finite in TLRS, for D=4
  - only the full physical observable is finite in D=4 in DR
    - this is the price to pay for not having solved the problem!
    - poles in $\frac{1}{\epsilon^n}$ for each elementary amplitude
    - need a « renormalization » scheme to be able to perform practical calculations (see later)

$$g_0 = g_R + \delta g_R$$

finite coupling constant

includes poles

Arbitrary scales and « universal » coupling constants

- **Energy scales**
  - Domain of validity of the Lagrangian : new physics scale Λ_{NP}

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \frac{c_i}{\Lambda_{NP}^{d_i - 4}} \mathcal{O}_i$$

    - ➥ Λ_{NP} can be uniquely determined if and only if we know $\mathcal{L}$
    - ➥ do not confuse Λ_{NP} with a cut-off in momentum space!

  - Physical scales
    - ➥ m_i : physical masses of the relevant degrees of freedom
    - ➥ Q_i : kinematical variables

❑ Arbitrary scales

➢ Regularization scale η

➥ not fixed by any experimental measurement

➥ in TLRS : scaling variable associated to the scaling properties in the UV/IR domains

$$k^2 \to \infty \quad , \quad \eta^2 \; k^2 \to \infty$$
$$k^2 \to 0 \quad , \quad \eta^2 \; k^2 \to 0$$

➥ in DR: dimensionality of the coupling constant at $\epsilon \neq 0$

$$D = 4 - \epsilon \qquad g \to \hat{g} = g \; \mu^{l \; \epsilon} \qquad \mu = \eta \; M_0$$

➥ η is dimensionless, once M_0 is fixed by a single measurement

ex. $M_0 = M_Z$

Rem. $\beta = \eta \dfrac{\partial g}{\partial \eta} \neq 0$ for $m = 0$ so $g(\eta)$ with η dimensionless

- ➢ **Renormalization point** M_i
  - ➥ given by the **kinematical conditions** to fix, from an **experimental measurement,** the bare parameters (m_0, g_0) of the Lagrangian
  - ➥ for a process $2 \to 2$ $\quad M_i \sim (\sqrt{s_0}, \sqrt{|t_0|})$

    $1 \to 2$ $\quad M_i \sim Q$

❑ « Universal » coupling constants

- ➢ Defined both in the **perturbative** as well as **non-perturbative** domains
- ➢ **Independent** of the chosen **renormalization scheme**
  - ➥ **bare coupling constant** g_0 in $\mathcal{L}$ (regularized)
    - depends on η but not on M_i $\qquad g_0(\eta)$
  - ➥ **physical coupling constant** g_M of the physical state
    - depends on M_i but not on η $\qquad g_M(M)$

❑ « Renormalized » coupling constants

- ➢ Defined in order to extract the divergences, if necessary

$$g_0 = g_R + \delta g_R$$

➥ δg_R incorporates divergences

➥ δg_R generates new terms in the Lagrangian (counterterms)

$$\mathcal{L} = \mathcal{L}_R + \delta \mathcal{L}_R$$

➥ counterterms fixed by « renormalization » conditions

- ➢ g_R is defined in perturbation theory only

- ➢ Two main renormalization schemes without the need to introduce any additional ad-hoc mass scale

➥ minimal subtraction : we include in δg_R the poles in $\frac{1}{\epsilon^n}$

this defines $g_R(\eta)$ (MS)

Redefinition rather than renormalization!

➥ **on-mass-shell** renormalization : one chooses for g_R the **physical coupling constant** g_M **(OMS)**

Renormalization in the sense of many-body theory

Rem. **similarities** between DR+MS and TLRS

➥ **TLRS :** $g_0(\eta)$ and poles in $\dfrac{1}{\epsilon^n}$ are **absent** by construction

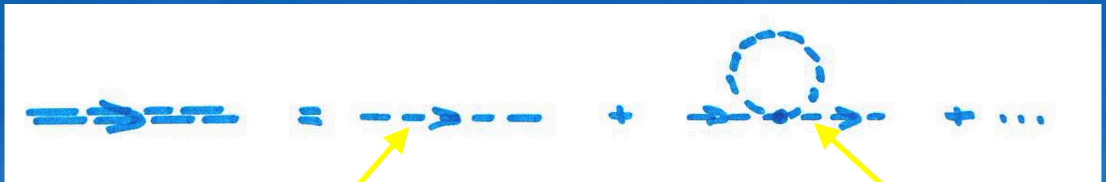
➥ **DR+MS :** $g_R(\eta)$ and we **remove** the poles by hand

➥ both coupling constants are **finite,** but $g_R(\eta)$ only defined in **perturbation theory**

Fine-tuning and hierarchy of the standard model and beyond

❑ Fine-tuning of the Higgs mass in the standard model

$$m_H^2 = m_0^2 + \delta m_H^2$$



bare mass

radiative correction

- ➢ Discussion **pre-1972** (and after!) $\delta m_H^2 \propto \Lambda_C^2$ $\qquad \Lambda_C \to \infty$
- ➢ **Fine-tuning** since $\Lambda_C \gg m_H$ so that m_H^2 is defined through the **cancellation** of two **very large numbers** : it is not natural!
- ➢ Should be **banned** from any **serious discussion**

  ➥ **violation** of gauge and Lorentz **invariances**

- In the case of DR+MS or TLRS

$$\delta m_H^2 \propto m_H^2 \;\; Log \; \eta^2$$

  - ➥ no other possible behavior from a dimensional point of view

- Since η is arbitrary, the question of fine-tuning has no « raison d'être » and $\delta m_H^2 \to 0$ when $m_H^2 \to 0$

❑ What happens beyond the standard model ?

- In the presence of a second physical mass scale, with $M_X \gg m_H$

  - ➥ so-called « hierarchy » problem

  - ➥ contribution to the physical mass of the Higgs

$\mu = \eta \; M_0$

$$\delta m_H^2 \propto M_X^2 \;\; Log \; \frac{\mu^2}{M_X^2}$$ in DR+MS

  - ➥ how to interpret this result?

- Since η is arbitrary, educated guess for its parametrization
  - $M_0 \equiv M_X \qquad \delta m_H^2 \propto M_X^2 \; Log \; \eta^2$
  - η of order 1 parametrized in terms of the mass scales in $\mathcal{L}$

$$\eta^2 = 1 + a \, \frac{m_H^2}{M_X^2}$$

  - a arbitrary with $a \ll \dfrac{M_X^2}{m_H^2}$

$$\delta m_H^2 \propto a \; m_H^2$$

- With a particular parametrization, no trace of any large cancellations
- The hierarchy problem is a false problem!
- This would not be the case if μ is interpreted as a physical scale

$$\mu \sim Q \qquad \delta m_H^2 \propto M_X^2 \; Log \; \frac{Q^2}{M_X^2} \gg m_H^2$$

The Landau pole and triviality of the scalar model

- How to interpret the Landau pole?
  - Running of the coupling constants as a function of the regularization scale η or the renormalization point M

$$\beta_\eta = \eta \frac{\partial}{\partial \eta} \, g_{0,R}(\eta)$$ independent of m

$$\beta_M = M \frac{\partial}{\partial M} \, g_M(M)$$ dependent on m

  - Rem. : for $M \gg m$ $\qquad \beta_M \simeq \beta_\eta$
  - In first order perturbation theory $\beta \simeq b_0 \, g^2$
  - What happens for $g_{0,R}$ and g_M ?

➢ For the **bare** (in TLRS) or **renormalized** (in DR+MS) coupling constants

$$g_{0,R}(\eta) = \frac{\bar{g}_{0,R}}{1 - b_0\ \bar{g}_{0,R}\ Log\ \eta} \qquad \bar{g}_{0,R} = g_{0,R}(\eta = 1)$$

➥ **Landau pole** for $\eta \sim \eta_c$ $\qquad Log\ \eta_c = \dfrac{1}{b_0\ \bar{g}_{0,R}}$

➥ how to **interpret** this pole?

- in a **perturbative calculation** of order N with a given **accuracy**

$$\mathcal{O}^N(Q,\eta) \simeq \sum_{n=0}^{N} g_{0,R}^n(\eta)\ \mathcal{O}_n(Q;\eta)$$

- for a **converged** calculation

$$|\mathcal{O}^{N+1} - \mathcal{O}^N| \leq accuracy$$

- since $\mathcal{O}$ is a **physical observable**

$$\mathcal{O}^N(Q;\eta) \simeq \mathcal{O}(Q) \simeq \mathcal{O}^N(Q)$$

- in this case, **one can choose** η in a domain **far away** from η_c , without any **loss of generality**

- the **presence** of the Landau pole, for g_0 or g_R is not the **sign of any weakness** of the standard model, **provided** we can perform a **perturbative calculation**

- how to **check** this: look at g_M !

Behavior of the physical coupling constant

- in perturbation theory, like $g_{0,R}(\eta)$

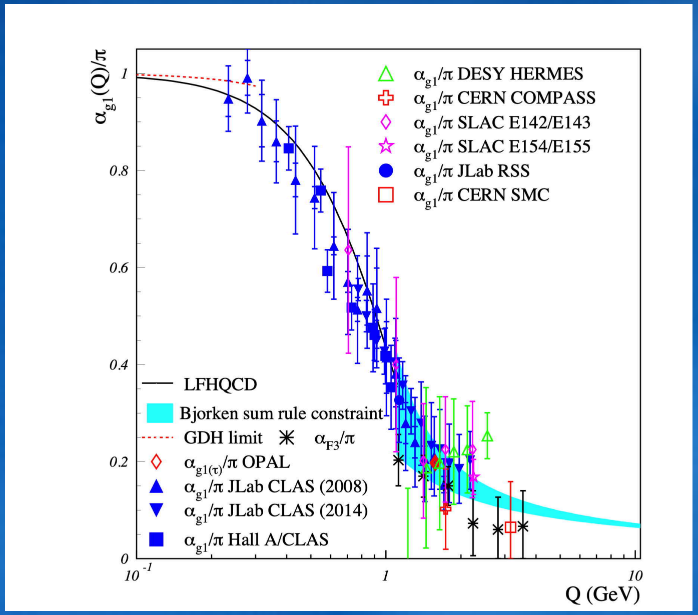
$$g_M(M) = \frac{g_M(M_0)}{1 - b_0 \; g_M(M_0) \; Log \; \frac{M}{M_0}}$$

for $M \gg m$

- **Landau pole** for $Log \frac{M_c}{M_0} = \frac{1}{b_0 \; g_M(M_0)}$

- Around the **physical condition** $Q \sim M_c$ **perturbation theory** is **not valid** anymore

Ex. α_s of QCD ($b_0 < 0$ in this case for asymptotic freedom)



(S. Brodsky, A. Deur, …)

➥ **effective strong coupling constant** extracted from the Bjorken sum rule

$$\frac{\alpha_{g_1}(Q)}{\pi} = 1 - \frac{6}{g_A}\int_0^1 dx\ g_1^{p-n}(x,Q)$$

➥ in perturbation theory

$$\alpha_{g_1}(Q) \propto \frac{1}{Log\ \frac{Q}{\Lambda_{QCD}}}$$

- **Nature has done for us** the **non-perturbative** calculation for $Q \sim \Lambda_{QCD}$ and the Landau pole has disappeared!

- To **check the validity of the Standard Model** at the Landau pole : need to do a **non-perturbative calculation** and a **physical measurement** at the critical scale

❑ Triviality of the scalar model

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$$

ϕ = scalar field (Higgs)

➢ **Standard analysis** in DR+MS at NLO

$$g_R(\mu) = \frac{g_R(\mu_0)}{1 - b_0\ g_R(\mu_0)\ Log\frac{\mu}{\mu_0}}$$

➥ if μ is **identified** with a **physical scale** Q

➥ the **only solution to avoid** the Landau pole at $Q = Q_c$
is to have $g_R(Q_0) \to 0$: **free** theory!

➢ When the right (dimensionless) **regularization scale** η is considered

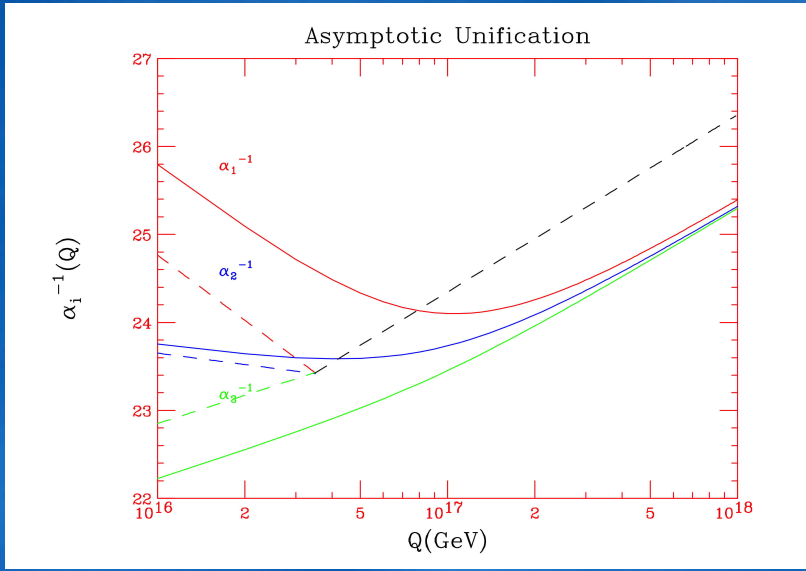
➥ the **Landau pole** can be **avoided**

➥ no **any constraint** on $\overline{g}_{0,R} = g_{0,R}(\eta_0 = 1)$

- What about the **physical coupling constant** g_M ?
  - if $M \to M_c$ **perturbation theory** is not **valid** anymore
  - we need
    - a **non-perturbative calculation**
    - a **physical measurement** in this **energy domain**
  - we can do this for QCD at **low energies**, and QCD is **not a trivial theory**!
  - **no a-priori constraints** on $g_M(M_0)$

Some immediate consequences

❑ **Unification** of the **coupling constants** in the Standard Model



➥ one should look at $g_M(M)$ and **not** at $g_R(\eta)$ in DR+MS for instance

➥ necessary **mass corrections** near the unification point

(M. Binger, S. Brodsky, 2004)

➢ The important **requirement** is not for the coupling constant to **cross** at a **single point,** but to **merge smoothly** to a unique coupling constant for $Q > M_{GUT}$

➢ This implies to **know** the **Lagrangian** beyond M_{GUT} !

❑ Analysis of the stability of the scalar vacuum at high energies

$$V(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!} \phi^4$$

- **Instability if** $\lambda < 0$
- **The relevant coupling constant to look at is not** $\lambda_R(\eta)$ **but** $\lambda_M(M)$
- **For the scalar potential** $M_i \sim (\sqrt{s_0}, \sqrt{|t_0|})$
  - ➥ **analysis in terms of two kinematical variables**
  - ➥ **with threshold effects for** $s_0 \geq 4m_H^2$

❑ Decoupling properties

- Decoupling : super heavy degrees of freedom should not influence in general the behavior of light ones
- Since the bare (in TLRS) or renormalized (in DR+MS) coupling constants refer to the regularization scale η associated to the behavior in the UV limit, their dependence on η should be mass-independent, as it is
- No need to restrict, by hand, the number of active quarks for $m < Q$: all physical degrees of freedom should be considered in the calculation of radiative corrections, for any Q
- The decoupling of heavy degrees of freedom is done at the level of the physical coupling constant $g_M(M)$ which dependence on M is mass-dependent

Conclusions

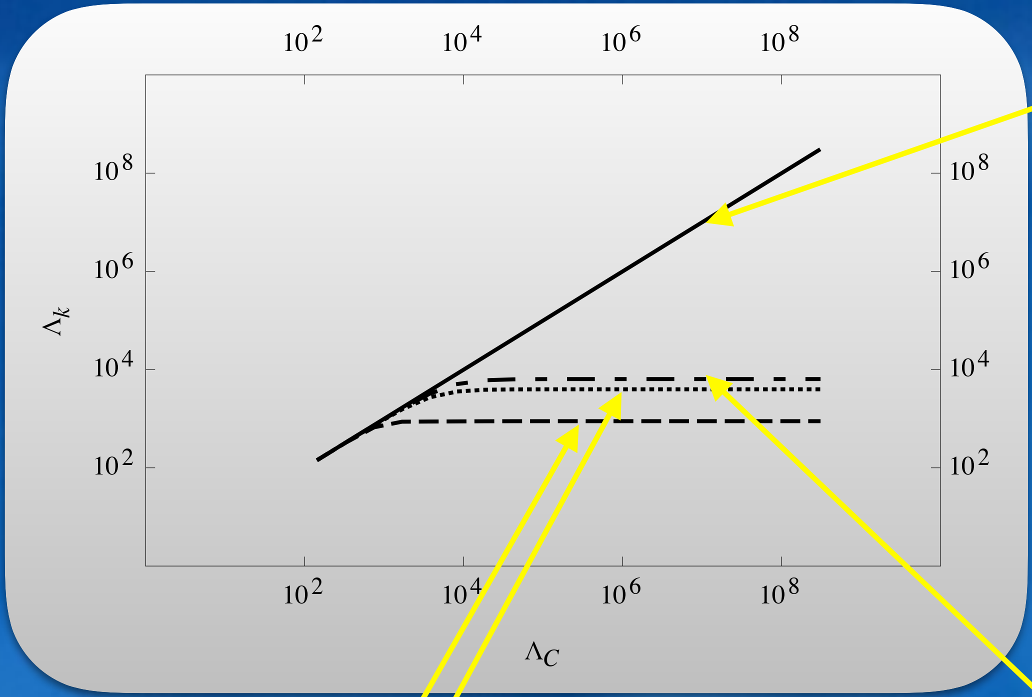
- One should clearly disentangle the internal coherence of the Lagrangian we start from (with $g_{0,R}(\eta)$) from the properties of the physical state realized in Nature (with $g_M(M)$)

- The Standard Model has its own coherence as a quantum field theory with local interactions

- Main concern : non-perturbative calculations needed near the Landau pole for $M \sim M_C$ far beyond presently planed accelerators

Thank you for your attention!

Relevant momenta in loop calculations



bare operator

TLRS

fully renormalized

$$\Sigma(p^2) = \int_0^{\Lambda_C^2} dk^2 \; \sigma(k^2, p^2)$$

$$\bar{\Sigma}(p^2) = \int_0^{\Lambda_k^2} dk^2 \; \sigma(k^2, p^2)$$

$$\frac{\bar{\Sigma}(p^2)}{\Sigma(p^2)} = 1 - \epsilon$$

$$\Sigma_R(p^2) = \Sigma(p^2) - \Sigma(m_H^2) - (p^2 - m_H^2) \left. \frac{d\Sigma(p^2)}{dp^2} \right|_{p^2 = m_H^2}$$