Comprendre l'Infiniment Grand
-
Introduction to Cosmology
-
Part II

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July 19, 2022

Summary of Part I

Equivalence Principle

- $m_i = m_g$: Uniform gravitational field ⇔ Uniform acceleration
  => to understand gravitation study uniform acceleration

- Photon trajectory is bent in an accelerated frame
  => bent in a gravitational field

- In an accelerated rocket:
  => same in a gravitational field

$$\Delta t_B = \left(1 - \frac{gh}{c^2}\right)\Delta t_A$$

A emits every 10 s -> B receives every 9.999… s

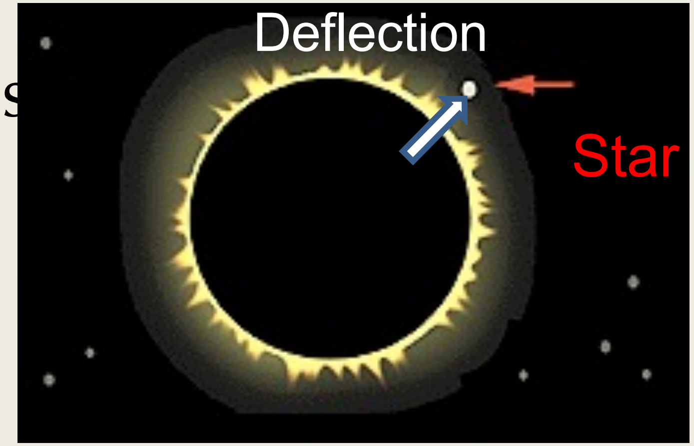
times run slower in a gravitational potential : $\Delta t_B = \left(1 - \frac{\Phi_A - \Phi_B}{c^2}\right)\Delta t_A$

$\Longrightarrow$ Gravitational Redshift observed with S2 star orbiting around the Black Hole, Sag .A* at the center of MW

Curved spacetime - Light rays are bent

- In 1919 : Arthur Eddington observes lightdeviation by the sun during a solar eclips

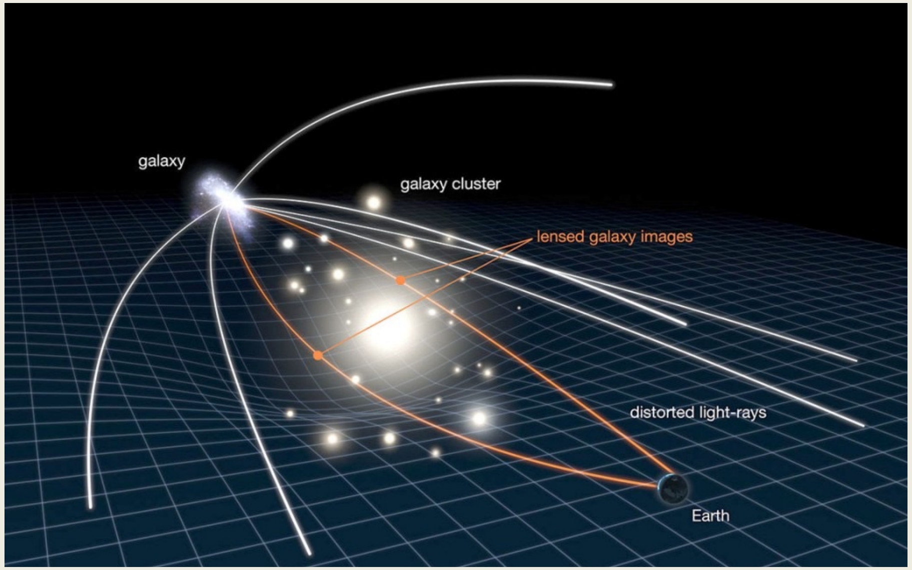
  - 1.75 arc second = 8.5 µrad as predicted by Einstein
  - Twice the deflection predicted by first computation (Eq. principle alone)



- GR : for a weak and static field, the metric is :

$$ds^2 = \underbrace{\left(1 + \frac{2\Phi(x)}{c^2}\right)}_{\text{Equivalence principle}} c^2 dt^2 - \underbrace{\left(1 - \frac{2\Phi(x)}{c^2}\right)}_{\text{GR}} (dx^2 + dy^2 + dz^2)$$

Curved spacetime - Gravitational lensing



- Strong gravitational lensing: modern proof of RG

- On July 11 2022 James Webb Space Telescope released this deep field

- Galaxies behind galaxy cluster SMACS 0723 ($z=0.39$, $R_{vir}=2.4$Mpc) are curved and warped

2.25 arcmin, 0.7Mpc at z=0.39

Cosmology - Part II

1. **Geometry of the Universe**
    - Cosmological principles
    - FLWR metric

2. **Expansion of the Universe**
    - Cosmological redshift
    - Friedman equation

1) Geometry of the Universe

Homogenous and isotropic

- **Cosmological principle**
  - Universe isotropic + homogeneous on large scales
  - Universe looks the same whoever and wherever you are

- **Isotropic** (on large scales)
  - CMB very isotropic
  - X ray background, radio galaxies

- **Homogeneous**
  - Test with 3D galaxy surveys
  - Only at large scales.... >Mpc

FLRW metric

Homogeneous and isotropic ⇒
Friedmann, Lemaitre, Robertson, Walker metric

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

- isotropic
- scale factor $R(t)$ due to expansion
- dimensionless scale factor : $a(t)=R(t)\ /\ R(t_0)$

  now $a(t_0) = 1$ **index 0, means today**

  in the past $a(t) < 1$

  Big Bang $a(t) = 0$

FLRW metric

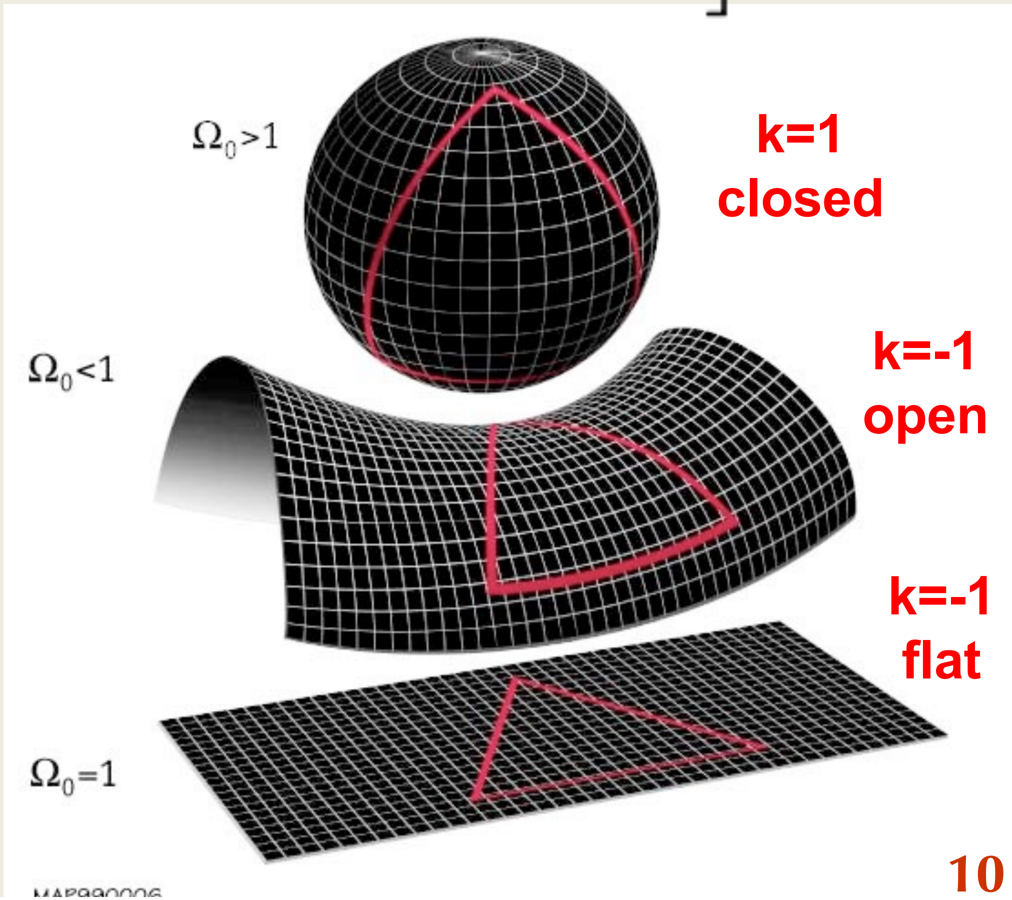
Friedmann, Lemaitre, Robertson, Walker metric

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

k = 1 : spherical geometry
or closed ($\sum\alpha > 180^o$)

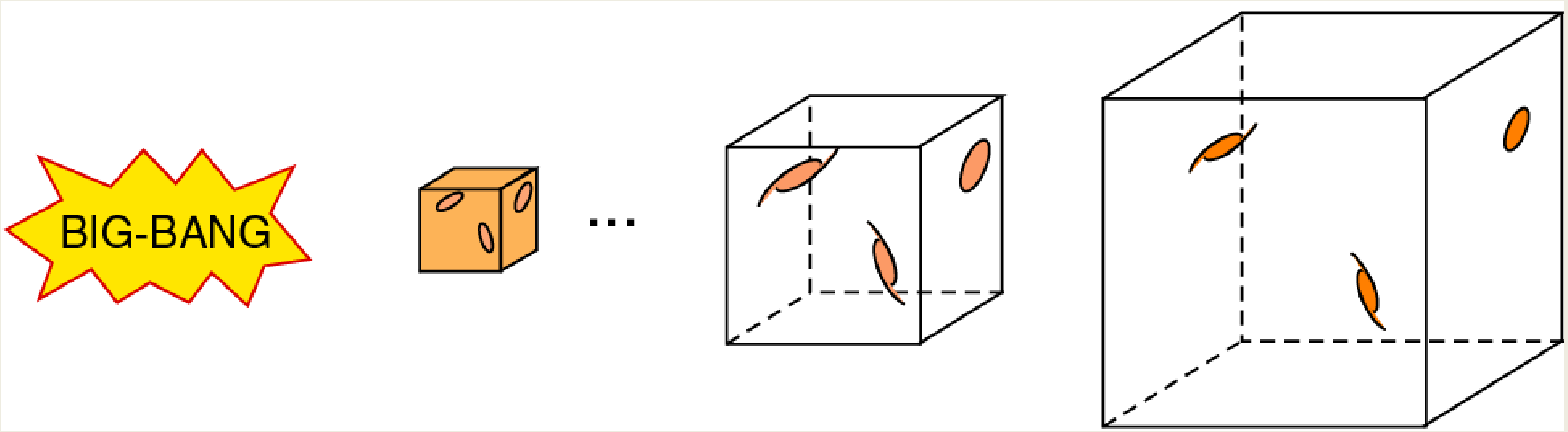
k = -1 : hyperbolic geometry
or open ($\sum\alpha < 180^o$)

k = 0 : flat geometry ($\sum\alpha = 180^o$)



The Big Bang

- Universe is expanding with H_0 (~ 70 Mpc/(km/s))
- If we go back in time,
  Universe more and more dense and hot: big bang



- Analogy with an inflated balloon
- Name invented by Fred Hoyle
- Fate of the Universe strongly related to the metric, i.e. k values (close, open or flat)

Comoving distance

- **Change of coordinates** r =sin χ (k=1, closed)

  r = χ (k=0, flat)

  r = sinh χ (k=-1, open)

$$ds^2 = dt^2 - R^2(t)\left[d\chi^2 + \left\{\begin{matrix}\sin^2\chi \\ \chi^2 \\ \sinh^2\chi\end{matrix}\right\}(d\theta^2 + \sin^2\theta d\phi^2)\right]\left\{\begin{matrix}\text{closed} \\ \text{flat} \\ \text{open}\end{matrix}\right\}$$

sin → spherical   sinh → hyperbolical

- **Distance:**
  - Galaxies remain at χ = cst (up to small local velocities)
  - Physical distance between 2 galaxies : R(t) × Δχ (Mpc)

    increases with the expansion
  - "comoving" distance : $R(t_0) \times \Delta\chi$ is fixed (comoving Mpc)

    = distance including the expansion up to $t=t_0$

    = independent from Universe expansion

2) Expansion of the Universe

Cosmological redshift

- Radial photon

$ds^2 = dt^2 - R^2(t)\, d\chi^2 = 0 \quad \Rightarrow \quad d\chi = dt/R$

$$\chi = \int_{t_e}^{t_r} \frac{dt}{R(t)} = \int_{t_e+\delta t_e}^{t_r+\delta t_r} \frac{dt}{R(t)}$$

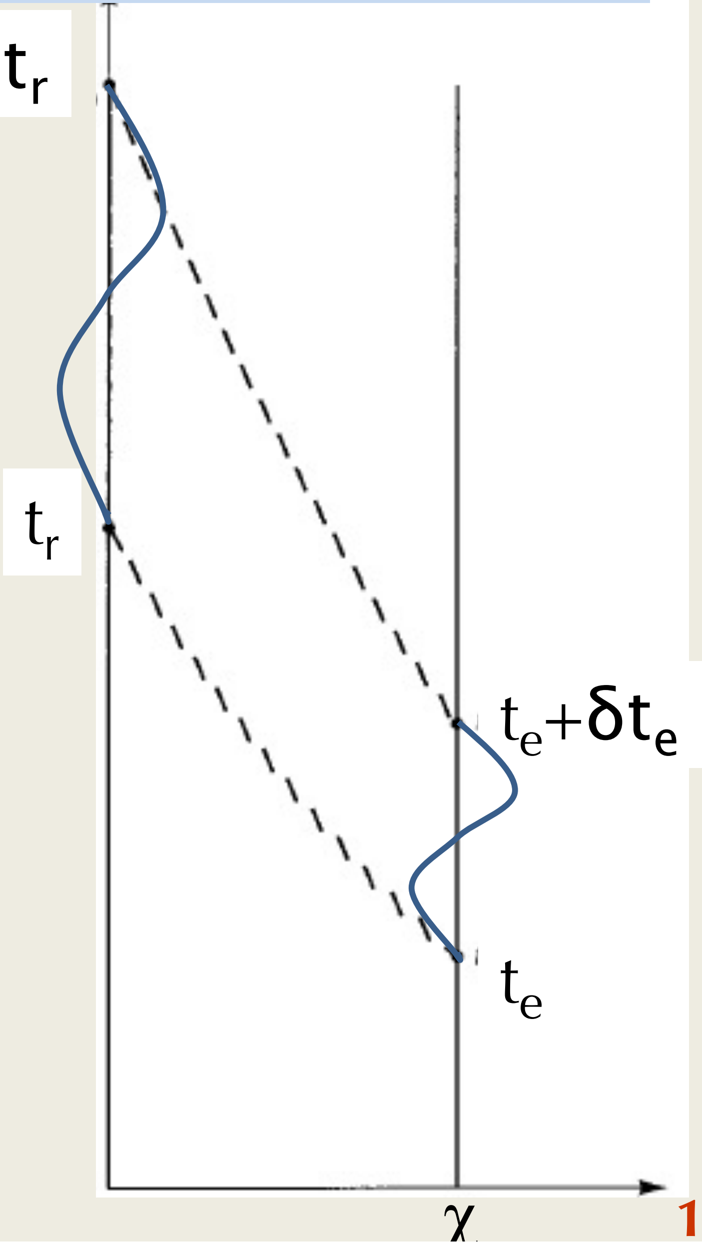
$$\Rightarrow \int_{t_e}^{t_e+\delta t_e} \frac{dt}{R(t)} = \int_{t_r}^{t_r+\delta t_r} \frac{dt}{R(t)}$$

$$\Rightarrow \frac{\delta t_e}{R(t_e)} = \frac{\delta t_r}{R(t_r)}$$

$$1 + z_e \equiv \frac{\lambda_r}{\lambda_e} = \frac{\delta t_r}{\delta t_e} = \frac{R(t_r)}{R(t_e)} \equiv \frac{1}{a(t_e)}$$

$$\boxed{1 + z = \frac{1}{a}}$$

« λ is dilating with the Universe »



Redshift: A fundamental concept in cosmology

- Measuring z → scale factor *a* when light emitted

- It is a cosmological redshift, $1+z = 1/a$ can be e.g. z=1000 (at CMB) cannot be interpreted as a simple Doppler effect

- In case of Hubble law ($v=H_0d$), it is locally interpreted as a Doppler effect

- z is also a measurement of time: e.g. CMB occurred at z = 1100 (i.e. when a=0.0009)

Hubble parameter

- Assume $t_e \sim t_0$ (locally) $\Rightarrow$ a ~ 1, small z

$$1 + z = \frac{1}{a} \qquad z = \frac{v}{c} = \frac{1-a}{a} = \frac{\dot{a}\Delta t}{a} \qquad \Rightarrow \qquad v = \frac{\dot{a}}{a}(c\Delta t)$$

$$v = \frac{\dot{a}}{a}D$$ Hubble law with $$H_0 = \frac{\dot{a}(t_0)}{a(t_0)} = H(t_0)$$

- Hubble parameter $H(t) \equiv \dfrac{\dot{a}(t)}{a(t)}$

- H_0 is not very precisely measured, we define

$$h \equiv \frac{H_0}{100\ (\mathrm{km/s})/\mathrm{Mpc}} \approx 0.7$$

- cosmological results in units like h^{-1}Mpc
  numerical result independent of h

Thermodynamic

- a volume V including a fixed number of particles (i.e. galaxies !)

$$d\,E = -\,P\,dV \qquad\qquad E = \rho\,V$$

- the physical volume is $V = a^3(t)\,V_{com}$ **(V$_{com}$ = comoving volume)**

$$d_t\,(\rho\,a^3\,V_{com}) = -P\,d_t\,(a^3\,V_{com}) \qquad \text{but } V_{com} = cst = V_0$$

$$\boxed{d_t\,[\rho(t)\,a^3(t)] = -P(t)\,d_t\,[a^3(t)]}$$

matter, radiation

- **Matter:** $d_t\,[\rho\, a^3] = -P\, d_t\,[a^3]$

  galaxies may be approximated as a pressure-less gas

  $d_t\,[\rho_m\, a^3] = 0$

  $\rho_m\,(t) = \rho_m\,(t_0)\, a^{-3}(t)$ $\varrho \propto 1/V$

- **Pure radiation** (black body) Stefan's law: $\rho_r = g\frac{\pi^2}{30}\frac{(k_B T)^4}{(\hbar c)^3}$

  Thermodynamics: $P_r = (1/3)\,\rho_r$

  $d_t\,[\rho\, a^3] = -(1/3)\,\rho\, d_t\,[a^3] \Rightarrow 4\rho a^3 d(a) + a^4 d(\rho) = 0$

  $\rho_r\,(t) = \rho_r\,(t_0)\, a^{-4}(t)$ a^{-3} for volume

  a^{-1} since $E \propto \lambda^{-1}$

  $T(t) = T(t_0)\, /\, a(t)$

vacuum

- "Vacuum is not empty"
  virtual particle-antiparticle pairs

- Results in a vacuum energy density
  constant in space and time

$$d_t\,[\rho\, a^3] = -P\, d_t\,[a^3] \quad \Rightarrow \quad \rho\, d_t\,[a^3] = -P\, d_t\,[a^3]$$

$$P_v = -\,\rho_v = cst < 0$$

- Vacuum pressure is negative !
- We will see that vacuum energy equivalent to cosmological constant or a form of dark energy

Analogy: Newtonian gravity → GRs

General Relativity

- Source mass M + test mass m: $F = G\, m\, M / r^2$
- Potential: $\Phi = -GM/r$

metric $g_{\mu\nu}$

- For a mass distribution ρ: $\nabla^2 \Phi = 4\pi G \rho$

Einstein eq.

- Equation of motion: $\vec{F} = -m\vec{\nabla}\Phi$

geodesic eq.

Friedman equation

- Einstein Eq => $\boxed{\left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = \frac{8\pi\rho}{3}}$ (Friedmann Eq.)
- Critical density today for which the Universe is flat (k=0)

t=t_0: $$\frac{8\pi\rho_c}{3} = \left(\frac{\dot{R}}{R}\right)_0^2 = \left(\frac{\dot{a}}{a}\right)_0^2 = H_0^2$$

$$\boxed{\rho_c = \frac{3H_0^2}{8\pi}} = 1.88 \times 10^{-29} h^2 \text{ g/cm}^3 \sim 5 \text{ protons / m}^3$$

note h^2 factor

- We introduce

$$\Omega_m \equiv \frac{\rho_m(t_0)}{\rho_c}, \qquad \Omega_r \equiv \frac{\rho_r(t_0)}{\rho_c}, \qquad \Omega_v \equiv \frac{\rho_v(t_0)}{\rho_c}$$

$\Omega_T = \Omega_m + \Omega_r + \Omega_v = \rho_0 / \rho_c$ (Ω_x, at t=t_0, should be $\Omega^0{}_x$)

Friedman equation

$\rho_0 = \rho_c\,\Omega_T$

- $$\left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = \frac{8\pi\rho}{3} \xrightarrow{t=t_0} \frac{k}{R_0^2} = \frac{8\pi\rho_0}{3} - H_0^2 = H_0^2(\Omega_T - 1)$$

$$\frac{8\pi\rho_c}{3} = H_0^2$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi\rho}{3} - H_0^2(\Omega_T - 1)\left(\frac{R_0}{R}\right)^2 = H_0^2\left[\frac{\rho(a)}{\rho_c} + (1-\Omega_T)a^{-2}\right]$$

- $$\begin{aligned}\rho(a) &= \rho_m(t_0)a^{-3} + \rho_r(t_0)a^{-4} + \rho_v(t_0) \\ &= \rho_c(\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_v)\ ,\end{aligned}$$

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_v + (1-\Omega_T)a^{-2}\right]}$$

Simplification: for a flat Universe ($k=0 \Rightarrow 1-\Omega_T = 0$)

Age of the Universe

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_v + (1-\Omega_T)a^{-2}\right]$$

- many quantities may be computed from this equation by expressing in terms of 'a' and $\dot{a}/a$

- e.g. the age of the universe : $dt = \frac{dt}{da}da = \frac{da}{\dot{a}} = \frac{da}{a(\dot{a}/a)}$

$$t = H_0^{-1}\int_0^1 \frac{da}{a\left[\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_v + (1-\Omega_T)a^{-2}\right]^{1/2}}$$

- $H_0 = 70(km/s)/Mpc = \frac{70km/s}{10^6 \times 3.262 \times 1an \times 300000km/s}$

$H_0^{-1} = 14.10^9$ years

Age of the Universe

- Note: - our Universe is flat ($k=0 \Rightarrow \Omega_T = 1$)
  - one may often neglect $\Omega_r = 9\ 10^{-5}$ (Ω_m=0.3, Ω_v=0.7)

- Simplification of the equation:

$$t = H_0^{-1} \int_0^a \frac{da}{a\left(\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_v\right)^{1/2}}$$

- Universe with just matter $\Omega_m \approx 1$

$$t(a) = H_0^{-1} \int_0^a \frac{da}{a^{-1/2}} = H_0^{-1} \int_0^a a^{1/2} da = \frac{2}{3} H_0^{-1} a^{3/2}$$

T ~9.10^9 years, incompatible with the age of the first galaxies

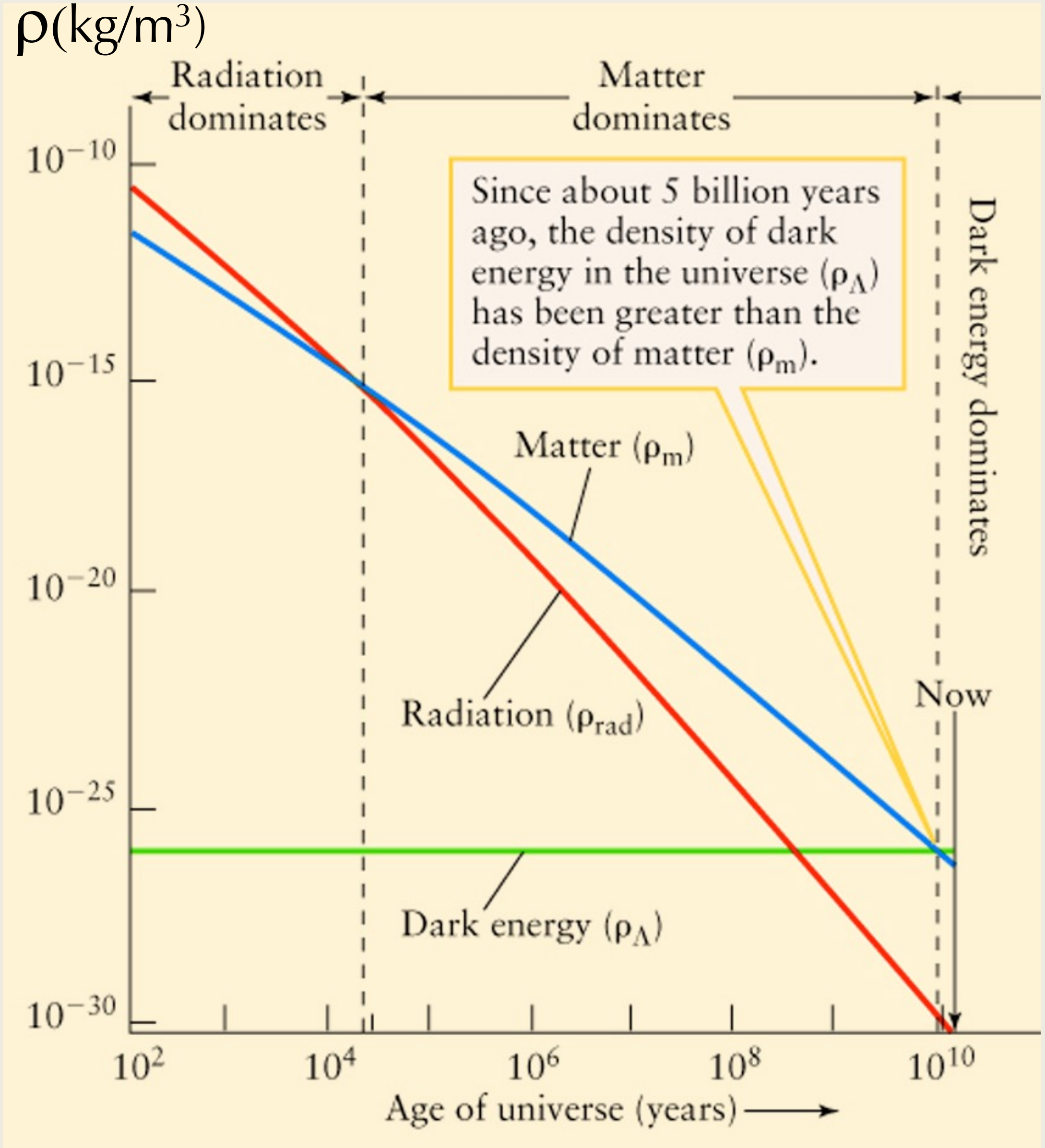
Epochs of the universe

$$\rho(a) = \rho_{crit}\left(\Omega_v + \frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4}\right)$$

- beginning
  ($'a'$ very small)
  radiation dominates

- then mater dominates

- "recently" vacuum
(or dark energy)
dominates



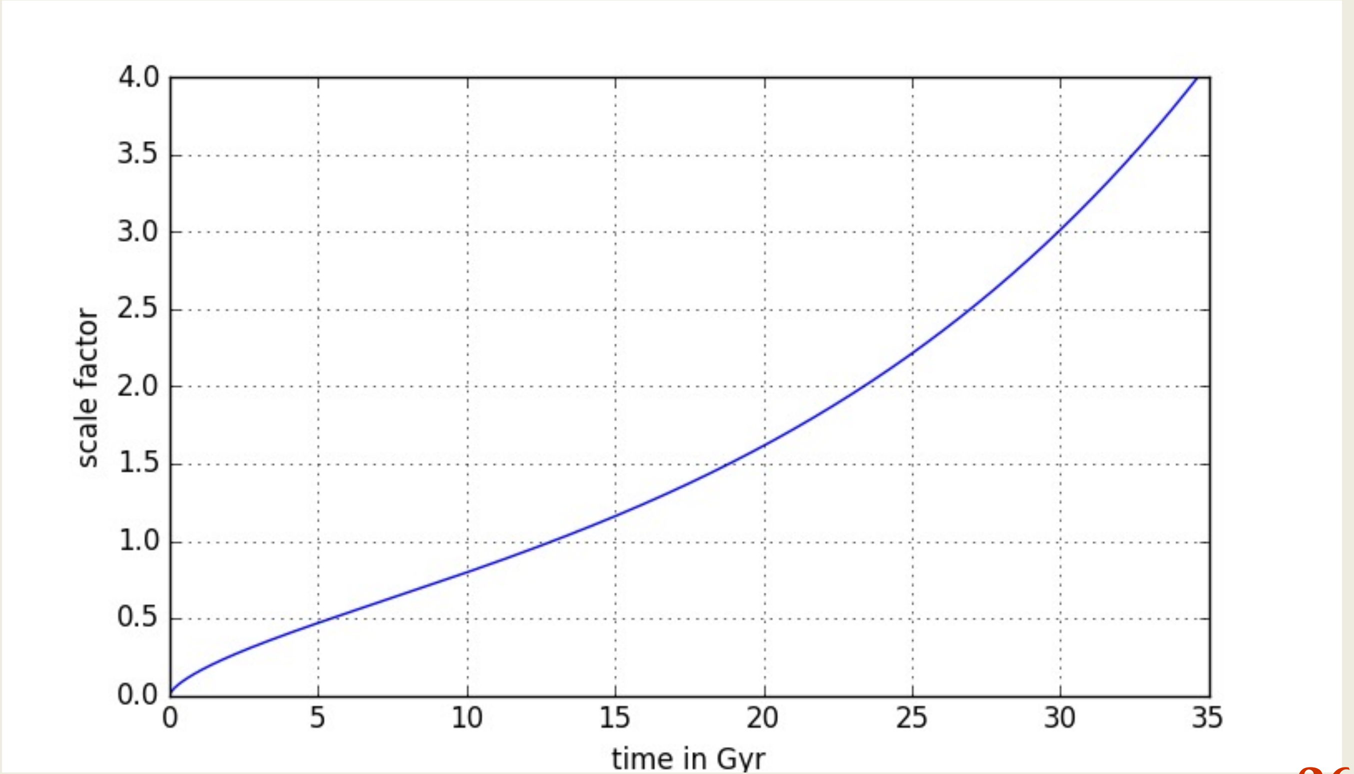
Evolution of a(t)
(flat universe)

k=0 ⇒ $\dot{a}^2 - \frac{8\pi\rho}{3}a^2 = 0$ $\rho(a) = \rho_{crit}\left(\Omega_v + \frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4}\right)$

when

- radiation dominates
  $a(t) \sim t^{1/2}$
- matter : $a(t) \sim t^{2/3}$
- vacuum:
  $a(t) \sim \exp(Ht)$

vacuum energy accelerates the expansion



Thermal history of the Universe

- at beginning: T and density are very large
  all particle species in equilibrium $\nu + \bar{\nu} \leftrightarrow e^+ + e^-$

  ν : neutrino

- when reaction rate $\Gamma(t) < \dot{a}(t)/a(t) = H(t)$

the reaction is no longer fast enough to maintain equilibrium / expansion: particle abundance is frozen

e.g.: T ~ 1 MeV, t ~ 1s, ν's decouple

- when T decreases particles may get bound :
  - T ~ 0.1 MeV, t ~ 3 mn : p+n → light nuclei
    primordial nucleosynthesis
  - T ~ 0.3 eV, t ~ 400 000 years: e + nuclei → atoms

History of the Universe