# Comprendre l'Infiniment Grand
# -
# Introduction to Cosmology

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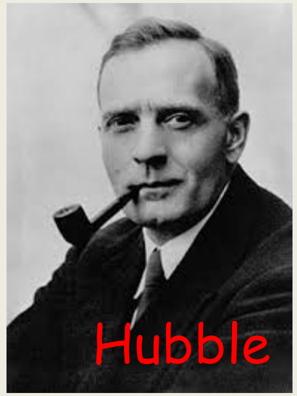
# Cosmology - Part I

1. **Introduction**
   - Hubble law
   - Content of the Universe
2. **Gravitation and General Relativity**
   - Equivalence principle
   - Tests of GR
   - Curved spacetime - Metric

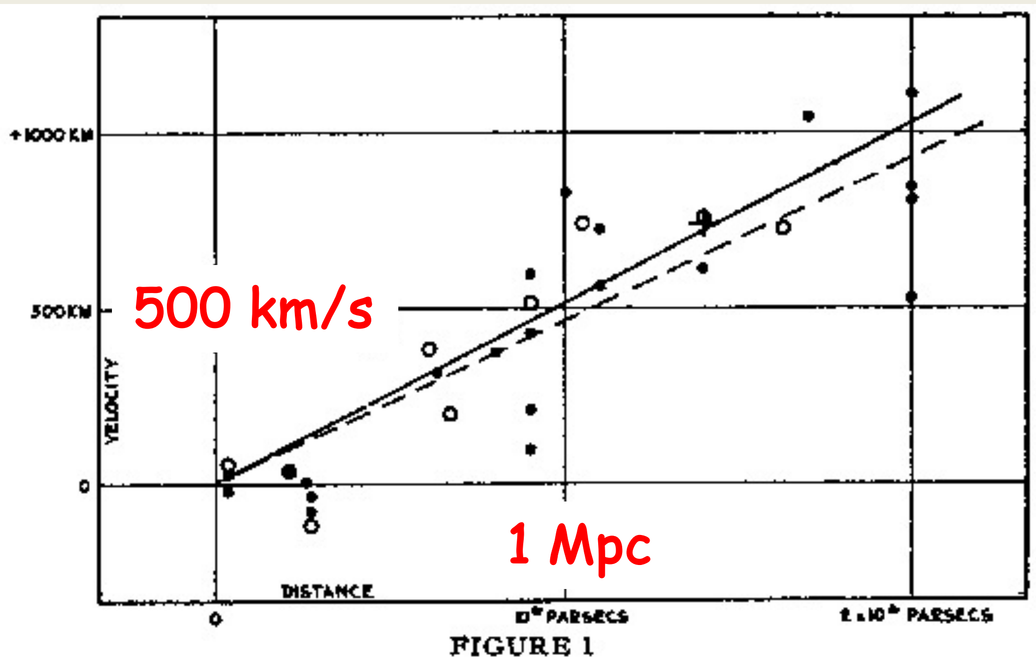
# 1) Introduction

# Expanding Universe


## History of the discovery

- ➢ 1914, Slipher: farther the « nebula » (galaxy) is from us, the more it seems to be escaping away
- ➢ 1927, Lemaître: solutions of Einstein General Relativity for a non static universe ⇒ velocity proportional to distance.



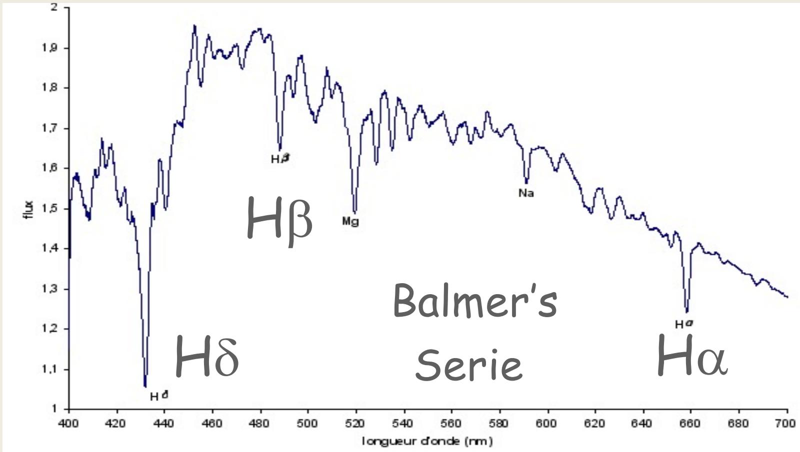

1 parsec= 3 light years

- ➢ 1929, Hubble: Relation distance – velocity thanks to cepheid in extragalactic "nebula "

$L_{obs} \propto L_0/R^2$

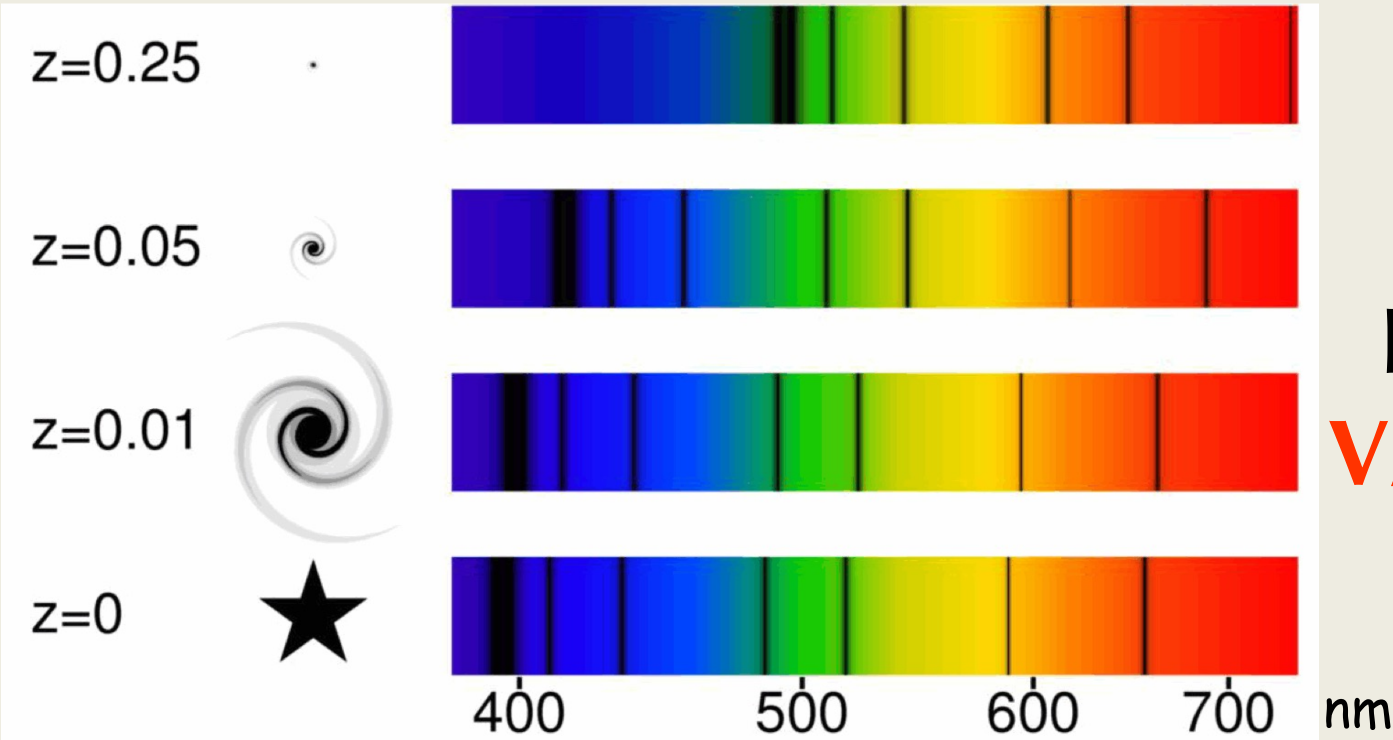
**Period of cepheid → $L_0$**

# How do we measure velocity?




**Stars spectra**

absorption lines

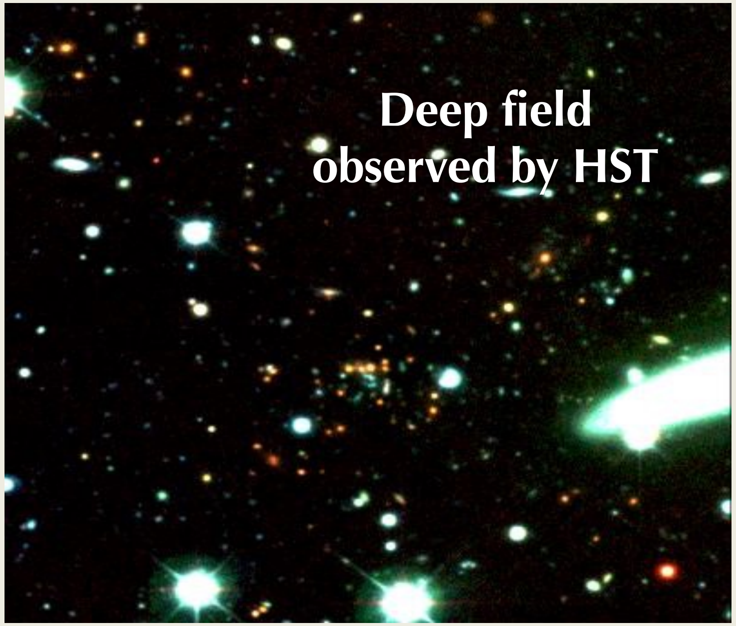



**Redshift**

Doppler effect

$V/c=(\lambda-\lambda_0)/\lambda_0=z$

# Expanding Universe




**Hubble's law**

$V = H_0 D$

- Measurement of the velocity of galaxies with **their redshift (z)**

Doppler effect : $v/c = (\lambda - \lambda_0)/\lambda_0 = z$

- Increasing z $\Rightarrow$ Back in time

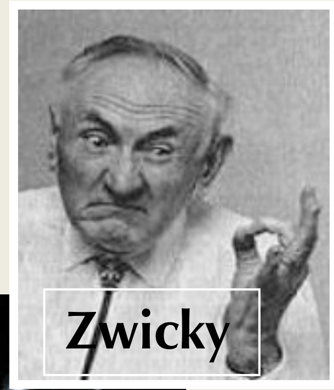
**What value of $H_0$?**

- Controversial and controverted measurement.

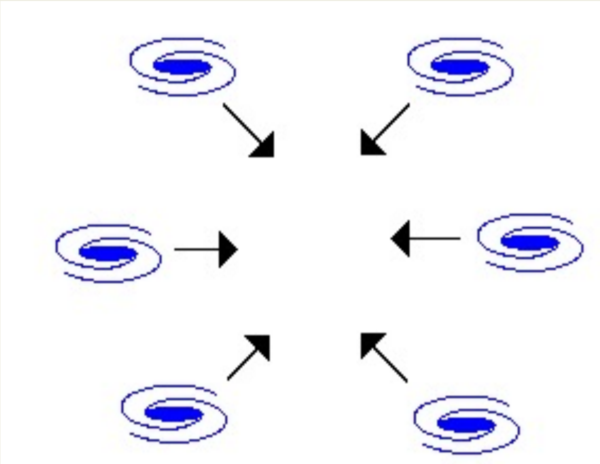
**What about gravitation?**

- It will slow the expansion of the universe for dark matter - Deceleration.
- It will accelerate the expansion of the universe for "repulsive" matter - Acceleration.

# Discovery of Dark Matter

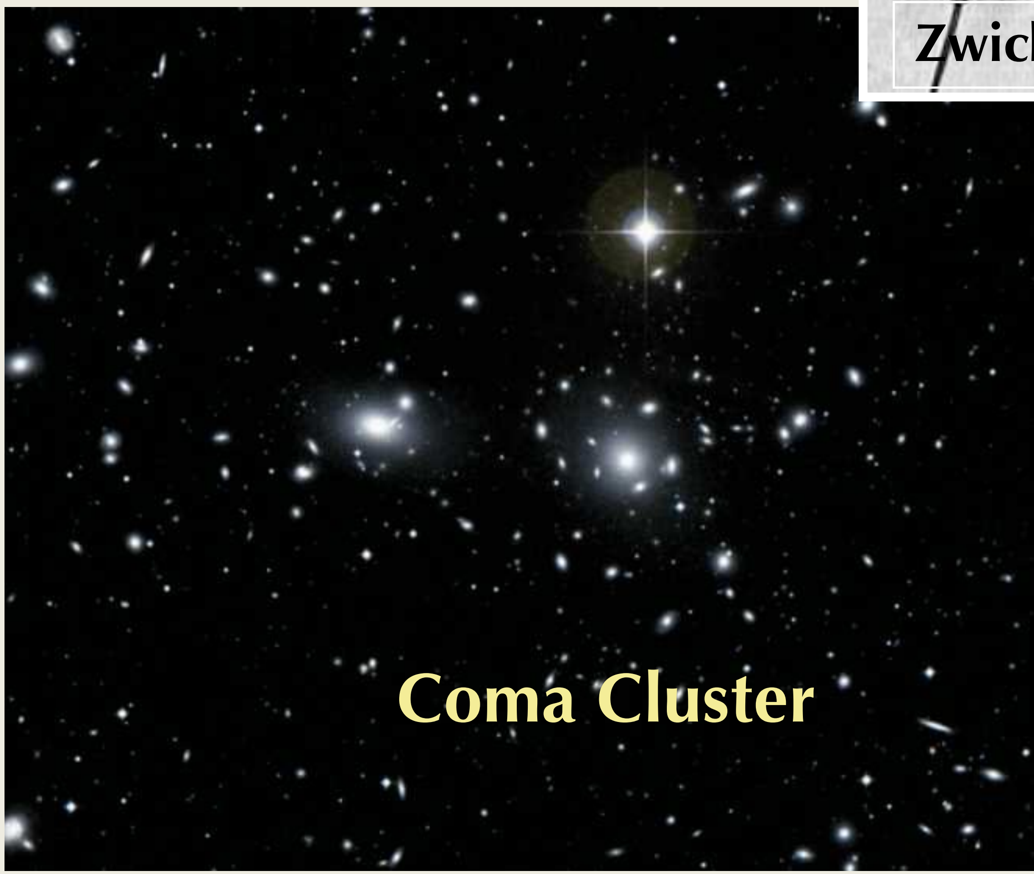



**Zwicky, 1933**



**“Invisible” matter**

- Galaxy cluster.
- Peculiar velocity of galaxies too high.
- Virial theorem.
- Visible galaxies are about 1-10% of the total mass.

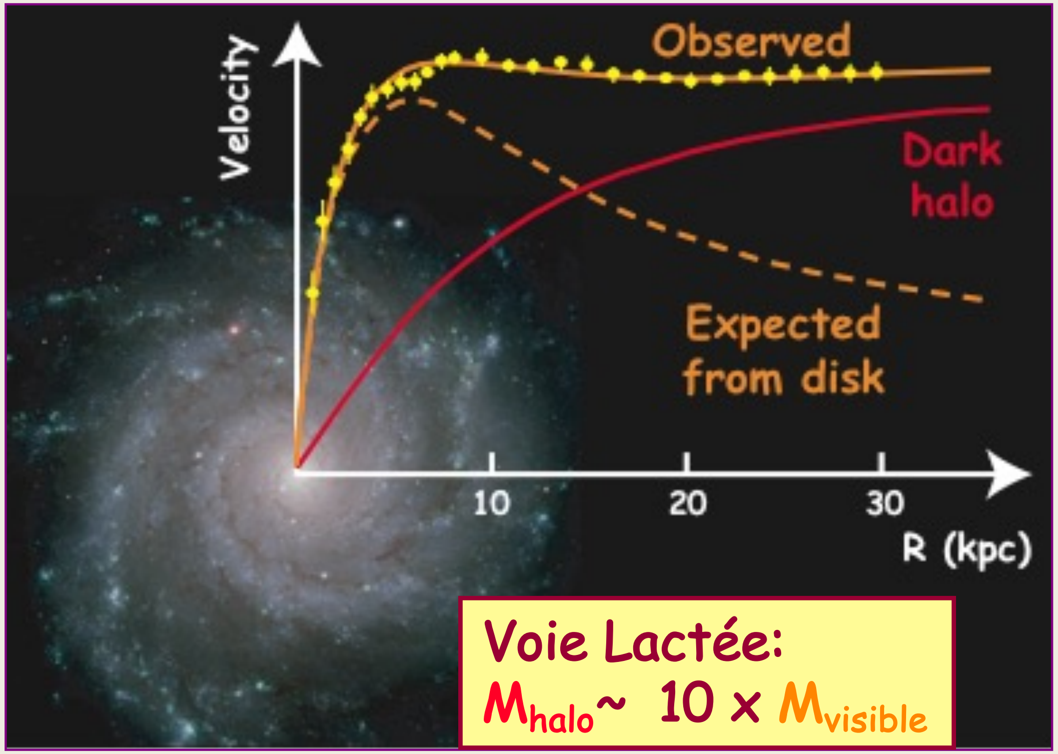



$$E_p + 2\,E_c = 0$$

$E_c = 1/2\ M\ v^2$ and $E_p = -\frac{1}{2}\ GM^2/R$

**$M = 2Rv^2/G$**

# 1970: how to weigh galaxies?




**Galactic rotation curves**

- Final proof by measuring the velocity of stars within galaxies
- Work of Vera Rubin and Kent Ford in the 70'

**Newton Law**

$$\boxed{\begin{gathered} E_c + E_p = 0 \\ V_{rot} = \sqrt{\frac{2GM}{R}} \end{gathered}}$$

**Constant rotation curve**

↓

**Halo of Dark Matter**

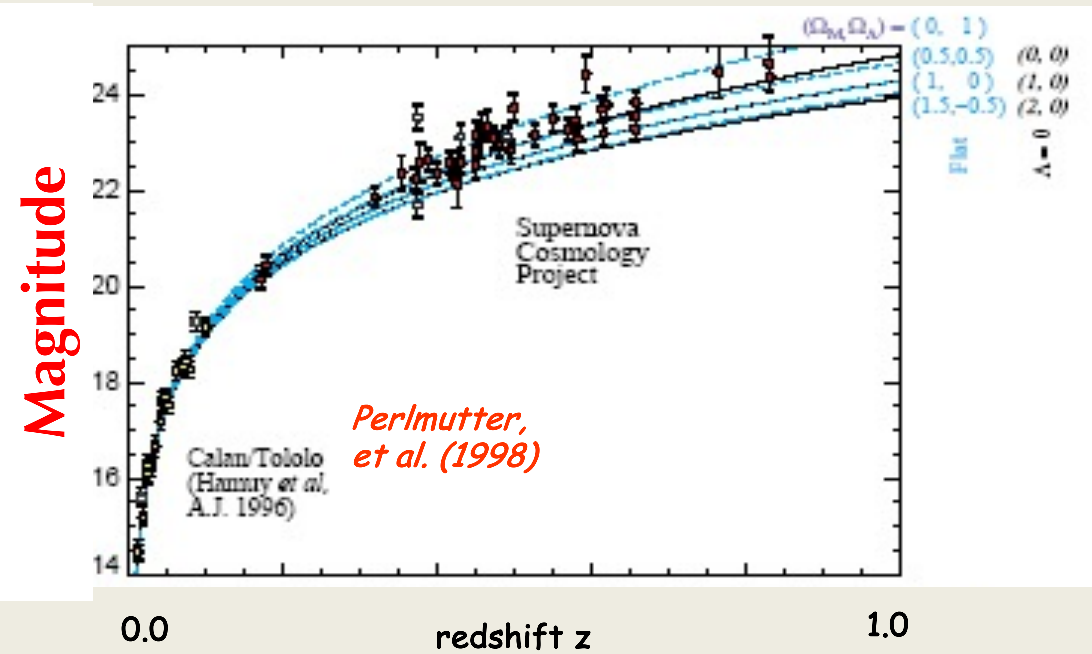
# Dark energy


**Discovery with supernovae**

- **In 1998,** Hubble diagram (magnitude ↔ z ) with standard candles (SN Ia)

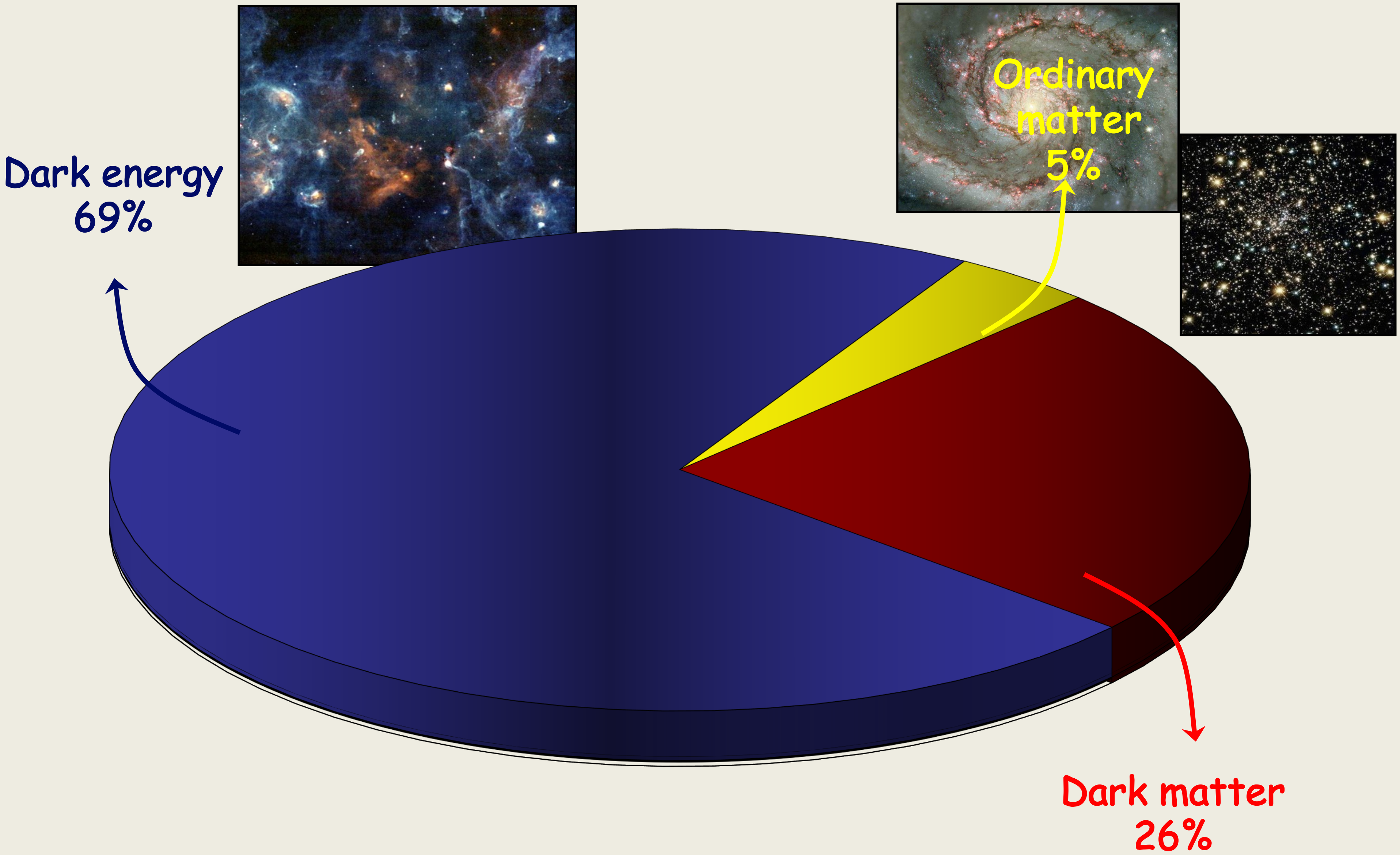
$$L_{obs} \propto L_0/R^2$$

- Acceleration of expanding Universe




- **~2/3 of Dark Energy** repulsive for gravitation
- **~1/3** "classical" matter

# Content of the Universe



# Summary - Content of the Universe

- Radiation 5 $10^{-5}$

  Cosmic microwave background (CMB) + neutrinos
- Ordinary matter (baryonic) ~5% ~ 1 proton / 4 $m^3$

  - galaxies (stars, interstellar gas, dust)
  - typical galaxy: $10^{12}$ $M_\odot$
  - $\langle\rho_{visible}\rangle = 10^{-31}$ g / $cm^3$ 0.2%
  - intergalactic gas
- Dark matter ~26%, many evidences:

  - star rotation curves in galaxies
  - galaxy rotation curves in clusters
  - structure development, …
- Dark energy ~69%

  - Acceleration of the Universe expansion (SNIa)

# 2) Gravitation and General Relativity

# Gravitation and relativity

- 1905 : Special Relativity

- Incompatible with Newton $F = \dfrac{G m_1 m_2}{\left| r_1(t) - r_2(t) \right|^2}$
- Instantaneous force, $r_1(t)$ et $r_2(t)$ at the *same t*
- Newton = approximation of a more fundamental theory

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}^2}$$

Coulomb law approximation of Maxwell eq.

- 1915 : General relativity

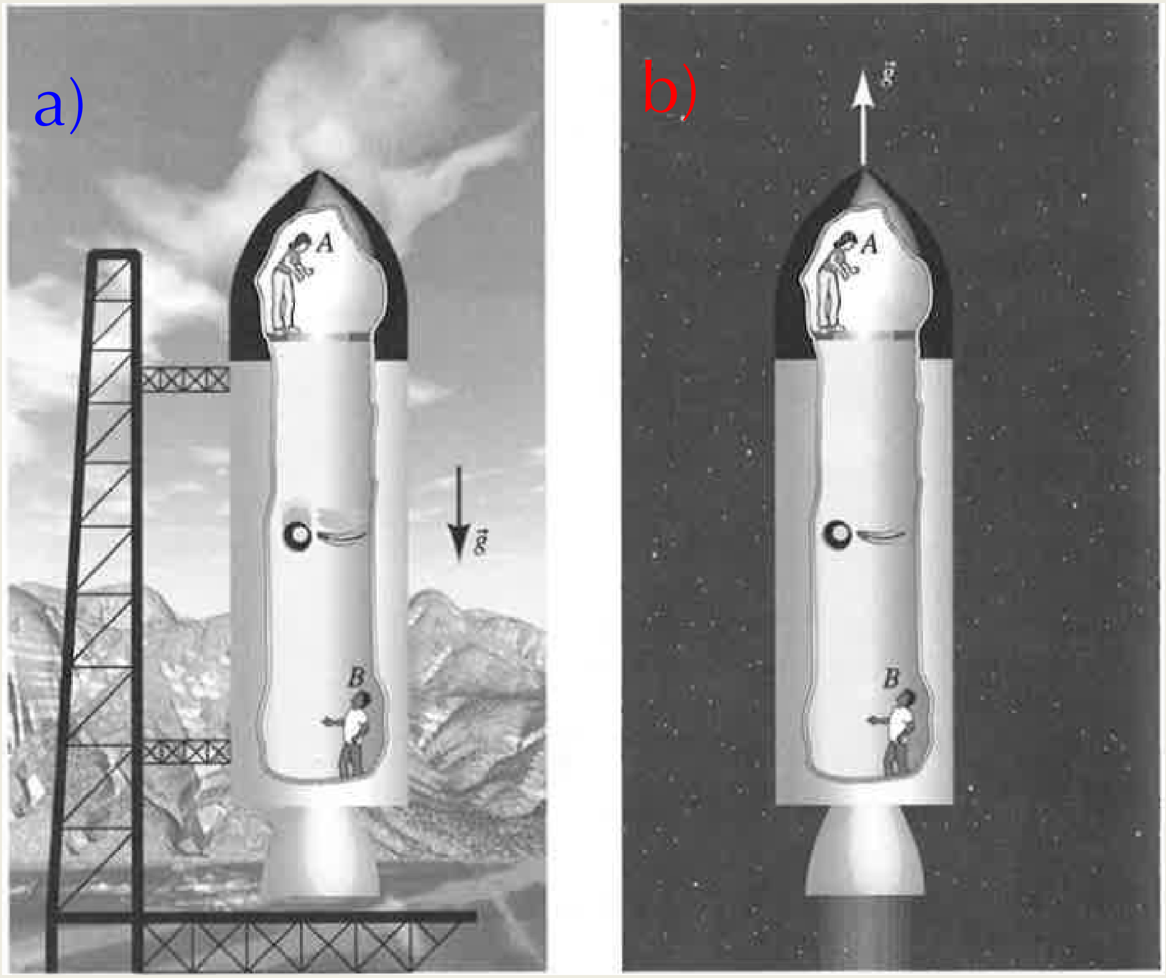
- Not just a new theory of gravitation
- But a revolution in our conception of space and time
- Gravitation = curvature of spacetime → Pure geometry

# Equivalence principle

a) $m_i a = m_g\, g \Rightarrow$
the lead ball and the feather experience the same Acceleration
$\Rightarrow m_i = m_g$ and $a=g$

b) they have the same constant speed but appear with the same acceleration

- uniform gravitational field
  = uniform acceleration




James B. Hartle

study effect of acceleration $\Rightarrow$ study gravitation

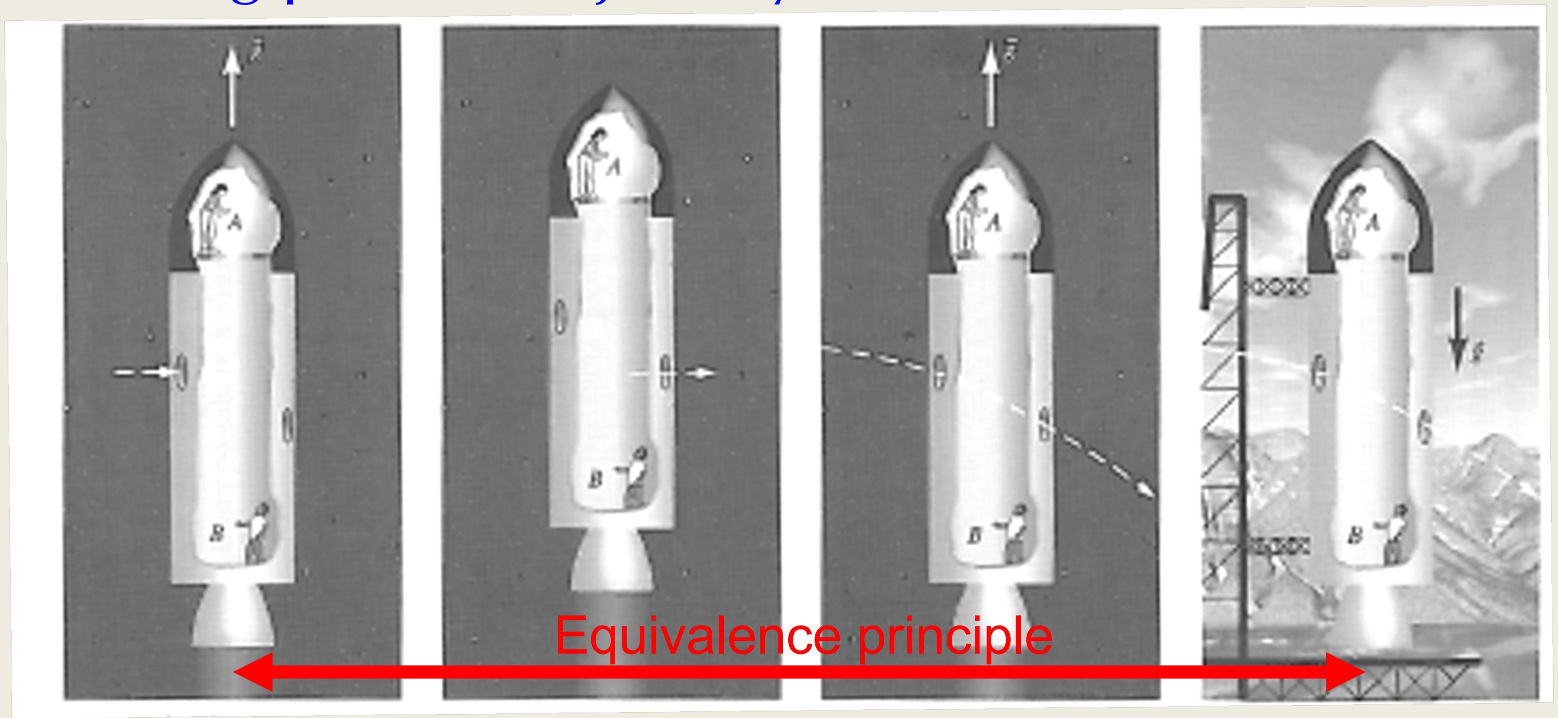
# Equivalence principle

**Equivalence Principle**: An experiment in a freely falling laboratory, small enough and over a sufficiently small duration, is indistinguishable from the same experiment in an inertial frame away from all sources of gravitation

**Gravity can be removed by free fall or conversely created by an acceleration**

# Light is falling !

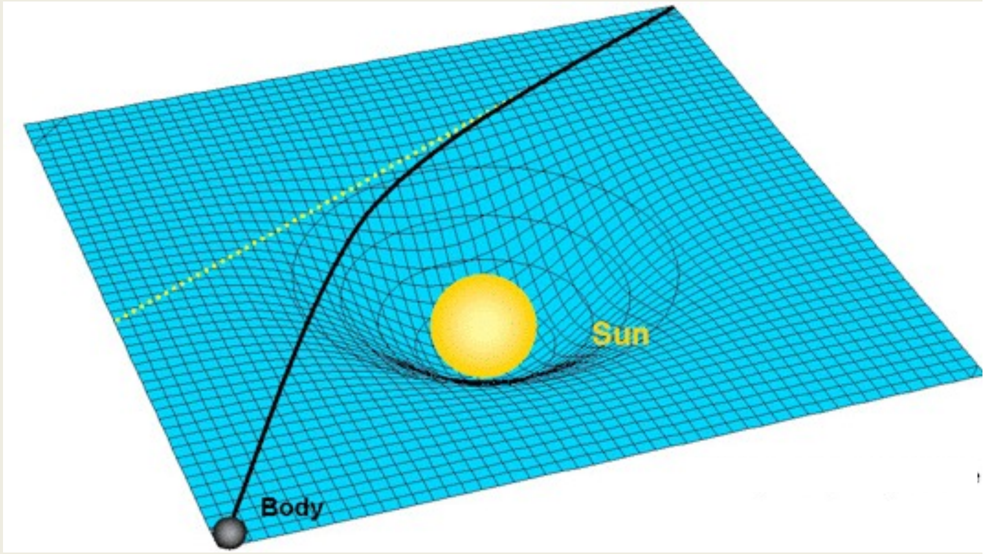
- Equivalence principle applies for all physical laws
  including photon trajectory




- $\Delta v = g\ \Delta t = g\ d/c \ll c \;\Rightarrow\;$ tiny effect on earth
  $\theta \sim \Delta v/c \sim gd / c^2 \qquad d=10\text{m} \;\Rightarrow\; \theta \sim 9.81\text{x}10 / (3\ 10^8)^2 = 10^{-15}$ !
  $\theta \sim 2GM / Rc^2 \sim 4\ \mu\text{rad}$ around sun!

# Curved spacetime

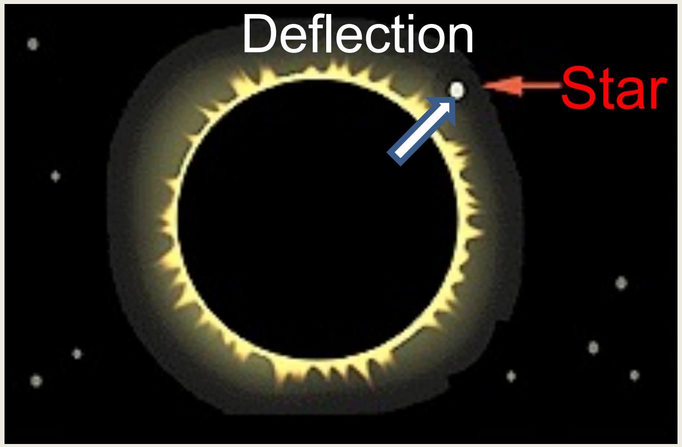
**light rays are bent**



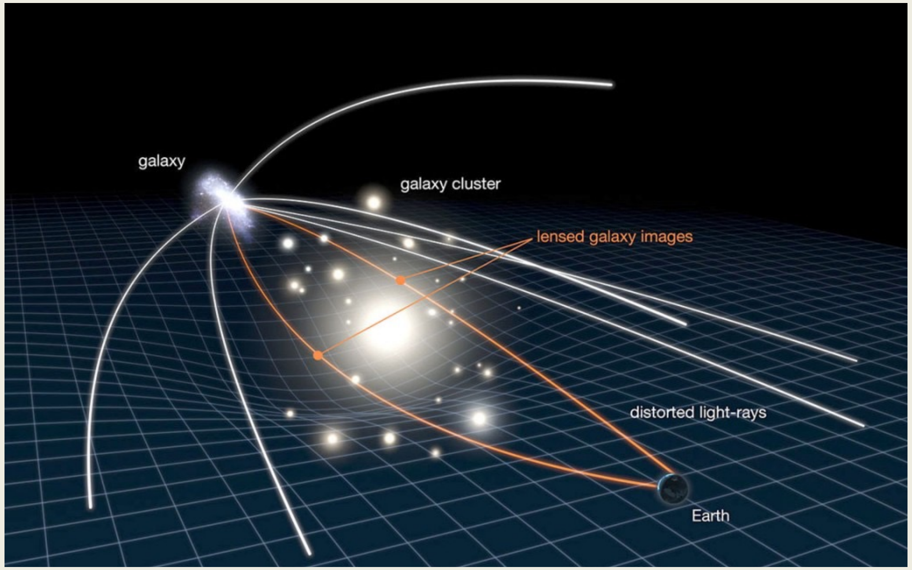

• 1915 : Einstein, General Relativity
mass curves spacetime
and bends light

• 1919 : Arthur Eddington observes light deviation by the sun during a solar eclipse :

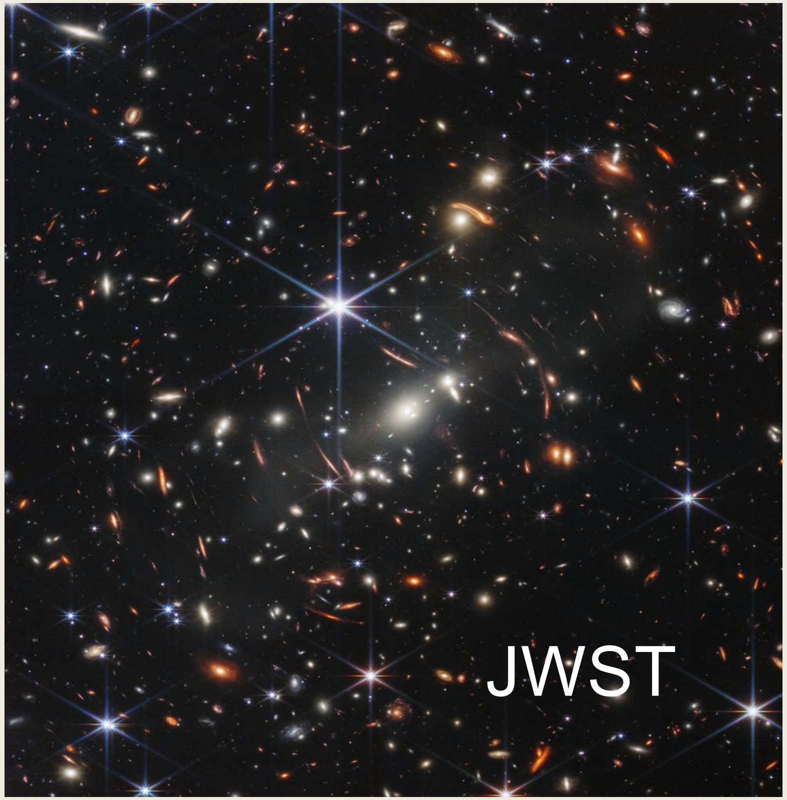
- 1.75 arc second = 8.5 µrad as predicted by Einstein
- Twice the deflection predicted by first computation (based on Eq. principle alone)



# Curved spacetime - Gravitational lensing




- Strong gravitational lensing: modern proof of RG

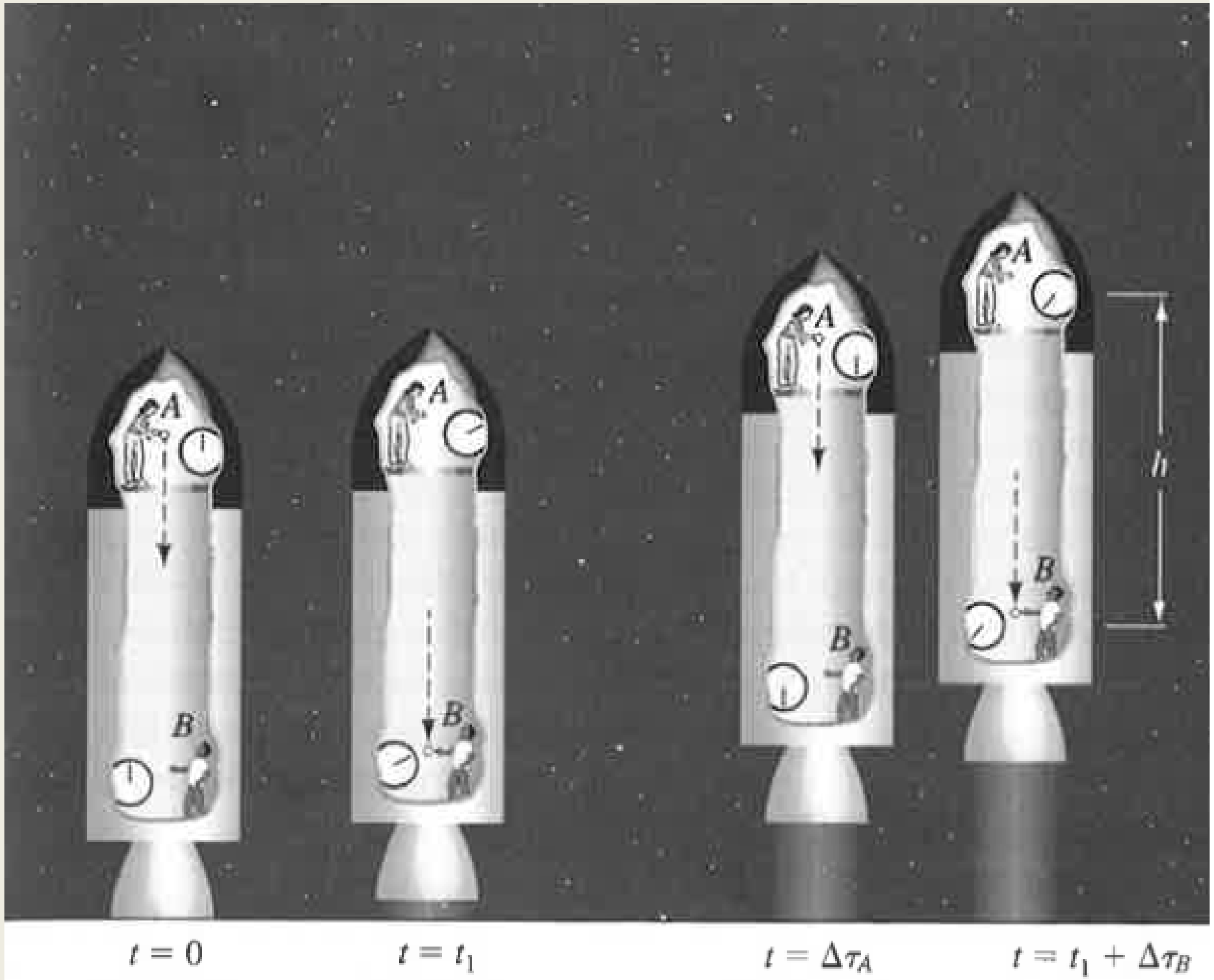



- July 11 2022 (yesterday): James Webb Space Telescope released this deep field

- Galaxies behind galaxy cluster SMACS 0723 are curved and warped

# Clocks and gravitation

- a rocket in deep space with acceleration +g
- A emits at t=0 and $\Delta\tau_A$
- B receives at $t=t_1$ and $t_1 + \Delta\tau_B$




- Propagation time : $(t_1 - 0)$
  acceleration $\Rightarrow$ faster
  $(t_1+\Delta\tau_B) - \Delta\tau_A < t_1 - 0$
  $\Rightarrow$ $\mathbf{\Delta\tau_B < \Delta\tau_A}$

- Calculation gives (totally classic): $\boxed{\Delta t_B = \left(1 - \frac{gh}{c^2}\right)\Delta t_A}$

# Clocks and gravitation

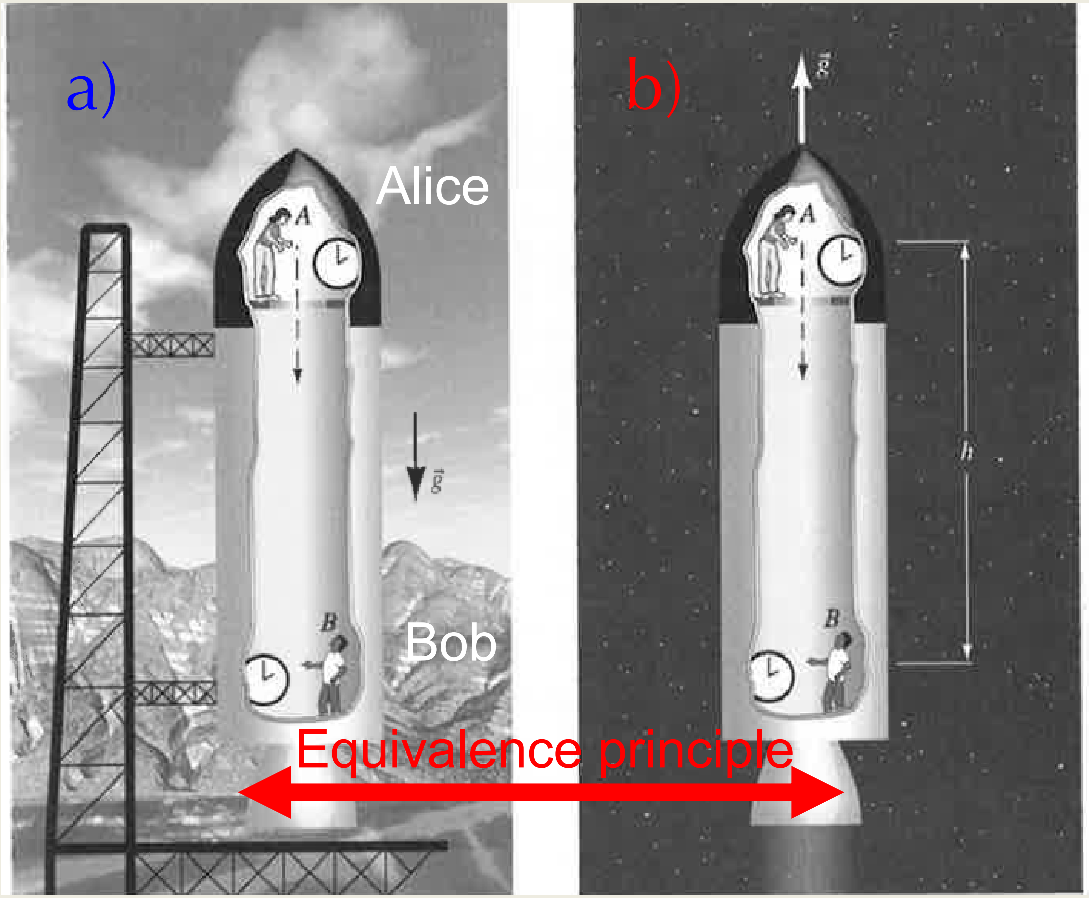
Equivalence Principle
(a) equivalent to (b)

$$\Delta t_B = \left(1 - \frac{gh}{c^2}\right)\Delta t_A$$

Times run slower in a gravitational field !

$h = z_A - z_B \Rightarrow gh \sim \Phi_A - \Phi_B$

$$\boxed{\Delta t_B = \left(1 - \frac{\Phi_A - \Phi_B}{c^2}\right)\Delta t_A}$$




Bob is younger than Alice….

# Gravitational "redshift"

$$\Delta t_B = \left(1 - \frac{\Phi_A - \Phi_B}{c^2}\right) \Delta t_A$$

- at the surface of a star: $\phi_A = -GM/R$

  far away: $\phi_B = 0$

$$\Delta t_\infty = \left(1 + \frac{GM}{Rc^2}\right) \Delta t_*$$

$$\nu_\infty = \left(1 - \frac{GM}{Rc^2}\right) \nu_* < \nu_*$$

$\Rightarrow$ gravitational redshift

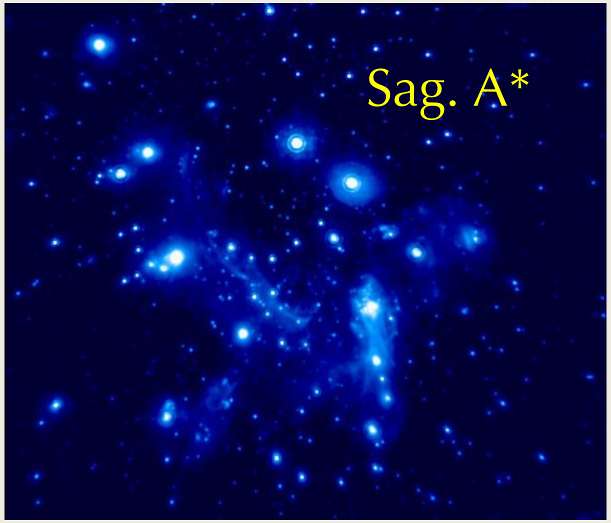
the photon looses energy going out of the potential well

Positive shift in wavelength $\Delta\lambda/\lambda > 0$

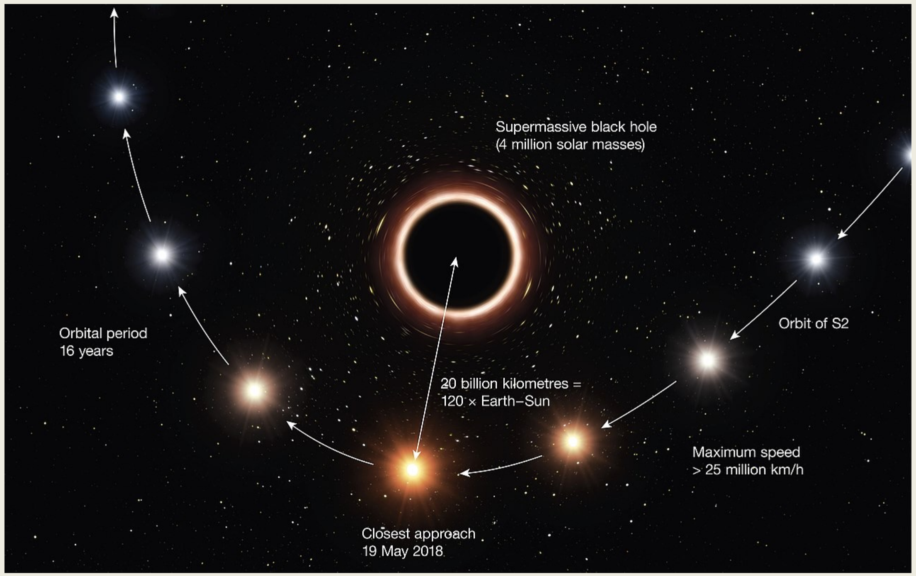
- very important for GPS : $\Delta\nu/\nu \sim 4.10^{-10}$

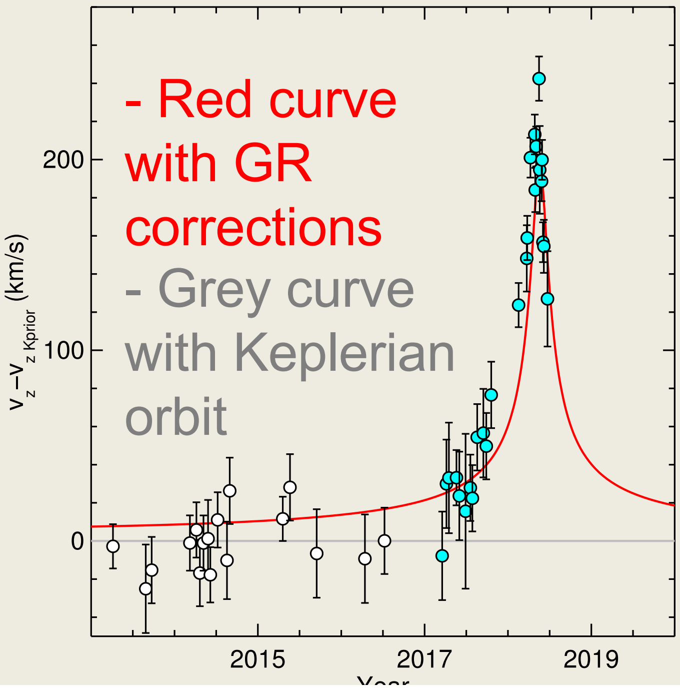
  after 1h : $10^{-10} \times 3600$ s error $\Rightarrow$ ~400 m error

# S2 star close to MW black Hole




- Close to source Sagittarius A*, BH in the Milky way (~4.10$^6$ solar mass)
- S2 star very close to the BH on May 19 2018
- Verification of Einstein shift (plot below)
- Redshift (c.$\Delta\lambda/\lambda$ $\rightarrow$ speed km/s), note sign!



# Interlude – Science in movies



**Planet of the Apes**

- Twin paradox in Special Relativity (SR)
- Lorentz boost
  $ct = \gamma(ct' + \beta x)$
  $\gamma = 1/(1 - \beta^2)^{1/2} > 1$
- Time dilatation $\mathbf{T = \gamma T'}$

**Interstellar**

- Strong gravitational field (GR)
- Proximity to a black hole (BH)
- $\mathbf{T = (1 + GM/(Rc^2)).T'}$

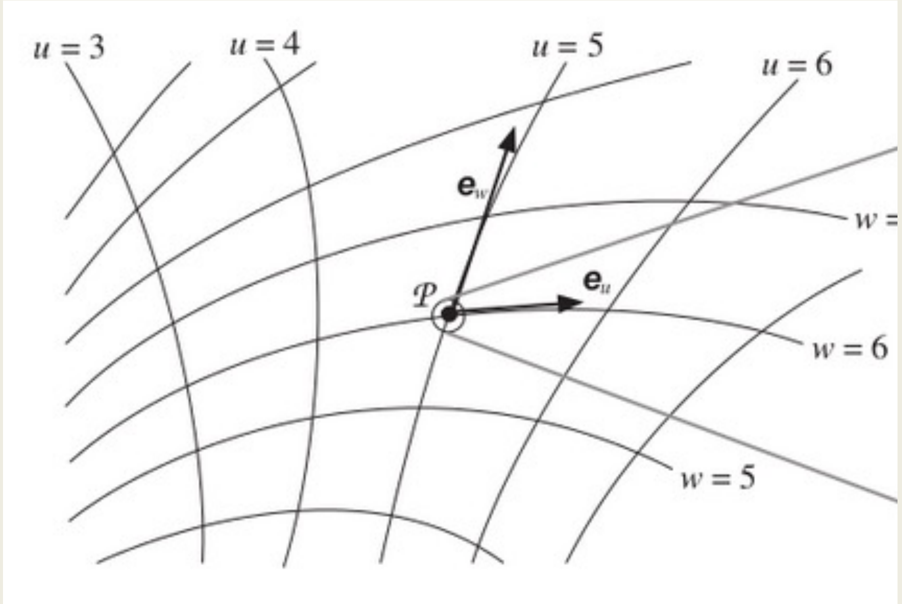
# From 3D space to 4D spacetime

1) In the usual 3D Euclidian

- Define a coordinate system

  $x^i$ = a labeling of space

  ex plan (x,y) or (r,ϕ)




- We can measure distances with a ruler: $dS^2 = \mathbf{g_{ij}(x)}\, dx^i\, dx^j$

- The metric $g_{ij}(x)$ alone totally defines the geometry

- but $dS^2 = dr^2 + r^2 d\phi^2$ and $dS^2 = dx^2 + dy^2$ : same geometry

  we mean $(dx)^2$ and not $d(x^2)$ ! length² not surface

2) We generalize to a non-Euclidian 4D spacetime

# Curved spacetime – Metric

- We generalize the 3D metrics to 4D in special relativity

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 = \eta_{\alpha\beta}dx^\alpha dx^\beta$$

- We generalize in GR with non constant terms $g_{\mu\nu}$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$

- The equivalence principle tells, $\Delta\tau_B = \left(1 - \frac{\Phi_A - \Phi_B}{c^2}\right)\Delta\tau_A$

- GR : for a weak and static field, the metric is :

$$ds^2 = \underbrace{\left(1 + \frac{2\Phi(x)}{c^2}\right)}_{\textbf{equivalence principle}} c^2dt^2 - \underbrace{\left(1 - \frac{2\Phi(x)}{c^2}\right)}_{\textbf{GR}} (dx^2 + dy^2 + dz^2)$$

fixed object $\Delta\tau^2 = \frac{ds^2}{c^2} = \left(1 + \frac{2\Phi}{c^2}\right)dt^2 \Rightarrow \Delta\tau_B = \left(1 - \frac{\Phi_A - \Phi_B}{c^2}\right)\Delta\tau_A$