Overview of $g-2$ of the charged leptons

ieri, oggi, domani

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OUTLINE

- Introduction: LFUV in the SM
- Ieri: a historical perspective
- Oggi: the situation today
- Domani: prospects for the (near) future

Introduction

General context:

Probes of the energy frontier, i.e. direct searches at colliders, have so far not provided evidence of physics beyond the SM

Indirect indications for possible BSM physics exist, either from the cosmic frontier (dark matter,...) or from the precision frontier (LHCb anomalies, muon $g-2$,...)

Is there LFUV beyond the one present in the SM, i.e. $h \to \ell^+\ell^-$?

The anomalous magnetic moments of charged leptons are sensitive to very subtle quantum effects that may also receive contributions from putative degrees of freedom beyond the SM

Such effects are necessarily quite small, so that high precision [i.e. sub-ppm] experiments are required

As a counterpart, a similar precision in the SM predictions for these observables is required in order to draw unambiguous conclusions from the experimental measurements

Leptonic sector of the three-family standard model

- charged leptons: $\ell = e^{\pm}, \mu^{\pm}, \tau^{\pm}$

$$q = \pm 1 \text{ (charge)} \qquad s = \frac{1}{2} \text{ (spin)}$$

differ only through their couplings to the Higgs:

this is the only source of LFU violation in the SM!

$$\begin{aligned} m_e &= 0.510\,998\,946\,1(3\,1)\ \mathrm{MeV} \\ m_\mu &= 105.658\,374\,5(2\,4)\ \mathrm{MeV} \\ m_\tau &= 1\,776.86(12)\ \mathrm{MeV} \end{aligned}$$

- this situation has dramatic consequences for the lifetimes:

$$\tau_e > 6.6 \cdot 10^{28}\, y, \quad \tau_\mu = 2.1969811(22) \cdot 10^{-6}\, s, \quad \tau_\tau = 290.3(5) \cdot 10^{-15}\, s$$

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- and for the magnetic moments:

$$\boldsymbol{\mu}_\ell = g_\ell \left(\frac{e_\ell}{2 m_\ell c} \right) \mathsf{S}\,, \; \mathsf{S} = \hbar \frac{\boldsymbol{\sigma}}{2}$$

Response of a charged lepton to an external (and static) electromagnetic field

$$\langle \ell; p'|\mathcal{J}_\rho(0)|\ell; p\rangle \equiv \bar{\mathrm{u}}(p')\Gamma_\rho(p',p)\mathrm{u}(p)$$
$$= \bar{\mathrm{u}}(p')\Big[F_1(k^2)\gamma_\rho + \frac{i}{2m_\ell}F_2(k^2)\sigma_{\rho\nu}k^\nu - F_3(k^2)\gamma_5\sigma_{\rho\nu}k^\nu + F_4(k^2)(k^2\gamma_\rho - 2m_\ell k_\rho)\gamma_5\Big]\mathrm{u}(p)$$

(uses only the conservation of the electromanetic current $\mathcal{J}_\rho$, $k_\mu \equiv p'_\mu - p_\mu$)

$F_1(k^2) \rightarrow$ Dirac form factor , $F_1(0) = 1$

$F_2(k^2) \rightarrow$ Pauli form factor $\rightarrow F_2(0) = a_\ell$

$F_3(k^2) \rightarrow$ $\not{P}$, $\not{T}$, electric dipole moment $\rightarrow F_3(0) = d_\ell/e_\ell$

$F_4(k^2) \rightarrow$ $\not{P}$, anapole moment

$$G_E(k^2) = F_1(k^2) + \frac{k^2}{4m_\ell^2}F_2(k^2),\; G_M(k^2) = F_1(k^2) + F_2(k^2)$$

in the SM, $F_2(k^2)$, $F_3(k^2)$, $F_4(k^2)$ are only induced by loops $\longrightarrow$ calculable!

At tree level, $g_\ell = g_\ell^{\rm Dirac} \equiv 2$.

The *anomalous* magnetic moment is induced at loop level:

$$a_\ell \equiv \frac{g_\ell - g_\ell^{\rm Dirac}}{g_\ell^{\rm Dirac}} (\equiv F_2(0))$$

a_ℓ probes all the degrees of freedom of the standard model,
and possibly beyond...

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... provided one can measure it with enough precision !...

... and predict its value in the standard model with a comparable accuracy!

leri: a historical perspective

Experimental aspects

- The electron case

1947: hf splitting in Na and Ga (0.2% discrepancy with the value $g_e^{\text{Dirac}} = 2$)

[P. Kusch, H. M. Foley, Phys. Rev. 73, 412 (1947), Phys. Rev. 74, 250 (1948)]

1958: first direct measurement of g_e for *free* electrons

[H. G. Dehmelt, Phys. Rev. 109, 381 (1958)]

1968 $\longrightarrow$ 1987: Penning trap type experiments $\longrightarrow$ single trapped electron (geonium)

$$\begin{aligned} a_{e^-}^{exp} &= 1\,159\,652\,188.4(4.3) \cdot 10^{-12} \;\; [3.7\,\mathrm{ppb}] \\ a_{e^+}^{exp} &= 1\,159\,652\,187.9(4.3) \cdot 10^{-12} \;\; [3.7\,\mathrm{ppb}] \end{aligned}$$

[R.S. van Dyck Jr. et al., PRL 59, 26 (1987)]

g_{e^-}/g_{e^+} = 1 + (0.5$\pm$2.1)$\times 10^{-12}$ probes CPT invariance

$\big(\rightarrow |M_{K^0} - M_{\overline{K}^0}|/M_{K^0} \leq 10^{-18}$ (90% CL)$\big)$

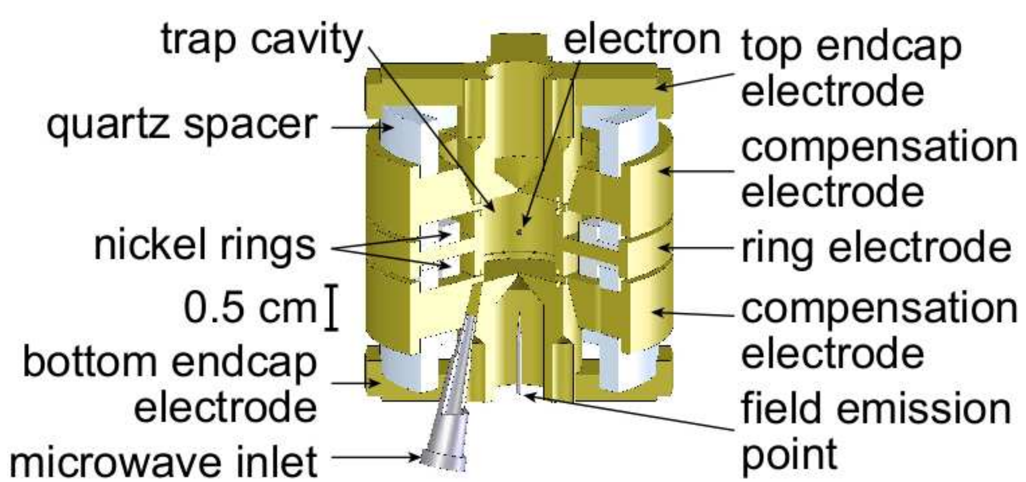
New series of high precision measurements conducted by the Harvard group (G. Gabrielse et al.)

$$a_e^{exp} = 1\,159\,652\,180.85(0.76) \cdot 10^{-12} \quad [0.66\,\mathrm{ppb}]$$

[Odom et al., PRL 97, 030801 (2006)]

$$a_e^{exp} = 1\,159\,652\,180.73(0.28) \cdot 10^{-12} \quad [0.24\,\mathrm{ppb}]$$

[D. Hanneke, S. Forgwell, G. Gabrielse, PRL 100, 120801 (2008)]



- The muon case

Recall: $\tau_\mu = 2.1969811(22) \cdot 10^{-6}\, s$

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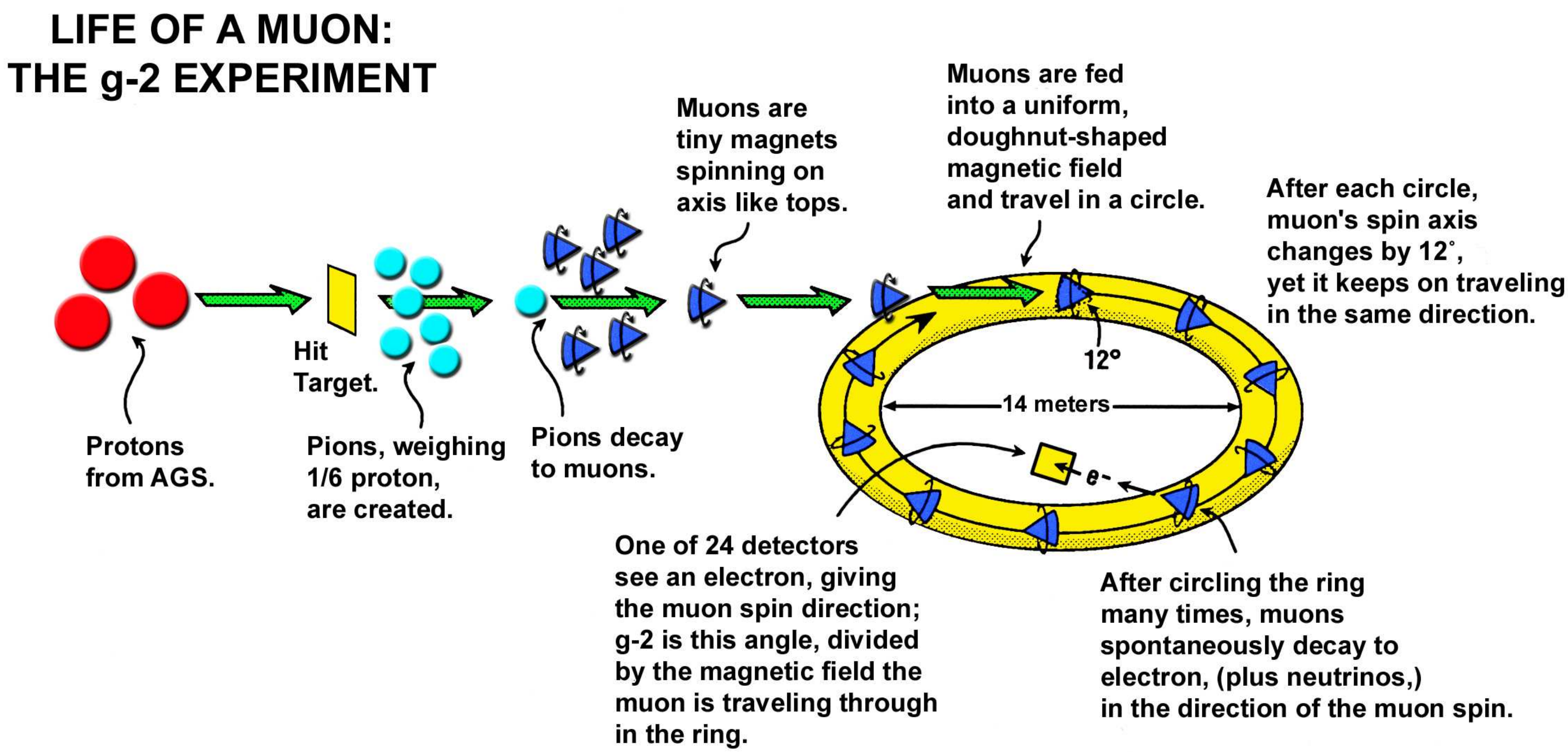
- muon storage ring experiment (CERN & BNL)



- muon storage ring experiment $[\vec{\beta}\cdot\vec{E} = \vec{\beta}\cdot\vec{B} = 0]$

$$\vec{\omega}_s - \vec{\omega}_c = -\frac{e}{m_\mu c}[a_\mu \vec{B} - (a_\mu - \frac{1}{\gamma^2 - 1})\vec{\beta}\times\vec{E}] - 2\,\vec{d}_\mu \cdot [\vec{\beta}\times\vec{B} + \vec{E}]$$

- $\gamma \sim 29.3$ [electrostatic focusing will not affect the spin]

[Muon g-2 Coll., H. N. Brown et al., Phys. Rev. Lett. 86, 2227 (2001)]

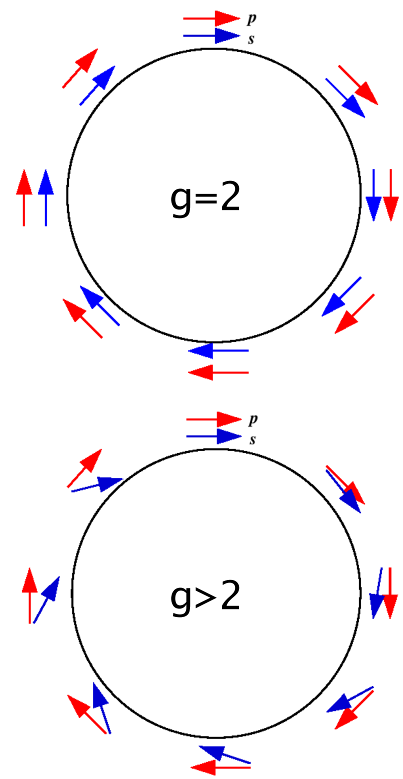
- $|d_\mu| < 1.9 \cdot 10^{-19}$ [95% CL] [Muon g-2 Coll., G. W. Bennett et al., Phys. Rev. D 80 (2009)]

Large advantage to using a storage ring

$$\omega_s = g\frac{eB}{2mc}$$

$$\omega_c = \frac{eB}{mc}$$

$$\begin{aligned}\omega_a &= \omega_s - \omega_c, \\ &= \frac{eB}{mc}\left(\frac{g}{2} - 1\right), \\ &= \frac{eB}{mc}\frac{g-2}{2}, \\ &= a_\mu \frac{eB}{mc},\end{aligned}$$



Measures a_μ directly

Factor of 800 in precision for free over experiments measuring g

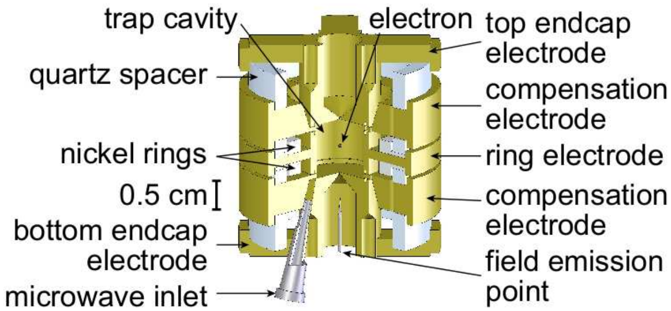
All recent experiments and proposals CERN II/III, BNL, FNAL, & J-PARC rely on this trick

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]
CERN I	1961	μ^+	11 450 000(220 000)	4300
CERN II	1962-1968	μ^+	11 661 600(3100)	270
CERN III	1974-1976	μ^+	11 659 100(110)	10
CERN III	1975-1976	μ^-	11 659 360(120)	10
BNL E821	1997	μ^+	11 659 251(150)	13
BNL E821	1998	μ^+	11 659 191(59)	5
BNL E821	1999	μ^+	11 659 202(15)	1.3
BNL E821	2000	μ^+	11 659 204(9)	0.73
BNL E821	2001	μ^-	11 659 214(9)	0.72

$$a_\mu^{\mathrm{exp}} = 11\,659\,208.9(5.4)(3.3) \cdot 10^{-10} \quad [0.54\,\mathrm{ppm}]$$

[G. W. Bennett *et al.* [Muon $g-2$ collaboration, BNL E821], Phys. Rev. D 73, 072003 (2006)]
[M. Tanabashi *et al.* [Particle Data Group], Phys. Rev. D 98, 030001 (2018)]

Experimentally measured to very high precision:



$$a_e^{\text{exp}} = 1\,159\,652\,180.91(0.26) \cdot 10^{-12}$$

$$\Delta a_e^{\text{exp}} = 2.6 \cdot 10^{-13} \text{ [0.24ppb]}$$

[D. Hanneke et al, Phys. Rev. Lett. 100, 120801 (2008)]

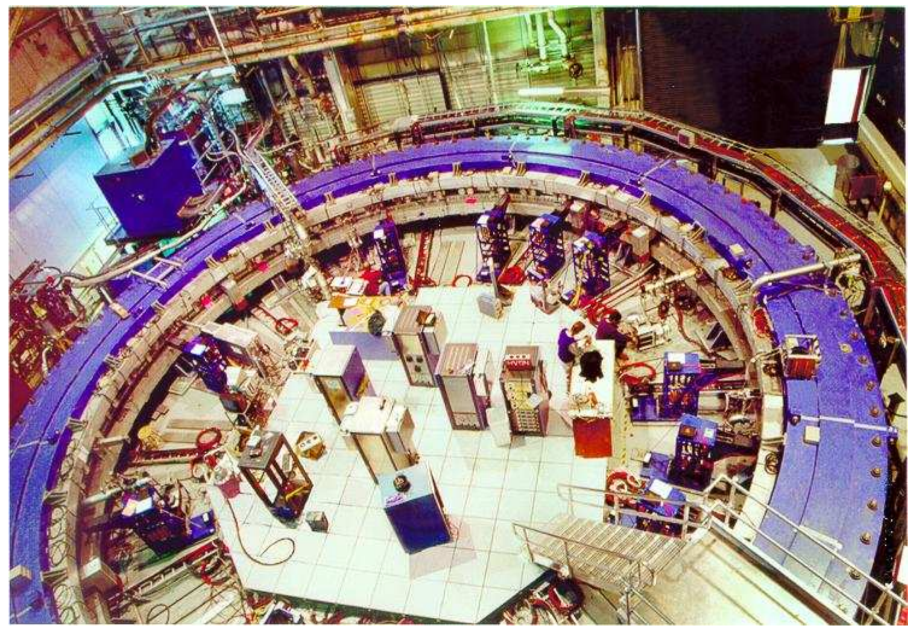
$$\tau_\mu = (2.19703 \pm 0.00004) \times 10^{-6} \text{ s}$$

$$\gamma \sim 29.3,\, p \sim 3.094 \text{ GeV/c}$$

$$a_\mu^{\text{exp}} = 11\,659\,208.9(6.3) \cdot 10^{-10}$$

$$\Delta a_\mu^{\text{exp}} = 6.3 \cdot 10^{-10} \text{ [0.54ppm]}$$

[G. W. Bennett et al, Phys Rev D 73, 072003 (2006)]



The tau case

$$\tau_\tau = 290.3(5) \cdot 10^{-15}\,\mathrm{s}$$

- $e^+e^- \to \tau^+\tau^-\gamma$

$$-0.052 < a_\tau^{exp} < +0.058 \text{ (L3, 1998, 95\% CL)}$$

[Phys. Lett. B 434, 169 (1998)]

$$-0.068 < a_\tau^{exp} < +0.065 \text{ (OPAL, 1998, 95\% CL)}$$

[Phys. Lett. B 431, 188 (1998)]

- $e^+e^- \to e^+e^-\tau^+\tau^-$

$$-0.052 < a_\tau^{exp} < +0.013 \text{ (DELPHI, 2004, 95\% CL)} \qquad a_\tau^{exp} = -0.018(17)$$

[Eur. Phys. J. C 35, 159 (2004)]

- Reanalysis of experiments $\longrightarrow$ $-0.007 < a_\tau^{exp} < +0.005$

[G. A. Gonzalez-Sprinberg, A. Santamaria, J. Vidal, Nucl. Phys. B 582, 3 (2000) [hep-ph/0002203]

Theoretical aspects

- QED contributions
- Weak contributions
- Strong interactions

What about the situation from the theory side?

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$$a_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{weak}}$$

a_μ^{QED}: loops with only photons and leptons $\longleftarrow$ fully perturbative

a_μ^{had}: loops with photons and leptons and at least one quark loop dressed with gluons $\longleftarrow$ fully non-perturbative

a_μ^{weak}: loops with also contributions from the electroweak sector $\longleftarrow$ perturbative with non-perturbative pieces

QED contributions : loops with only photons and leptons

$\longrightarrow$ can be computed in perturbation theory:

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

$A_1^{(2n)} \longrightarrow$ mass-independent (universal) contributions (one-flavour QED)

$A_2^{(2n)}(m_\ell/m_{\ell'}),\ A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''}) \longrightarrow$

mass-dependent (non-universal) contributions (multi-flavour QED)

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mass-dependent (non-universal) contributions (multi-flavour QED)

For the electron, $A_1^{(2n)}$ matter, whereas $A_2^{(2n)}$ and $A_3^{(2n)}$ are suppressed by powers of m_e^2/m_μ^2 and m_e^2/m_τ^2 (times logarithms)

QED contributions : loops with only photons and leptons

$\longrightarrow$ can be computed in perturbation theory:

$$a_\ell{}^{\mathrm{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

$A_1^{(2n)} \longrightarrow$ mass-independent (universal) contributions (one-flavour QED)

$A_2^{(2n)}(m_\ell/m_{\ell'}),\ A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''}) \longrightarrow$

mass-dependent (non-universal) contributions (multi-flavour QED)

For the muon, $A_1^{(2n)}$ are negligible, whereas $A_2^{(2n)}(m_\mu/m_e)$ are enhanced by powers of $\ln(m_\mu/m_e)$ $\left(\pi^2 \ln \frac{m_\mu}{m_e} \sim 50\right)$

QED contributions : loops with only photons and leptons

$\longrightarrow$ can be computed in perturbation theory:

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

Expressions for $A_1^{(2)}$, $A_1^{(4)}$, $A_2^{(4)}$, $A_1^{(6)}$, $A_2^{(6)}$, $A_3^{(6)}$ known analytically

J. Schwinger, Phys. Rev. 73, 416L (1948)

C. M. Sommerfield, Phys. Rev. 107, 328 (1957); Ann. Phys. 5, 26 (1958)

A. Petermann, Helv. Phys. Acta 30, 407 (1957)

H. Suura and E. Wichmann, Phys. Rev. 105, 1930 (1955)

A. Petermann, Phys. Rev. 105, 1931 (1955)

H. H. Elend, Phys. Lett. 20, 682 (1966); Err. Ibid. 21, 720 (1966)

M. Passera, Phys. Rev. D 75, 013002 (2007)

S. Laporta, E. Remiddi, Phys. Lett. B265, 182 (1991); B356, 390 (1995); B379, 283 (1996)

S. Laporta, Phys. Rev. D 47, 4793 (1993); Phys. Lett. B343, 421 (1995)

$\longrightarrow$ no uncertainties in $A_1^{(2)}$, $A_1^{(4)}$, $A_1^{(6)}$

$\longrightarrow$ precision $A_2^{(4)}$, $A_2^{(6)}$, $A_3^{(6)}$ only limited by precision in $m_\ell/m_{\ell'}$

order $(\alpha/\pi)^4$: 891 diagrams

$A_1^{(8)}$ has also been evaluated! (a_e)

S. Laporta, Phys. Lett. B 772, 232 (2017)

$$A_1^{(8)} = -1.912\,245\,764\,926\,445\,574\,152\,647\,167\,439\,830\,054\,060\,873\,390\,658\,725\,345\,171\,329\ldots$$

Good agreement with earlier numerical evaluations

$$A_1^{(8)} = -1.912\,98(84)$$

T. Aoyama et al.,, Phys. Rev. D 91, 033006 (2015)

Mass-dependent contributions (a_μ)

only a few diagrams are known analytically $\longrightarrow$ numerical evaluation

Automated generation of diagrams, systematic numerical evaluation of multi-dimensional integrals over Feynman-parameter space

$$A_2^{(8)}(m_e/m_\mu) = 9.161\,970\,703(373)\cdot 10^{-4} \quad A_2^{(8)}(m_e/m_\tau) = 7.429\,24(118)\cdot 10^{-6}$$

$$A_3^{(8)}(m_e/m_\mu, m_e/m_\tau) = 7.468\,7(28)\cdot 10^{-7}$$

$$A_2^{(8)}(m_\mu/m_e) = 132.685\,2(60) \quad A_2^{(8)}(m_\mu/m_\tau) = 0.042\,34(12)$$

$$A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 0.062\,72(4)$$

Independent check of mass-dependent contributions

A. Kataev, Phys. Rev. D 86, 013019 (2012)

A. Kurz, T. Liu, P. Marquard, M. Steinhauser, Nucl. Phys. B 879, 1 (2014)

A. Kurz, T. Liu, P. Marquard, A. V. Smirnov, V. A. Smirnov, M. Steinhauser, Phys. Rev. D 92, 073019 (2015)]

Agreement at the level of accuracy required by present and future experiments for a_μ

order $(\alpha/\pi)^5$: 12 672 diagrams...

6 classes, 32 gauge invariant subsets

Five of these subsets are known analytically

S. Laporta, Phys. Lett. B 328, 522 (1994)
J.-P. Aguilar, D. Greynat, E. de Rafael, Phys. Rev. D 77, 093010 (2008)

Complete numerical results have been published

T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 78, 053005 (2008); D 78, 113006 (2008); D 81, 053009 (2010); D 82, 113004 (2010); D 83, 053002 (2011); D 83, 053003 (2011); D 84, 053003 (2011); D 85, 033007 (2012); Phys. Rev. Lett. 109, 111807 (2012); Phys. Rev. Lett. 109, 111808 (2012)

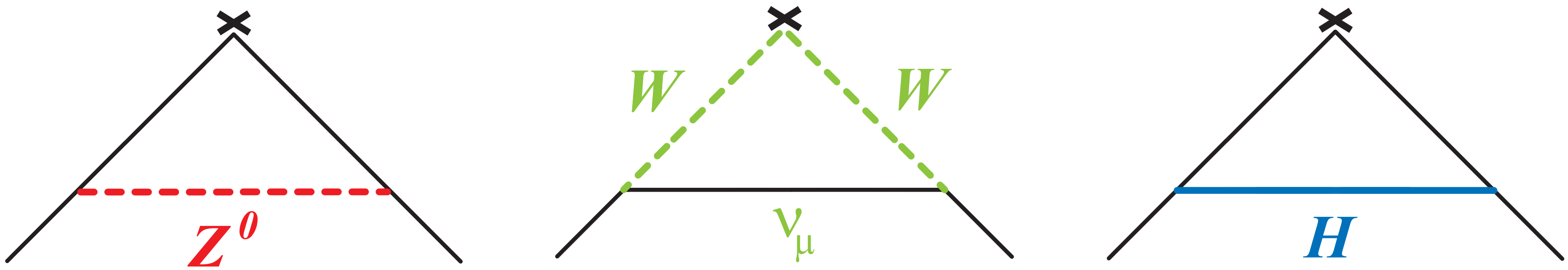
No systematic cross-checks even for mass-dependent contributions

For efforts toward an independent numerical evaluation of $A_1^{(10)}$ (a_e), see however

S. Volkov, arXiv:1905.08007[hep-ph], arXiv:1909.08015[hep-ph]

Contributions from weak interactions

- Weak contributions : W, Z,... loops



$$
\begin{aligned}
a_\mu^{\text{weak(1)}} &= \frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\left[\frac{5}{3}+\frac{1}{3}\left(1-4\sin^2\theta_W\right)^2+\mathcal{O}\left(\frac{m_\mu^2}{M_Z^2}\log\frac{M_Z^2}{m_\mu^2}\right)+\mathcal{O}\left(\frac{m_\mu^2}{M_H^2}\log\frac{M_H^2}{m_\mu^2}\right)\right] \\
&= 19.48\times 10^{-10}
\end{aligned}
$$

W.A. Bardeen, R. Gastmans and B.E. Lautrup, Nucl. Phys. B46, 315 (1972)

G. Altarelli, N. Cabbibo and L. Maiani, Phys. Lett. 40B, 415 (1972)

R. Jackiw and S. Weinberg, Phys. Rev. D 5, 2473 (1972)

I. Bars and M. Yoshimura, Phys. Rev. D 6, 374 (1972)

M. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D 6, 2923 (1972)

Two-loop bosonic contributions

$$a_\mu^{\text{weak(2);b}} = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \frac{\alpha}{\pi} \cdot \left[-5.96 \ln \frac{M_W^2}{m_\mu^2} + 0.19 \right] = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \left(\frac{\alpha}{\pi} \right) \cdot (-79.3)$$

A. Czarnecki, B. Krause, W. J. Marciano, Phys. Rev. Lett. 76, 3267 (1996)

Two-loop fermionic contributions

A. Czarnecki, B. Krause, W. J. Marciano, Phys. Rev. D 52, R2619 (1995)
M. K., S. Peris, M. Perrottet, E. de Rafael, JHEP11, 003 (2002)
A. Czarnecki, W.J. Marciano, A. Vainshtein, Phys. Rev. D 67, 073006 (2003). Err.-ibid. D 73, 119901 (2006)

$$\begin{aligned} a_\mu^{\text{weak}} &= (154 \pm 1) \cdot 10^{-11} \\ a_e^{\text{weak}} &= (0.0297 \pm 0.0005) \cdot 10^{-12} \end{aligned}$$

Updated a few years ago: $a_\mu^{\text{weak}} = (153.6 \pm 1.0) \cdot 10^{-11}$

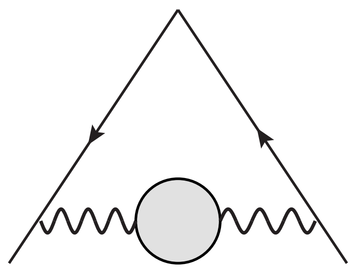
C. Gnendiger, D. Stöckinger, H. Stöckinger-Kim, Phys. Rev. D 88, 053005 (2013)

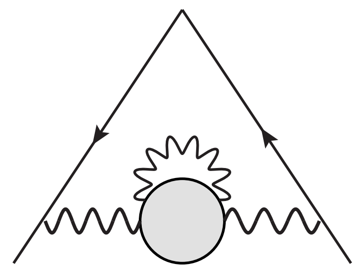
Recent numerical evaluation: $a_\mu^{\text{weak}} = (152.9 \pm 1.0) \cdot 10^{-11}$

T. Ishikawa, N. Nakazawa and Y. Yasui, Phys. Rev. D 99, 073004 (2019)

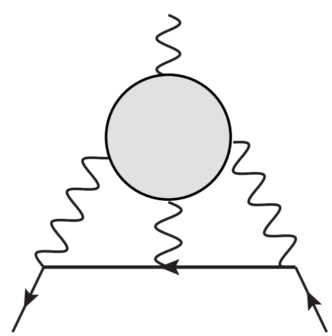
Contributions from strong interactions

- hadronic vacuum polarization





- (virtual) hadronic light-by-light (HLxL



$\longrightarrow$ non-perturbative regime of QCD

Hadronic vacuum polarization

- Occurs first at order $\mathcal{O}(\alpha^2)$

- Can be expressed as (optical theorem)

$$a_\ell^{\text{HVP-LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4M_\pi^2}^{\infty} \frac{dt}{t} K(t) R^{\text{had}}(t) \quad K(t) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)\frac{t}{m_\ell^2}}$$

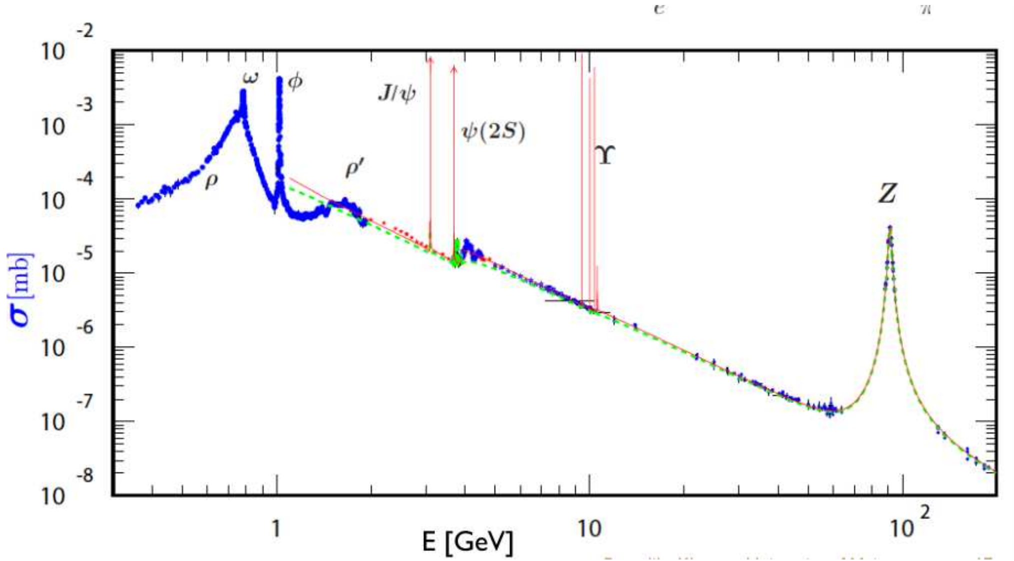
C. Bouchiat, L. Michel, J. Phys. Radium 22, 121 (1961)
L. Durand, Phys. Rev. 128, 441 (1962); Err.-ibid. 129, 2835 (1963)
M. Gourdin, E. de Rafael, Nucl. Phys. B 10, 667 (1969)

- $K(s) > 0$ and $R^{\text{had}}(s) > 0 \Longrightarrow a_\ell^{\text{HVP-LO}} > 0$

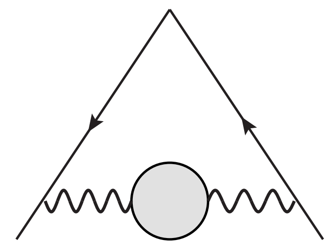
- $K(s) \sim m_\ell^2/(3s)$ as $s \to \infty \Longrightarrow$ the (non perturbative) low-energy region dominates

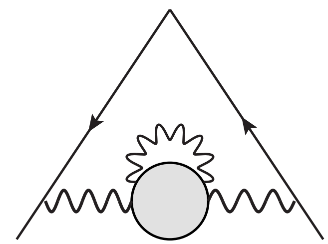
Hadronic vacuum polarization

- Can be evaluated using available experimental data



$\longrightarrow$





- Combination of ~ 39 exclusive channels

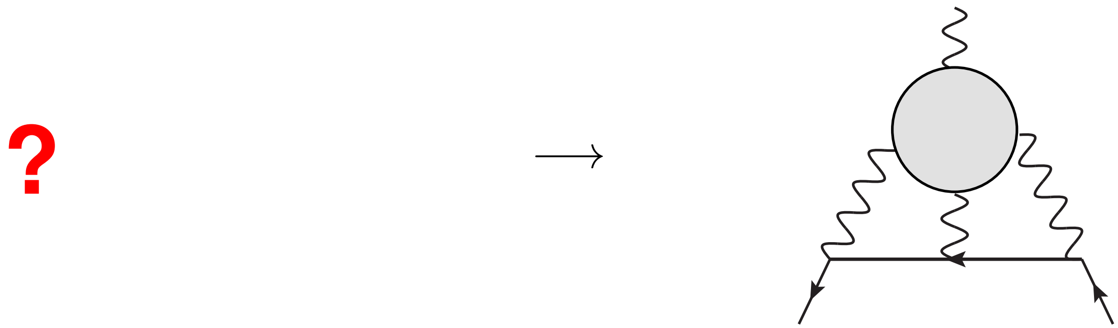
$\longrightarrow$ Scan experiments (e.g. @ VEPP)

$\longrightarrow$ ISR experiments (e.g. @ DAΦNE, B-factories, BEPC)

Hadronic light-by-light

- Occurs at order $\mathcal{O}(\alpha^3)$

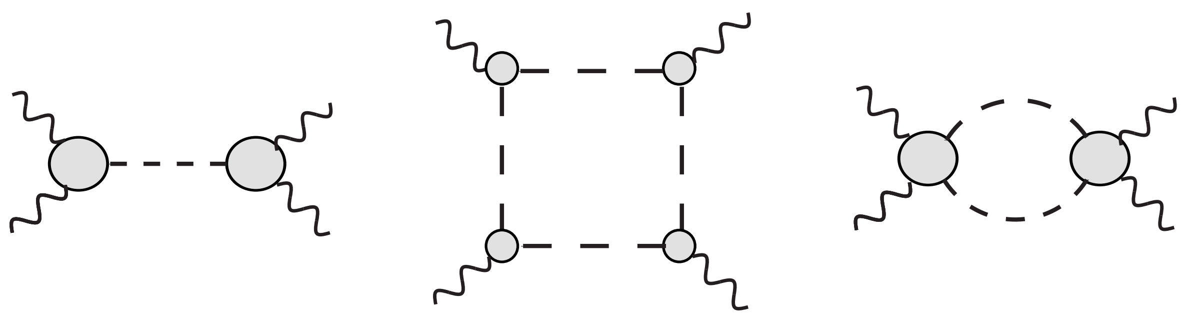
- Not related, as a whole, to an experimental observable...



- Involves the fourth-rank vacuum polarization tensor

$$\text{F.T. } \langle 0|T\{VVVV\}|0\rangle \longrightarrow \Pi_{\mu\nu\rho\sigma}(q_1, q_2, q_3, q_4) \quad q_1 + q_2 + q_3 + q_4 = 0$$

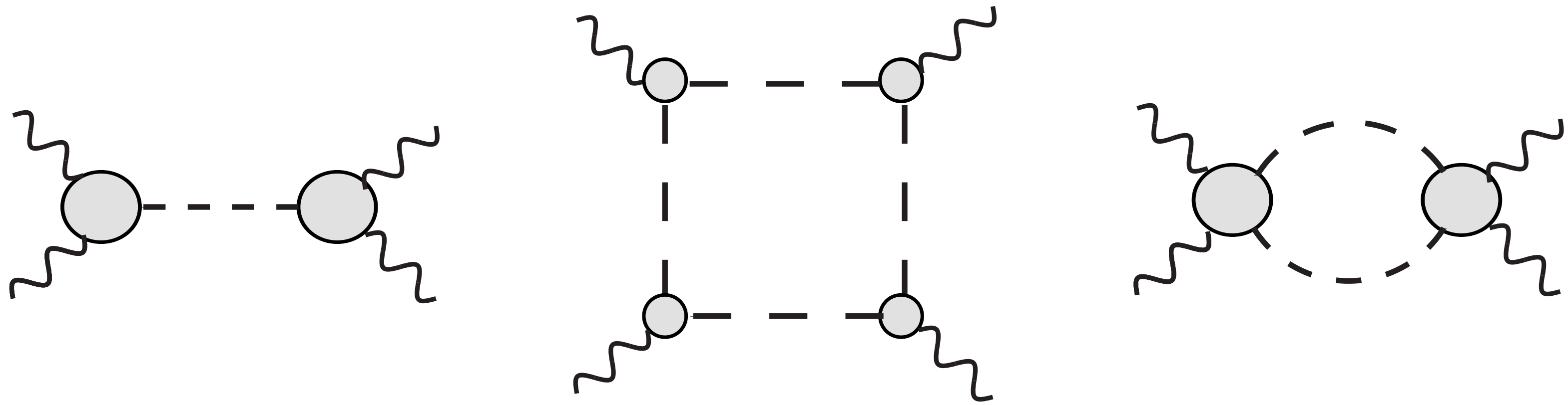
- Many individual contributions have been identified...



Hadronic light-by-light

- More recently: dispersive approaches

— for $\Pi_{\mu\nu\rho\sigma} \longrightarrow$



$$\Pi = \Pi^{\pi^0,\eta,\eta' \text{ poles}} + \Pi^{\pi^\pm,K^\pm \text{ loops}} + \Pi^{\pi\pi} + \Pi^{\text{residual}}$$

G. Colangelo, M. Hoferichter, M. Procura, P. Stoffer, JHEP09, 091 (2014); JHEP09, 074 (2015)

Needs input from data (transition form factors,...)

G. Colangelo, M. Hoferichter, B. Kubis, M. Procura, P. Stoffer, Phys. Lett. B 738, 6 (2014)

A. Nyffeler, arXiv:1602.03398 [hep-ph]

— for $F_2^{\text{HLxL}}(k^2)$

only pion pole with VMD form factor (two-loop graph) reconstructed this way so far

V. Pauk and M. Vanderhaeghen, Phys. Rev. D 90, 113012 (2014)

Oggi: the present situation

QED contributions : loops with only photons and leptons
$\longrightarrow$ can be computed in perturbation theory:

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5$$

	$\ell = e$	$\ell = \mu$
$C_\ell^{(2)}$	0.5	0.5
$C_\ell^{(4)}$	$-0.328\,478\,444\,00\ldots$	$0.765\,857\,425(17)$
$C_\ell^{(6)}$	$1.181\,234\,017\ldots$	$24.050\,509\,96(32)$
$C_\ell^{(8)}$	$-1.911\,321\,390\ldots$	$130.878\,0(61)$
$C_\ell^{(10)}$	$6.595(223)$	$750.72(93)$

n	1	2	3	4	5
$(\alpha/\pi)^n$	$2.32\ldots\cdot 10^{-3}$	$5.39\ldots\cdot 10^{-6}$	$1.25\ldots\cdot 10^{-8}$	$2.91\ldots\cdot 10^{-11}$	$6.76\ldots\cdot 10^{-14}$

A few comments about the QED contributions

- Uncertainties on the coefficients $C_\mu^{(2n)}$ not relevant for a_μ at the present (and future) level of precision

$$\Delta C_\mu^{(4)} \cdot (\alpha/\pi)^2 \sim 0.9 \cdot 10^{-13} \qquad \Delta C_\mu^{(6)} \cdot (\alpha/\pi)^3 \sim 0.04 \cdot 10^{-13}$$
$$\Delta C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 1.8 \cdot 10^{-13} \qquad \Delta C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 0.7 \cdot 10^{-13} \qquad \Delta a_\mu^{\mathrm{exp}} = 6.3 \cdot 10^{-10}$$

- Order $\mathcal{O}(\alpha^4)$ and even order $\mathcal{O}(\alpha^5)$ relevant for a_μ at the present (and future) level of precision

$$C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 3.8 \cdot 10^{-9} \qquad C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 0.5 \cdot 10^{-10}$$

- Drastic increase with n in the coefficients $C_\mu^{(2n)}$ [$\pi^2 \ln(m_\mu/m_e) \sim 50$!]
- Estimate of $\mathcal{O}(\alpha^6)$ contributions with these enhancement factors

$$\delta a_\mu \sim A_2^{(6)}(m_\mu/m_e; \mathrm{LxL}) \left[\frac{2}{3}\ln\frac{m_\mu}{m_e} - \frac{5}{9}\right]^3 \cdot 10 \left(\frac{\alpha}{\pi}\right)^6 \sim 0.6 \cdot 10^4 \cdot \left(\frac{\alpha}{\pi}\right)^6 \sim 1 \cdot 10^{-12}$$

- No sign of substantial contribution to a_μ from higher order QED

QED contributions : loops with only photons and leptons

$\longrightarrow$ can be computed in perturbation theory:

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

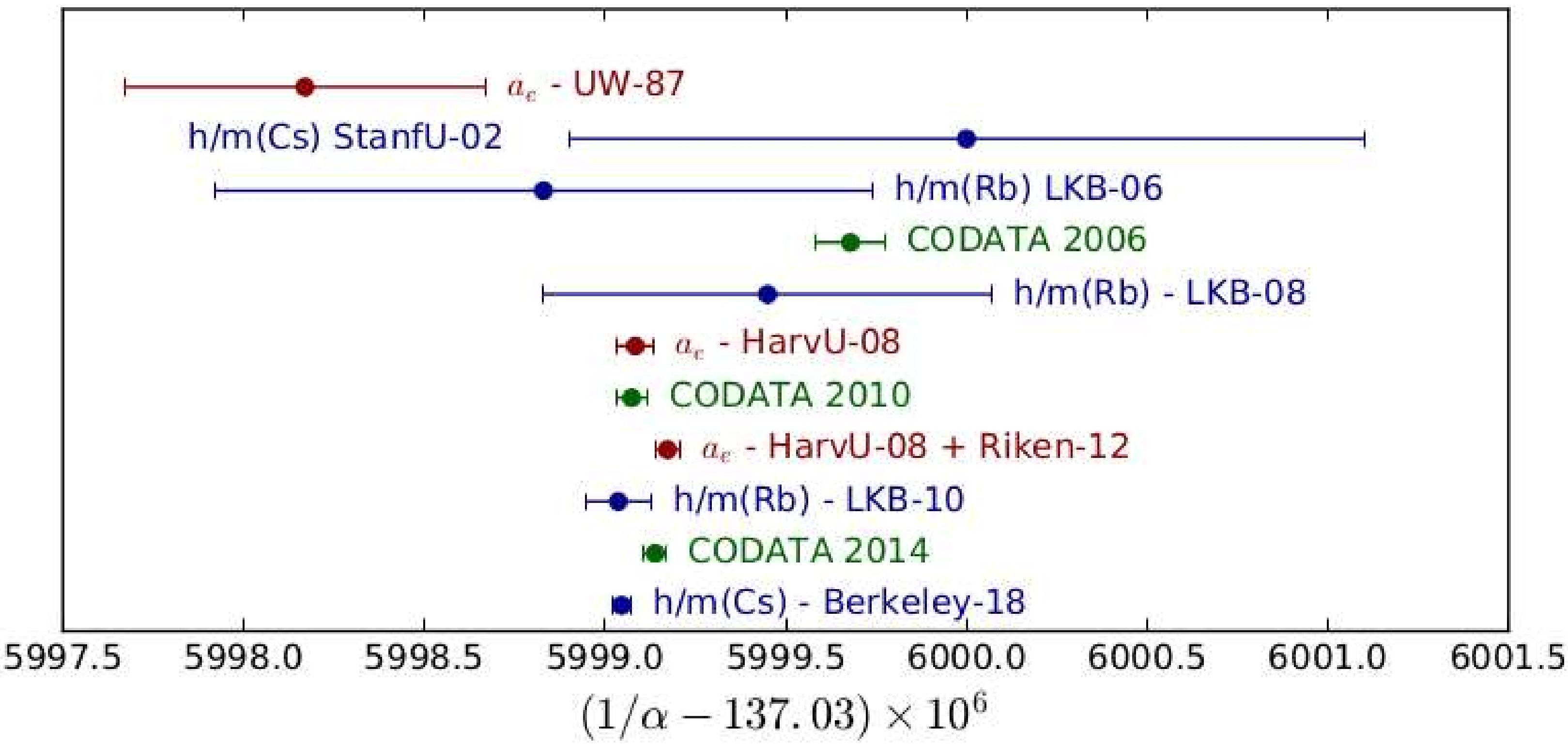
$\rightarrow$ requires an input for the fine structure constant α that matches the experimental accuracy on a_ℓ

$$\alpha^{-1}[Rb\,11] = 137.035\,999\,037(91) \qquad [0.66\text{ppb}]$$

R. Bouchendira, P. Cladé, S. Ghelladi-Khélifa, F. Nez, F. Biraben, Phys. Rev. Lett. 106, 080801 (2011)

$$\alpha^{-1}[Cs\,18] = 137.035\,999\,046(27) \qquad [0.20\text{ppb}]$$

R. H. Parker, C. Yu, W. Zhong, B. Estey, H. Müller, Science 360, 191 (2018)



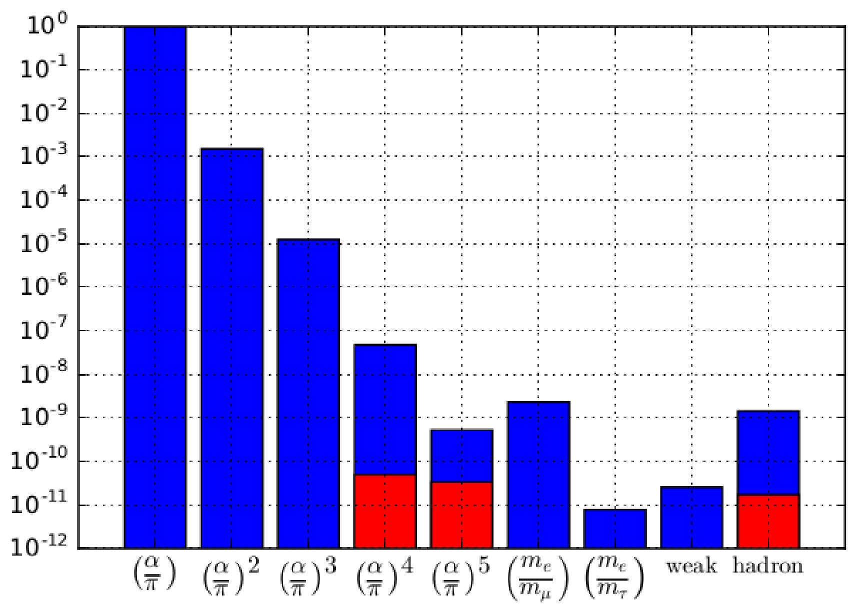
P. Cladé, F. Nez, F. Biraben, S. Guellati-Khelifa, CR Physique 20, 77 (2019)

$$a_e^{\mathrm{QED}}(Cs) = 1\,159\,652\,179.874(0)_{\alpha^4}(7)_{\alpha^5}(213)_{\alpha(Cs18)} \cdot 10^{-12}$$

Aoyama et al., Phys. Rev. Lett. 109, 111807 (2012)
R. H. Parker, C. Yu, W. Zhong, B. Estey, H. Müller, Science 360, 191 (2018)

$$a_e^{\mathrm{exp}} - a_e^{\mathrm{QED}} = +0.86(36) \cdot 10^{-12}$$

Strong and weak interaction contributions still to be added...



P. Cladé, F. Nez, F. Biraben, S. Guellati-Khelifa, CR Physique 20, 77 (2019)

$$a_\mu^{\rm QED}(Rb) = 1\,165\,847\,189.51(9)_{\rm mass}(19)_{\alpha^4}(7)_{\alpha^5}(77)_{\alpha(Rb11)} \cdot 10^{-12}$$

Aoyama et al., Phys. Rev. Lett. 109, 111807 (2012)

$$a_\mu^{\rm exp} - a_\mu^{\rm QED} = 737.0(6.3) \cdot 10^{-10}$$

QED provides more than 99.99% of the total value, without uncertainties at this level of precision

The missing part has to be provided by weak and strong interactions (or else, new physics...)

Hadronic vacuum polarization

$a_\mu^{\text{HVP-LO}} \cdot 10^{10}$, e^+e^-

$692.3(4.2)$	M. Davier et al., Eur. Phys. J. C 71, 1515 (2011)
$694.9(4.3)$	K. Hagiwara et al., J. Phys. G 38, 085003 (2011)
$690.75(4.72)$	F. Jegerlehner, R. Szafron, Eur. Phys. J. C 71, 1632 (2011)
$688.07(4.14)$	F. Jegerlehner, EPJ Web Conf. 166, 00022 (2018)
$693.1(3.4)$	M. Davier et al., Eur. Phys. J. C 77, 827 (2017)
$693.26(2.46)$	A. Keshavarzi et al., Phys. Rev. D 97, 114025 (2018)
$693.9(4.0)$	M. Davier et al., arXiv:1908.00921
$692.78(2.42)$	A. Keshavarzi et al., arXiv:1911.00367

$a_\mu^{\text{HVP-NLO}} \cdot 10^{10}$, e^+e^-

$-9.84(7)$	K. Hagiwara et al., J. Phys. G 38, 085003 (2011)
$-9.93(7)$	F. Jegerlehner, EPJ Web Conf. 166, 00022 (2018)
$-9.82(4)$	A. Keshavarzi et al., Phys. Rev. D 97, 114025 (2018)
$-9.83(4)$	A. Keshavarzi et al., arXiv:1911.00367

$a_\mu^{\text{HVP-NNLO}} \cdot 10^{10}$, e^+e^-

$1.24(1)$	A. Kurz et al., Phys. Lett. B 734, 144 (2014)
$1.22(1)$	F. Jegerlehner, EPJ Web Conf. 166, 00022 (2018)

Hadronic light-by-light

- Updated estimate

$$a_\mu^{\text{HLxL}} = +(10.3 \pm 2.9) \cdot 10^{-10}$$

F. Jegerlehner, arXiv:1705.002633 [hep-ph]

Present situation

$$a_e^{\rm exp} - a_e^{\rm SM} = -0.88(36) \cdot 10^{-12} \qquad [2.5\sigma]$$

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = +26.1(7.9) \cdot 10^{-10} \qquad [3.3\sigma]$$

$$a_\tau^{\rm exp} - a_\tau^{\rm SM} = ??? - 117721(5) \cdot 10^{-8} \; [42{\rm ppm}]$$

[S. Eidelman, M. Passera, Mod. Phys. Lett. A 22, 159 (2007)]
[S. Narison, Phys Lett B 513 (2001); err. B 526, 414 (2002)]

Domani: prospects for the (near) future

- The electron case

- new measurement of h/M_{Rb} by LKB group

- proposal for a measurement of a_e at the level of 20ppt

- proposal to measure h/M_{Cs} at the level of $10 - 20$ppt

- The electron case

- new measurement of h/M_{Rb} by LKB group

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$\rightarrow$ may require $\mathcal{O}(\alpha^6)$ corrections...

- The muon case

Improvements of the experimental situation

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]
CERN I	1961	μ^+	11 450 000(220 000)	4300
CERN II	1962-1968	μ^+	11 661 600(3100)	270
CERN III	1974-1976	μ^+	11 659 100(110)	10
CERN III	1975-1976	μ^-	11 659 360(120)	10
BNL E821	1997	μ^+	11 659 251(150)	13
BNL E821	1998	μ^+	11 659 191(59)	5
BNL E821	1999	μ^+	11 659 202(15)	1.3
BNL E821	2000	μ^+	11 659 204(9)	0.73
BNL E821	2001	μ^-	11 659 214(9)	0.72

$$a_\mu^{\rm exp} = 11\,659\,208.9(5.4)(3.3) \cdot 10^{-10} \ \ [0.54\,\mathrm{ppm}]$$

G. W. Bennett *et al.* [Muon $g-2$ collaboration, BNL E821], Phys. Rev. D 73, 072003 (2006)
M. Tanabashi *et al.* [Particle Data Group], Phys. Rev. D 98, 030001 (2018)

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M. Tanabashi *et al.* [Particle Data Group], Phys. Rev. D 98, 030001 (2018)

$$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{th}} = 26.1(7.9) \cdot 10^{-10} \quad [\sim 3.5\sigma]$$

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M. Tanabashi *et al.* [Particle Data Group], Phys. Rev. D 98, 030001 (2018)

$$a_\mu^{\text{exp}} - a_\mu^{\text{th}} = 26.1(7.9) \cdot 10^{-10} \quad [\sim 3.5\sigma]$$

This is not the end of the story...

Experimental situation and prospects

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]
CERN I	1961	μ^+	11 450 000(220 000)	4300
CERN II	1962-1968	μ^+	11 661 600(3100)	270
CERN III	1974-1976	μ^+	11 659 100(110)	10
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BNL E821	2001	μ^-	11 659 214(9)	0.72
FNAL E989	2019	μ^+	???	$\sim$ 0.35
FNAL E989	2022?	μ^+	???	$\sim$ 0.14
J-PARC E34	???	μ^+	???	$\sim$ 0.45

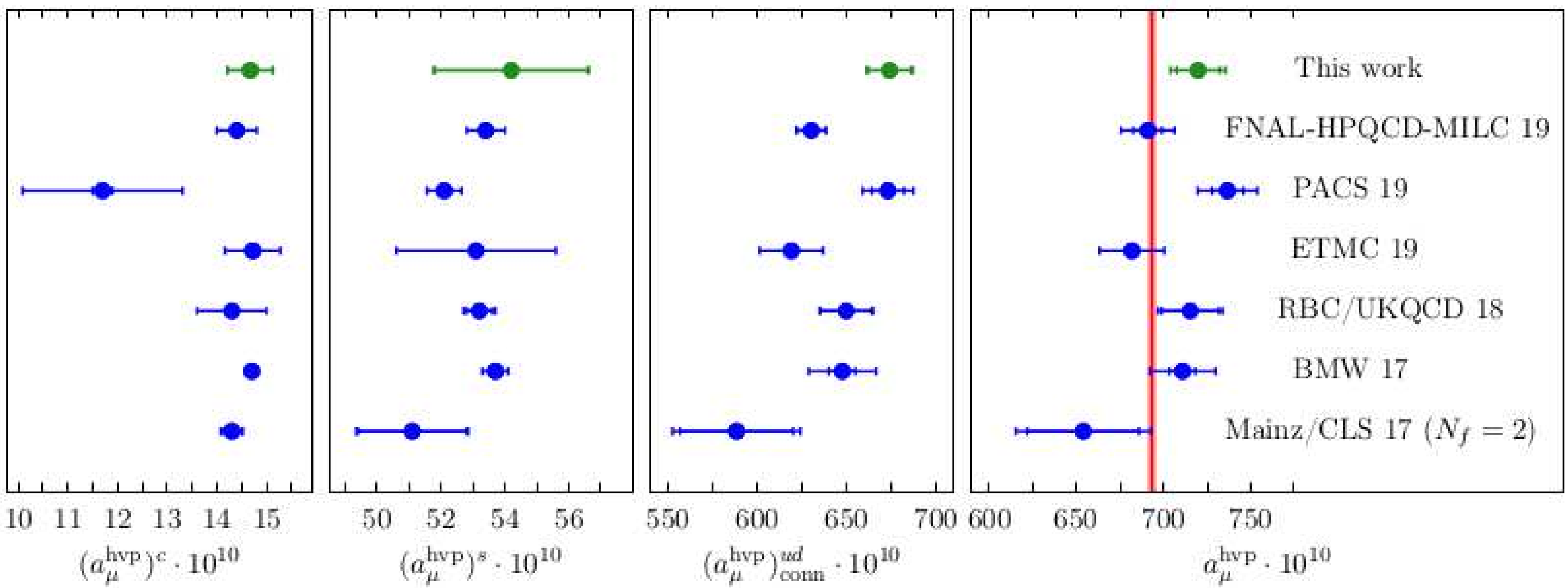
A. Keshavarzi, EPJ Web Conf. **212**, 05003 (2019)
M. Abe *et al.*, Prop. Theor. Exp. Phys. 2019, 053C02 (2019)

What about the situation at the theory side?

What are the prospects for further improvements?

Hadronic vacuum polarization

• Alternative for the (near?) future: Lattice QCD:
several groups have produced results recently



A. Gérardin *et al.*, Phys. Rev. D 100, 014510 (2019)
C. T. H. Davies *et al.*, arXiv:1902.04223 [hep-lat]
E. Shintani and Y. Kuramashi, arXiv:1902.00885 [hep-lat]
D. Giusti *et al.*, Phys. Rev. D 99, 114502 (2019)
T. Blum *et al.*, Phys. Rev. Lett. 121, 022003 (2018)
S. Borsanyi *et al.*, Phys. Rev. Lett. 121, 022002 (2018)
M. Della Morte *et al.*, JHEP 10, 020 (2017)

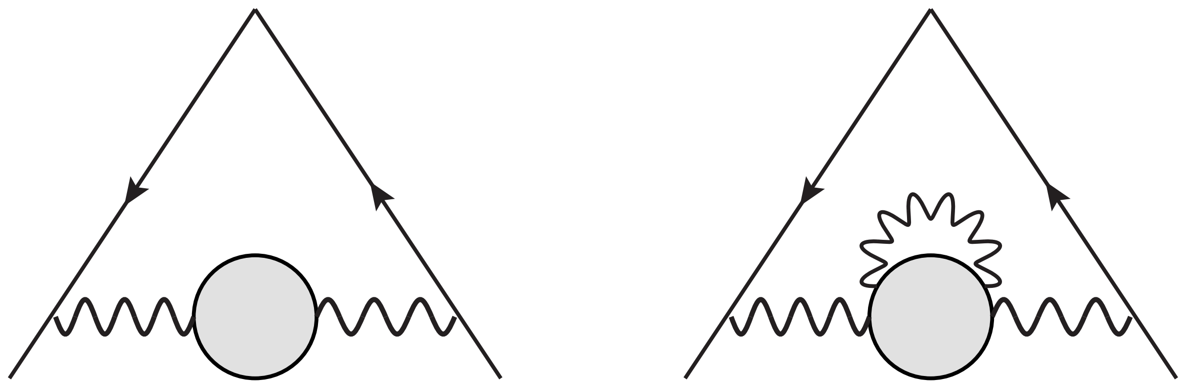
present precision $\sim 2.5\%$
work in progress

Review of lattice calculations for a_μ: H. B. Meyer and H. Wittig, Prog. Part. Nucl. Phys. 104, 46 (2019)

Hadronic vacuum polarization

• Alternative for the (near?) future: Lattice QCD:
several groups have produced results recently

comparison with data at the sub-percent level:
isospin breaking effects (radiative corrections)



(Experimentalists don't live in the theoretician's paradise)

Hadronic vacuum polarization

• Possibility to measure HVP in the space-like region (from Bhabha or μe scattering)?

C. M. Carloni-Calame, M. Passera, L. Trentadue, G.Venanzoni, Phys. Lett. B 476, 325 (2015)

G. Abbiendi et al., Eur. Phys. J. C 77, 139 (2017)

• $a_\mu^{\mathrm{HVP}} = \frac{\alpha}{\pi} \int_0^1 dx(1-x)\Delta\alpha_{\mathrm{had}}(\frac{x^2}{x-1}m_\mu^2)$

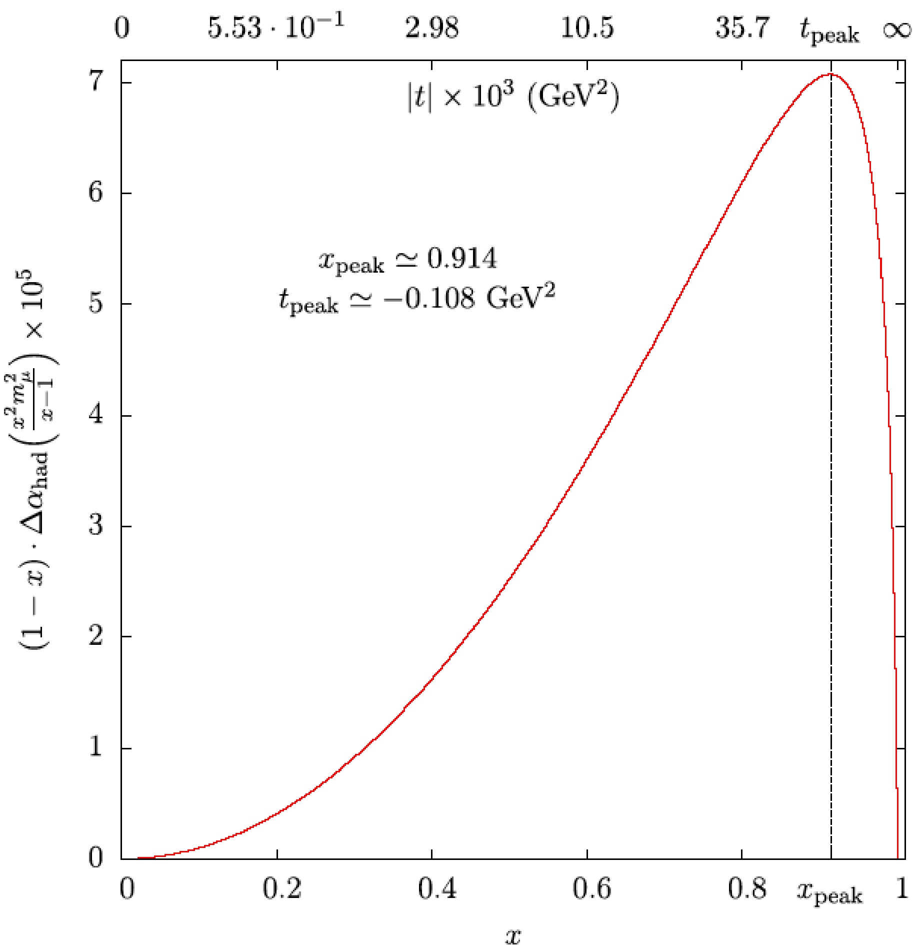
$t = \frac{x^2 m_\mu^2}{x-1} \quad 0 \le -t < +\infty \quad 0 \le x < 1$

• a_μ^{HVP} given by the integral

• measurement of $\Delta\alpha_{\mathrm{had}}$ in the space-like region

• contribution at small t enhanced

• a 0.3% error can be achieved in 2y of data taking with 1.3x10^7 μ/s (CERN)



MUonE proposal $\longrightarrow$ data taking in 2021-2014

G. Venanzoni, arXiv:1811.11466 [hep-ex]

Hadronic light-by-light

- Also on the agenda of lattice QCD, see e.g.

T. Blum et al., Phys. Rev. D 93, 014503 (2016); Phys. Rev. Lett 118, 022005 (2017)

N. Asmussen et al., arXiv:1609.08454 [hep-lat]; arXiv:1510.08384 [hep-lat]

Goal: evaluation of HLxL with a reliable uncertainty of $\sim 10\%$

- The tau case

- $\gamma\gamma \to \tau\tau$ with heavy-ion collisions at LHC $\longrightarrow \; -0.0082 < a_\tau < 0.0046$ at 68% CL with 2 nb^{-1}

[L. Beresford and J. Liu, arXiv:1908.05180 [hep-ph]]

- $\tau^- \to \ell^- \nu_\tau \bar{\nu}_\ell \gamma$ at B factories $\longrightarrow \; \Delta a_\tau \sim 0.012$

[S. Eidelman, D. Epifanov, M. Fael, L. Mercolli, M. Passera, JHEP 1603, 140 (2016)]

- spin precession in a bent crystal

[A. S. Fomin, A. Y. Korchin, A. Stocchi, S. Barsuk and P. Robbe, JHEP 1903, 156 (2019)]

- ...

Thanks for your attention!