Charmless B-meson Decays

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LHCb

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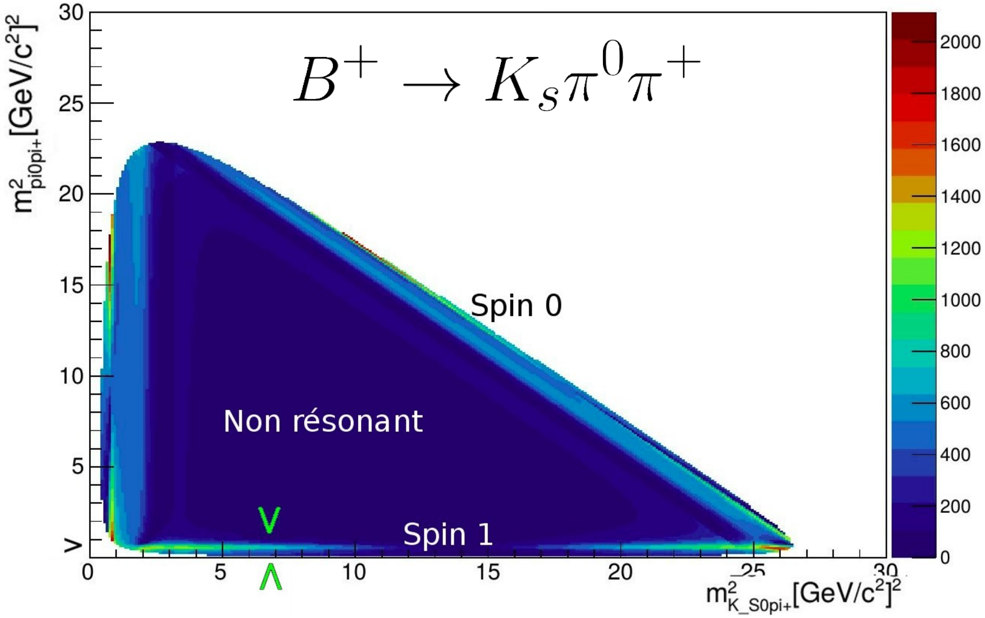
3-body decays



B → hhh pseudo-scalars = 2 degrees of freedom

Phase space usually described as Dalitz Plane (DP)

Resonances appear as structures

Isobar model:

Used to parametrise the amplitude

$$A = \sum c_i F_i(m_{13}^2, m_{23}^2)$$

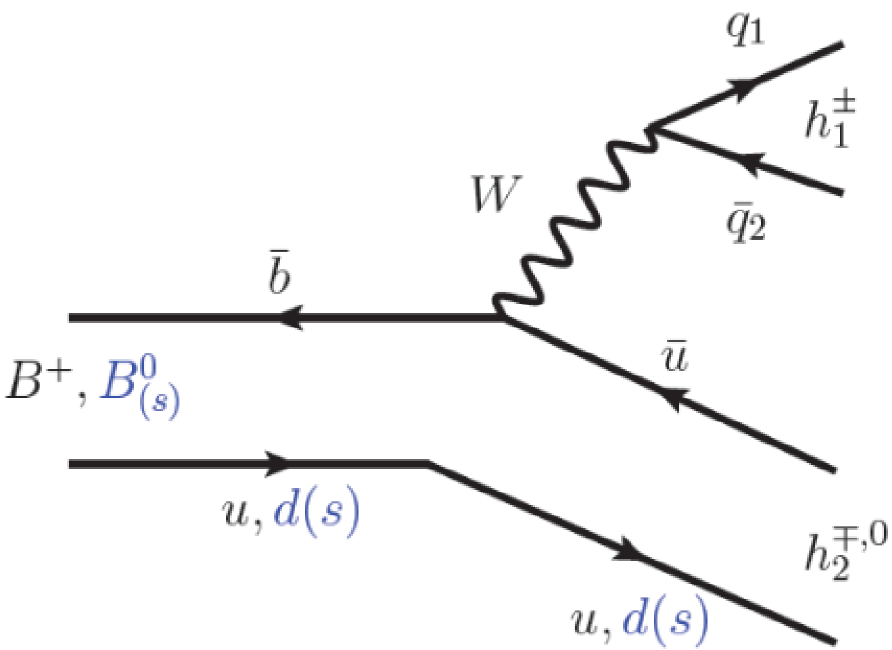
$$\bar{A} = \sum \bar{c}_i \overline{F}_i(m_{13}^2, m_{23}^2)$$

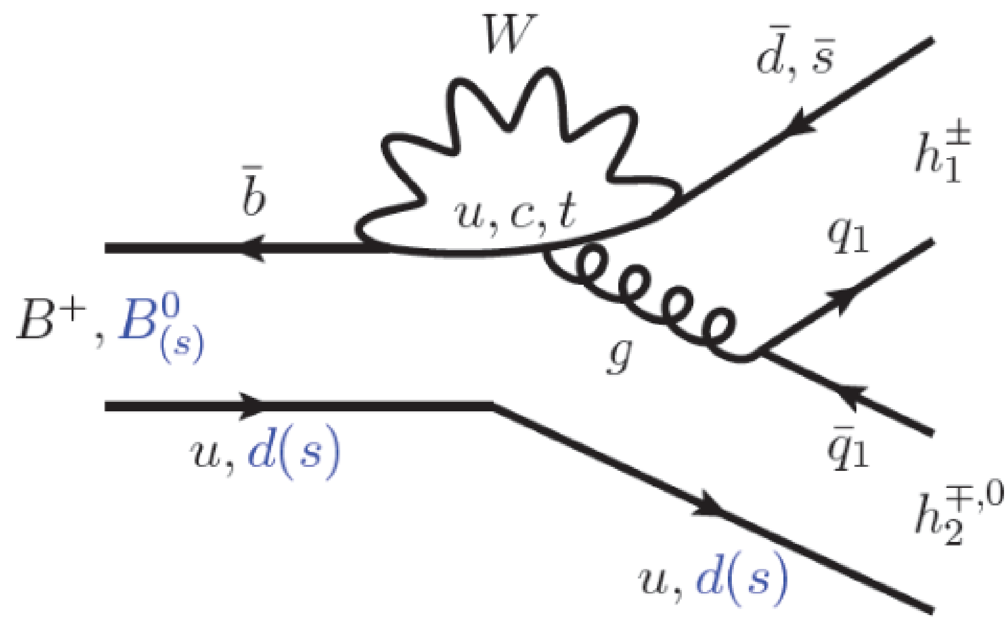
F_i strong dynamics

C_i complex coefficients

Strong and weak dynamics give access to Branching Fractions (BF), CP asymmetries (A_{cp}),...

Why charmless B decays ?





Contributions from both tree and penguin processes

Amplitudes can be comparable → sensitive to CP violation

Loops involved → probe new physics

Give access to many observables: BFs, A_{cp}, CKM phases, polarization fractions...

Recent LHCb Charmless studies

3-body decays but not only !

- Amplitude analysis of $B^0_s \to K^0_S K^\pm \pi^\mp$ decays [JHEP 06 (2019) 114]

- Measurement of CP asymmetries in charmless four-body Λ^0_b and Ξ^0_b decays [Eur. Phys. J. C79 (2019) 745]

- Amplitude analysis of the $B^0_{(s)} \to K^{*0}\overline{K}^{*0}$ decays and measurement of the branching fraction of the $B^0 \to K^{*0}\overline{K}^{*0}$ decay [JHEP 07 (2019) 032]

- Study of the $B^0 \to \rho(770)^0 K^*(892)^0$ decay with an amplitude analysis of $B0 \to (\pi^\pm\pi^\mp)(K^+\pi^-)$ decays [JHEP 05 (2019) 026]

- First measurement of the CP-violating phase $\phi^{d\bar{d}}_s$ in $B^0_s \to (K^+\pi^-)(K^-\pi^+)$ decays [JHEP 03 (2018) 140]

- Amplitude analysis of the decay $B^0 \to K^0_S \pi^+\pi^-$ and first observation of CP asymmetry in $B^0 \to K^*(892)^-\pi^+$ [Phys. Rev. Lett. 120 261801]

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Will be discussed today

Amplitude analysis of $B^0 \to K_s \pi^+ \pi^-$ decays

First observation of CP asymmetry in $B^0 \to K^*(892)^+ \pi^-$ using 3 fb^{-1}

- Could help with the "Kπ puzzle" → contains intermediate states such as $B^0 \to K^{*-} \pi^+$

- Time integrated → CKM phases not accessible

- BUT **direct CP asymmetries between flavour-specific (FS)** states such as $B^0 \to K^{*-} \pi^+$ and $B^0 \to K^{*+} \pi^-$ are **measured**

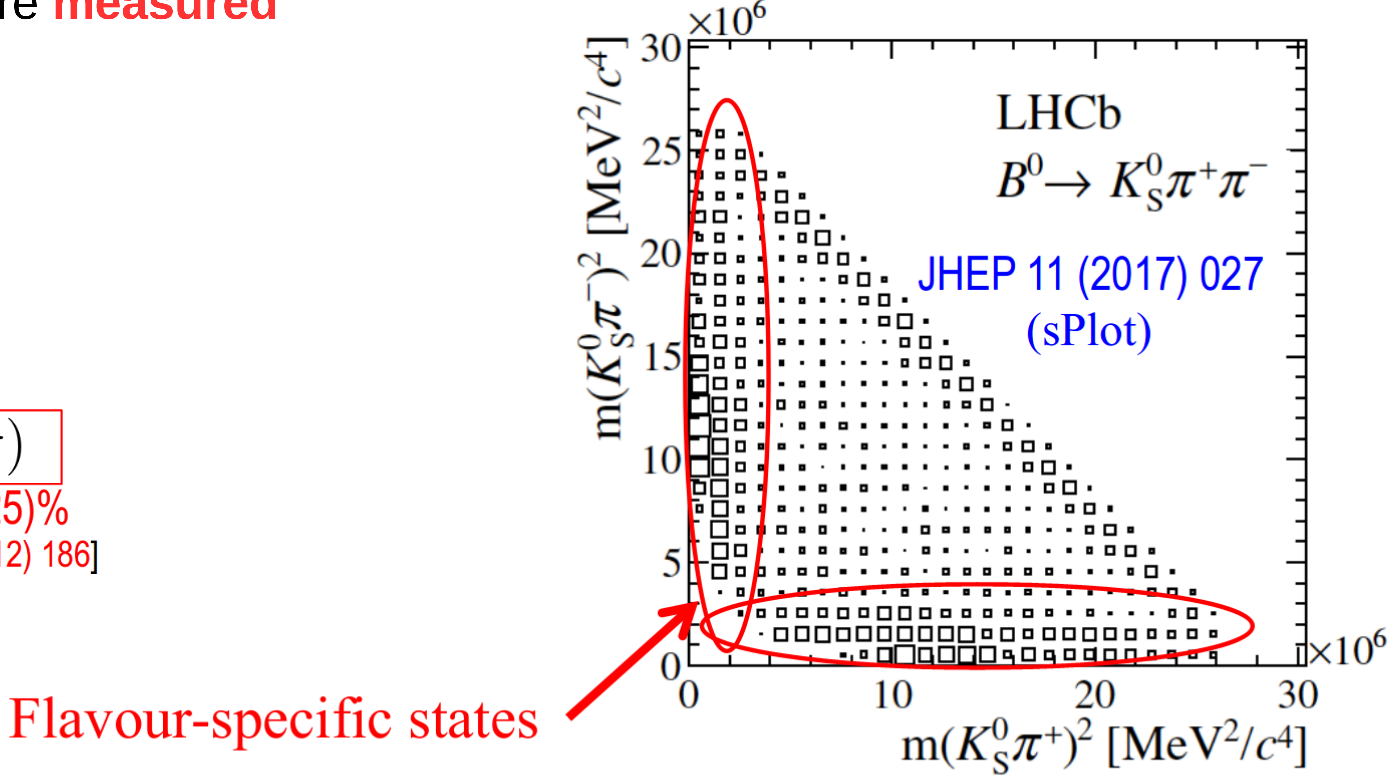
$$\mathcal{A}_{\mathrm{raw}} = \frac{|\bar{c}_j|^2 - |c_j|^2}{|\bar{c}_j|^2 + |c_j|^2}$$

$$\mathcal{A}_{CP} = \mathcal{A}_{\mathrm{raw}} - \mathcal{A}_{\Delta}$$

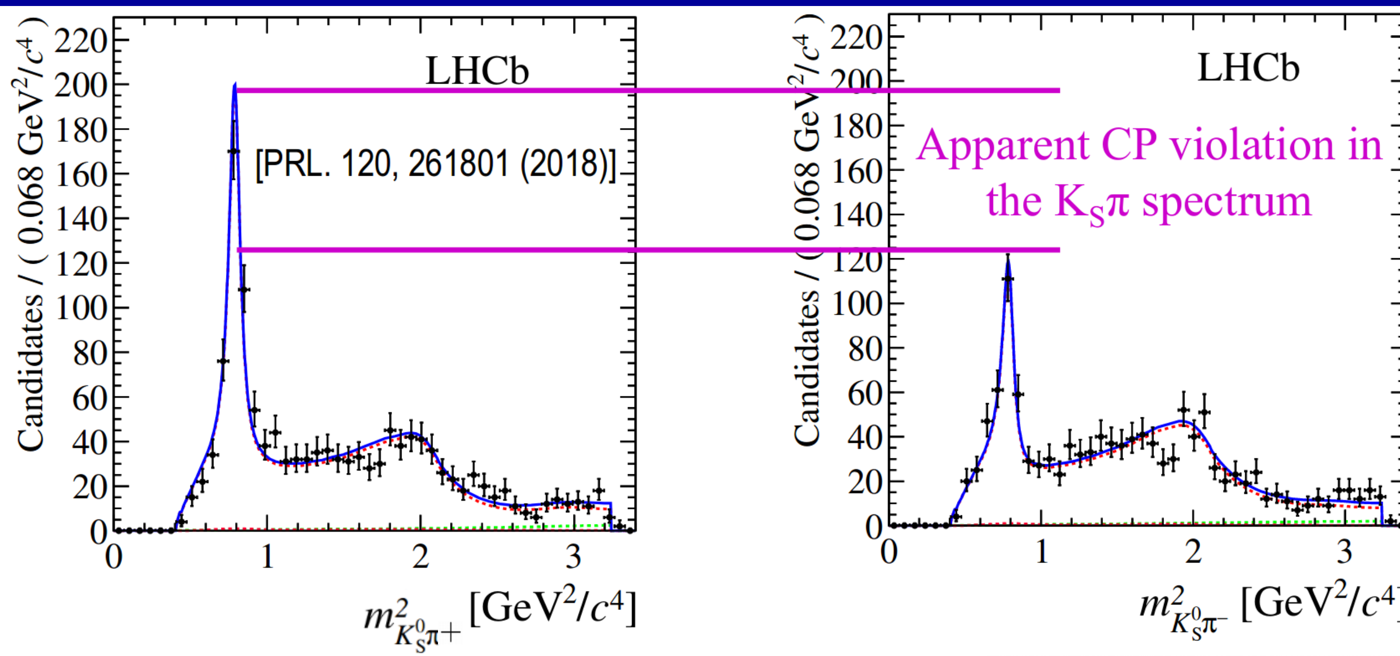
$$\mathcal{A}_{\Delta} = \mathcal{A}_P(B^0) + \mathcal{A}_D(\pi)$$

(-0.35 ± 0.81)% (0 ± 0.25)%

[D_s^+, PRL 110 (2013) 221601, PLB 713 (2012) 186]



Results



Not included by B-factories in this mode

$\mathcal{F}(K^*(892)^-\pi^+)$	=	$9.43 \pm 0.40 \pm 0.33 \pm 0.34$ %
$\mathcal{F}((K\pi)_0^-\pi^+)$	=	$32.7 \pm 1.4 \pm 1.5 \pm 1.1$ %
$\mathcal{F}(K_2^*(1430)^-\pi^+)$	=	$2.45\,^{+0.10}_{-0.08} \pm 0.14 \pm 0.12$ %
$\mathcal{F}(K^*(1680)^-\pi^+)$	=	$7.34 \pm 0.30 \pm 0.31 \pm 0.06$ %
$\mathcal{F}(f_0(980)K^0_S)$	=	$18.6 \pm 0.8 \pm 0.7 \pm 1.2$ %
$\mathcal{F}(\rho(770)^0K^0_S)$	=	$3.8\,^{+1.1}_{-1.6} \pm 0.7 \pm 0.4$ %
$\mathcal{F}(f_0(500)K^0_S)$	=	$0.32\,^{+0.40}_{-0.08} \pm 0.19 \pm 0.23$ %
$\mathcal{F}(f_0(1500)K^0_S)$	=	$2.60 \pm 0.54 \pm 1.28 \pm 0.60$ %
$\mathcal{F}(\chi_{c0}K^0_S)$	=	$2.23\,^{+0.40}_{-0.32} \pm 0.22 \pm 0.13$ %
$\mathcal{F}(K^0_S\pi^+\pi^-)^{\mathrm{NR}}$	=	$24.3 \pm 1.3 \pm 3.7 \pm 4.5$ %

$$\mathcal{A}_{CP}(K^*(892)^-\pi^+) = -0.308 \pm 0.060 \pm 0.011 \pm 0.012$$

$$\mathcal{A}_{CP}(f_0(980)K^0_S) = 0.28 \pm 0.27 \pm 0.05 \pm 0.14$$

[PRL. 120, 261801 (2018)]

Stat syst. model

First observation of CP violation in $B^0 \to K^*(892)\pi$ with ~ 6σ

Consistent with SM predictions* and previous world average: **A(K*(892)π)=-0.23±0.06** (HFLAV)

*[JHEP09 (2008) 038; PRD78 034011 (2008), PRD91, 014011 (2015)]

Results in agreement with the measurements from the B factories (for components that can be compared)

Belle PRD 79 (2009) 072004, BaBar PRD 80 (2009) 112001

Amplitude analysis of $B^0_s \to K^0_S K^{\pm}\pi^{\mp}$ decays

Using 3 fb^{-1}

- In principle Both B^0_s and $\overline{B}^0_s$ decay to $K^0_S K^+ \pi^-$ and $K^0_S K^- \pi^+$:
  - $B^0_s \to K^0_S K^+ \pi^-$
  - $B^0_s \to K^0_S K^- \pi^+$
  - $\overline{B}^0_s \to K^0_S K^+ \pi^-$
  - $\overline{B}^0_s \to K^0_S K^- \pi^+$

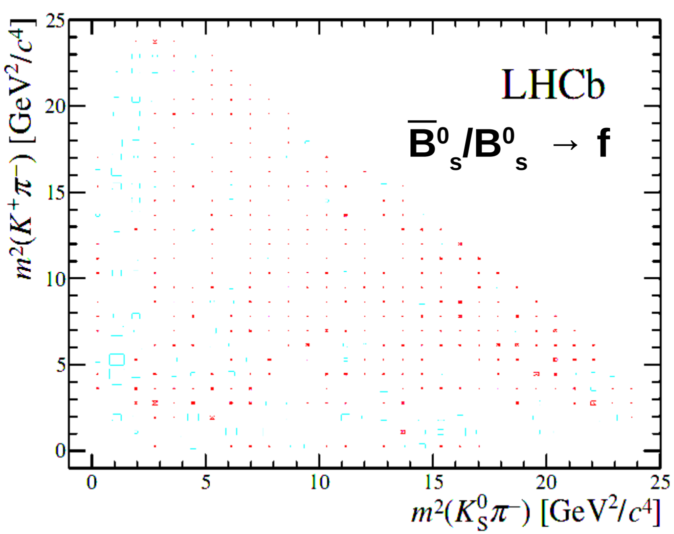
  4 different DP if B^0_s and $\overline{B}^0_s$ are tagged
- Expect large interference effects and possibly large CP violation

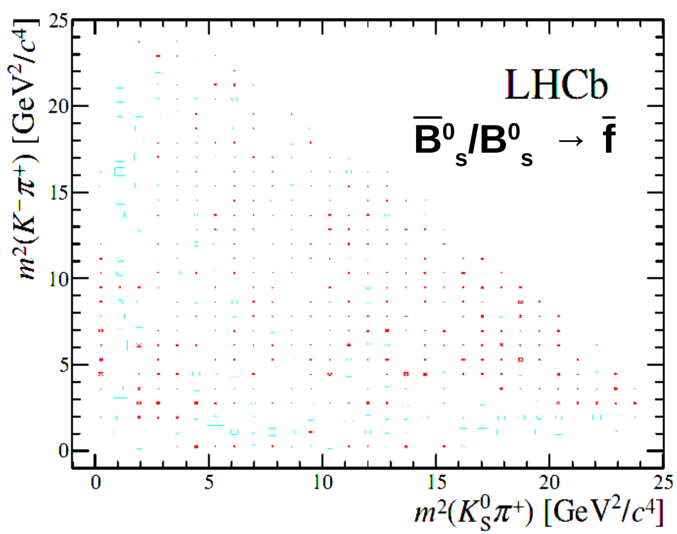
Using run I data → statistics do not allow to use flavour tagging (power between 3% and 7% in LHCb)

→ Time integrated → incoherent sum of B^0_s and $\overline{B}^0_s$

- First DP analysis

- Provides a basis for future studies



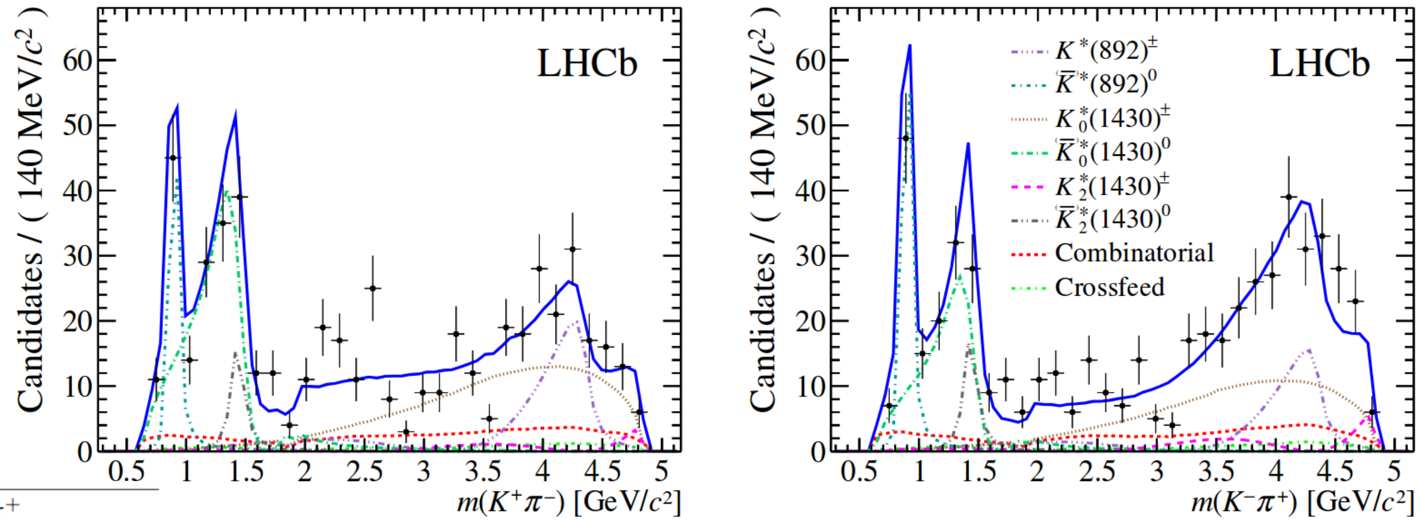


Two independent final states:
- $K^0_S K^+\pi^-$ denoted f
- $K^0_S K^-\pi^+$ denoted $\bar{f}$

Fitted simultaneously

[HEP 06 (2019) 114]

Results



$K_S^0K^+\pi^-$		$K_S^0K^-\pi^+$	
Resonance	Fit fraction (%)	Resonance	Fit fraction (%)
$K^*(892)^-$	15.6 ± 1.5	$K^*(892)^+$	13.4 ± 2.0
$K_0^*(1430)^-$	30.2 ± 2.6	$K_0^*(1430)^+$	28.5 ± 3.6
$K_2^*(1430)^-$	2.9 ± 1.3	$K_2^*(1430)^+$	5.8 ± 1.9
$K^*(892)^0$	13.2 ± 2.4	$\overline{K}^*(892)^0$	19.2 ± 2.3
$K_0^*(1430)^0$	33.9 ± 2.9	$\overline{K}_0^*(1430)^0$	27.0 ± 4.1
$K_2^*(1430)^0$	5.9 ± 4.0	$\overline{K}_2^*(1430)^0$	7.7 ± 2.8

No significant CP violation observed

$$\mathcal{B}\left(B_s^0 \to K^*(892)^\pm K^\mp\right) = (18.6 \pm 1.2 \pm 0.8 \pm 4.0 \pm 2.0) \times 10^{-6}$$
$$\mathcal{B}\left(B_s^0 \to K_0^*(1430)^\pm K^\mp\right) = (31.3 \pm 2.3 \pm 0.7 \pm 25.1 \pm 3.3) \times 10^{-6}$$
$$\mathcal{B}\left(B_s^0 \to K_2^*(1430)^\pm K^\mp\right) = (10.3 \pm 2.5 \pm 1.1 \pm 16.3 \pm 1.1) \times 10^{-6}$$
$$\mathcal{B}\left(B_s^0 \to \overset{(-)}{K}{}^*(892)^0 \overset{(-)}{K}{}^0\right) = (19.8 \pm 2.8 \pm 1.2 \pm 4.4 \pm 2.1) \times 10^{-6}$$
$$\mathcal{B}\left(B_s^0 \to \overset{(-)}{K}{}_0^*(1430)^0 \overset{(-)}{K}{}^0\right) = (33.0 \pm 2.5 \pm 0.9 \pm 9.1 \pm 3.5) \times 10^{-6}$$
$$\mathcal{B}\left(B_s^0 \to \overset{(-)}{K}{}_2^*(1430)^0 \overset{(-)}{K}{}^0\right) = (16.8 \pm 4.5 \pm 1.7 \pm 21.2 \pm 1.8) \times 10^{-6}$$

Stat syst model *

Consistent with theoretical predictions and previous LHCb results

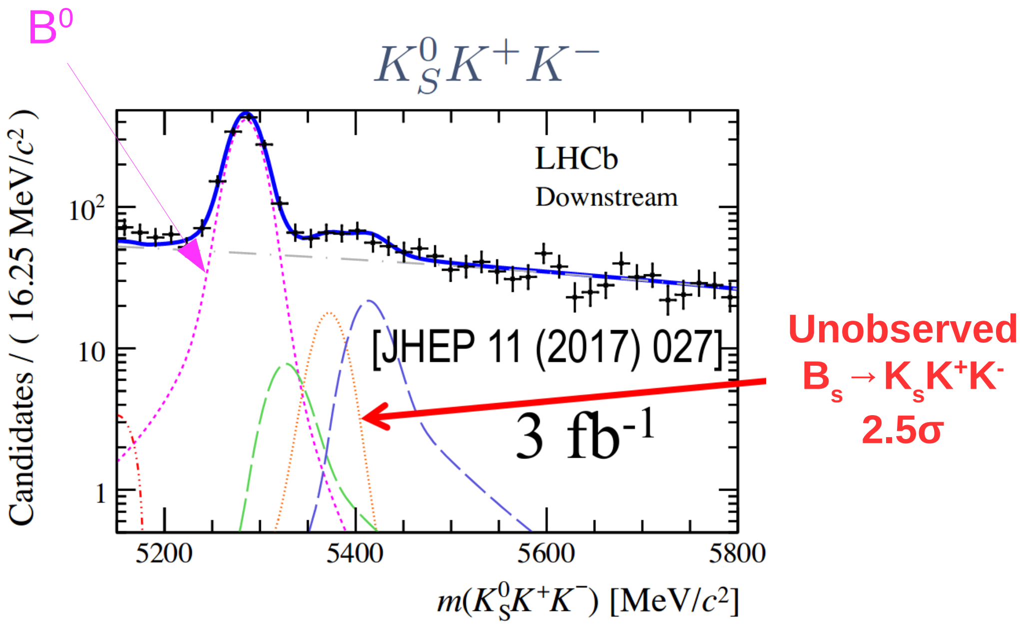
[Phys. Rev. D 89, 074025]

[New J. Phys.16(2014) 123001]
[JHEP01(2016) 012]

* uncertainties on BF($B_s^0 \to K^0K^\pm\pi^\mp$), BF($K^* \to K\pi$) and, in the case of $K^{*0}(1430)$, the uncertainty of the proportion of the $(K\pi)^{*0}$ component due to the $K^{*0}(1430)$ resonance

Branching fraction measurements of $B^0_{(s)} \to K_S h^+ h'^-$

Using 3 fb^{-1}, all modes observed but **$B^0_s \to K_s K^+ K^-$**



B type / Decay mode	B^0	B^0_s
$K_s \pi^+ \pi^-$	Favoured	Supressed
$K_s K^\pm \pi^\mp$	Supressed	Favoured
$K_s K^+ K^-$	Favoured	Supressed

Dataset divided into:

- 4 final states
- 2 K_S reconstruction categories
- 3 data-taking periods

→ **24 invariant-mass distributions**

Five BFs are measured relative to that of $B^0 \to K_s \pi^+ \pi^-$

All are compatible with previous results

$$\frac{\mathcal{B}(B^0_s \to K^0_S K^+ K^-)}{\mathcal{B}(B^0 \to K^0_S \pi^+ \pi^-)} \in [0.008 - 0.051] \text{ at } 90\% \text{ confidence level}$$

[JHEP 11 (2017) 027]

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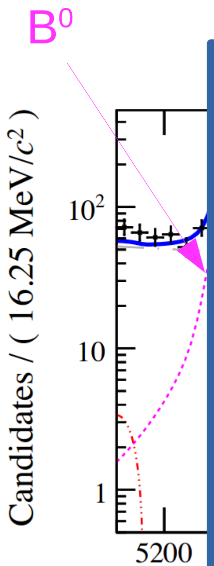

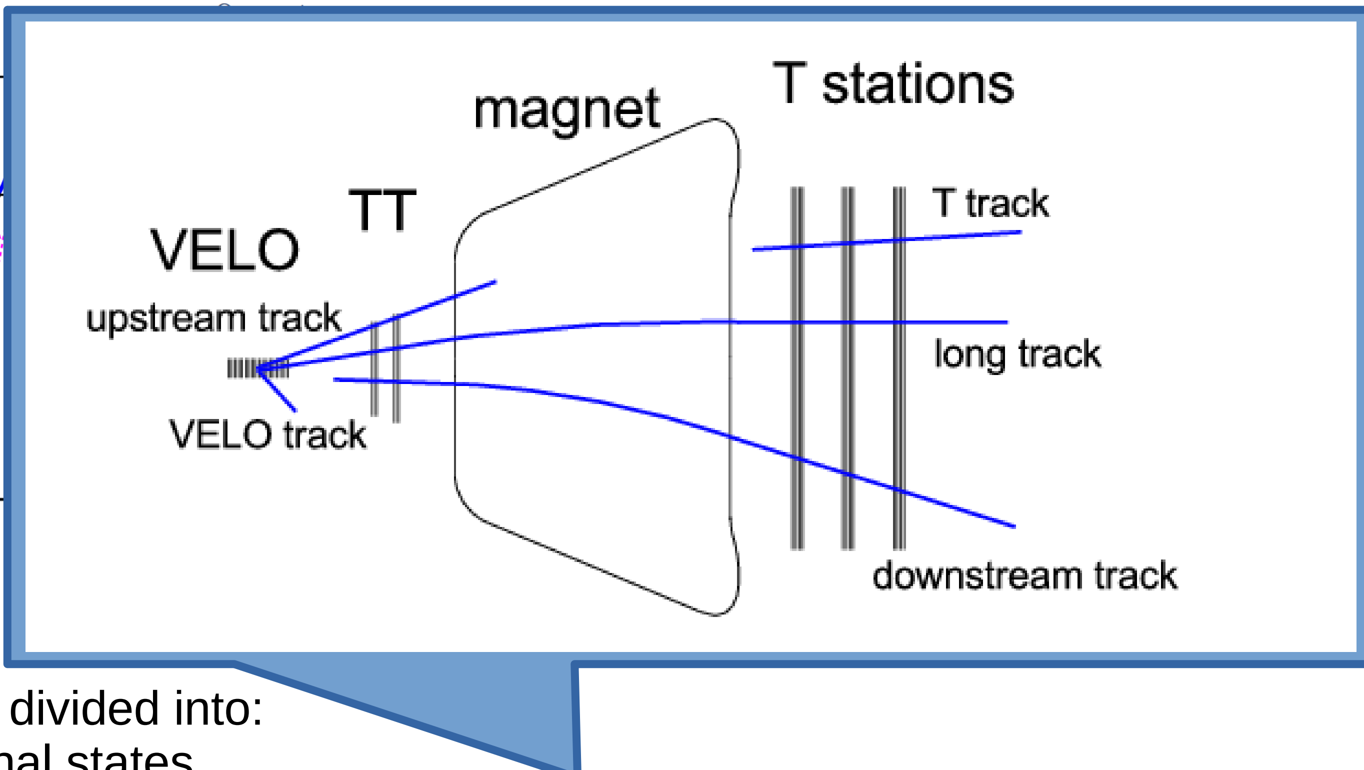

	B^0_s
ed	Supressed
ed	Favoured
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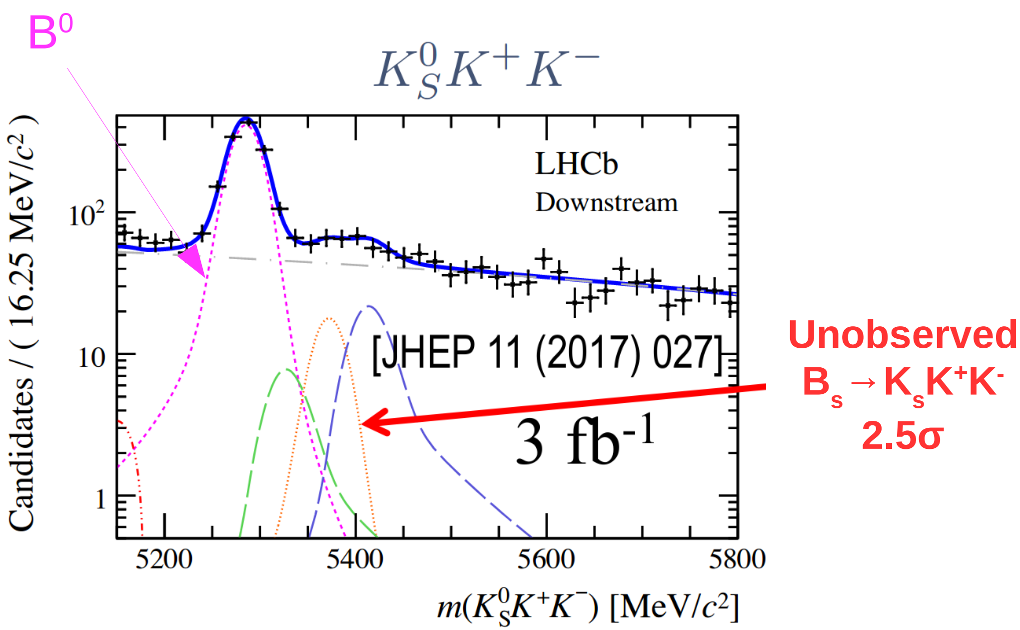

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$K_sK^\pm\pi^\mp$	Supressed	Favoured
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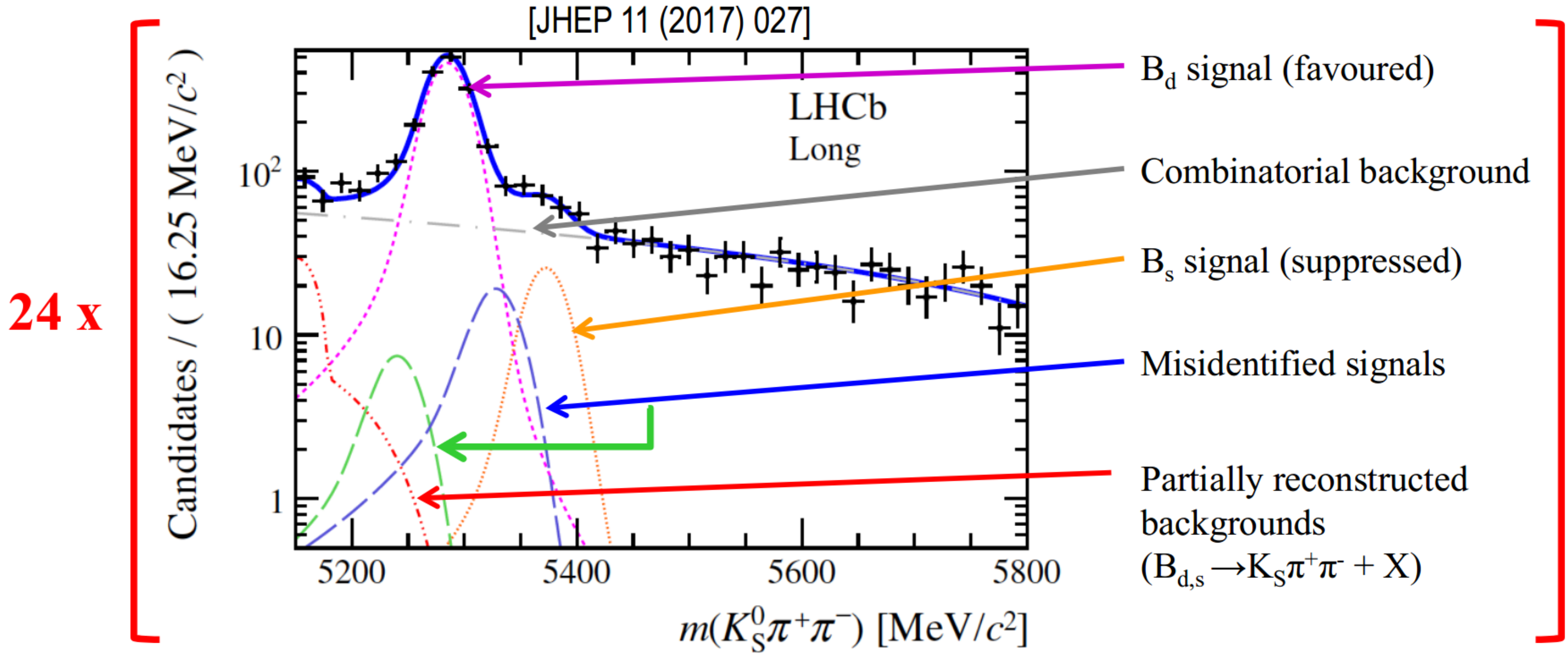
[JHEP 11 (2017) 027]

Analysis strategy



- Shapes taken from Monte-Carlo, except for combinatorial background

- B_d and B_s masses and widths from fit to data

- Gaussian constraints on yields of misidentified signal and partially reconstructed background

- Fast Monte-Carlo developed for partially reconstructed background modelling

Current Status

2016, DD – LHCb unofficial

Analysis updated using
run I + 2016 data ~ 5 fb^{-1}

Goals :
- Use run I + run II data ~ 9 fb^{-1}
- search for $B_s \to K_S K^+ K^-$

- Silmultaneous fit :
  - 4 final states
  - 2 K_s reconstruction categories
  - 6 data-taking periods

→ **42 invariant-mass distributions**

Analysis ongoing in Paris, Clermont, Bogota and Warwick

Current Status

Goals :
- Use run I + run II data ~ 9 fb^{-1}
- search for $B_s \to K_S K^+ K^-$
- Silmultaneous fit :
    - 4 final states
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    - 6 data-taking periods

→ **42 invariant-mass distributions**

2016, DD – LHCb unofficial

Analysis performed using run I + 2016 data ~ 5 fb^{-1}

Extraction of the CKM angle γ using charmless 3-body B-meson decays

Why measuring the CKM parameters?

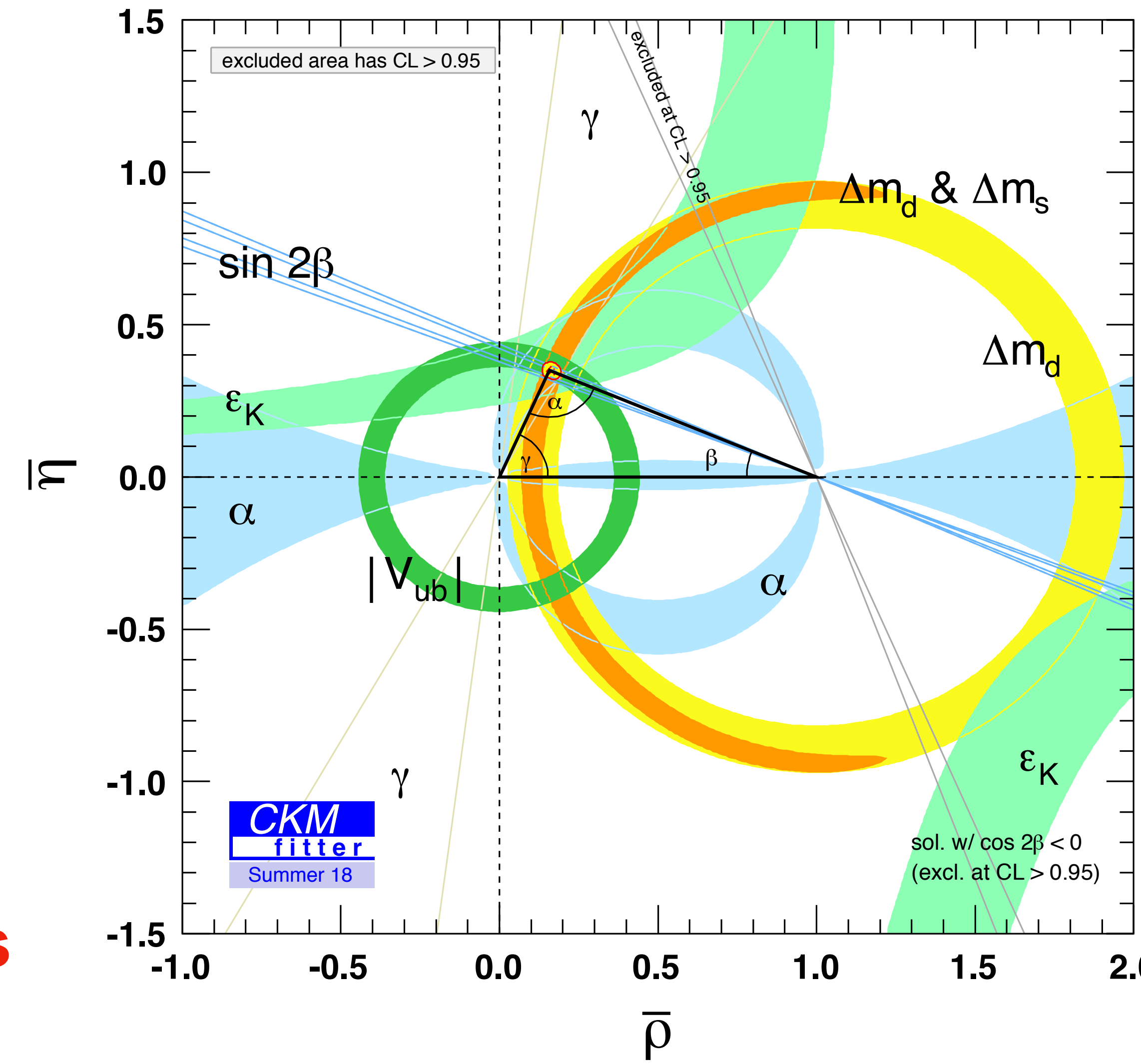
CKM parameters

- Precision measurements
- Test the unitarity of the CKM matrix

$\Rightarrow$ **Over-constrain the triangle**

Measure γ

- From tree decays $\rightarrow$ **precise value**
- From loop decays $\rightarrow$ **probe for new physics**



$$\alpha = [86.4^{+4.5}_{-4.3}]^\circ$$

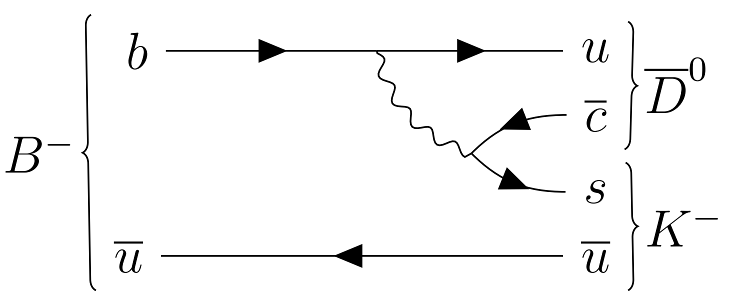
$$\beta = [22.14^{+0.69}_{-0.67}]^\circ$$

Least precise parameter $\longleftarrow$ $\gamma = [72.1^{+5.4}_{-5.7}]^\circ$

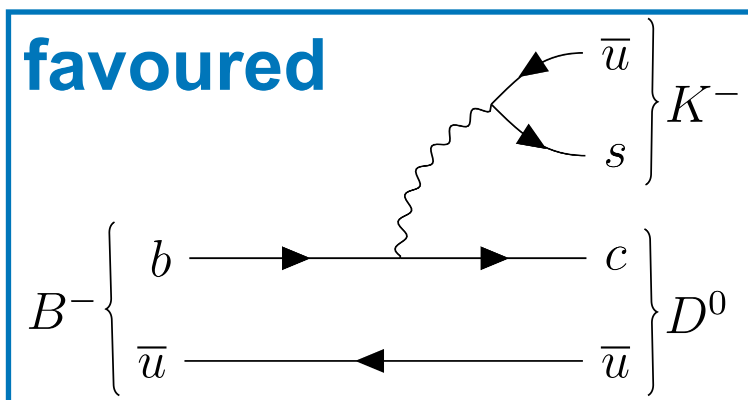
γ from tree decays

CP violation measurement requires interference

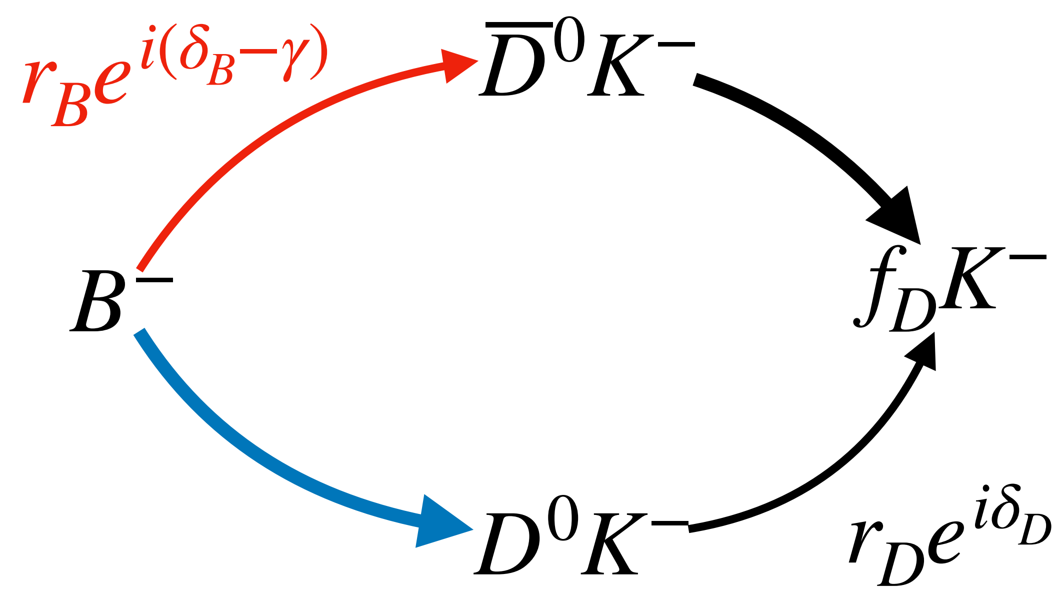
suppressed



favoured



$$\frac{\mathscr{A}_{b\to u}}{\mathscr{A}_{b\to c}} = r_B e^{i(\delta_B-\gamma)}$$



$$r_B = \left|\frac{\mathscr{A}(B^- \to \overline{D}^0 K^-)}{\mathscr{A}(B^- \to D^0 K^-)}\right| \approx \frac{1}{3}\left|\frac{V_{cs}V_{ub}^*}{V_{us}V_{cb}^*}\right| \approx 0.1$$

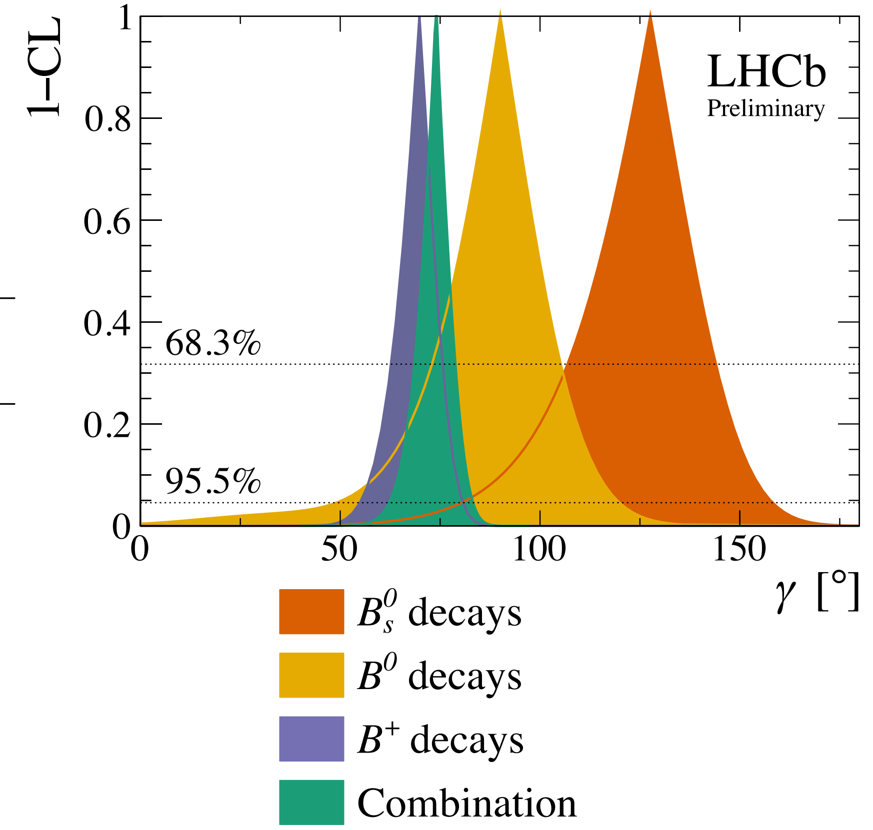
- **Theoretically clean** $(\delta\gamma/\gamma \propto 10^{-7})$
- **Experimentally challenging** (small branching fractions)

$\Rightarrow$ the best experimental precision is obtained by **combining several measurements**.

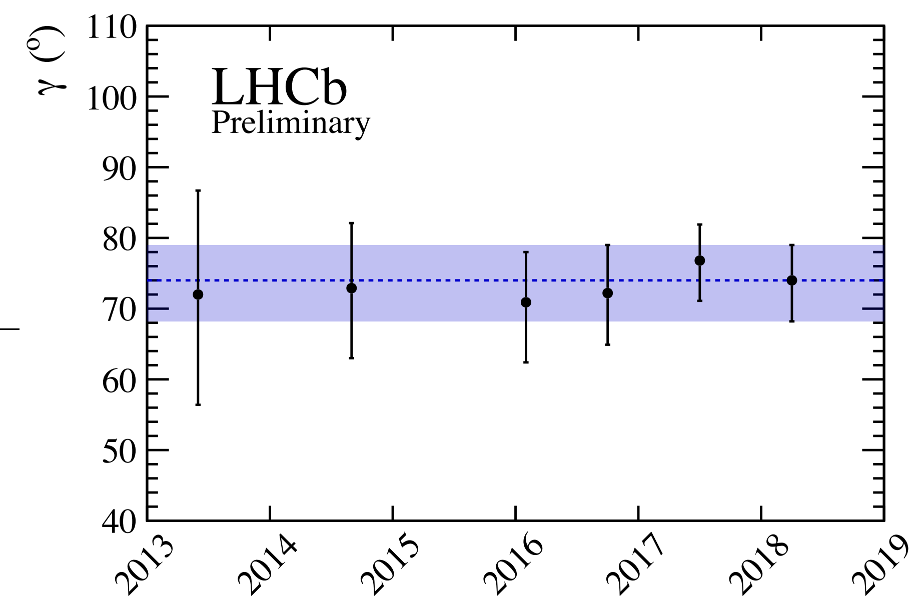
LHCb γ combination

LHCb-CONF-2018-002

- 98 experimental observables, 40 free parameters in the fit
- hadronic parameters (r_B, δ_B) also extracted along with γ
- Frequentist treatment



B decay	D decay	Method	Ref.	Dataset†	Status since last combination [3]
$B^+ \to DK^+$	$D \to h^+h^-$	GLW	[14]	Run 1 & 2	Minor update
$B^+ \to DK^+$	$D \to h^+h^-$	ADS	[15]	Run 1	As before
$B^+ \to DK^+$	$D \to h^+\pi^-\pi^+\pi^-$	GLW/ADS	[15]	Run 1	As before
$B^+ \to DK^+$	$D \to h^+h^-\pi^0$	GLW/ADS	[16]	Run 1	As before
$B^+ \to DK^+$	$D \to K^0_S h^+h^-$	GGSZ	[17]	Run 1	As before
$B^+ \to DK^+$	$D \to K^0_S h^+h^-$	GGSZ	[18]	Run 2	New
$B^+ \to DK^+$	$D \to K^0_S K^+\pi^-$	GLS	[19]	Run 1	As before
$B^+ \to D^*K^+$	$D \to h^+h^-$	GLW	[14]	Run 1 & 2	Minor update
$B^+ \to DK^{*+}$	$D \to h^+h^-$	GLW/ADS	[20]	Run 1 & 2	Updated results
$B^+ \to DK^{*+}$	$D \to h^+\pi^-\pi^+\pi^-$	GLW/ADS	[20]	Run 1 & 2	New
$B^+ \to DK^+\pi^+\pi^-$	$D \to h^+h^-$	GLW/ADS	[21]	Run 1	As before
$B^0 \to DK^{*0}$	$D \to K^+\pi^-$	ADS	[22]	Run 1	As before
$B^0 \to DK^+\pi^-$	$D \to h^+h^-$	GLW-Dalitz	[23]	Run 1	As before
$B^0 \to DK^{*0}$	$D \to K^0_S\pi^+\pi^-$	GGSZ	[24]	Run 1	As before
$B^0_s \to D_s^{\mp}K^{\pm}$	$D_s^+ \to h^+h^-\pi^+$	TD	[25]	Run 1	Updated results
$B^0 \to D^{\mp}\pi^{\pm}$	$D^+ \to K^+\pi^-\pi^+$	TD	[26]	Run 1	New



LHCb combination

$$\gamma = (74.0^{+5.0}_{-5.8})^\circ$$

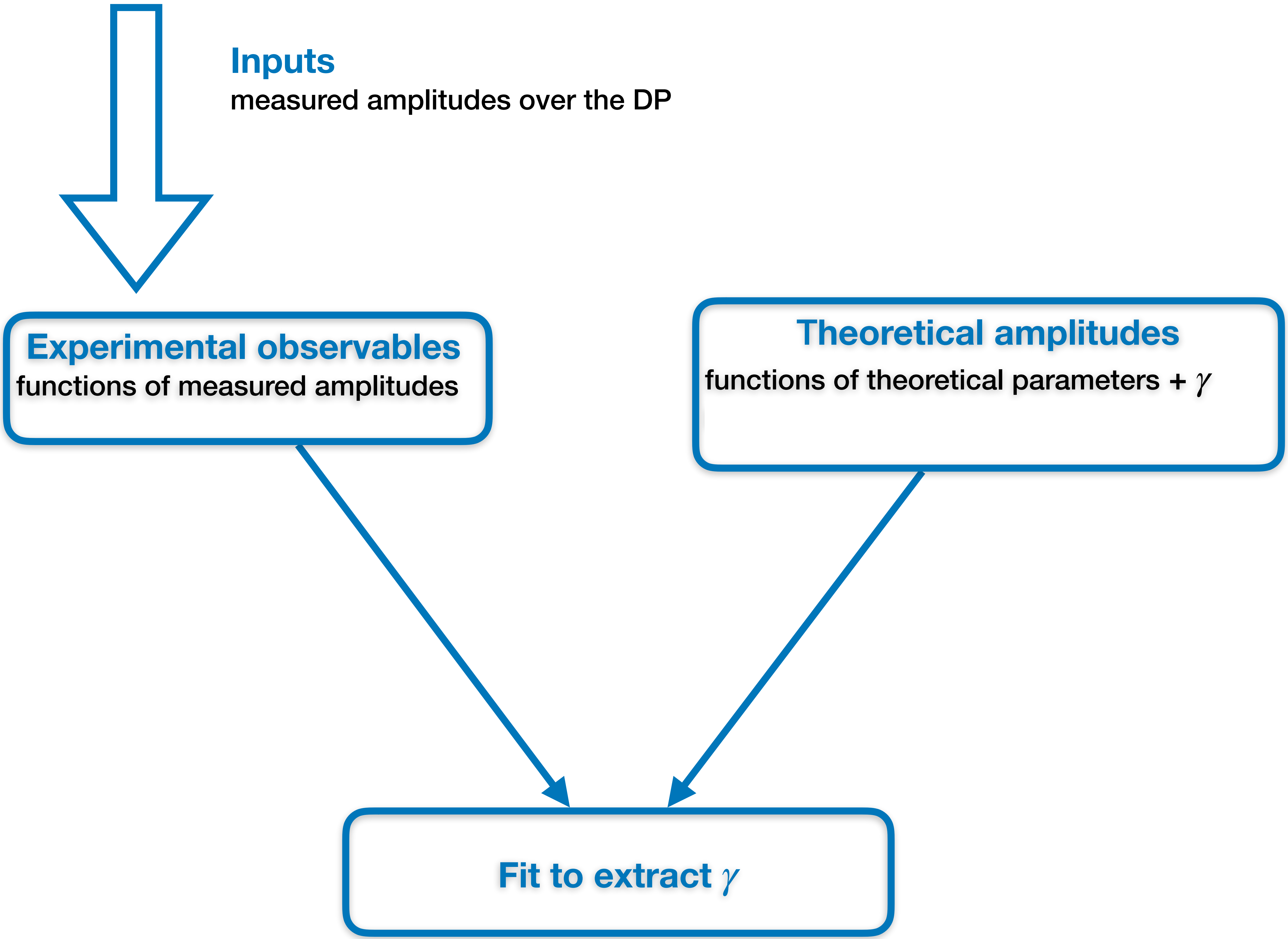
Run 2 measurements were performed with an integrated luminosity of 2fb^{-1} @13TeV. Analyses with the full 6fb^{-1} dataset still to come.

- In agreement with world averages (CKMfitter, UTfit, HFLAV).
- Supersedes the previous LHCb measurement.
- Most precise determination of γ from a single experiment to date.

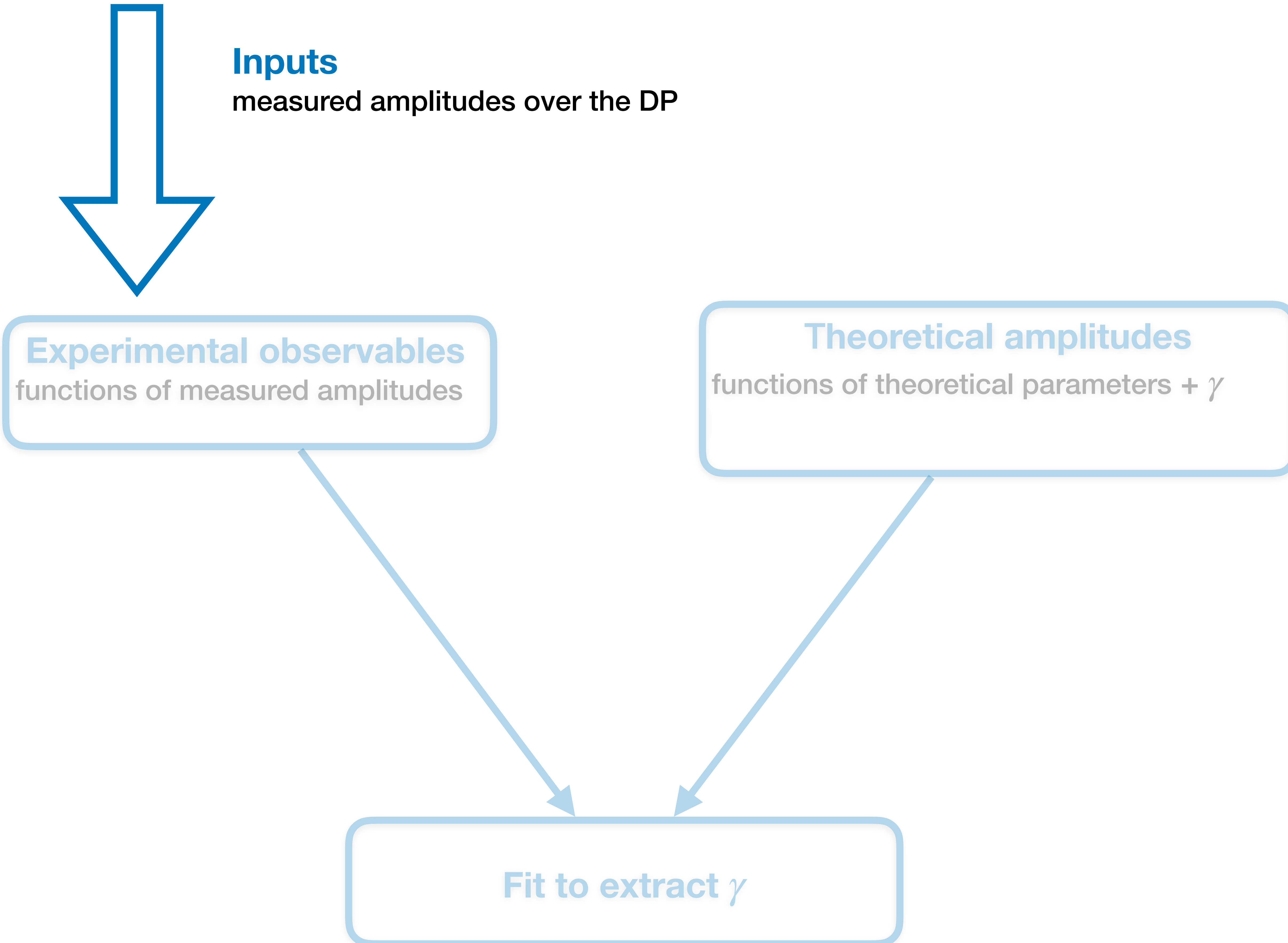
Extraction of the CKM angle γ using charmless 3-body decays of B mesons

- theoretical method developed by B. Bhattacharya, M. Imbeault and D. London Phys. Lett. B728 (2014) 206-209
- combine information coming from several charmless modes
- potentially sensitive to new physics
- goal of the study: extract γ with its uncertainty

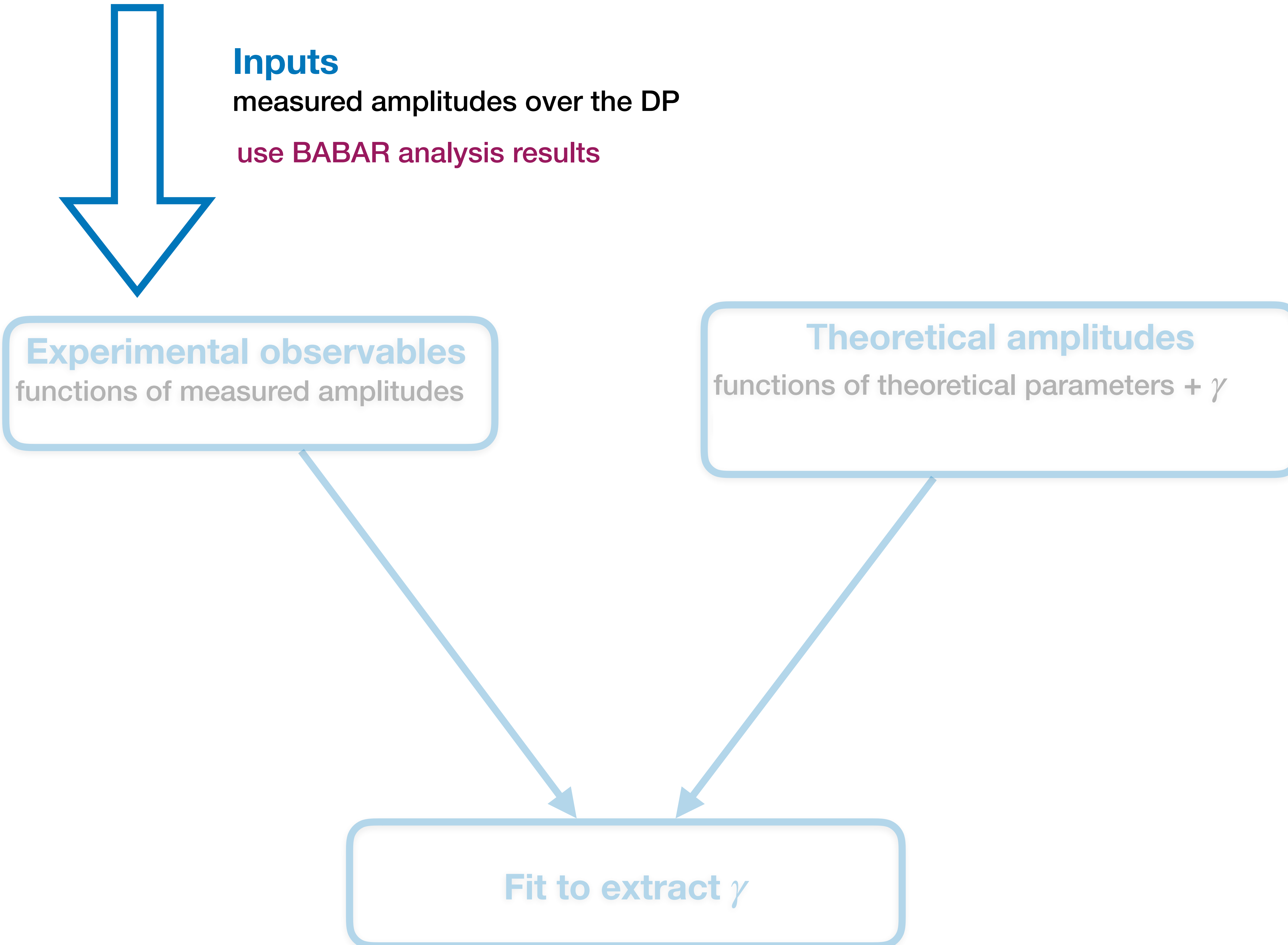
In a nutshell



In a nutshell



In a nutshell



Experimental inputs

BABAR amplitude-analysis results of 5 decay modes

1) $B^0 \to K_S^0 K_S^0 K_S^0$ 2) $B^0 \to K^+ \pi^0 \pi^-$ 3) $B^+ \to K^+ \pi^+ \pi^-$
4) $B^0 \to K_S^0 K^+ K^-$ 5) $B^0 \to K_S^0 \pi^+ \pi^-$

1) Phys. Rev. D85 (2012) 054023   2) Phys. Rev. D83 (2011) 112010   3) Phys. Rev. D78 (2009) 112004
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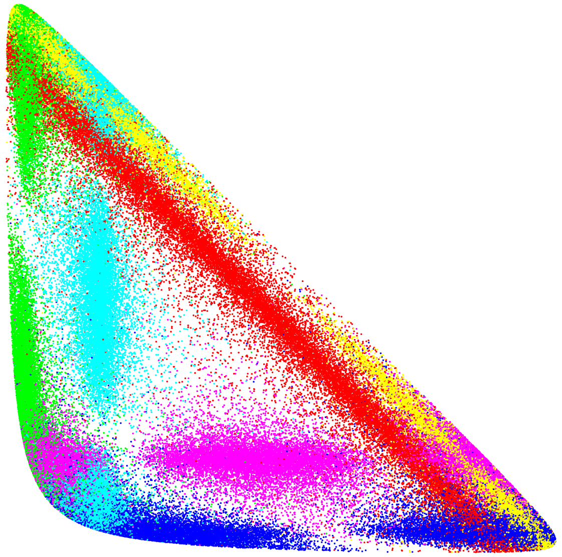
Description of the amplitude in the DP: **the isobar model**

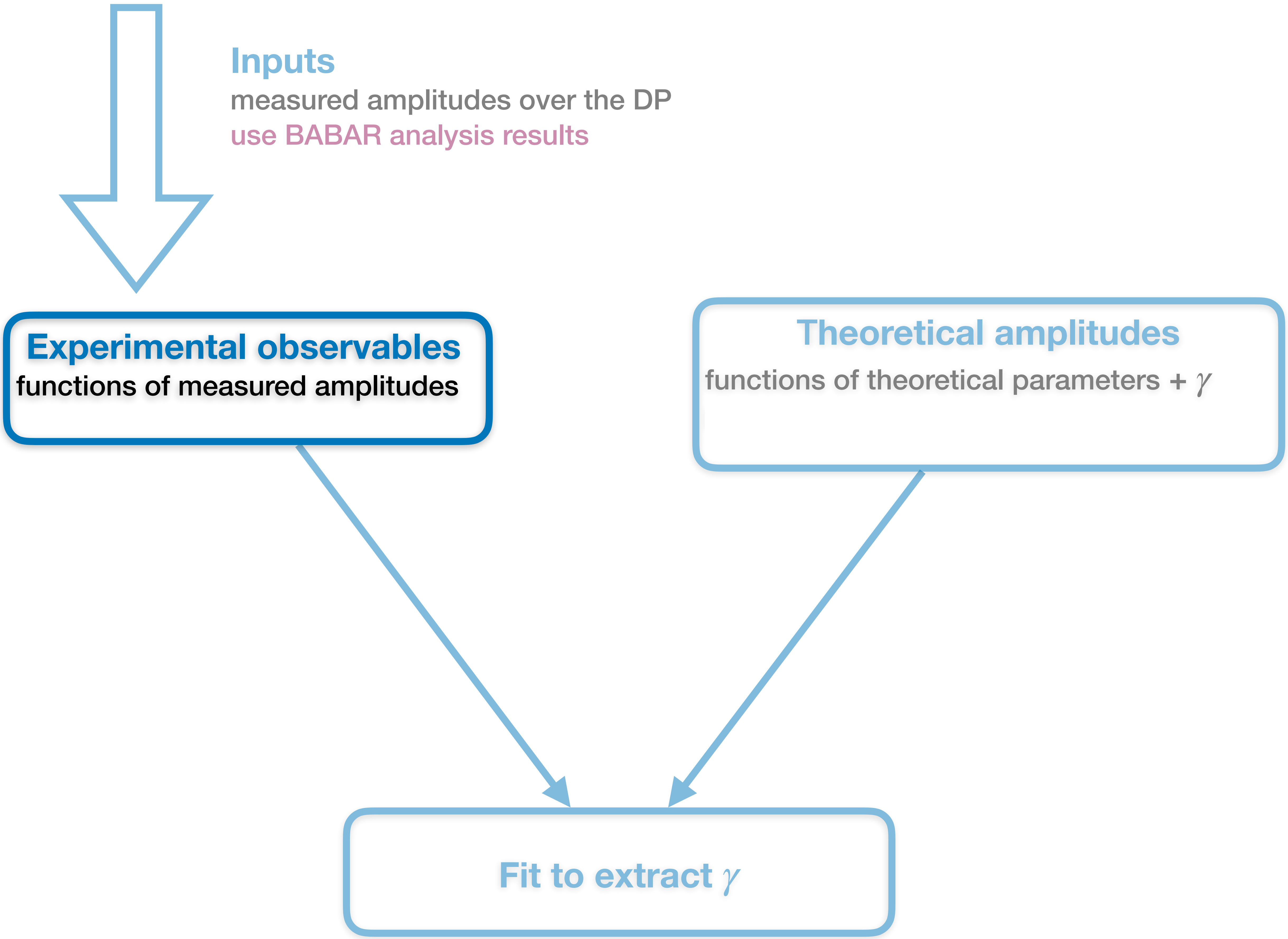
Total amplitude of the decay modelled as a coherent sum of partial amplitudes.

$$\mathscr{A}(m_{12}^2, m_{23}^2) = \sum_{j=1}^{N} c_j e^{i\phi_j} F_j(m_{12}^2, m_{23}^2)$$



Isobar parameters
Weak and strong interactions

Lineshape
Strong dynamics


Inputs
measured amplitudes over the DP
use BABAR analysis results
Experimental observables
functions of measured amplitudes
Theoretical amplitudes
functions of theoretical parameters + γ
Fit to extract γ

Observables

From the experimental amplitudes we construct
momentum-dependent observables

$$X(m_{13}^2, m_{23}^2) \stackrel{\text{def}}{=} |\mathscr{A}(m_{13}^2, m_{23}^2)|^2 + |\overline{\mathscr{A}}(m_{13}^2, m_{23}^2)|^2$$ **branching ratio**

$$Y(m_{13}^2, m_{23}^2) \stackrel{\text{def}}{=} |\mathscr{A}(m_{13}^2, m_{23}^2)|^2 - |\overline{\mathscr{A}}(m_{13}^2, m_{23}^2)|^2$$ **direct A_{CP}**

$$Z(m_{13}^2, m_{23}^2) \stackrel{\text{def}}{=} \mathrm{Im}[\mathscr{A}^*(m_{13}^2, m_{23}^2)\overline{\mathscr{A}}(m_{13}^2, m_{23}^2)]$$ **indirect A_{CP}**

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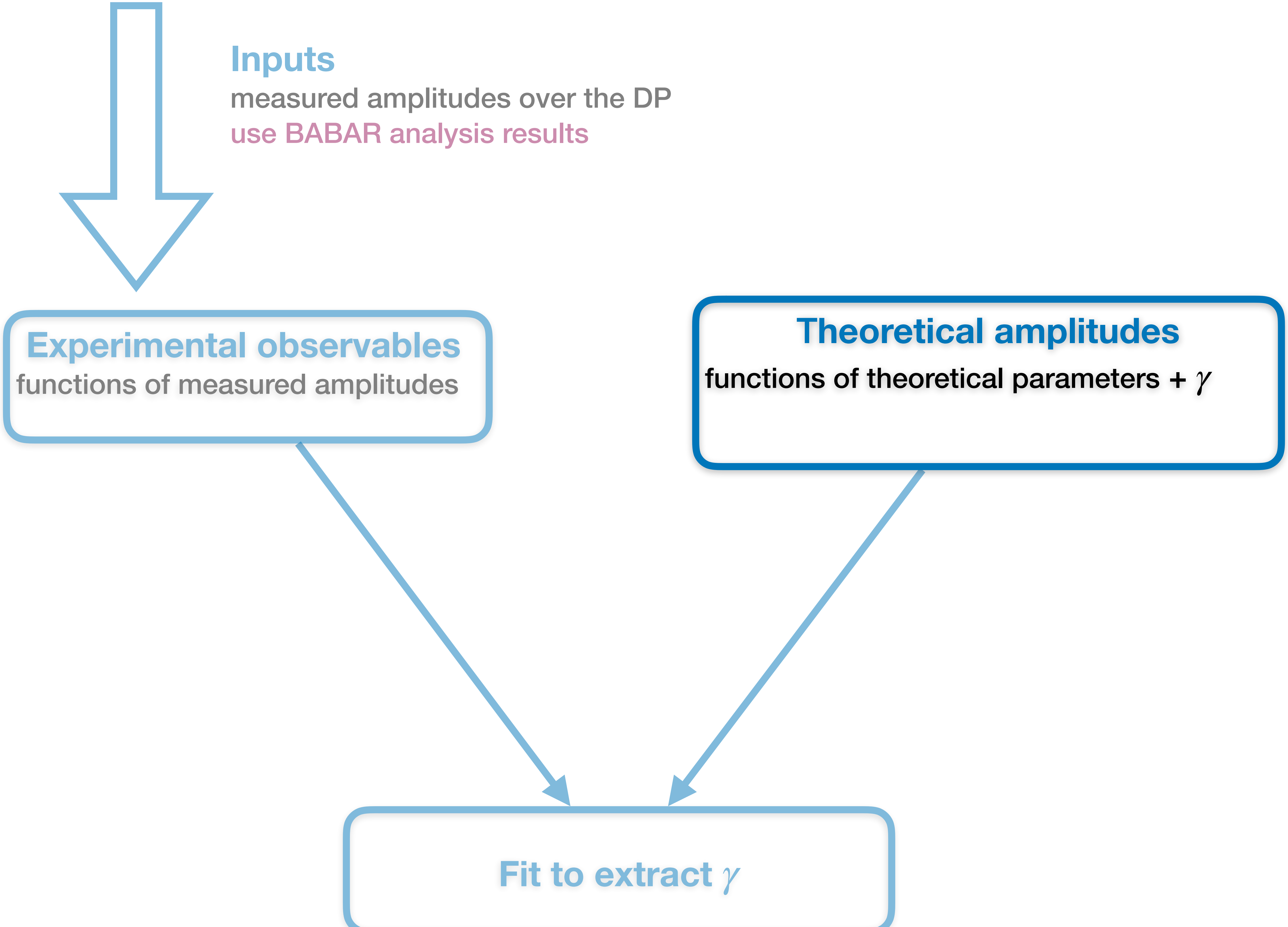
13 observables for the 5 modes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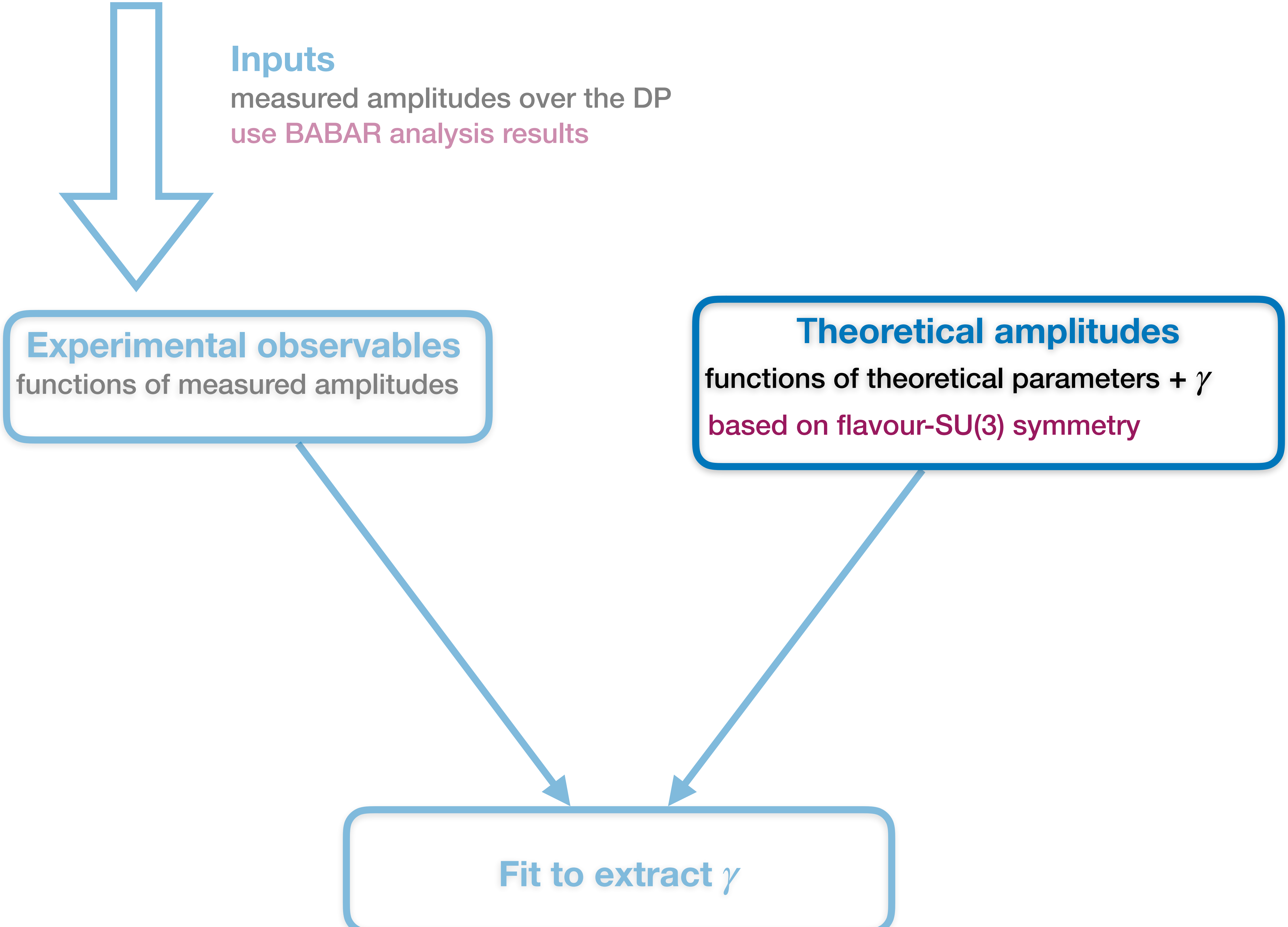
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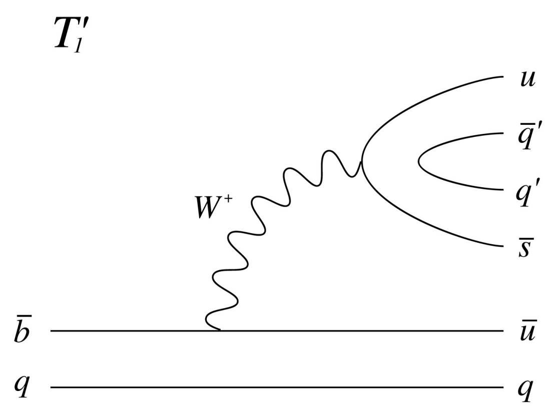
Theoretical amplitudes

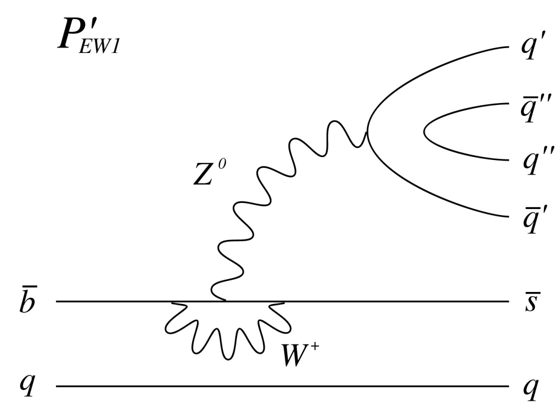
$B \to Khh$ modes: ***several diagrams***

tree



+

electroweak penguin



+...

γ from 3-body decays: $N_{\text{obs}} < N_{\text{params}}$

Phys. Rev. D.84.034040

Theoretical amplitudes

$B \to Khh$ modes: ***several diagrams***

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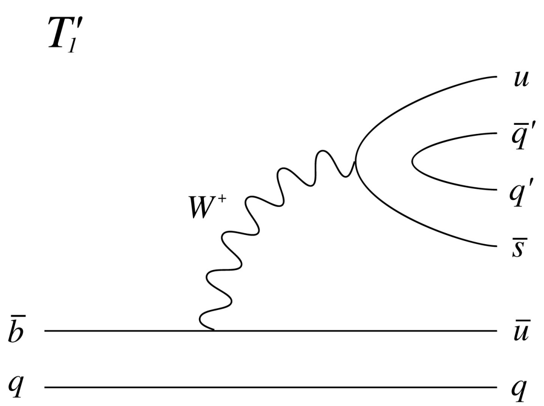

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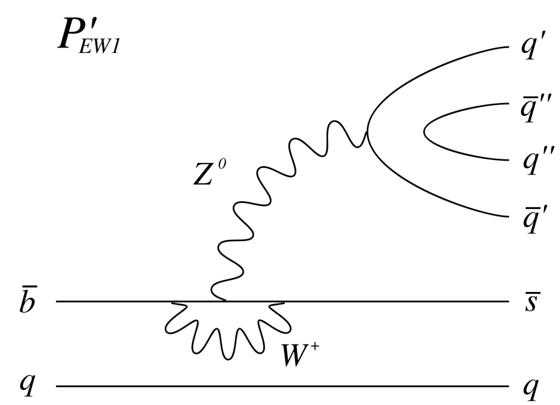

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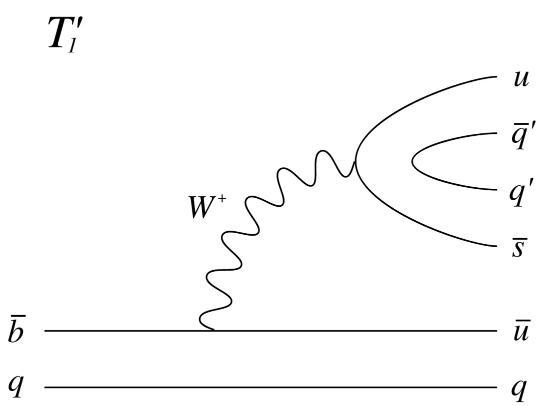

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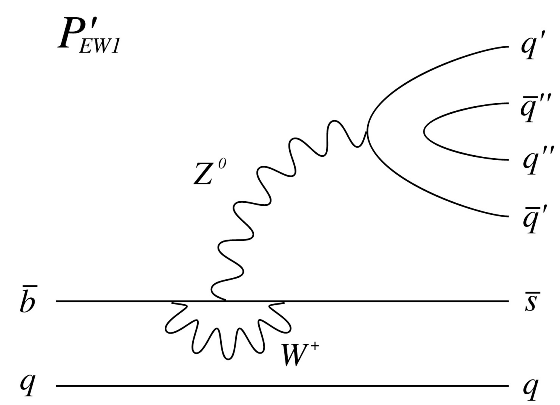

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Flavour-SU(3)

Quark masses are the same
3 identical particles in the final state }

Tree and **penguin** diagrams are proportional: $P_{\mathrm{EW}}^{(c)} = \kappa T^{(c)}$ with $\kappa \approx 0.5$

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Theoretical amplitudes

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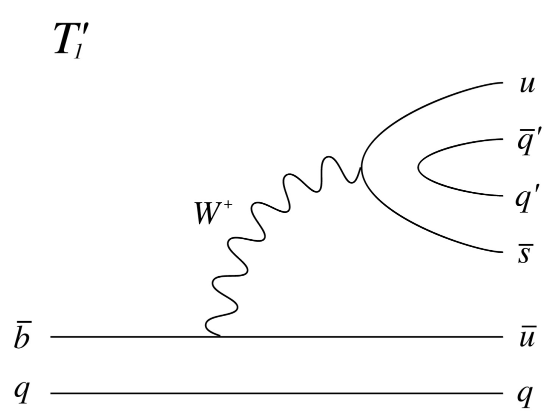

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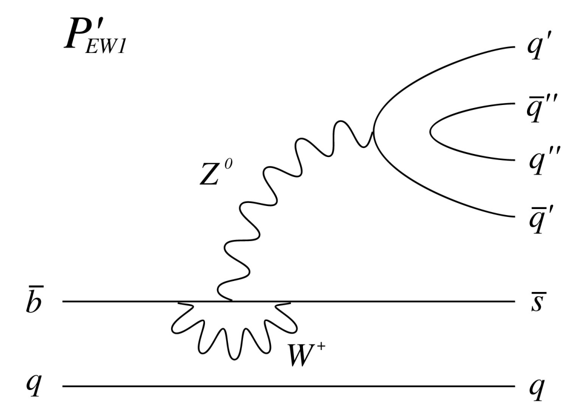

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Flavour-SU(3)

Quark masses are the same
3 identical particles in the final state } ⇒ **symmetrised amplitudes**

Tree and **penguin** diagrams are proportional: $P_{\mathrm{EW}}^{(c)} = \kappa T^{(c)}$ with $\kappa \approx 0.5$

Phys. Rev. D.84.034040

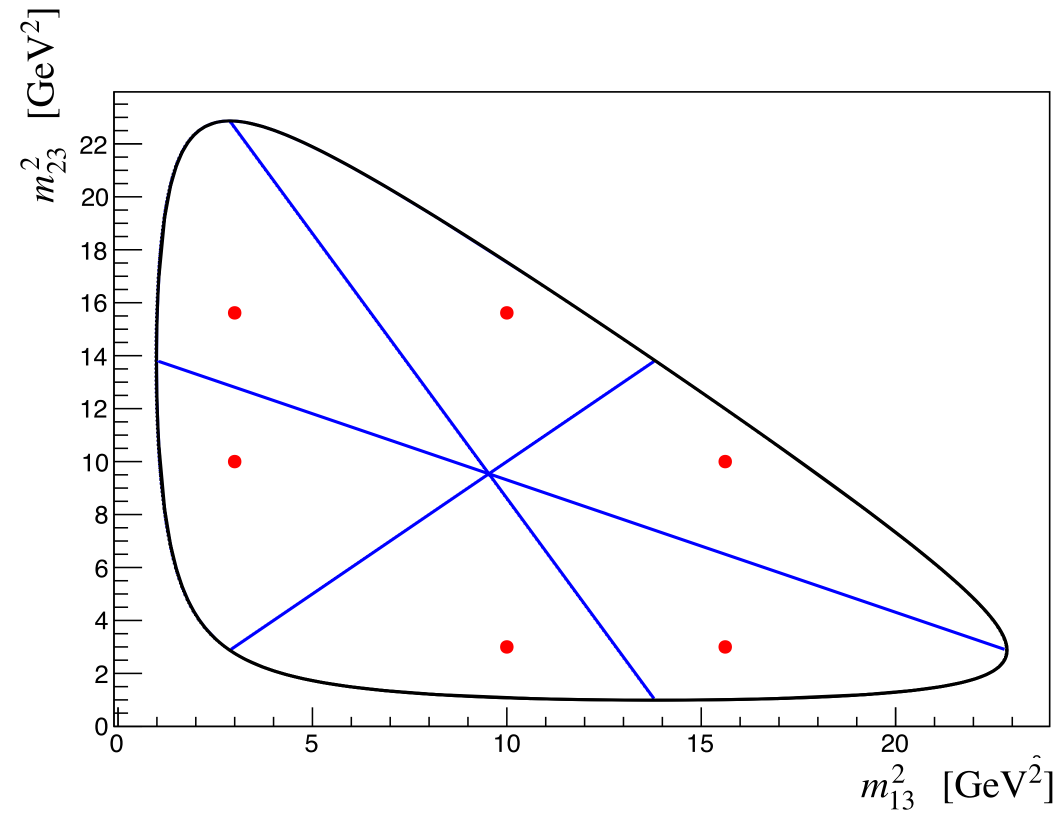
Amplitude symmetrisation

Different final-state symmetrisations are possible (fully symmetric, totally antisymmetric, mixed states)

Choice for this work

Fully-symmetrised amplitudes

$$\mathscr{A}_{\rm fs}(m_{12}^2, m_{23}^2) = \frac{1}{\sqrt{6}}\Big(\mathscr{A}(m_{12}^2, m_{13}^2) + \mathscr{A}(m_{12}^2, m_{23}^2) + \mathscr{A}(m_{13}^2, m_{23}^2) + \mathscr{A}(m_{13}^2, m_{12}^2) + \mathscr{A}(m_{23}^2, m_{12}^2) + \mathscr{A}(m_{23}^2, m_{13}^2)\Big)$$



Fully symmetric DP divided into **6 regions containing the same information**

Both theoretical and experimental amplitudes must be symmetrised in the same way

Theoretical expressions for the amplitudes

$$2A_{\rm fs}(B^0 \to K^+\pi^0\pi^-) = Be^{i\gamma} - \kappa C$$

$$\sqrt{2}A_{\rm fs}(B^0 \to K^0\pi^+\pi^-) = -De^{i\gamma} - \tilde{P}'_{\rm uc}e^{i\gamma} - A + \kappa D$$

$$A_{\rm fs}(B^0 \to K^0K^0\overline{K}^0) = \alpha_{\rm SU(3)}(\tilde{P}'_{\rm uc}e^{i\gamma} + A)$$

$$\sqrt{2}A_{\rm fs}(B^0 \to K^+K^0K^-) = \alpha_{\rm SU(3)}(-Ce^{i\gamma} - \tilde{P}'_{\rm uc}e^{i\gamma} - A + \kappa B)$$

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- 5 effective diagrams: A, B, C, D and $\tilde{P}'_{\rm uc}$

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- 5 effective diagrams: A, B, C, D and $\tilde{P}'_{\rm uc}$
- 1 weak phase: γ
- 1 parameter related to flavour SU(3) breaking: $\alpha_{\rm SU(3)}$

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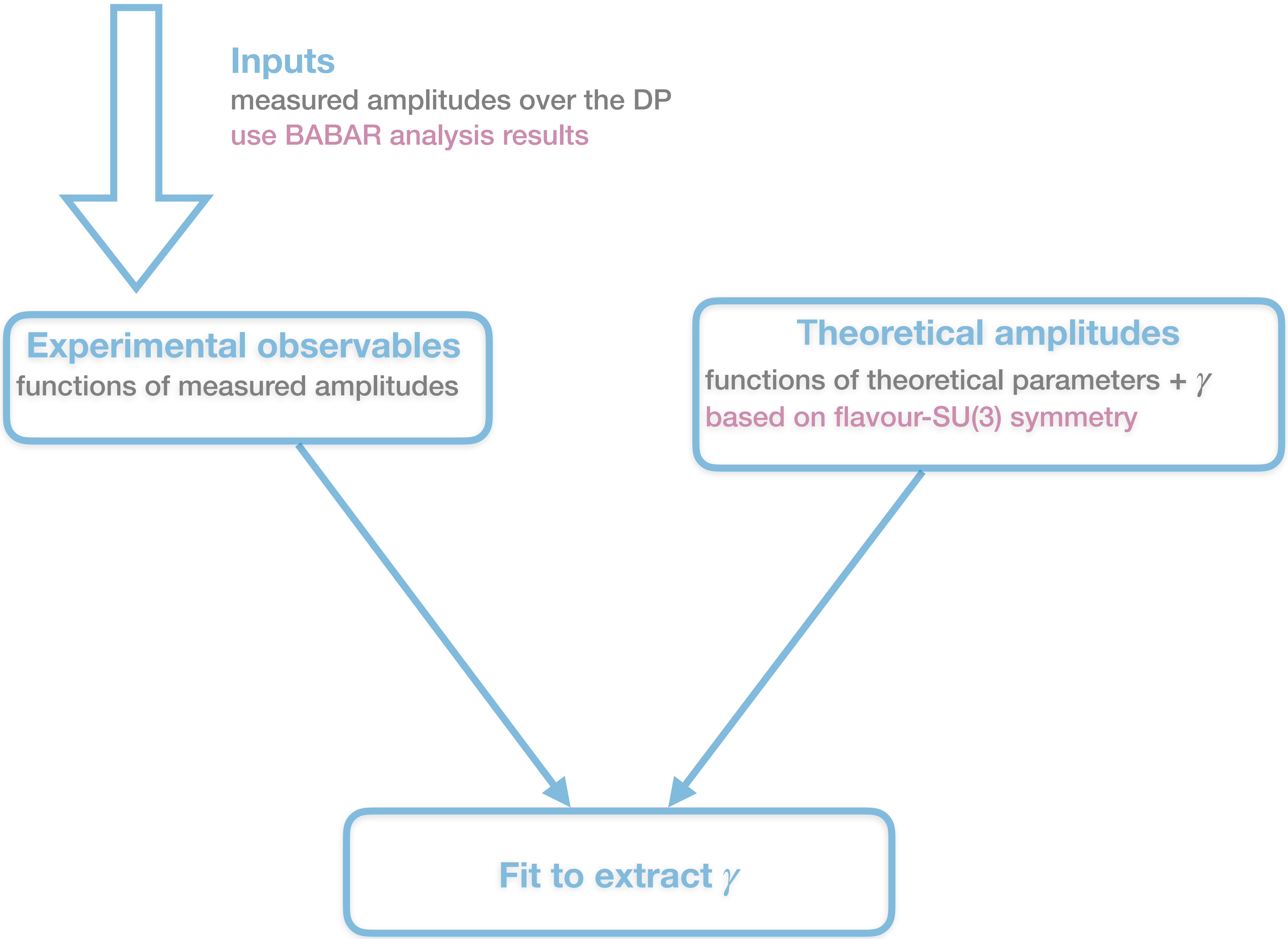
Parameter counting

	th. params.	exp. observables
4 modes	10	11
5 modes	11	13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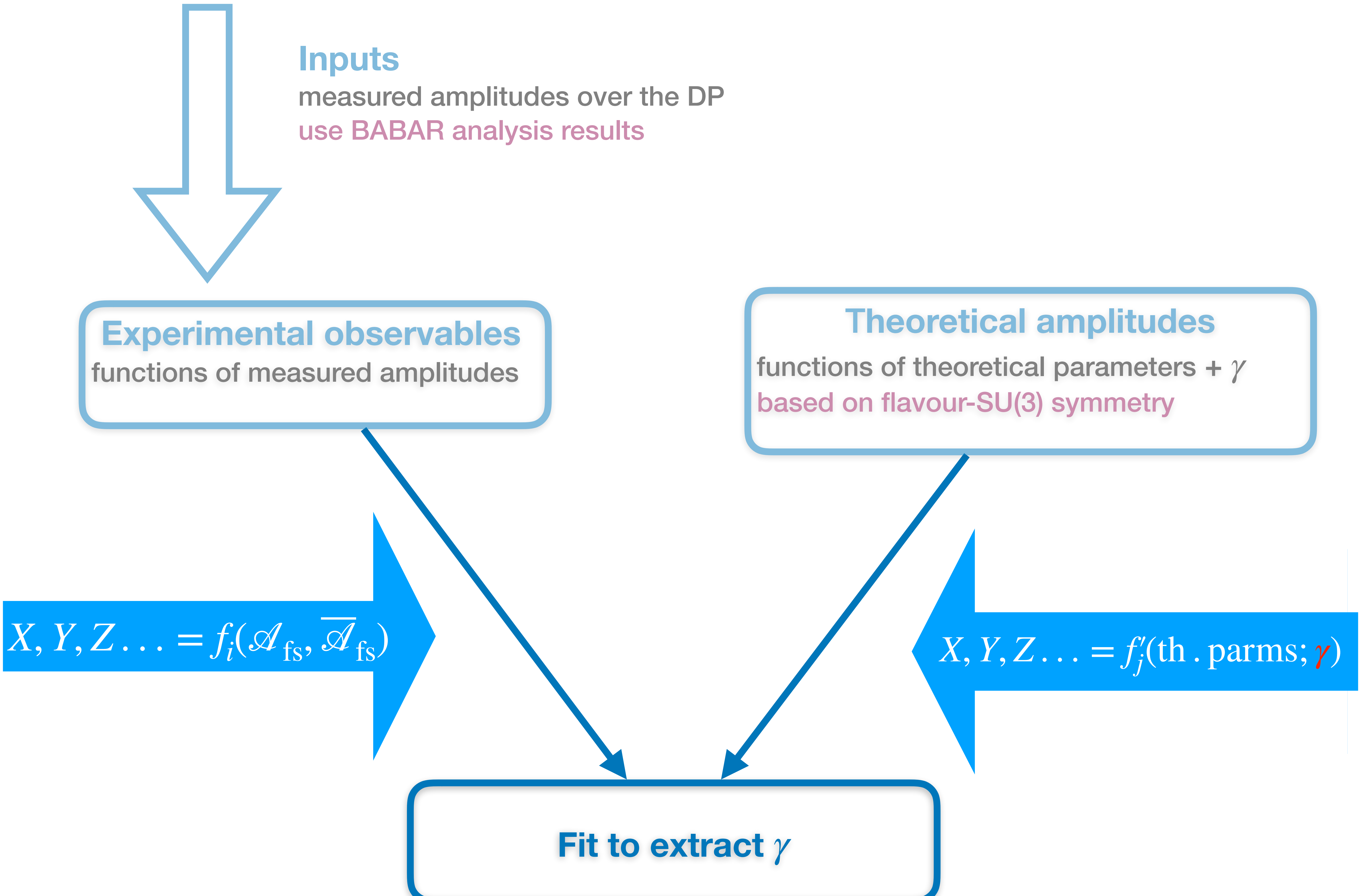
Inputs
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functions of measured amplitudes
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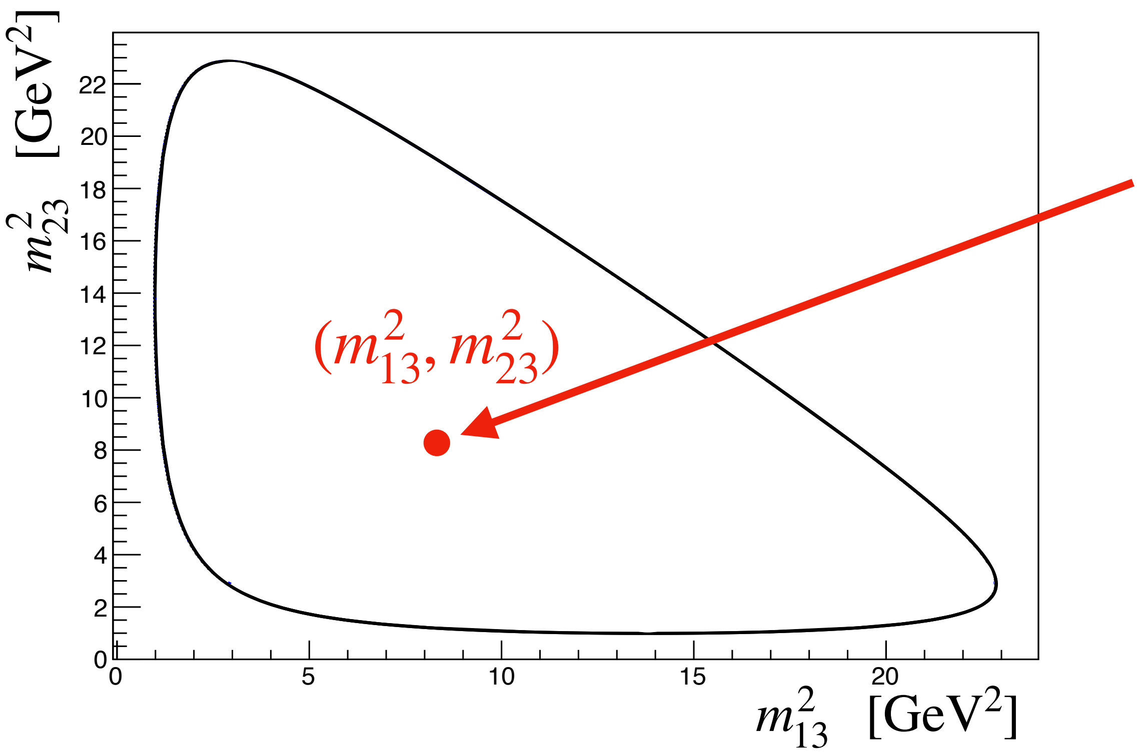
functions of theoretical parameters + γ
based on flavour-SU(3) symmetry
$X, Y, Z \ldots = f_i(\mathscr{A}_{\rm fs}, \overline{\mathscr{A}}_{\rm fs})$
$X, Y, Z \ldots = f'_j(\rm th\,.\,parms; \gamma)$
Fit to extract γ

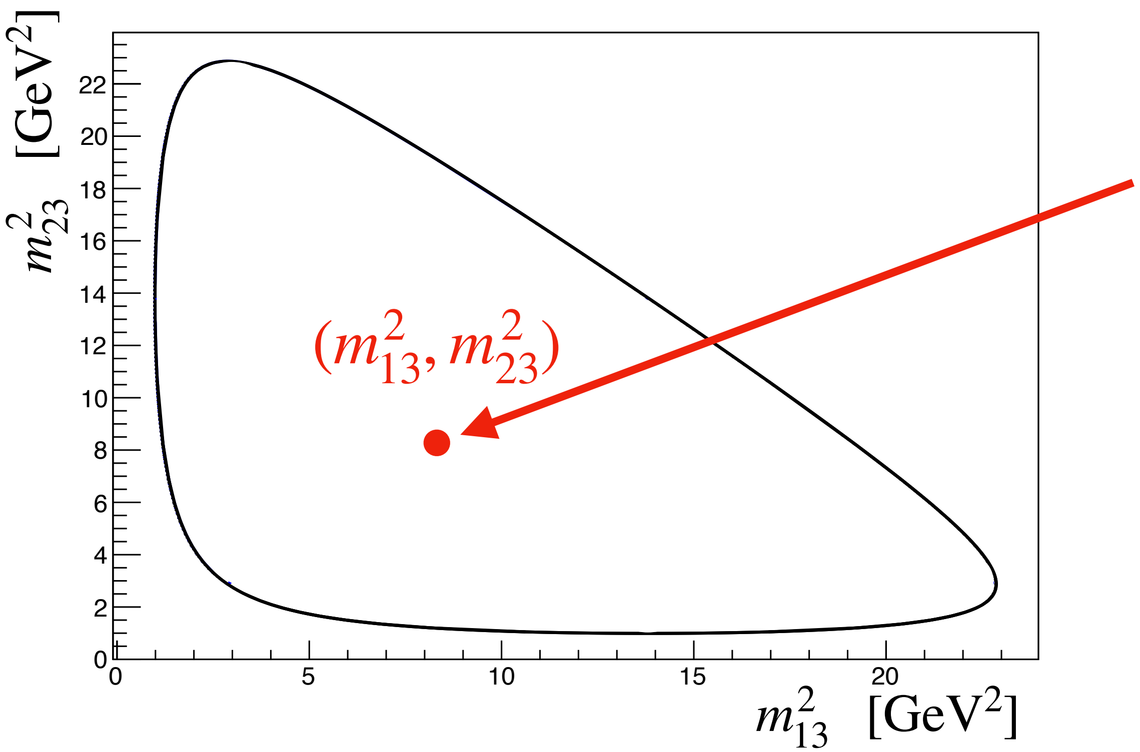
Extracting γ using one DP point



- Observables (X, Y, Z) for all the modes.
- Covariance matrix including the correlations.
- Scan on γ: fix γ to consecutive values and evaluate the other parameters minimising a χ^2 function.

Cov matrix: 11x11 (13x13)

Extracting γ using one DP point



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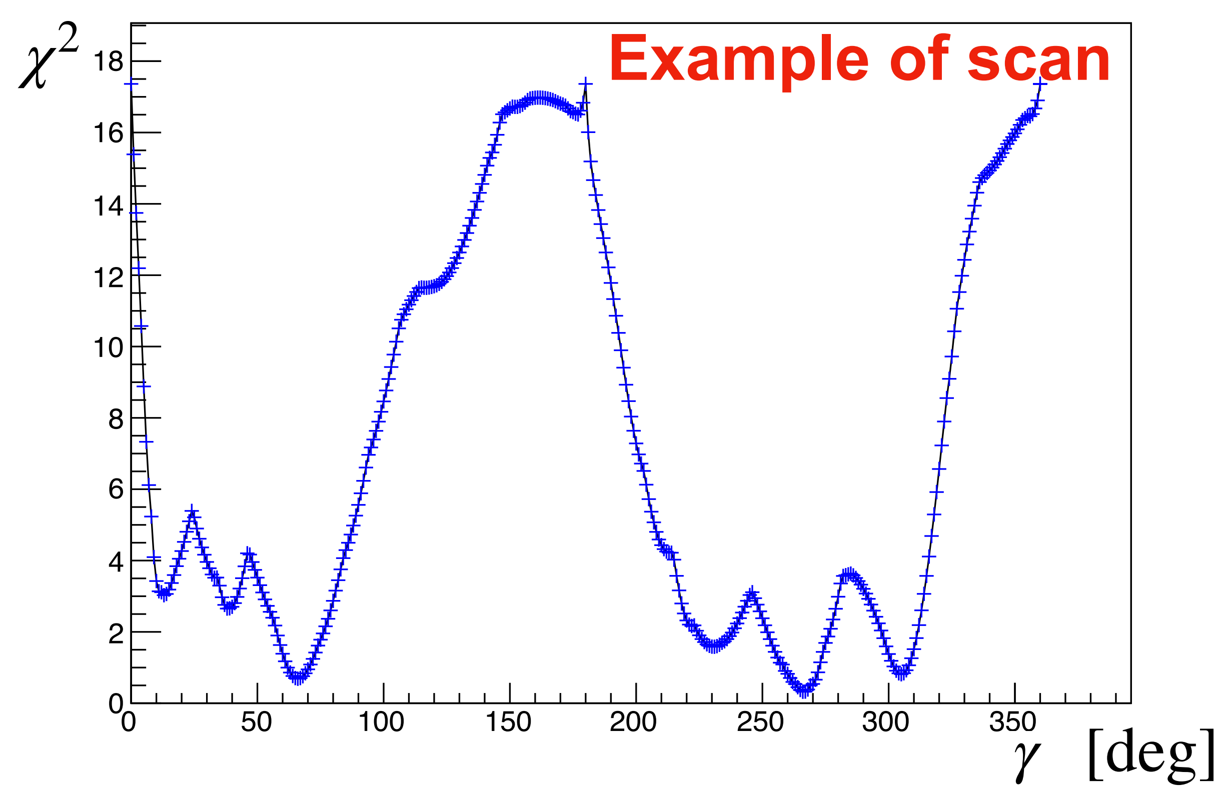

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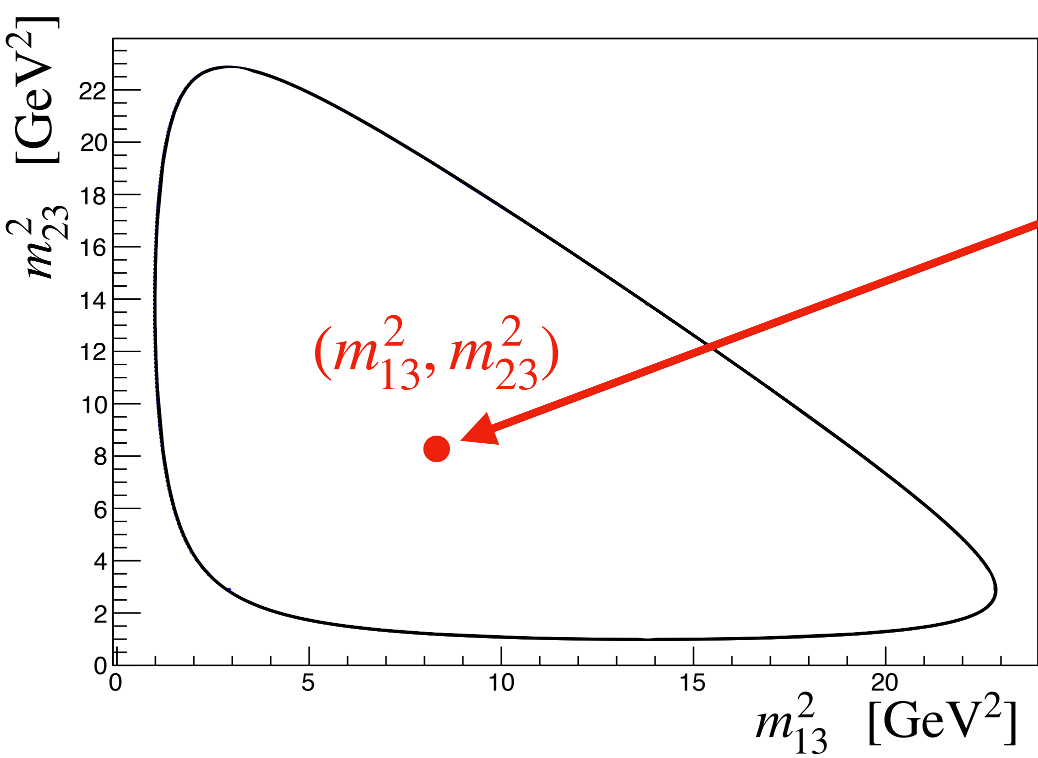

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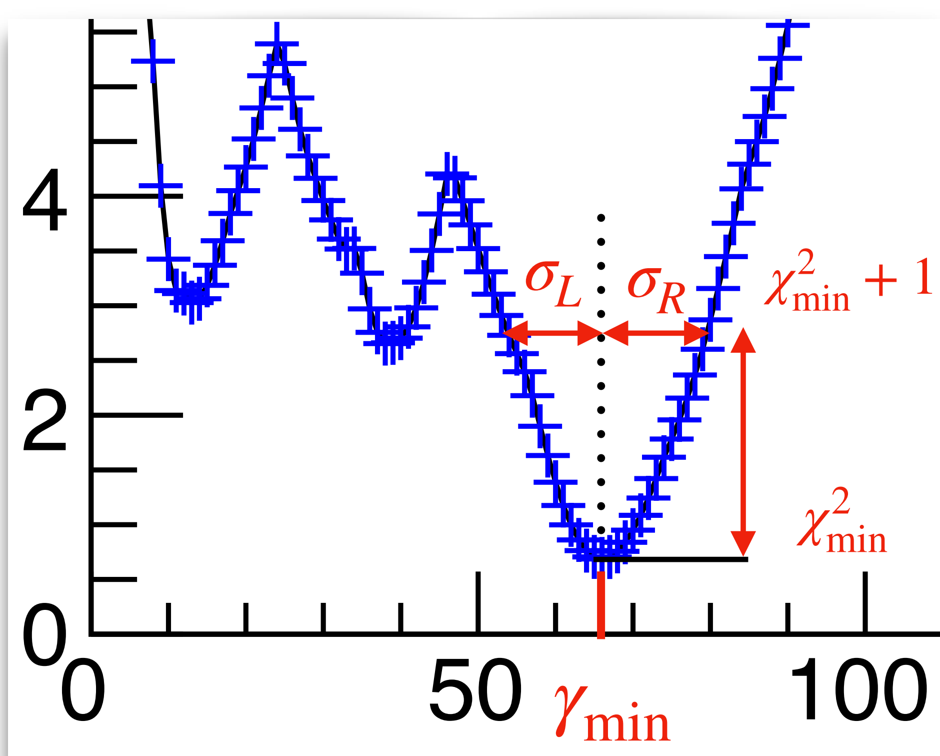
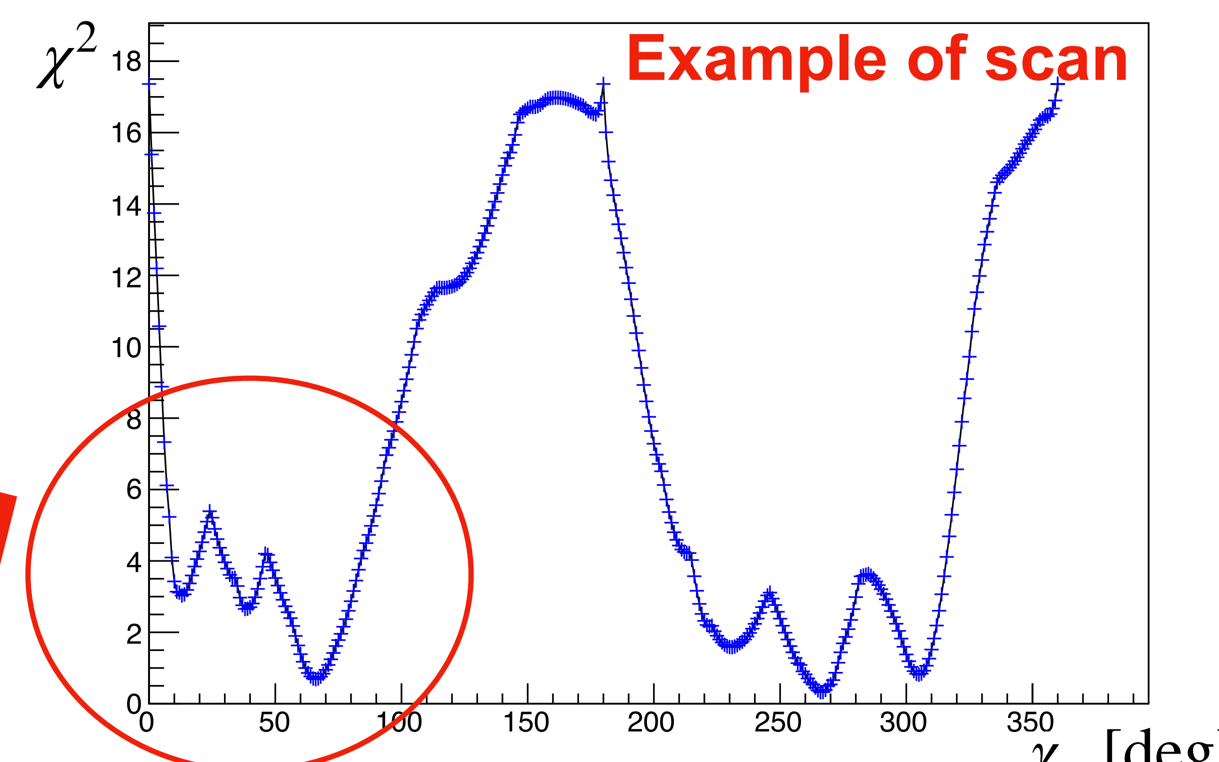


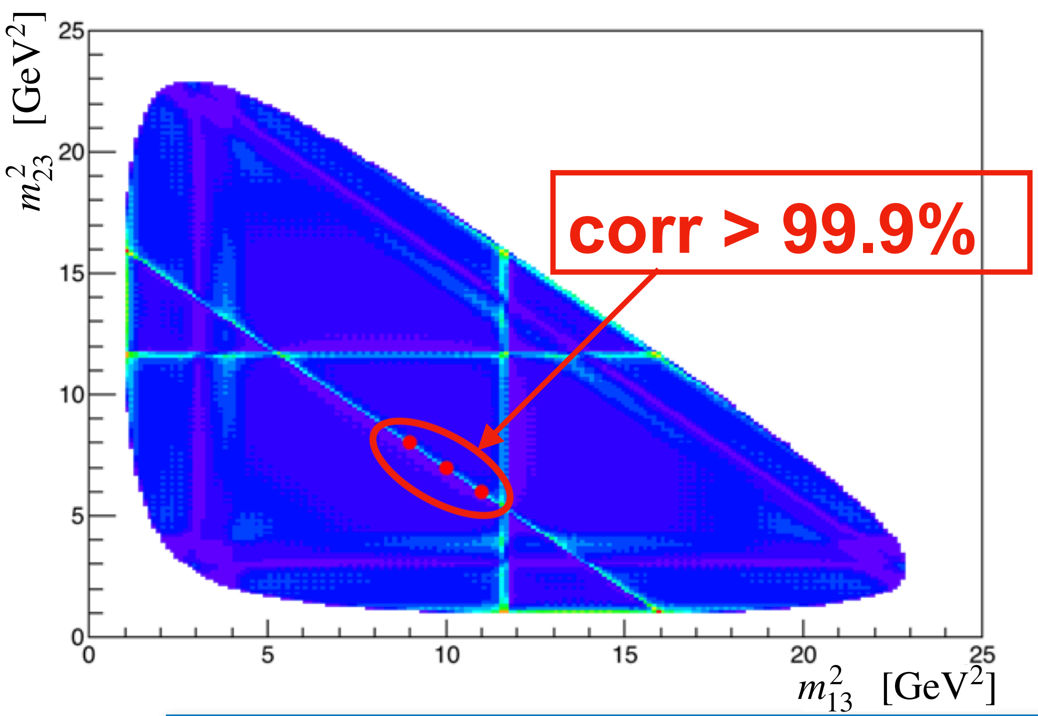


Combining several points

The use of several points allows:

- Using the maximum amount of information.
- Improving the validity of flavour SU(3) hyp.

Extract γ using the maximum number of points in the DP.



But, due to very high correlations between certain points we are limited to the use of 3 points simultaneously

Cov matrix: 33x33 (39x39)

In practice

- Several hundred combinations of 3 points randomly scattered over the DP.
- For each set of points:
  - γ scan (500 fits with random initial parameters).
  - Extract minima and statistical uncertainties
- Combine results of all the scans.

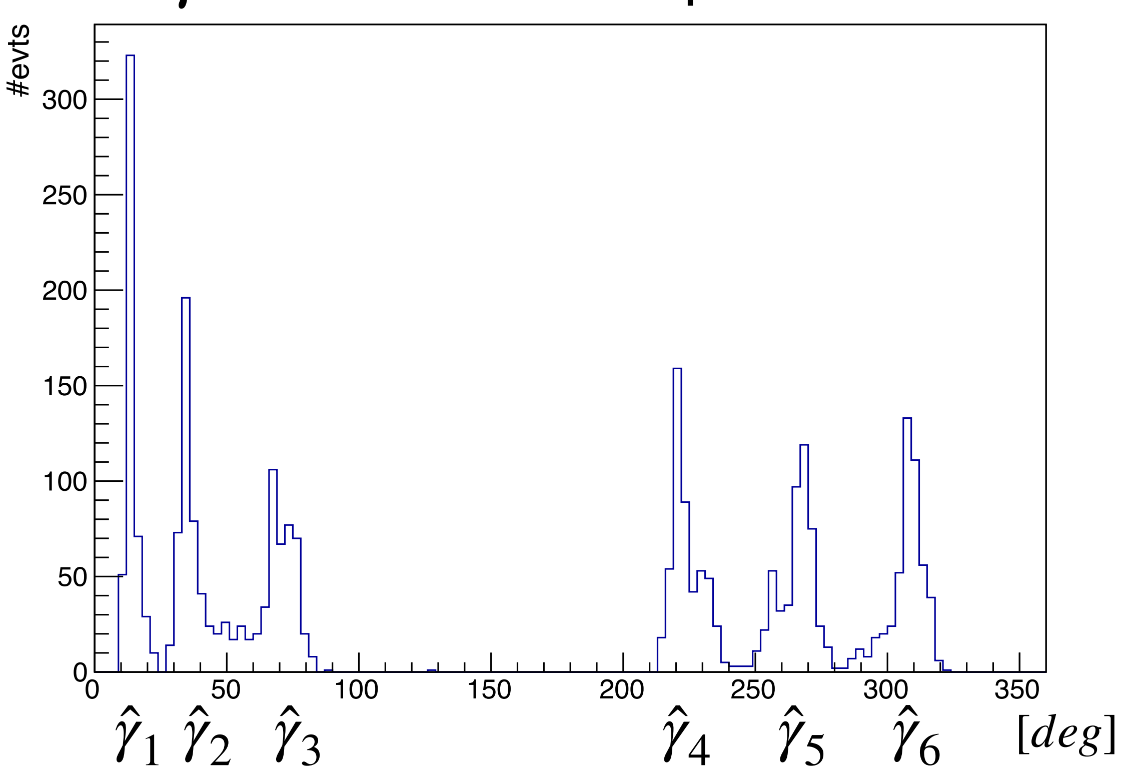
Baseline results

Minimum	central value $\hat{\gamma}$	"statistical" uncertainties σ_L	σ_R
γ_1	$12.9°$	$4.3°$	$8.4°$
γ_2	$36.6°$	$6.1°$	$6.6°$
γ_3	$68.9°$	$8.6°$	$8.6°$
γ_4	$223.2°$	$7.5°$	$10.9°$
γ_5	$266.4°$	$10.8°$	$9.2°$
γ_6	$307.5°$	$8.1°$	$6.9°$

$\gamma_{\text{WorldAverage}} = \left(73.5^{+4.2}_{-5.1}\right)^{\circ}$

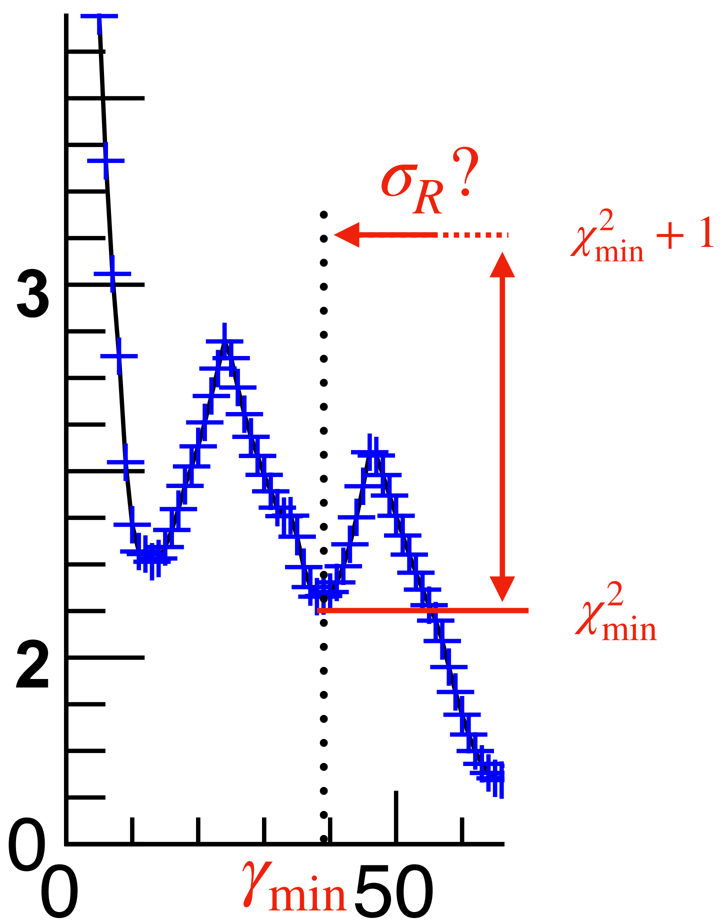
- use 4 modes ($\alpha_{SU(3)} = 1$)
- 501 sets of random 3-points combinations
- for each set: hundreds of fits randomising initial values of parameters
- fit convergence = 100%

$\hat{\gamma}$ for the different 3-point sets



Systematic uncertainties

Influence of "poorly resolved" minima



Poorly resolved minimum: the statistical uncertainty cannot be extracted.

⇒ **not included** in the average for the baseline result

$$\mathrm{Syst} = |\hat{\gamma}^{\,\mathrm{baseline}} - \hat{\gamma}^{\,\mathrm{all}}|$$

Baseline combination

Including the poorly resolved minima

Systematic uncertainties

Influence of $SU(3)_f$ breaking

The baseline fit **does not** take into account $SU(3)_f$ breaking.

$\Rightarrow$ Extract γ again with the 5 modes, letting $\alpha_{SU(3)}$ float in the fit.

Minimum	$\hat{\gamma}$	σ_L	σ_R
γ_1	$11.9°$	$5.8°$	$9.1°$
γ_2	$39.2°$	$6.3°$	$6.7°$
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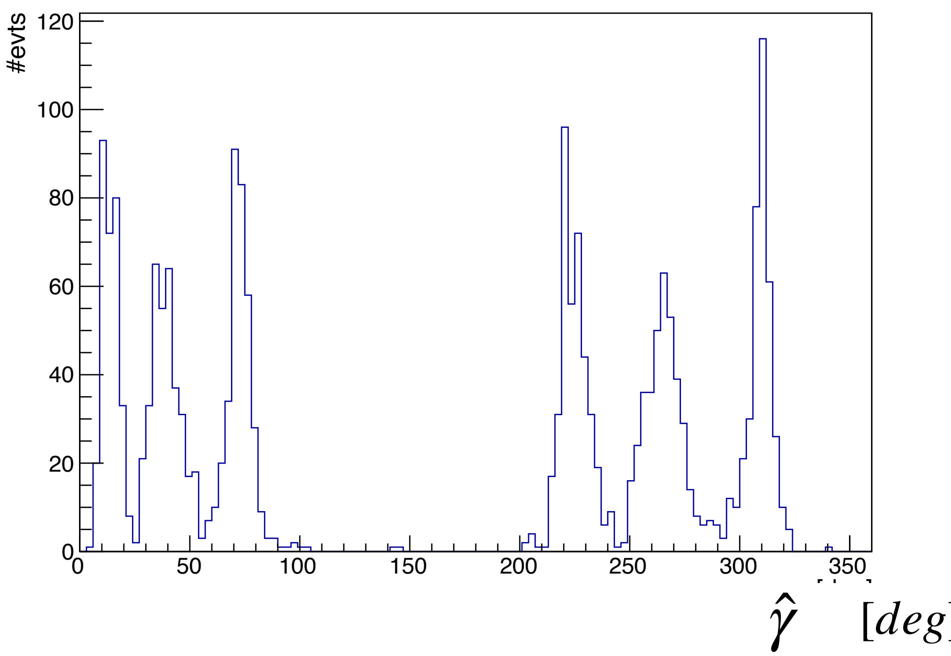

results compatible with the baseline model

$$\text{Syst} = |\hat{\gamma}^{\text{baseline}} - \hat{\gamma}^{\text{5modes}}|$$

Summary of systematic uncertainties

Minimum	Poorly resolved minima	Flavour SU(3) breaking
γ_1	0.8°	1.0°
γ_2	0.3°	2.6°
γ_3	0.2°	2.4°
γ_4	0.7°	0.7°
γ_5	1.4°	1.3°
γ_6	0.7°	0.9°

$\Rightarrow$**"Statistical" error dominates**

Summary and results

Using BABAR results we found **6 possible values for** γ:

$$\gamma_1 = [\;12.9^{+8.4}_{-4.3}\;(\text{stat}) \pm 1.3\;(\text{syst})]^\circ$$
$$\gamma_2 = [\;36.6^{+6.6}_{-6.1}\;(\text{stat}) \pm 2.6\;(\text{syst})]^\circ$$
$$\gamma_3 = [\;68.9^{+8.6}_{-8.6}\;(\text{stat}) \pm 2.4\;(\text{syst})]^\circ$$
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$$\gamma_5 = [266.4^{+9.2}_{-10.8}\;(\text{stat}) \pm 1.9\;(\text{syst}]^\circ$$
$$\gamma_6 = [307.5^{+6.9}_{-8.1}\;(\text{stat}) \pm 1.1\;(\text{syst})]^\circ$$

- Uncertainty < 11°
  (BABAR results only!)
- Statistical uncertainty dominates
- Solutions well separated

$$\gamma_{\text{WorldAverage}} = \left(73.5^{+4.2}_{-5.1}\right)^\circ$$

It is **possible to extract γ with an acceptable uncertainty** with this method.

PhysRevD.99.114011

Perspectives

- **Add information from other symmetry states**
  - totally anti-symmetric states
  - mixed states

  } **May help to decrease the statistical uncertainties and reduce the number of solutions.**

- The amplitude analyses of these modes can also be done at LCHb or Belle II with **more data**.

- Interesting longer term possibility: **dedicated analysis** in a single experiment (LHCb, BELLE II...) or even a joint analysis(?)

Backup

Impact of $SU(3)_f$ breaking

First test

$$\mathscr{A}_{\rm fs}(B^0 \to K^+K^0K^-) = \alpha_{\rm SU(3)}\mathscr{A}_{\rm fs}(B^+ \to K^+\pi^+\pi^-)$$

$\alpha_{SU(3)} = 1$ if $SU(3)_f$ is conserved

We define the ratio $R(m_{13}^2, m_{23}^2)$

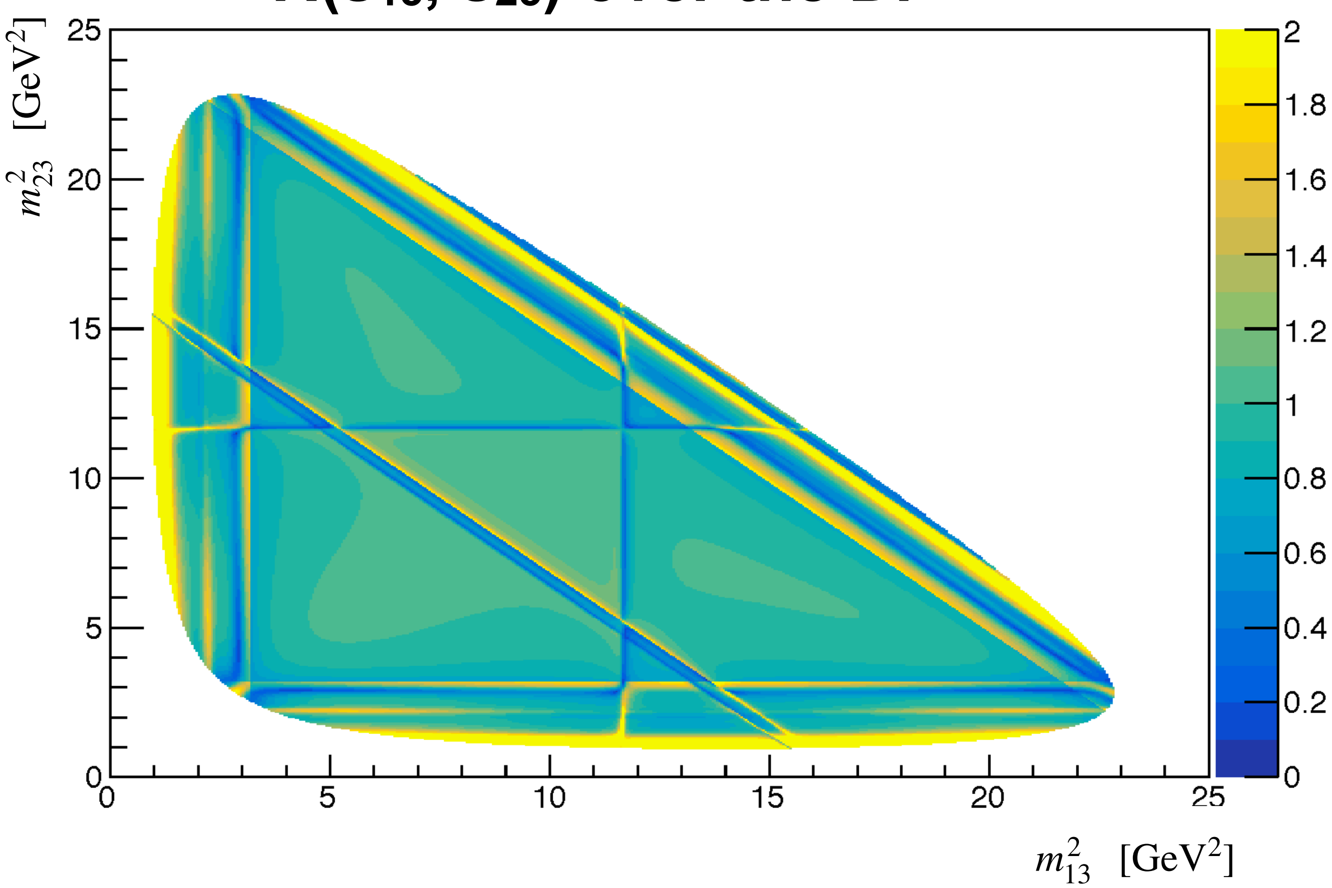
$$R(m_{13}^2, m_{23}^2) = \left| \frac{\mathscr{A}_{\rm fs}(B^+ \to K^+\pi^+\pi^-) + \mathscr{A}_{\rm fs}(B^- \to K^-\pi^-\pi^+)}{\mathscr{A}_{\rm fs}(B^0 \to K^+K_S^0K^-) + \mathscr{A}_{\rm fs}(\overline{B}^0 \to K^+K_S^0K^-)} \right|$$

$R(m_{13}^2, m_{23}^2) \in \mathbb{R}$

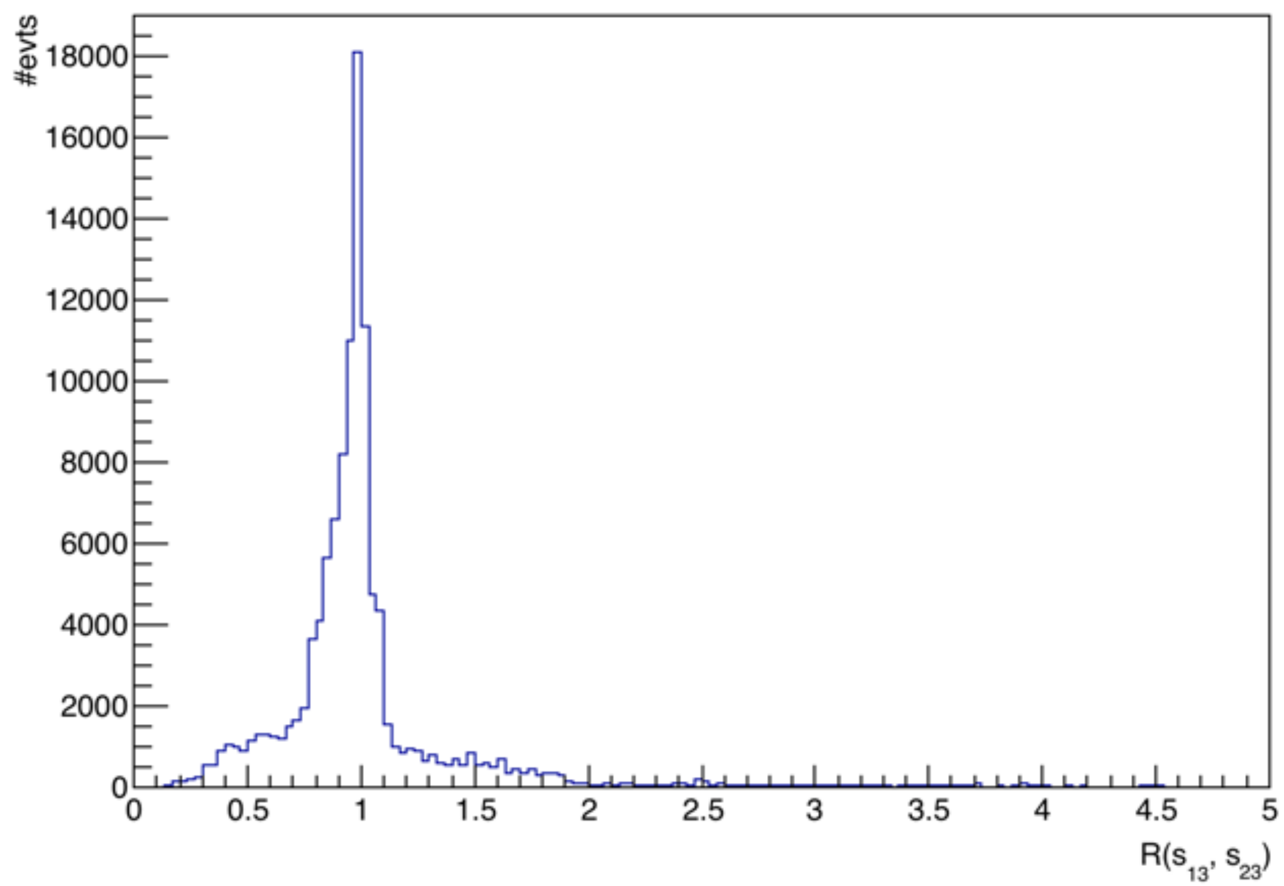
The impact of $SU(3)_f$ breaking

First test

R(s$_{13}$, s$_{23}$) over the DP



Histogram of the values of R(s$_{13}$, s$_{23}$)



$R(m^2_{13}, m^2_{23})$ varies over the DP, especially near resonances.

$$\langle R(m^2_{13}, m^2_{23})\rangle = 1.03 \approx 1$$

Flavour SU(3) symmetry is conserved when averaging over many points in the DP.

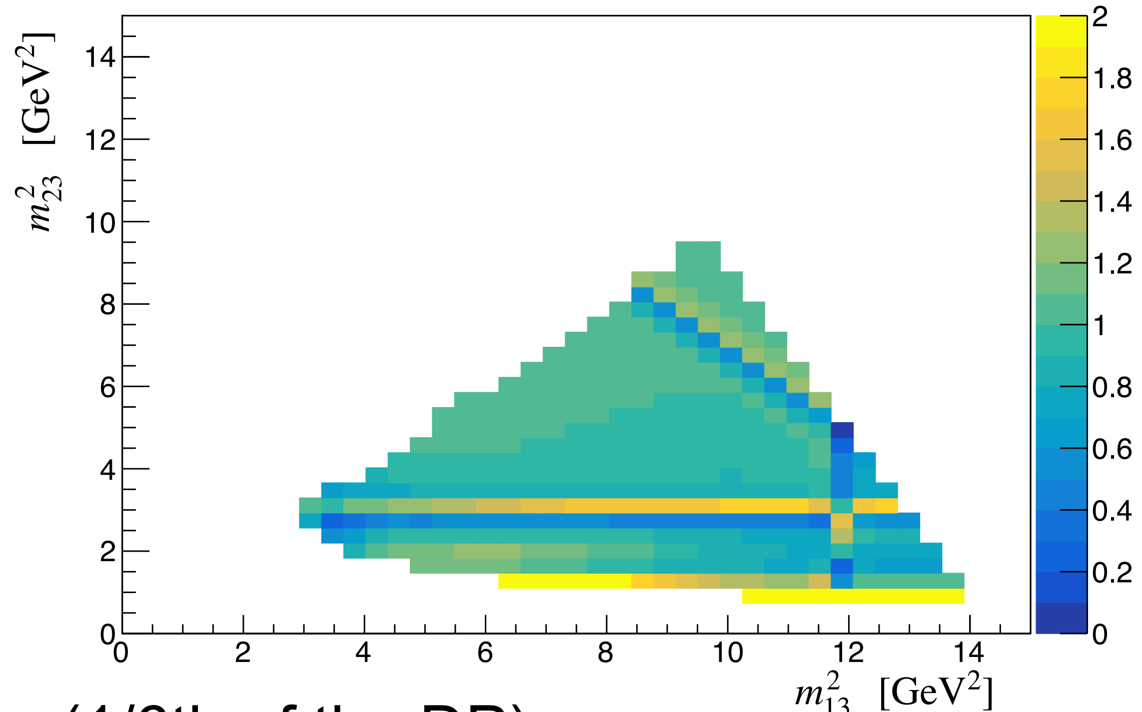
The impact of $SU(3)_f$ breaking

Second test

Extract $\alpha_{SU(3)}$ from fits at several single points (≈ 400) over the DP fixing γ to the values of the 6 minima found previously.

x 6

(very similar results for all minima)



(1/6th of the DP)

γ_i	$\langle \alpha_{SU(3)} \rangle_i$
12°	1.06
37°	1.06
68°	1.05
223°	1.06
266°	1.05
307°	1.05

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$$B \to PPP$$

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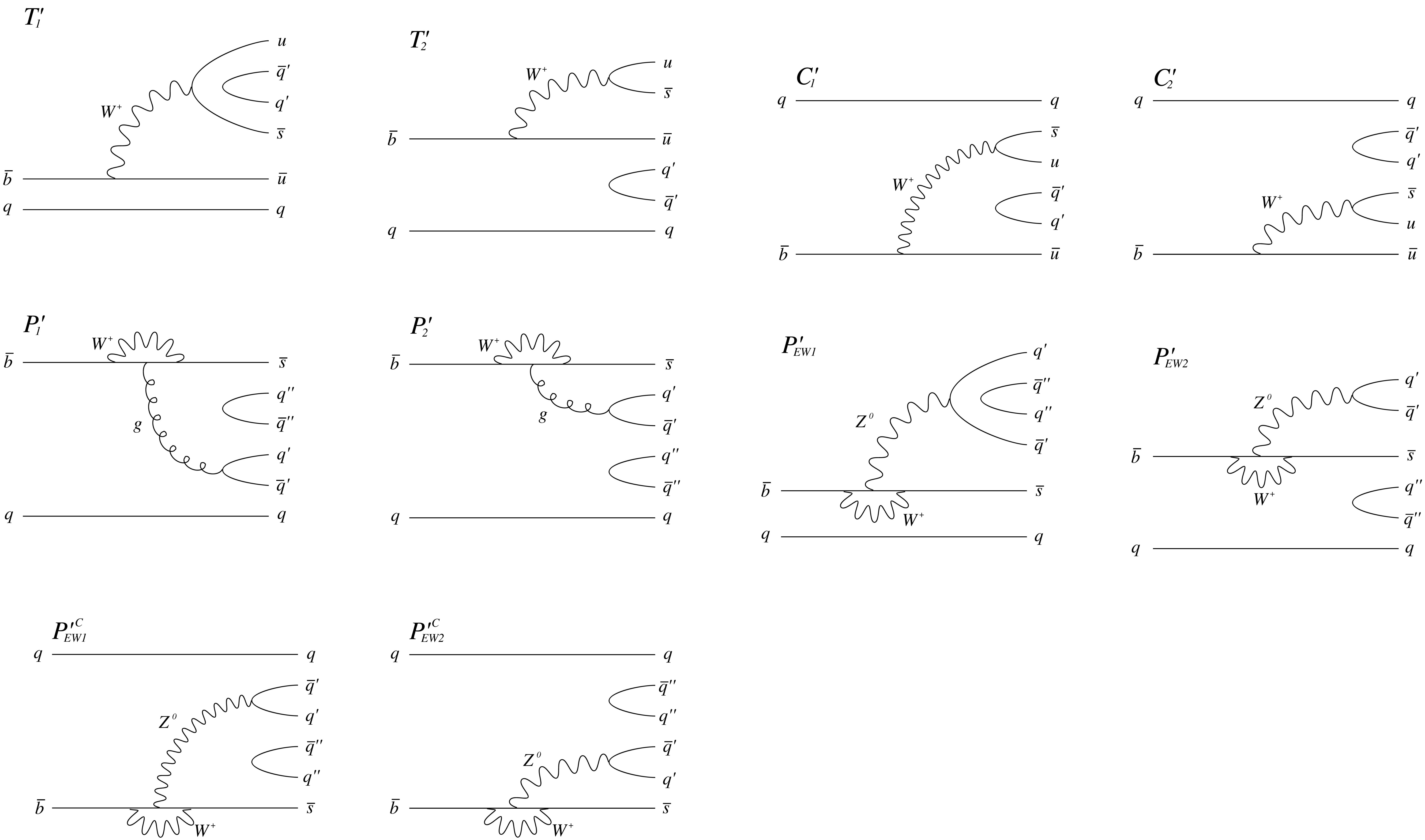
Model independent (no hadronic input needed) and data driven

Diagrams: $B \to K\pi\pi$



Sources of $SU(3)_f$ breaking

$$P'_{\mathrm{EW},1,2} = \kappa\, T'_{1,2}$$

$$\kappa \equiv -\frac{3}{2}\frac{|\lambda_t^{(s)}|}{|\lambda_u^{(s)}|}\frac{c_9+c_{10}}{c_1+c_2} \quad \begin{cases} \lambda_p^{(s)} = V_{pb}^* V_{ps} \\ c_i \text{ :Wilson coefficients} \end{cases}$$

assumptions:

- $SU(3)_f$ symmetry holds
- $\frac{c_1}{c_2} = \frac{c_9}{c_{10}}$ (holds to ≈ 5%)

$\bar{b} \to \bar{s}$: dominant contribution is P'_{tc}
(EW penguin and tree diagrams are suppressed)
⇒ error relative to $SU(3)_f$ breaking subdominant

$B \to K\pi\pi$ and $B \to K\bar{K}K$

$B \to K\bar{K}K$: $s\bar{s}$ pair in the final state
$B \to K\pi\pi$: $u\bar{u}/d\bar{d}$ pair in the final state

$m_{u,d} \neq m_s$

⇒ $SU(3)_f$-breaking effect is considered to be the same for each diagrams ($\alpha_{SU(3)}$)

π's and K's

π's and K's assumed to the identical particles while it's not the case

⇒ also included in $\alpha_{SU(3)}$

Observables as functions of the theoretical parameters

$$X^{th}_{K^+\pi^+\pi^-}(s_1, s_2) = a^2 + (\kappa b)^2 + c^2 + 2ac\cos\phi_c\cos\gamma - 2\kappa ab\cos\phi_b - 2\kappa bc\cos(\phi_b - \phi_c)\cos\gamma$$

$$Y^{th}_{K^+\pi^+\pi^-}(s_1, s_2) = -2\left(ac\sin\phi_c + \kappa bc\sin(\phi_b - \phi_c)\right)\sin\gamma \ ,$$

$$X^{th}_{K^0_S K^+K^-}(s_1, s_2) = (\alpha_{\mathrm{SU}(3)})^2 X^{th}_{K^+\pi^+\pi^-} \ ,$$

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$$X^{th}_{K^0_S\pi^+\pi^-}(s_1, s_2) = a^2 + (\kappa d)^2 + d^2 + 2ad\cos\phi_d\cos\gamma - 2\kappa ad\cos\phi_d - 2\kappa d^2\cos\gamma \ ,$$

$$Y^{th}_{K^0_S\pi^+\pi^-}(s_1, s_2) = -2ad\sin\phi_d\sin\gamma \ ,$$

$$Z^{th}_{K^0_S\pi^+\pi^-}(s_1, s_2) = \left(-d^2\cos\gamma - ad\cos\phi_d + \kappa d^2\right)\sin\gamma \ ,$$

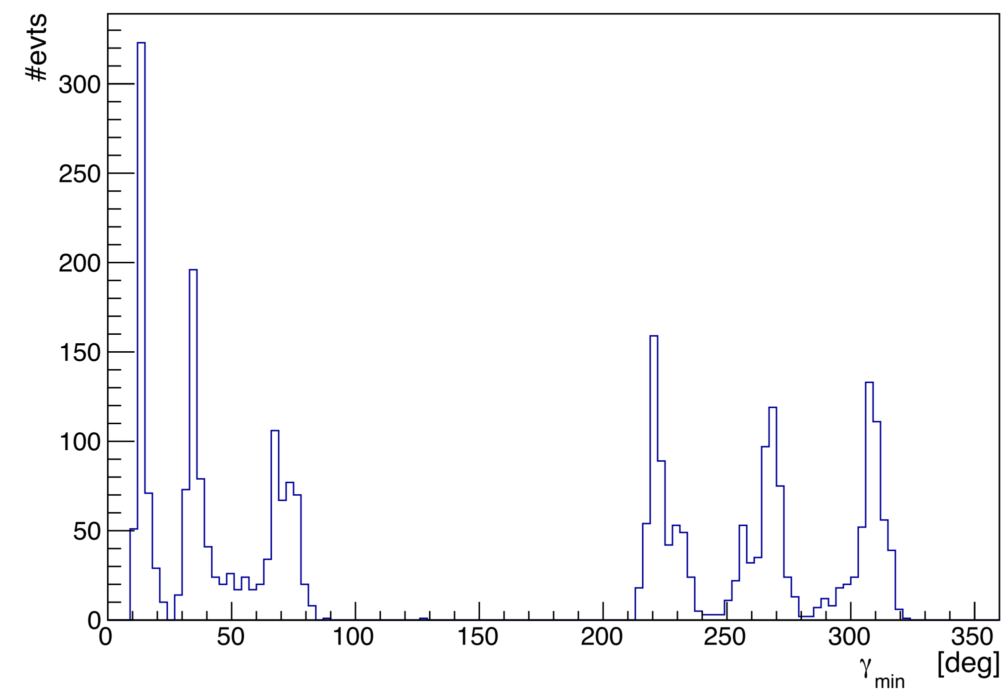
$$X^{th}_{K^+\pi^+\pi^0}(s_1, s_2) = \frac{1}{2}\left(b^2 + \kappa^2 c^2 - 2\kappa bc\cos\gamma\cos(\phi_b - \phi_c)\right) \ ,$$

$$Y^{th}_{K^+\pi^+\pi^0}(s_1, s_2) = \kappa bc\sin\gamma\sin(\phi_b - \phi_c) \ ,$$

$$X^{th}_{K^0_S K^0_S K^0_S}(s_1, s_2) = 2(\alpha_{\mathrm{SU}(3)})^2 a^2 \ .$$

Baseline results

- $\alpha_{\mathrm{SU(3)}} = 1$
- 501 sets of random 3-points combinations.
- 500 fits randomising the initial values of the parameters per set.
- Fit convergence = 100%.



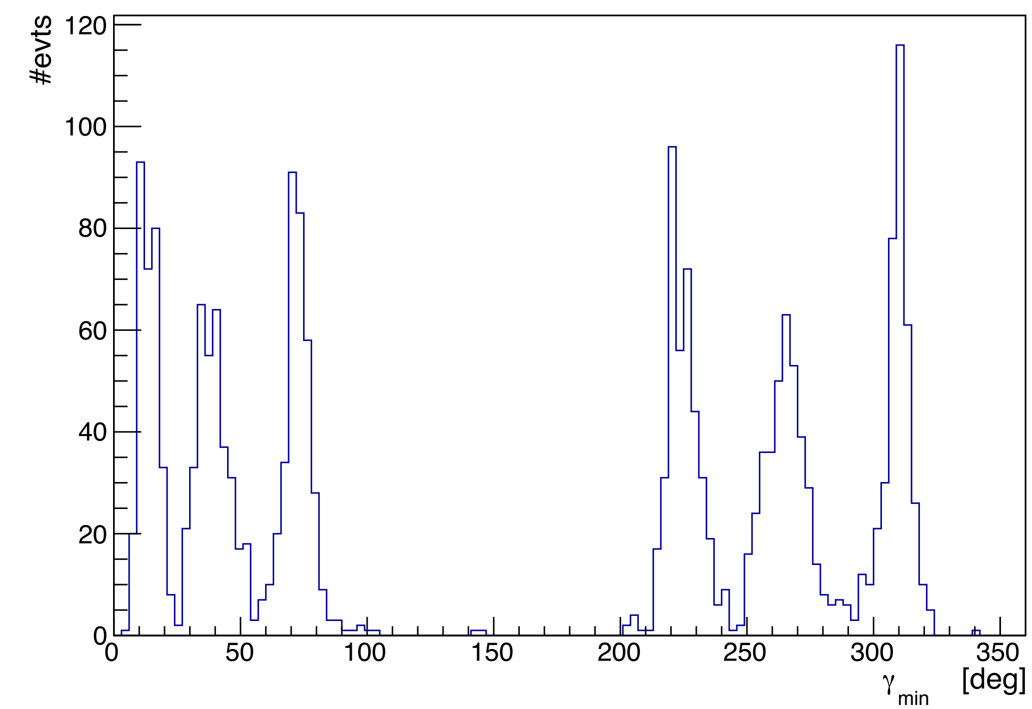
	μ	σ_L	σ_R	χ^2
Minimum 1	12.9°	4.3°	8.4°	3.61
Minimum 2	36.6°	6.1°	6.6°	1.99
Minimum 3	68.9°	8.6°	8.6°	2.07
Minimum 4	223.2°	7.5°	10.9°	2.15
Minimum 5	266.4°	10.8°	9.2°	1.40
Minimum 6	307.5°	8.1°	6.9°	1.74

	Count	Fraction (%)
Minimum 1	484	96.6
Minimum 2	474	94.6
Minimum 3	461	92.0
Minimum 4	499	99.6
Minimum 5	487	97.2
Minimum 6	488	97.4

rates at which the different minima are obtained

Extraction of γ with 5 modes

- $\alpha_{\mathrm{SU(3)}}$ free to vary in the fit
- 401 sets of random 3-points combinations.
- 500 fits randomising the initial values of the parameters per set.
- Fit convergence ≳ 80%.



	μ	σ_L	σ_R	χ^2	$\lvert\mu - \mu^{all}\rvert$	$\lvert\mu^{4\,\mathrm{modes}} - \mu^{5\,\mathrm{modes}}\rvert$
Minimum 1	11.9°	5.8°	9.1°	3.53	1.3	1.0
Minimum 2	39.2°	6.3°	6.7°	2.50	1.2	2.6
Minimum 3	71.3°	9.5°	9.3°	2.58	0.4	2.4
Minimum 4	223.9°	7.4°	9.5°	2.92	0.1	0.7
Minimum 5	265.0°	11.0°	10.0°	2.19	1.2	1.3
Minimum 6	308.4°	8.8°	7.0°	2.49	0.6	0.9

	Count	Fraction (%)
minimum 1	306	76.3
minimum 2	329	82.0
minimum 3	372	92.3
minimum 4	383	95.5
minimum 5	378	94.3
minimum 6	391	97.5

rates at which the different minima are obtained

Amplitude symmetrisations

3 particles in the final state (1,2,3) ⇒ 6 symmetrisations

$$|S\rangle \propto (|123\rangle + |132\rangle + |312\rangle + |321\rangle + |231\rangle + |213\rangle),$$

Fully symmetric

$$|A\rangle \propto (|123\rangle - |132\rangle + |312\rangle - |321\rangle + |231\rangle - |213\rangle),$$

Fully antisymmetric

$$\left.\begin{aligned} |M_1\rangle &\propto (2|123\rangle + 2|132\rangle - |312\rangle - |321\rangle - |231\rangle - |213\rangle, \\ |M_2\rangle &\propto (|312\rangle - |321\rangle - |231\rangle + |213\rangle), \\ |M_3\rangle &\propto (-|312\rangle - |321\rangle + |231\rangle + |213\rangle), \\ |M_4\rangle &\propto (2|123\rangle - 2|132\rangle - |312\rangle + |321\rangle - |231\rangle + |213\rangle). \end{aligned}\right\}$$

Mixed

2 identical particles in the final state ⇒ $|A\rangle = 0$

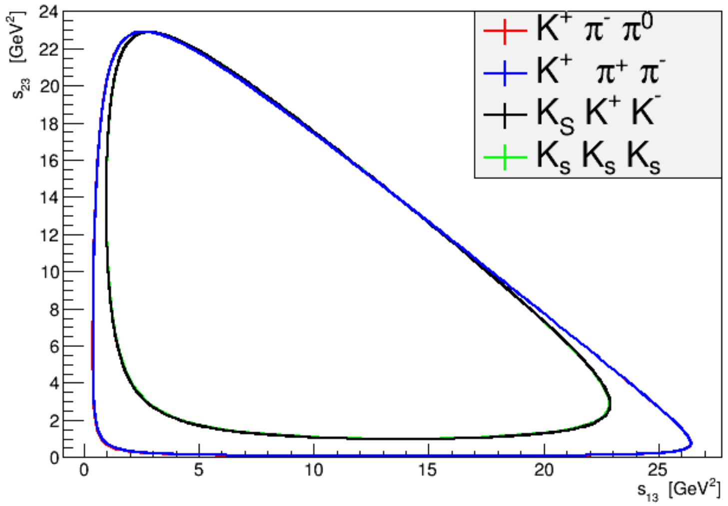
3 identical particles in the final state ⇒ $|A\rangle = 0$ and $|M_i\rangle = 0$

Each final-state symmetry has its own set of diagrams ⇒ $T_1^{\mathrm{FS}} \neq T_1^{\mathrm{AS}}$

Fully symmetric: spin 1 resonances disappear

Fully antisymmetric: S = 0, S = 2 resonances disappear

DP mappings



Goal: find a mapping so that $K\pi\pi$ planes are used completely

Idea: use a mapping into a square

$\Rightarrow$ use the cosines of helicity angles c_{ij}

$\Rightarrow$ **need to define fully-symmetric states in terms of c_{ij}**

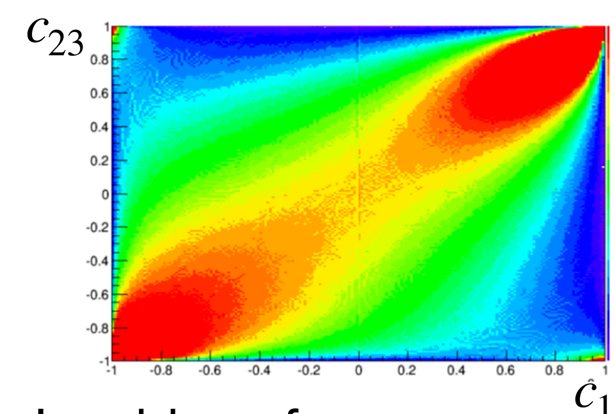
relations between m_{ij}^2 and c_{ij} non linear

$$c_{12} = \frac{M^2(m_1^2 - m_2^2 - s_{12}) + (m_2^2 - s_{12})(m_3^2 - s_{12}) - m_1^2(m_3^2 + s_{12}) + 2s_{12}s_{23}}{s_{12}\sqrt{\frac{m_1^4 + (m_2^2 - s_{12})^2 - 2m_1^2(m_2^2 + s_{12})}{s_{12}}}\sqrt{\frac{M^4 + (m_3^2 - s_{12})^2 - 2M^2(m_3^2 + s_{12})}{s_{12}}}}$$

c_{12}, c_{13}, c_{23} not related by a relation like $m_{12}^2 + m_{13}^2 + m_{23}^2 = M^2 + \sum_{i=0}^{3} m_i^2$

$c_{ij} = -c_{ji}$

m_{ij}^2 from c_{ij} not analytical

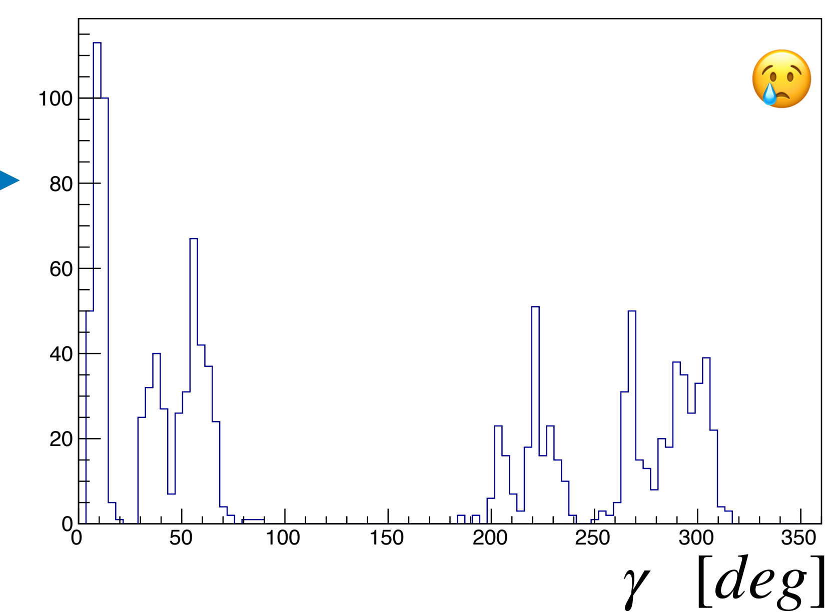


Jacobian of $(s_{13}, s_{23}) \rightarrow (c_{12}, c_{23})$
$(B^0 \rightarrow K_S^0 K_S^0 K_S^0)$

How? use $K_S^0 K_S^0 K_S^0$: the symmetrisation with the mass must be exactly identical to the symmetrisation with the c_{ij}: **different possibilities**

"easiest" one

$$A_{fs} \propto A(c_{21}, c_{23}, c_{31}) + A(c_{23}, c_{21}, c_{13}) + A(c_{12}, c_{31}, c_{23})$$
$$+ A(c_{32}, c_{13}, c_{21}) + A(c_{31}, c_{12}, c_{32}) + A(c_{13}, c_{32}, c_{12})$$



actually harder as it looks...

To a point (c_{13}, c_{23}) is associated a different point (m_{13}^2, m_{23}^2) per mode (since the masses of the particles differ)...

Thoughts on a dedicated analysis

- Enough of data to perform time-dependent DP analyses
- Observables for each mode

} ⇒ measure γ with a simultaneous fit using several charmless modes as inputs

- Measure γ directly from the data?
  ⇒ need a relation between the parameters of the amplitude model and the theoretical parameters
- Isobar model?
  ⇒ not required by the method
  ⇒ could use alternative parametrisations, eg. QMI?

These are only ideas ⇒ phenomenological work needed before being able to measure γ this way