# Relic neutrino detection through angular correlations in inverse $\beta$ -decay

Evgeny Akhmedov

Max-Planck Institute für Kernphysik, Heidelberg

(arXiv:1905.10207, JCAP 1909 (2019) 031)



Evgeny Akhmedov

#### **Relic neutrinos**

Produced in the hot and dense early Universe and decoupled from cosmic plasma at  $t \sim 1$  sec after the BB ( $T_{\nu} \sim 2$  MeV). Cooled down in the course of Universe expansion. At present have nearly Fermi-Dirac spectrum with

 $T_{\nu 0} \simeq 1.945 \,\mathrm{K} \simeq 1.676 \times 10^{-4} \,\mathrm{eV}$ 

and are in mass eigenstates (flavour eigenstates have lost their coherence). Average number density per mass eigenstate and per spin degree of freedom:

$$n_{\nu 0} = \frac{3\zeta(3)}{4\pi^2} T_{\nu 0}^3 \simeq 56 \text{ cm}^{-3}.$$

Average momentum:

$$\langle p_{\nu 0} \rangle \simeq 3.151 T_{\nu 0} \simeq 5.314 \times 10^{-4} \,\mathrm{eV}.$$

In the standard  $3\nu$  picture: at least 2 out of 3 relic neutrino species are non-relativistic now.

Evgeny Akhmedov

## **Relic neutrinos**

Should carry very important information about the Universe and its evolution!

- Are relic neutrino still there (are they stable enough against decay and annihilation)?
- What is their composition? Are there sterile components?
- What are their energy distributions? Are there contributions from decays of heavy relics? Are there non-thermal components?
- What are velocity and spin distributions of relic vs? Are there CvB anisotropies (like in CMB)? Gravitational clustering?

A wealth of information (probably much more than in CMB!) May also shed light on Dirac vs. Majorana neutrino nature.

<u>But:</u> Because of very low energies and very weak interactions, extremely difficult to detect A number of approaches suggested so far.

All but one are either based on flawed considerations or fall short of the required sensitivity by many orders of magnitude.

The only currently known approach that may work: *Neutrino capture on*  $\beta$ *-decaying nuclei*.

# **Relic neutrino capture on** $\beta$ **-decaying nuclei**

 $\beta$ -decay:

$$\diamond A(Z,N) \rightarrow A(Z+1,N-1) + e^- + \bar{\nu}_e$$

Neutrino capture on  $\beta$ -decaying nuclei (inverse  $\beta$ -decay):

$$\diamond \quad \nu_e + A(Z,N) \rightarrow A(Z+1,N-1) + e^-$$

Exothermic reaction (no energy threshold). Neutrinos of  $q_{\nu} = 0$  can be captured with finite rate:

$$\lim_{v_{\nu}\to 0} v_{\nu}\sigma_{\nu} = const. \neq 0.$$

Can be used for detection of relic neutrinos! (Weinberg, 1962).

# **Relic neutrino capture on** $\beta$ **-decaying nuclei**

Main problem: electrons from the accompanying usual  $\beta$ -decay are *much* more abundant. For  $\beta$  decays with  $E_0 - \delta \leq E_e \leq E_0$  ( $\delta \ll E_0$ ):

$$\frac{\Gamma_c}{\Gamma_d} \simeq 6\pi^2 \frac{n_\nu}{\delta^3} \simeq \frac{n_\nu}{56 \text{ cm}^{-3}} \frac{2.54 \times 10^{-11}}{[\delta \text{ (eV)}]^3} \,,$$

independently of the  $Q_{\beta}$ -value ( $Q_{\beta} = E_0 - m_e$ ).

Weinberg's (1962) assumptions: massless neutrinos;  $C\nu B$  is degenerate with large chemical potential  $\mu_{\nu}$ :  $\xi_{\nu} = \mu_{\nu}/T_{\nu} \gtrsim 10^5$ .

⇒ Energies of electrons from relic  $\nu$  capture exceed  $E_0$  by up to  $\mu_{\nu}$  ( $\gtrsim 10 \text{ eV}$ ), can be distinguished for moderate energy resolution.

<u>But:</u> Current constraints  $|\xi_{\nu}| \lesssim 0.07$  rule this out.

# **Relic neutrino capture on** $\beta$ **-decaying nuclei**

Taking into account  $m_{\nu} \neq 0$ : a gap  $\sim 2m_{\nu}$  between the spectra of electrons from  $\nu$  capture and  $\beta$ -decay

(Cocco, Mangano & Messina, 2007, 2009).



(From Y.-F. Li, 1504.03966)

The only currently known potentially feasible approach!

 $\diamond$  Main problem: Requires an extremely high energy resolution  $\Delta \lesssim m_{\nu}$ .

# **PTOLEMY:** $\nu + {}^{3}\mathbf{H} \rightarrow {}^{3}\mathbf{He} + e^{-}$



(From Betti et al., 1902.05508)

PTOLEMY: 100 g of <sup>3</sup>H;  $\mathcal{O}(10)$  events/yr. Aim at  $\Delta = 0.05$  eV.

<u>But</u>: For normal mass ordering and  $m_1 \leq 0.01 \text{ eV}$  even  $\sim 10$  times smaller energy resolution may be necessary! (N.B: the weight of  $\nu_e$  in  $\nu_3$  is  $|U_{e3}|^2 \simeq 2 \times 10^{-2}$ ).

The idea: Use time dependence of angular correlations to distinguish  $\nu$  capture from  $\beta$ -decay (EA, arXiv:1905.10207)

 $\beta$ -processes exhibit a number of angular correlations. An example: correlation between the spin of the parent nucleus  $\vec{s}_N$  and electron direction  $\vec{v}_e$  in expts. with polarized targets ( $\not{P}$ ).

Similarly, angular correlation between  $\vec{s}_N$  and  $\vec{v}_{\nu}$ .

How can this be used?

# **Time-varying angular correlations**

Solar system moves w.r.t.  $C\nu B$  rest frame with velocity  $-\vec{u}$  ( $u \sim 10^{-3}$ )  $\Rightarrow$  neutrino wind: relic  $\nu$ s have a preferred arrival direction  $\vec{u}$ .

Angular correlation:

 $\Gamma_c \propto \left(1 + \beta \vec{u} \cdot \vec{s}_N\right).$ 

For a polarized nuclear target with fixed direction of polarization  $\vec{s}_N$  in the lab frame:

Because of the Earth's rotation, the angle between  $\vec{u}$  and  $\vec{s}_N$  will change during the day. Periodic variations with period  $T_0 \simeq 23h \ 56m \ 4s$  (sidereal day).

 $\Rightarrow$  Time dependence of the total relic neutrino capture rate.

 $C\nu B$  caprture on polarized tritium (including time variations) considerd by Lisanti, Safdi & Tully (2014) within the approach based on the separation of spectra.

#### **Earth rotation effect**



 $\vec{u} \cdot \vec{s}_N = u[\cos \theta_u \cos \theta_N + \sin \theta_u \sin \theta_N \cos(\phi_u(t) - \phi_N)]$  $\phi_u(t) = (2\pi/T_0)t + \phi_0 \qquad (T_0 \simeq 24 \ h).$ 

Polarization of large targets – a difficult task. Can one use angular correlations in expts. with unpolarized targets?

- Measure polarization of daughter nuclei with  $J_f \neq 0$  (e.g. measuring circular polarization of de-excitation  $\gamma$ -rays for transitions to  $A_f^*$ , as in the Goldhaber, Grodzins and Sunyar experiment).
- Make use of β-ν angular correlations ⇒ Time dependent forward-backward asymmetry of electrons w.r.t. a fixed direction in the lab frame in expts. with unpolarized targets.
   Can work even for pure Fermi 0<sup>π</sup> → 0<sup>π</sup> transitions!

(But  $\beta$ - $\nu$  correlation may be strongly suppressed for mixed Fermi–Gamow-Teller  $\beta$ -transitions, such as the  $1/2^+ \rightarrow 1/2^+$  transition  $\nu_e + {}^3\text{H} \rightarrow {}^3\text{He} + e^-$ ).

# $\mathbf{C}\nu\mathbf{B}$ detection in inverse $\beta$ -decay

Neutrino detection in the inverse  $\beta^-$  decay process

$$\nu_j(q) + A_i(p) \to A_f(p') + e^-(k)$$
,

 $C\nu B$  neutrinos: Produced as chiral states; now are in helicity states (in  $C\nu B$  rest frame)

Produced:		Now:
left-chiral	$\longrightarrow$	left-helical
right-chiral	$\longrightarrow$	right-helical

Dirac neutrinos: right-helical states are  $\bar{\nu}$ . Cannot be captured in  $\beta^-$ -processes, but can be detected through inverse  $\beta^+$ -decay.

Majorana neutrinos: for non-rel.  $\nu$ s both left-helical and right-helical states can participate in inverse  $\beta^-$ -processes through their left-chirality components  $\Rightarrow$  $\Gamma_c^{\text{Maj}} \simeq 2\Gamma_c^{\text{Dir}}$  (Long, Lunardini & Sabancilar, 2014).

# $\mathbf{C}\nu\mathbf{B}$ detection in inverse $\beta$ -decay

Helicity is not Lorentz-invariant  $\Rightarrow$  relic neutrinos that are in helicity eigenstates in C $\nu$ B rest frame may not have definite helicity in the lab frame.  $\Rightarrow$  Consider  $\nu$  capture process for arbitrary direction of  $\nu$  spin.

For pure Gamow-Teller transition  $1^{\pi} \rightarrow 0^{\pi}$  (e.g.  ${}^{64}\text{Co} \rightarrow {}^{64}\text{Ni}$ ,  ${}^{80}\text{Br} \rightarrow {}^{80}\text{Kr}$ )

$$\diamondsuit \ v_j d\sigma_j = \frac{G_\beta^2}{2} |U_{ej}|^2 \frac{1}{(2\pi)^2} \frac{(\varepsilon_\mu \varepsilon_\nu^* X^{\mu\nu})}{4E_e E_j} |M_{\rm GT}|^2 F(Z, E_e) E_e \sqrt{E_e^2 - m_e^2} \, d\Omega_e \,.$$

Energy conservation:

$$E_e = E_0 + E_j \,,$$

 $E_0$ : is total energy release in the corresponding  $\beta^-$ -decay;  $E_0 = Q_\beta + m_e$ .  $\varepsilon_\mu$ : polarization 4-vector of the parent nucleus.

$$X^{\mu\nu} = [\bar{u}_e(k)\gamma^{\mu}(1-\gamma_5)u_j(q)][\bar{u}_j(q)\gamma^{\nu}(1-\gamma_5)u_e(q)].$$

# **Electron asymmetry (unpolarized target)**

 $\beta$ – $\nu$  correlation:

$$\frac{d\sigma}{d\Omega_e} = const.(1 + \alpha \vec{u} \cdot \vec{v_e}),$$

Leads to time dependent forward–backward asymmetry of electron emission with respect to a fixed direction  $\vec{\xi}$  in the lab frame. Time dependence: maximized when  $\vec{\xi} \perp$  to the Earth's rotation axis.

In the geocentric spherical coordinates:

$$\vec{u} = u \big( \cos \phi_u(t) \sin \theta_u \,, \sin \phi_u(t) \sin \theta_u \,, \cos \theta_u \big) \,,$$
$$\vec{v}_e = v_e \big( \cos \phi_e \sin \theta_e \,, \sin \phi_e \sin \theta_e \,, \cos \theta_e \big) \,,$$
$$\vec{\xi} = \big( \cos \phi_\xi \,, \sin \phi_\xi \,, 0 \big) \,,$$

where

$$\phi_u(t) = \frac{2\pi}{T_0}t + \phi_0 \,,$$

 $T_0 \simeq 24 \,\mathrm{h}$  is the sidereal day.

$$\vec{u} \cdot \vec{v}_e = uv_e \left\{ \cos \theta_u \cos \theta_e + \sin \theta_u \sin \theta_e \cos[\phi_u(t) - \phi_e] \right\}.$$

Forward-backward asymmetry of the electron emission with respect to  $\vec{\xi}$ :

$$\diamondsuit \quad \mathcal{A} \equiv \frac{\sigma_{\uparrow} - \sigma_{\downarrow}}{\frac{1}{2}[\sigma_{\uparrow} + \sigma_{\downarrow}]} = \alpha u v_e \sin \theta_u \cos[\phi_u(t) - \phi_{\xi}].$$

Electron spin asymmetry with respect to a fixed direction in the lab frame can be considered similarly.

# **Time dependence of the signal**

For polarized targets – time variation of the total signal rate:

 $\Gamma_c \propto 1 + \beta \vec{u} \cdot \vec{s}_N$  .

For unpolarized targets: time-varying angular correlation

$$\frac{d\Gamma_c}{d\Omega_e} \propto 1 + \alpha \vec{u} \cdot \vec{v}_e \; .$$

(and similarly  $\vec{u} \cdot \vec{s}_e$  correlation).  $\Rightarrow$  Forward-backward anisotropy w.r.t. a fixed direction in lab frame.

Coeffs.  $\alpha$ ,  $\beta$ : depend on  $v_j$  and on neutrino nature (Dir./Maj.); typically O(1). But:  $u = O(10^{-3})$ .

The problem: how to separate a small periodic signal from large fluctuating backgrounds. Very challenging!

# **Signal from noise separation**

Main background: electrons from competing  $\beta$  decay of target nuclei.

For extraction of periodic signal, main problem comes from large statistical fluctuations of the background (due to quantum nature of the underlying processes) and <u>not</u> directly from the high average background level.

The problem of separation of signals from noisy backgrounds: Routinely encountered in astronomy, acoustics, radiodetection, engineering, medical applications such as electrocardiography, magnetic resonance imaging, etc. – well studied. Enormous literature exists!

S-from-N separation: Especially efficient when signal has a known periodicity.

This work: A simple approach to signal-from-noise separation based on the Fourier analysis and filtering in the frequency domain.

# **Signal from noise separation**

Consider

$$f(t) = s(t) + n(t)$$

s(t) – periodic signal of zero average, n(t) – "noise" due to statistical fluctuations of the background (constant average background subtracted). By definition  $\overline{s(t)} = \overline{n(t)} = 0 \implies$  signal and noise strenghts characterized by their variances  $\overline{s(t)^2}$  and  $\overline{n^2(t)}$ . Take signal in the form

$$s(t) = A_0 \sin(\omega_0 t + \varphi)$$

with period  $T_0 = 2\pi/\omega_0$  and constant  $A_0$  and  $\varphi$ . Signal autocorrelation function:

$$R_{ss}(\tau) = \overline{s(t)s(t+\tau)} \equiv \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} s(t)s(t+\tau)dt \,,$$

and similarly for noise  $(T \rightarrow \infty \text{ limit})$ .

N.B.: the signal and noise variances satisfy  $\overline{s(t)^2} = R_{ss}(0)$ ,  $\overline{n^2(t)} = R_{nn}(0)$ .

#### **Power spectra**

The signal and noise power spectra  $S(\omega)$  and  $N(\omega)$ :

$$S(\omega) = \int_{-\infty}^{\infty} d\tau \, e^{-i\omega\tau} R_{ss}(\tau) \,, \qquad N(\omega) = \int_{-\infty}^{\infty} d\tau \, e^{-i\omega\tau} R_{nn}(\tau) \,.$$

For sinusoidal signal:

$$R_{ss}(\tau) = \frac{A_0^2}{2} \cos \omega_0 \tau \,.$$

Signal power spectrum:

$$\diamondsuit \quad S(\omega) = \frac{\pi}{2} A_0^2 \left\{ \delta(\omega - \omega_0) + \delta(\omega + \omega_0) \right\}.$$

Noise n(t) is due to statistical fluctuations of background  $\Rightarrow$  white noise:

$$\diamondsuit \quad N(\omega) = N_0 = const.$$

(flat power spectrum). The autocorrelation function :

$$R_{nn}(\tau) = N_0 \delta(\tau).$$

 $N_0 = (\text{number of bckgr. events})/(\text{unit time}).$ 

$$R_{ss}(\tau) = \frac{A_0^2}{2} \cos \omega_0 \tau \,.$$

 $R_{nn}(\tau) = N_0 \delta(\tau)$ 

In this approach: formally possible to separate arbitrarily weak periodic signal from noise [Consider their autocorrelation functions at any  $\tau \neq 0$  for which  $R_{ss}(\tau)$  does not vanish]. But: this would only be the case for infinite *T*.

<u>Also:</u> The total noise power obtained by integrating the flat power spectrum over the whole freq.cy interval  $(-\infty, \infty)$  is infinite. The resolution: any realistic measurement actually is characterized by a finite interval of frequencies.

Realistic case: finite total observation time T and finite measurement bandwidth. The signal power spectrum for finite T:

$$S(\omega) = \frac{A_0^2}{T} \Big\{ \frac{\sin^2 \left[ (\omega - \omega_0) \frac{T}{2} \right]}{(\omega - \omega_0)^2} + \frac{\sin^2 \left[ (\omega + \omega_0) \frac{T}{2} \right]}{(\omega + \omega_0)^2} \Big\}.$$

Peaks at  $\omega = \pm \omega_0$  of width  $\sim 4\pi/T$  (interval  $[-2\pi/T, 2\pi/T]$  around the center of each peak contains about 90% of its strength). Height:  $S(\omega_0) = S(-\omega_0) \simeq (A_0^2/4)T$ .



The bandwidth of the experiment:  $\Omega = [-\omega_m, \omega_m]$ .

If time binning (with bin length  $\Delta t$ ) is involved:  $\omega_m$  related to the Nyquist linear frequency  $f_N \equiv (2\Delta t)^{-1}$  by

$$\omega_m = 2\pi f_{\rm N} = \frac{\pi}{\Delta t} \,.$$

 $[\omega_m]$  is the highest available frequency because  $\Delta t$  is the shortest time interval spanned].

In expts. allowing real-time detection:

 $\Delta t \ \rightarrow \ ({\rm mean \ time \ interval \ between \ two \ consecutive \ events}) \,,$  That is,

$$1/\Delta t =$$
 (mean bckgr. event rate)  $= N_0$ ,

$$\diamondsuit \quad \omega_m = \pi N_0 \,.$$

$$\overline{s(t)^2} = \int_{\Omega} \frac{d\omega}{2\pi} S(\omega) , \qquad \overline{n(t)^2} = \int_{\Omega} \frac{d\omega}{2\pi} N(\omega) .$$

For the noise  $(N(\omega) = N_0 = const.)$ :

$$\overline{n(t)^2} = \int_{-\omega_m}^{\omega_m} \frac{d\omega}{2\pi} N_0 = N_0^2.$$

Without noise suppression: S-to-N ratio  $\rho_0 \equiv \overline{s(t)^2}/\overline{n(t)^2}$  is

$$\rho_0 = \frac{\int_\Omega \frac{d\omega}{2\pi} S(\omega)}{\int_\Omega \frac{d\omega}{2\pi} N(\omega)} = \frac{A_0^2/2}{N_0^2}.$$

N.B.:  $(A_0/\sqrt{2})T$  is the r.m.s. number of signal events,  $N_0T$  is the total number of bckgr. events.

- Very small quantity  $((S/B)^2)$
- Does not depend on the total running time of the expt. T
- Does not depend on the detector mass

# Filtering in frequency domain

Passband filtering: make use of the fact that the signal is concentrated in the narrow intervals in the frequency domain whereas the noise power is constant per unit bandwidth.



To improve the signal-to-noise ratio: integrate the signal and noise power spectra over the intervals  $\Delta \omega = [-2\pi/T, 2\pi/T]$  around the points  $\omega = \pm \omega_0$ .

# **Noise suppression**

Improved signal-to-noise ratio:

$$\diamondsuit \quad \rho = \frac{\int_{\Delta\omega} \frac{d\omega}{2\pi} S(\omega)}{\int_{\Delta\omega} \frac{d\omega}{2\pi} N(\omega)} \,.$$

We have:

$$\int_{\Delta\omega} \frac{d\omega}{2\pi} S(\omega) \simeq \frac{A_0^2}{4\pi} \int_{-\pi}^{\pi} \frac{\sin^2 x}{x^2} dx \simeq \frac{A_0^2}{4} \cdot 0.9028 ,$$
$$\int_{\Delta\omega} \frac{d\omega}{2\pi} N(\omega) \simeq \frac{2N_0}{T} .$$
$$\Rightarrow \quad \rho \simeq \frac{0.9028}{8} \frac{A_0^2 T}{N_0} .$$

Coincides (up to about a factor of  $\sim 1/2$ ) with the ratio of the peak value of the signal power to the (flat) value of the noise power.

# **Improved S-to-N ratio**

Compare with the original ("un-filtered") S-to-N ratio  $\rho_0$ :

$$\rho_0 = \frac{A_0^2/2}{N_0^2} \longrightarrow \rho = \frac{0.9028}{8} \frac{A_0^2 T}{N_0}.$$

Improvement factor:  $0.225N_0T$  ( $\gg 1$ ). [ $N_0T$  is the total # of bckgr. events ]. The improved S-to-N ratio:

- Increases linearly with the running time of the experiment T
- Increases linearly with the mass of the detector M ( $A_0 \propto M$ ,  $N_0 \propto M$ ).

Alternatively, compare  $\rho$  with S/B rather than with un-filtered  $\rho_0$ :

$$\rho \simeq \frac{(A_0/\sqrt{2})T}{N_0 T} \cdot \left[ 0.225(A_0 T/\sqrt{2}) \right].$$

First factor: the S/B ratio; [...] – enhancement factor, increases with signal statistics.

Noise reduction by simple frequency-domain filtering: greatly improves S-to-N, still may not be enough unless the total number of signal events is very large!

⇒ More sophisticated methods of noise reduction may be necessary. E.g. measure noise power spectrum in freq.cy regions where the signal is practically absent (away from  $\omega = \pm \omega_0$ ) to predict its value in the regions of  $\omega$  where the signal is concentrated and cancel (subtract) it there (Samsing, 2015).

But: The cancellation will be incomplete because noise spectrum is in reality only approximately flat.

Other methods can be used, e.g. studying cross-correlation of the data with test functions in the form of the expected time-dependent signal – similarly to approach used in gravitational wave detection.

⇒ Extensive signal and noise simulations in the conditions of a particular experiment will be necessary.

### Conclusions

- New approach for  $C\nu B$  detection proposed.
- Based on neutrino capture one beta-decaying nuclei (as suggested by Weinberg and by Cocco et al.), but uses a different method of discrimination between the relic ν capture and the usual β-decay.
- Employs periodic time dependence of angular correlations in the  $C\nu B$  signal due to the peculiar motion of the solar system and rotation of the Earth about its axis. Does not depend crucially on energy resolution.
- Angular correlation coeffs. found for the special case of pure 1<sup>π</sup> → 0<sup>π</sup> allowed G.-T. transitions. Advantage of pure transitions (F. and G.-T.): correlation coeffs. do not depend on nuclear matrix elements.
- For polarized nuclear targets: time variation of the total  $\nu$  capture rate.
- For unpolarized nuclear targets: time-dependent correlation between preferred direction of neutrino arrival and electron direction.

### **Conclusions – contd.**

- Main challenge: Separation of weak periodic signal from large fluctuating background.
- Nuclids with larger  $Q_{\beta}$ -values may be preferable (lead to larger absolute detection rates).
- Selection of suitable nuclides should take into account
  - lifetime and availability
  - **•** type of  $\beta$  transition
  - (neutrino capture)/( $\beta$  decay) rate ratio
  - feasibility of target polarization (for expts. w/ polarized nuclei).

...

 $\diamond$  Extensive simulations of signal and background will be necessary to clarify if the proposed C $\nu$ B detection method is viable and competitive with the approach based on the separation of the spectra.

# **Backup slides**

Several suggestions so far.

I. Coherent detection through mechanical effects.

• Order  $\sim G_F$  forces – coherent effects (Opher, 1974; Lewis, 1980). Predicted measurable effects due to neutrino reflection or refraction.

Several suggestions so far.

I. Coherent detection through mechanical effects.

• Order  $\sim G_F$  forces – coherent effects (Opher, 1974; Lewis, 1980). Predicted measurable effects due to neutrino reflection or refraction.

<u>Wrong.</u> In reality order  $\sim G_F$  effects are due to coherent forward scattering ( $\Delta p = 0$ ), produce only potential (like in the MSW eff.) but no forces. Non-zero momentum transfer would break order  $\sim G_F$  coherence (Cabibbo and Maiani, 1982; Langacker, Leveille and Sheiman, 1983).

Several suggestions so far.

I. Coherent detection through mechanical effects.

• Order  $\sim G_F$  forces – coherent effects (Opher, 1974; Lewis, 1980). Predicted measurable effects due to neutrino reflection or refraction.

<u>Wrong.</u> In reality order  $\sim G_F$  effects are due to coherent forward scattering ( $\Delta p = 0$ ), produce only potential (like in the MSW eff.) but no forces. Non-zero momentum transfer would break order  $\sim G_F$  coherence (Cabibbo and Maiani, 1982; Langacker, Leveille and Sheiman, 1983).

• Order  $\sim G_F$  angular momentum transfer from C $\nu$ B to polarized electrons due to "neutrino wind" (Stodolsky, 1975).

Several suggestions so far.

I. Coherent detection through mechanical effects.

• Order  $\sim G_F$  forces – coherent effects (Opher, 1974; Lewis, 1980). Predicted measurable effects due to neutrino reflection or refraction.

<u>Wrong.</u> In reality order  $\sim G_F$  effects are due to coherent forward scattering ( $\Delta p = 0$ ), produce only potential (like in the MSW eff.) but no forces. Non-zero momentum transfer would break order  $\sim G_F$  coherence (Cabibbo and Maiani, 1982; Langacker, Leveille and Sheiman, 1983).

• Order  $\sim G_F$  angular momentum transfer from C $\nu$ B to polarized electrons due to "neutrino wind" (Stodolsky, 1975).

Requires non-zero lepton asymmetry in  $C\nu B$  ( $N_{\nu} \neq N_{\bar{\nu}}$ ); proportional to the peculiar velocity of the solar system ( $\sim 10^{-3}$ )  $\Rightarrow$  Correct but too small to be observable (Langacker, Leveille and Sheiman, 1983; Duda et al., 2001).

#### How to detect them? – contd.

#### II. Order $\sim G_F^2$ coherent effects.

Scattering on targets loosely filled with small (~ 1 mm) pellets. Employs macroscopic size of CνB de Broglie wavelength (λ<sub>D</sub> ~ 2.4 mm) (Zeldovich & Khlopov, 1981; Shvartsman, Braginsky, Gershtein, Zeldovich & Khlopov, 1982; Duda, Gelmini & Nussinov, 2001).
 Predicted acceleration:

$$a \sim (10^{-34} - 10^{-28}) \text{ cm/s}^2$$

Smallest acceleration currently measured:  $a \simeq 5 \times 10^{-14} \text{ cm/s}^2$ .
### How to detect them? – contd.

### II. Order $\sim G_F^2$ coherent effects.

Scattering on targets loosely filled with small (~ 1 mm) pellets. Employs macroscopic size of CνB de Broglie wavelength (λ<sub>D</sub> ~ 2.4 mm) (Zeldovich & Khlopov, 1981; Shvartsman, Braginsky, Gershtein, Zeldovich & Khlopov, 1982; Duda, Gelmini & Nussinov, 2001).
 Predicted acceleration:

$$a \sim (10^{-34} - 10^{-28}) \text{ cm/s}^2$$

Smallest acceleration currently measured:  $a \simeq 5 \times 10^{-14} \text{ cm/s}^2$ .

• Coherent radiative  $\nu$  scattering on electrons in conductors ( $\nu e \rightarrow \nu e \gamma$ ) (Loeb & Starkman, 1990). Predicted small but observable effects.

<u>But:</u> missed a factor  $(E_{\nu}/\omega_p)^4 \sim 10^{-20}$  which makes the effect unobservable (S. Bahcall & A. Gould, 1991).

### How to detect them? – contd.

III. Resonant annihilation of UHE CR neutrinos on relic neutrinos of opposite helicity into  $Z^0$  boson (Weiler, 1982):

$$\nu_{\rm CR} + \nu_{\rm C\nu B} \to Z^0 \to \dots$$

Resonance energy:

$$E_{\nu_{\rm CR}i} = \frac{M_Z^2}{2m_{\nu i}} \simeq 4.2 \times 10^{22} \,\mathrm{eV} \left(\frac{0.1 \,\mathrm{eV}}{m_{\nu i}}\right).$$

- ⇒ Absorption dips in the spectra of UHE neutrinos; production of particles above the GZK cutoff.
- But: No sources of such UHE neutrinos known.

### **Order** $\sim G_F$ angular momentum transfer

 $C\nu$ B-induced torque due to coherent order  $\sim G_F$  interaction with particles a (a = e, p, n) in anisotropic (e.g. spin-polarized) targets (Stodolsky, 1975).

$$\vec{\tau} = \langle \dot{\vec{\sigma}} \rangle = \frac{\Delta E}{2} (\hat{\vec{v}}_a \times \langle \vec{\sigma} \rangle) ,$$

$$\Delta E = 2\sqrt{2}G_F u N_a^{\text{tot}} \sum_i (n_{\nu_i} - n_{\bar{\nu}_i}) g_A^{ai} K(p, m_i) \qquad (u = u_{\text{Earth}}),$$

$$K(p,m_i) = \frac{(E+m_i)}{4E} \left[ 1 + \frac{p}{E+m_i} \right]^2 \simeq \begin{cases} 1, & p \gg m_i, \\ \frac{1}{2}, & p \ll m_i. \end{cases}$$

For transversely polarized electrons: spin precession around  $\vec{v}_e$  with  $\omega = \dot{\phi} = \Delta E$ . Assuming  $\Delta n_{\nu}/n_{\nu} \sim 1$  (actually excluded by the data), for a single electron:

$$\Delta E \sim 10^{-38} \text{ eV} \sim 10^{-23} \frac{\text{rad}}{\text{s}}.$$

Could be enhanced by  $\sim 10^{27}$  in a  $\sim 1 t$  ferromagnet.  $\Rightarrow \Delta E \sim 10^{-11} \text{ eV}$  – still many orders of magn. smaller than the magnet interaction with any feasible magn. field (Langacker et al., 1983).

## Elastic scattering of relic neutrinos ( $\sim G_F^2$ )

For relativistic relic neutrinos (with  $\langle p_{\nu 0} \rangle \simeq 3.15 T_{\nu 0}$ ):

$$\sigma_{\nu N}^{\rm D,M} \simeq \frac{G_F^2 E_{\nu}^2}{\pi} \simeq 5 \times 10^{-63} \text{ cm}^2.$$

For non-relativistic neutrinos:

$$\sigma_{\nu N}^{\rm D} \simeq \frac{G_F^2 m_{\nu}^2}{\pi} \simeq 10^{-56} (m_{\nu}/{\rm eV})^2 \ {\rm cm}^2 \,,$$

$$\sigma_{\nu N}^{\rm M} \simeq \frac{G_F^2 p_{\nu}^2}{\pi} \simeq 5 \times 10^{-63} \ {\rm cm}^2 \,.$$

### **Coherent neutrino scattering**

The force acting on a target pellet of size  $r \lesssim \lambda_D (\simeq 2 \text{mm})$ :

 $F \simeq (\sigma_{\nu N} n_{\nu} u) \Delta p \cdot k_L^2 N_0^2$ 

$$N_0 = \frac{\rho}{Am_p} \frac{4}{3} \pi r^3, \quad \Delta p \simeq \begin{cases} m_\nu u, & non - rel., \\ \langle p_\nu \rangle u, & rel. \end{cases}, \quad k_L = \begin{cases} 2Z - N, & \nu_e, \\ -N, & \nu_{\mu,\tau} \end{cases}$$

Acceleration:

$$a \simeq \frac{F}{N_0 A m_p} \sim (10^{-34} - 10^{-28}) \text{ cm/s}^2$$

Smallest acceleration currently measured:  $a \simeq 5 \times 10^{-14} \text{ cm/s}^2$ .

### $\mathbf{C}\nu\mathbf{B}$ detection in inverse $\beta$ -decay

For electron and neutrino in definite spin states:

$$u_e(k)\bar{u}_e(k) = \frac{1}{2}(\not\!\!k + m_e)(1 + \gamma_5 \not\!\!S_e),$$

$$u_j(q)\bar{u}_j(q) = \frac{1}{2}(\not q + m_j)(1 + \gamma_5 \not S_j).$$

 $S_e^{\mu}$  and  $S_j^{\mu}$  – electron and neutrino spin 4-vectors:

$$S_e^{\mu} = \left(\frac{\vec{k} \cdot \vec{s}_e}{m_e}, \ \vec{s}_e + \frac{(\vec{k} \cdot \vec{s}_e)\vec{k}}{m_e(E_e + m_e)}\right),$$

$$S_j^{\mu} = \left(\frac{\vec{q} \cdot \vec{s}_j}{m_j}, \ \vec{s}_j + \frac{(\vec{q} \cdot \vec{s}_j)\vec{q}}{m_j(E_j + m_j)}\right)$$

 $\vec{s}_e$  and  $\vec{s}_j$ : unit vectors in the direction of the electron and neutrino spin in their rest frames.

Evgeny Akhmedov

### $\mathbf{C}\nu\mathbf{B}$ detection in inverse $\beta$ -decay

If neutrinos are in helicity eigenstates and  $\vec{s}_j \parallel \vec{s}_N$ , then  $\vec{B} \cdot \vec{s}_N = -B^0 \Rightarrow$ the transition amplitude vanishes [in agreement with angular momentum conservation (initial state with total angular momentum J + 1/2 cannot decay into a nucleus with spin J - 1 and electron)].

Notation:

$$\begin{split} \diamondsuit \quad \varepsilon_{\mu}\varepsilon_{\nu}^{*}X^{\mu\nu} &= 2E_{e}E_{j}\big[(1-\vec{v_{e}}\cdot\vec{s_{e}}) - K_{e}(\vec{v_{e}}\cdot\vec{s_{N}}) + \frac{m_{e}}{E_{e}}(\vec{s_{e}}\cdot\vec{s_{N}})\big] \\ & \times\big[(1-\vec{v_{j}}\cdot\vec{s_{j}}) + K_{j}(\vec{v_{j}}\cdot\vec{s_{N}}) - \frac{m_{j}}{E_{j}}(\vec{s_{j}}\cdot\vec{s_{N}})\big]. \end{split}$$

$$\begin{aligned} \diamondsuit \quad \varepsilon_{\mu} \varepsilon_{\nu}^{*} X^{\mu\nu} &= 2E_{e}E_{j} \left[ (1 - \vec{v}_{e} \cdot \vec{s}_{e}) - K_{e} (\vec{v}_{e} \cdot \vec{s}_{N}) + \frac{m_{e}}{E_{e}} (\vec{s}_{e} \cdot \vec{s}_{N}) \right] \\ & \times \left[ (1 - \vec{v}_{j} \cdot \vec{s}_{j}) + K_{j} (\vec{v}_{j} \cdot \vec{s}_{N}) - \frac{m_{j}}{E_{j}} (\vec{s}_{j} \cdot \vec{s}_{N}) \right]. \end{aligned}$$

$$\begin{aligned} \diamondsuit \quad \varepsilon_{\mu} \varepsilon_{\nu}^{*} X^{\mu\nu} &= 2E_{e}E_{j} \left[ (1 - \vec{v}_{e} \cdot \vec{s}_{e}) - K_{e} (\vec{v}_{e} \cdot \vec{s}_{N}) + \frac{m_{e}}{E_{e}} (\vec{s}_{e} \cdot \vec{s}_{N}) \right] \\ & \times \left[ (1 - \vec{v}_{j} \cdot \vec{s}_{j}) + K_{j} (\vec{v}_{j} \cdot \vec{s}_{N}) - \frac{m_{j}}{E_{j}} (\vec{s}_{j} \cdot \vec{s}_{N}) \right]. \end{aligned}$$

$$\begin{array}{ll} \vec{v}_e \cdot \vec{s}_e \,, & \vec{v}_e \cdot \vec{s}_N \,, & \vec{s}_e \cdot \vec{s}_N \,, & \vec{v}_j \cdot \vec{s}_j \,, \\ \\ \vec{v}_e \cdot \vec{v}_j \,, & \vec{v}_e \cdot \vec{s}_j \,, & \vec{v}_j \cdot \vec{s}_e \,, & \vec{s}_e \cdot \vec{s}_j \,, & \vec{v}_j \cdot \vec{s}_N \,, & \vec{s}_j \cdot \vec{s}_N \,. \end{array}$$

$$\begin{aligned} & \diamondsuit \quad \varepsilon_{\mu} \varepsilon_{\nu}^{*} X^{\mu\nu} = 2E_{e}E_{j} \left[ (1 - \vec{v}_{e} \cdot \vec{s}_{e}) - K_{e}(\vec{v}_{e} \cdot \vec{s}_{N}) + \frac{m_{e}}{E_{e}}(\vec{s}_{e} \cdot \vec{s}_{N}) \right] \\ & \times \left[ (1 - \vec{v}_{j} \cdot \vec{s}_{j}) + K_{j}(\vec{v}_{j} \cdot \vec{s}_{N}) - \frac{m_{j}}{E_{j}}(\vec{s}_{j} \cdot \vec{s}_{N}) \right]. \end{aligned}$$

$$\vec{v}_e \cdot \vec{s}_e , \quad \vec{v}_e \cdot \vec{s}_N , \quad \vec{s}_e \cdot \vec{s}_N , \quad \vec{v}_j \cdot \vec{s}_j ,$$

$$\vec{v}_e \cdot \vec{v}_j , \quad \vec{v}_e \cdot \vec{s}_j , \quad \vec{v}_j \cdot \vec{s}_e , \quad \vec{s}_e \cdot \vec{s}_j , \quad \vec{v}_j \cdot \vec{s}_N , \quad \vec{s}_j \cdot \vec{s}_N .$$

 $\varepsilon_{\mu}\varepsilon_{\nu}^{*}X^{\mu\nu}$  contains only six of them [terms  $\vec{v}_{e} \cdot \vec{v}_{j}$ ,  $\vec{v}_{e} \cdot \vec{s}_{j}$ ,  $\vec{v}_{j} \cdot \vec{s}_{e}$ ,  $\vec{s}_{e} \cdot \vec{s}_{j}$ are not there.

$$\begin{aligned} & \diamondsuit \quad \varepsilon_{\mu} \varepsilon_{\nu}^{*} X^{\mu\nu} = 2E_{e}E_{j} \left[ (1 - \vec{v}_{e} \cdot \vec{s}_{e}) - K_{e}(\vec{v}_{e} \cdot \vec{s}_{N}) + \frac{m_{e}}{E_{e}}(\vec{s}_{e} \cdot \vec{s}_{N}) \right] \\ & \times \left[ (1 - \vec{v}_{j} \cdot \vec{s}_{j}) + K_{j}(\vec{v}_{j} \cdot \vec{s}_{N}) - \frac{m_{j}}{E_{j}}(\vec{s}_{j} \cdot \vec{s}_{N}) \right]. \end{aligned}$$

$$\begin{array}{ll} \vec{v}_e \cdot \vec{s}_e \,, & \vec{v}_e \cdot \vec{s}_N \,, & \vec{s}_e \cdot \vec{s}_N \,, & \vec{v}_j \cdot \vec{s}_j \,, \\ \\ \vec{v}_e \cdot \vec{v}_j \,, & \vec{v}_e \cdot \vec{s}_j \,, & \vec{v}_j \cdot \vec{s}_e \,, & \vec{s}_e \cdot \vec{s}_j \,, & \vec{v}_j \cdot \vec{s}_N \,, & \vec{s}_j \cdot \vec{s}_N \,. \end{array}$$

 $\varepsilon_{\mu}\varepsilon_{\nu}^{*}X^{\mu\nu}$  contains only six of them [terms  $\vec{v}_{e} \cdot \vec{v}_{j}$ ,  $\vec{v}_{e} \cdot \vec{s}_{j}$ ,  $\vec{v}_{j} \cdot \vec{s}_{e}$ ,  $\vec{s}_{e} \cdot \vec{s}_{j}$ are not there.

Terms  $\vec{v}_e \cdot \vec{s}_e$ ,  $\vec{v}_e \cdot \vec{s}_N$ ,  $\vec{s}_e \cdot \vec{s}_N$ : do not depend on the neutrino variables.

$$\begin{aligned} & \diamond \quad \varepsilon_{\mu}\varepsilon_{\nu}^{*}X^{\mu\nu} = 2E_{e}E_{j}\left[\left(1-\vec{v_{e}}\cdot\vec{s_{e}}\right)-K_{e}(\vec{v_{e}}\cdot\vec{s_{N}})+\frac{m_{e}}{E_{e}}(\vec{s_{e}}\cdot\vec{s_{N}})\right] \\ & \times\left[\left(1-\vec{v_{j}}\cdot\vec{s_{j}}\right)+K_{j}(\vec{v_{j}}\cdot\vec{s_{N}})-\frac{m_{j}}{E_{j}}(\vec{s_{j}}\cdot\vec{s_{N}})\right]. \end{aligned}$$

$$\begin{array}{ll} \vec{v}_e \cdot \vec{s}_e \,, & \vec{v}_e \cdot \vec{s}_N \,, & \vec{s}_e \cdot \vec{s}_N \,, & \vec{v}_j \cdot \vec{s}_j \,, \\ \\ \vec{v}_e \cdot \vec{v}_j \,, & \vec{v}_e \cdot \vec{s}_j \,, & \vec{v}_j \cdot \vec{s}_e \,, & \vec{s}_e \cdot \vec{s}_j \,, & \vec{v}_j \cdot \vec{s}_N \,, & \vec{s}_j \cdot \vec{s}_N \,. \end{array}$$

 $\varepsilon_{\mu}\varepsilon_{\nu}^{*}X^{\mu\nu}$  contains only six of them [terms  $\vec{v}_{e} \cdot \vec{v}_{j}$ ,  $\vec{v}_{e} \cdot \vec{s}_{j}$ ,  $\vec{v}_{j} \cdot \vec{s}_{e}$ ,  $\vec{s}_{e} \cdot \vec{s}_{j}$ are not there.

Terms  $\vec{v}_e \cdot \vec{s}_e$ ,  $\vec{v}_e \cdot \vec{s}_N$ ,  $\vec{s}_e \cdot \vec{s}_N$ : do not depend on the neutrino variables.

**Term**  $\vec{v}_j \cdot \vec{s}_j$ : **not useful** [expts. on helicity measurements cannot distinguish between absorption of a L-helical neutrino and production of a R-helical (anti)neutrino]. But:  $\vec{v}_j \cdot \vec{s}_j$  affects the total capture rate and may be important for studying Dirac vs. Majorana  $\nu$  nature].

Terms relevant for  $C\nu B$  capture on polarized targets:  $\vec{v}_j \cdot \vec{s}_N$ ,  $\vec{s}_j \cdot \vec{s}_N - do$  not depend on  $\vec{v}_e$  and  $\vec{s}_e$ . Summing over  $s_e$  and averaging over the directions of  $\vec{v}_e$ :

$$\diamondsuit \quad \int \frac{d\Omega_e}{4\pi} \sum_{s_e} \varepsilon_\mu \varepsilon_\nu^* X^{\mu\nu} = 4E_e E_j \Big\{ (1 - \vec{v}_j \cdot \vec{s}_j) + \big( K_j \vec{v}_j - \frac{m_j}{E_j} \vec{s}_j \big) \cdot \vec{s}_N \Big\}.$$

[N.B.: For helical neutrinos  $\Rightarrow 4E_e E_j (1 - \lambda_j v_j)(1 - \vec{s}_j \cdot \vec{s}_N)$ .]

For fixed direction of  $\vec{s}_N$  in the Earth frame – will vary with time due to variations of the directions of  $\vec{v}_j$  and  $\vec{s}_j$ .

 $\Rightarrow$  In expts. with polarized nuclear targets time variations of the C $\nu$ B signal can be studied by merely measuring the total rate of electron production.

Measuring the angular and/or spin distributions of electrons will be useful for experiments with unpolarized nuclei.

## **Unpolarized nuclear targets**

Polarization of large targets – a difficult task. Can one use angular correlations in expts. with unpolarized targets?

- Make use of polarization of daughter nuclei with  $J_f \neq 0$  (e.g. in expts. of Goldhaber, Grodzins and Sunyar type; for  $0^{\pi} \rightarrow 1^{\pi}$  replace  $\vec{s}_N \rightarrow -\vec{s}_N$ ).
- More practical approach: measure  $\beta \nu$  angular correlations (or correl. between the direction of incoming  $\nu$  and the spin of e).

Averaging over the polarizations of the parent nucleus:

$$\frac{1}{3}\sum_{\lambda}\varepsilon_{\mu}(\lambda)\varepsilon_{\nu}^{*}(\lambda)X^{\mu\nu} = 2\left[A^{0}B^{0} - \frac{1}{3}\vec{A}\cdot\vec{B}\right].$$

[The same result can be obtained by taking  $\int (d\Omega_{\vec{s}_N}/4\pi)\varepsilon_{\mu}\varepsilon_{\nu}^* X^{\mu\nu}$ ].

### **Unpolarized targets**

When the direction of the spin of the produced electrons is observed while the directions of their momenta are not:

$$\int \frac{d\Omega_e}{4\pi} \frac{1}{3} \sum_{\lambda} \varepsilon_{\mu}(\lambda) \varepsilon_{\nu}^*(\lambda) X^{\mu\nu} = 2E_e \left\{ B^0 + \frac{1}{9} \left( 1 + 2\frac{m_e}{E_e} \right) (\vec{s_e} \cdot \vec{B}) \right\}$$

$$= 2E_e E_j \Big\{ 1 - \vec{v}_j \cdot \vec{s}_j + \frac{1}{9} \Big( 1 + 2\frac{m_e}{E_e} \Big) \Big[ \vec{v}_j \cdot \vec{s}_e - \frac{E_j}{E_j + m_j} (\vec{v}_j \cdot \vec{s}_j) (\vec{v}_j \cdot \vec{s}_e) - \frac{m_j}{E_j} \vec{s}_e \cdot \vec{s}_j \Big] \Big\}.$$

When neither the spin state nor the direction of the produced electron is observed:

$$\int \frac{d\Omega_e}{4\pi} \frac{1}{3} \sum_{\lambda, s_e} \varepsilon_\mu(\lambda) \varepsilon_\nu^*(\lambda) X^{\mu\nu} = 4E_e E_j \left( 1 - \vec{v_j} \cdot \vec{s_j} \right),$$

- the usual inclusive squared amplitude for neutrino capture on unpolarized nuclei.

### $\beta - \nu$ angular correlation

If the spin state of the produced electron is not measured, sum over  $s_e$ :

$$\frac{1}{3}\sum_{\lambda,s_e}\varepsilon_{\mu}(\lambda)\varepsilon_{\nu}^{*}(\lambda)X^{\mu\nu} = 4E_e\left(B^0 - \frac{1}{3}\vec{B}\cdot\vec{v}_e\right).$$

In a more detailed form:

$$\Rightarrow \quad 4E_e E_j \left[ 1 - \vec{v}_j \cdot \vec{s}_j - \frac{1}{3} \vec{v}_e \cdot \vec{v}_j + \frac{1}{3} \frac{E_j}{E_j + m_j} (\vec{v}_j \cdot \vec{s}_j) (\vec{v}_e \cdot \vec{v}_j) + \frac{1}{3} \frac{m_j}{E_j} \vec{v}_e \cdot \vec{s}_j \right].$$

Describes anisotropies of electron emission with respect to the directions of  $\vec{v}_j$  and  $\vec{s}_j$  which change with time in the lab frame.

For helical neutrinos:

$$\Rightarrow 4E_e E_j (1 - \lambda_j v_j) \Big\{ 1 + \frac{1}{3} \frac{\lambda_j}{v_j} \vec{v}_e \cdot \vec{v}_j \Big\}.$$

In the relativistic limit:

=

$$\Rightarrow \begin{cases} 8E_e E_j \left\{ 1 - \frac{1}{3} \vec{v}_e \cdot \vec{v}_j \right\}, & \lambda_j = -1, \\ 0, & \lambda_j = +1 \end{cases} \Rightarrow a = -\frac{1}{3}.$$



# Solar v fluxes from the Sudbury Neutrino Observatory SNO



#### C. Weinheimer, ISAPP School Heidelberg



### Solar v fluxes from the **Sudbury Neutrino Observatory SNO**



#### C. Weinheimer, ISAPP School Heidelberg

### Lab frame and angular averaging

### **Implications for relic** $\nu$ **capture: lab frame**

Standard cosmology predicts isotropic neutrino velocity distributions in  $C\nu B$  rest frame. As neutrinos are in helicity eigenstates in that frame – neutrino spin distributions are also isotropic. That is, angular averaging gives

$$\langle ec v_j 
angle \; = \; 0 \, , \qquad \langle ec s_j 
angle \; = \; 0 \, .$$

The Sun moves with a velocity  $-\vec{u}$  ( $u \ll 1$ ) with respect to the C $\nu$ B rest frame  $\Rightarrow$  neutrino variables in the lab frame obtained by a Lorentz boost. To  $\mathcal{O}(u)$ :

$$E'_{j} = E_{j} + \vec{u} \cdot \vec{q}, \qquad \vec{q}' = \vec{q} + \vec{u}E_{j}, \qquad \vec{v}'_{j} = \vec{v}_{j}(1 - \vec{u} \cdot \vec{v}_{j}) + \vec{u}.$$

In the regime  $u \ll v_j$  (in the standard 3-flavour neutrino picture corresponds to hierarchical neutrino mass spectrum) in this limit

$$\frac{\vec{v}_{j}'}{v_{j}'} = \frac{1}{v_{j}} \left\{ \vec{v}_{j} + \left[ \vec{u} - \frac{(\vec{u} \cdot \vec{v}_{j})}{v_{j}^{2}} \vec{v}_{j} \right] \right\} \equiv \frac{1}{v_{j}} \left( \vec{v}_{j} + \vec{u}_{\perp j} \right),$$

 $(\vec{u}_{\perp j})$  is the component of  $\vec{u}$  orthogonal to  $\vec{v}_j$ ).

$$\vec{s}_{j}' = \vec{s}_{j} + \frac{(\vec{q} \cdot \vec{s}_{j})\vec{u} - (\vec{u} \cdot \vec{s}_{j})\vec{q}}{E_{j} + m_{j}}.$$

If in the C $\nu$ B rest frame neutrinos are in the exact helicity eigenstates,

 $\Rightarrow$ 

$$\vec{s}_{j} = \lambda_{j} \frac{\vec{v}_{j}}{v_{j}}, \qquad (\lambda_{j} = \pm 1)$$
$$\vec{s}_{j}' = \lambda_{j} \left\{ \frac{\vec{v}_{j}}{v_{j}} + v_{j} \frac{E_{j}}{E_{j} + m_{j}} \vec{u}_{\perp j} \right\}$$

N.B.: Although helicity is not a Lorentz-invariant quantity, in the regime  $u \ll v_j$ (valid for hierarchical  $\nu$  masses and no  $\nu_s$ ) and to first order in u neutrinos are also helical in the lab frame with the same  $\lambda_j$  (corrections are  $\mathcal{O}(u^2)$ ).

[This would not be in general correct for  $u \gtrsim v_j$   $(m_j \gtrsim 0.5 \text{ eV})$  even if  $u \ll 1$ . E.g. for  $v_j \ll u \ll 1$  we would have  $\vec{v}'_j \simeq \vec{u}$ ,  $\vec{s}'_j \simeq \vec{s}_j$  and  $\vec{s}'_j \cdot \vec{v}'_j / v'_j \simeq \vec{s}_j \cdot \vec{u} / u$  ( $\neq \lambda_j$  in general)].

From  $\langle \vec{v}_j \rangle = 0$ ,  $\langle \vec{s}_j \rangle = 0$ :  $\langle E'_j \rangle = E_j$ ,  $\langle \vec{q'} \rangle = \vec{u}E_j$ ,  $\langle \vec{v}'_j \rangle = \left(1 - \frac{v_j^2}{3}\right)\vec{u}$ .

$$\langle \vec{s}_j' \rangle = \lambda_j v_j \frac{2}{3} \frac{E_j}{E_j + m_j} \vec{u},$$

$$\left\langle \vec{v}_{j}^{\,\prime}\cdot\vec{s}_{j}^{\,\prime}
ight
angle =\lambda_{j}v_{j}=\left\langle \vec{v}_{j}\cdot\vec{s}_{j}
ight
angle .$$

### **Polarized targets**

Upon averaging over the angular distributions of the  $C\nu B$  neutrinos:

$$\diamondsuit \quad \left\langle \frac{\varepsilon_{\mu}\varepsilon_{\nu}^{*}X^{\mu\nu}}{4E_{e}E_{j}^{\prime}} \right\rangle = \frac{1}{2E_{e}} \left( A^{0} - \vec{A} \cdot \vec{s}_{N} \right) \mathcal{F}_{j}(\lambda_{j}) \,,$$

$$\mathcal{F}_j(\lambda_j) \equiv \left\{ 1 - \lambda_j v_j + \left( 1 - \frac{2}{3} \lambda_j v_j - \frac{v_j^2}{3} \right) \vec{u} \cdot \vec{s}_N \right\}.$$

For L-helical  $\nu$ s:

$$\mathcal{F}_j(\lambda_j = -1) = 1 + v_j + \left(1 + \frac{2}{3}v_j - \frac{v_j^2}{3}\right)\vec{u} \cdot \vec{s}_N \,.$$

For R-helical  $\nu$ s: need to distinguish between Dirac and Majorana neutrinos.  $\mathcal{F}_{j}^{\mathrm{Dir}}(\lambda_{j}=1)=0;$ 

$$\mathcal{F}_{j}^{\text{Maj}}(\lambda_{j}=1) = 1 - v_{j} + \left(1 - \frac{2}{3}v_{j} - \frac{v_{j}^{2}}{3}\right)\vec{u} \cdot \vec{s}_{N} \,.$$

Vanishes in the limit  $v_j \rightarrow 1$  (no difference between Dir. and Maj.  $\nu$ s).

Standard cosmology predicts  $C\nu B$  to contain equal numbers of L-helical and R-helical neutrinos.  $\Rightarrow$  Summing over the helicities:

$$\mathcal{F}_{j} \equiv \sum_{\lambda_{j}=\pm 1} \mathcal{F}_{j}(\lambda_{j}) = \begin{cases} 1 + v_{j} + \left(1 + \frac{2}{3}v_{j} - \frac{v_{j}^{2}}{3}\right)\vec{u}\cdot\vec{s}_{N}, & \text{Dirac neutrinos}, \\ 2\left[1 + \left(1 - \frac{v_{j}^{2}}{3}\right)\vec{u}\cdot\vec{s}_{N}\right], & \text{Majorana neutrinos} \end{cases}$$

In the limit of non-relativistic neutrinos:

$$\mathcal{F}_{j} \simeq \begin{cases} 1 + v_{j} + \left(1 + \frac{2}{3}v_{j}\right)\vec{u}\cdot\vec{s}_{N}, & \text{Dirac neutrinos}, \\ \\ 2\left(1 + \vec{u}\cdot\vec{s}_{N}\right), & \text{Majorana neutrinos} \end{cases}$$

For highly relativistic neutrinos:

$$\mathcal{F}_{j}^{\mathrm{Dir}} \simeq \mathcal{F}_{j}^{\mathrm{Maj}} \simeq 2\left(1 + \frac{2}{3}\vec{u}\cdot\vec{s}_{N}\right).$$

### Non-relativistic case:

The results for Dirac and Majorana neutrinos differ. the total detection cross section for Majorana  $\nu$ s is about twice as large as that for Dirac  $\nu$ s (Long, Lunardini & Sabancilar, 2014).  $\Rightarrow$  Can in principle be used to discriminate between Dir. amd Maj. neutrinos.

<u>But</u>: complicated by the fact that the local C $\nu$ B density at the Earth may differ from  $n_{\nu 0}$  due to gravitational clustering effects, not precisely known.

The degeneracy between unknown local  $n_{\nu}$  relic and Dirac/Majorana  $\nu$  nature can in principle be lifted by measuring time-dependent angular correlations in relic  $\nu$  capture.

In practice, however, this will be very difficult because the difference between the angular correlatiopn coeffs. contains an extra factor  $v_j$  compared to the main  $\vec{u} \cdot \vec{s}_N$  term.

### **Unpolarized targets**

Averaging over the polarizations of the parent nuclei we find

$$\left\langle \frac{1}{3} \sum_{\lambda} \frac{\varepsilon_{\mu}(\lambda) \varepsilon_{\nu}^{*}(\lambda) X^{\mu\nu}}{4E_{e}E_{j}^{\prime}} \right\rangle = \frac{1}{2E_{e}} \left\{ A^{0}(1-\lambda_{j}v_{j}) - \frac{1}{3} \left(1 - \frac{2}{3}\lambda_{j}v_{j} - \frac{v_{j}^{2}}{3}\right) \vec{u} \cdot \vec{A} \right\}.$$

If the spin state of the produced electron is not measured – sum over  $s_e$ :

$$\tilde{\mathcal{F}}_{j}(\lambda_{j}) \equiv \left\langle \frac{1}{3} \sum_{\lambda, s_{e}} \frac{\varepsilon_{\mu}(\lambda) \varepsilon_{\nu}^{*}(\lambda) X^{\mu\nu}}{4E_{e}E_{j}^{\prime}} \right\rangle = 1 - \lambda_{j} v_{j} - \frac{1}{3} \left( 1 - \frac{2}{3} \lambda_{j} v_{j} - \frac{v_{j}^{2}}{3} \right) \vec{u} \cdot \vec{v}_{e} \,.$$

Summing over helicities:

$$\tilde{\mathcal{F}}_{j} \equiv \sum_{\lambda_{j}=\pm 1} \tilde{\mathcal{F}}_{j}(\lambda_{j}) = \begin{cases} 1+v_{j} - \frac{1}{3} \left(1 + \frac{2}{3}v_{j} - \frac{v_{j}^{2}}{3}\right) \vec{u} \cdot \vec{v}_{e} , & \text{Dirac neutrinos }, \\ 2 \left[1 - \frac{1}{3} \left(1 - \frac{v_{j}^{2}}{3}\right) \vec{u} \cdot \vec{v}_{e} \right], & \text{Majorana neutrinos }. \end{cases}$$

### **Unpolarized targets**

For highly relativistic neutrinos:

$$\widetilde{\mathcal{F}}_{j}^{\mathrm{Dir}} \simeq \widetilde{\mathcal{F}}_{j}^{\mathrm{Maj}} \simeq 2 \Big[ 1 - \frac{2}{9} \vec{u} \cdot \vec{v}_{e} \Big].$$

In the limit of non-relativistic neutrinos:

$$\tilde{\mathcal{F}}_{j} \simeq \begin{cases} 1 + v_{j} - \frac{1}{3} \left( 1 + \frac{2}{3} v_{j} \right) \vec{u} \cdot \vec{v}_{e} , & \text{Dirac neutrinos} , \\ \\ 2 \left[ 1 - \frac{1}{3} \vec{u} \cdot \vec{v}_{e} \right] , & \text{Majorana neutrinos} \end{cases}$$

On top of the overall factor  $\simeq 2$  for Dirac neutrinos, different angular correlation coeffs. for non-relat. Dirac and Majorana  $\nu$ s:

$$1 - \frac{1}{3} \left( 1 - \frac{v_j}{3} \right) \vec{u} \cdot \vec{v}_e , \quad \text{Dirac neutrinos} , \\ 1 - \frac{1}{3} \vec{u} \cdot \vec{v}_e , \quad \text{Majorana neutrinos} .$$

Difference  $\sim v_j \vec{u} \cdot \vec{v}_e$  – extra small factor  $v_j$ .

# **Comparison with** $\nu$ capture on polarized <sup>3</sup>H

Compare our results for pure Gamow-Teller  $1^{\pi} \rightarrow 0^{\pi}$  transitions with those for  $\nu$  capture on polarized tritium (mixed Fermi–Gamow-Teller  $1/2^+ \rightarrow 1/2^+$  transition) in the case when spins of the electron and daughter <sup>3</sup>He are not observed (Lisanti et al., 2014):

$$1 - \vec{v}_j \cdot \vec{s}_j + A(1 - \vec{v}_j \cdot \vec{s}_j)\vec{v}_e \cdot \vec{s}_N + BK_j\vec{v}_j \cdot \vec{s}_N - B\frac{m_j}{E_j}\vec{s}_j \cdot \vec{s}_N + aK_j\vec{v}_e \cdot \vec{v}_j - a\frac{m_j}{E_j}\vec{v}_e \cdot \vec{s}_j \,.$$

Correlation coeffs. *A*, *B* and *a* depend on the ratio of the F. and G.-T. nuclear matrix elements. Numerically

 $A \simeq -0.095$ ,  $B \simeq 0.99$ ,  $a \simeq -0.087$ .

To compare with our results, sum our squared amplitude over  $s_e \Rightarrow$ 

$$1 - \vec{v}_j \cdot \vec{s}_j - (1 - \vec{v}_j \cdot \vec{s}_j) \vec{v}_e \cdot \vec{s}_N + K_j \vec{v}_j \cdot \vec{s}_N - \frac{m_j}{E_j} \vec{s}_j \cdot \vec{s}_N - K_j (\vec{v}_j \cdot \vec{s}_N) (\vec{v}_e \cdot \vec{s}_N) + \frac{m_j}{E_j} (\vec{v}_e \cdot \vec{s}_N) (\vec{s}_j \cdot \vec{s}_N) .$$

The first five terms have the same structure; the two sets coincide if one takes A = -1 and B = 1 in the tritium result. Terms bilinear in  $\vec{s}_N$  are absent in the tritium case. Reason: the special nature of the  $1/2^+ \rightarrow 1/2^+$  transition, for which the tensor alignment  $\propto [J(J+1) - 3\langle (\vec{J} \cdot \vec{s}_N)^2 \rangle]$  vanishes (Jackson et al., 1957).

Upon averaging over the directions of  $\vec{s}_N$  the last two terms in G.-T. case would reproduce those in the tritium case but with a = -1/3.

### To $\mathcal{O}(u)$ in the lab frame:

$$\begin{split} E'_{j} &= E_{j} + \vec{u} \cdot \vec{q}, \qquad \vec{q}' = \vec{q} + \vec{u}E_{j}, \qquad \vec{v}_{j}' = \vec{v}_{j}(1 - \vec{u} \cdot \vec{v}_{j}) + \vec{u}. \\ \vec{s}_{j}' &= \vec{s}_{j} + \frac{(\vec{q} \cdot \vec{s}_{j})\vec{u} - (\vec{u} \cdot \vec{s}_{j})\vec{q}}{E_{j} + m_{j}}. \\ & \downarrow \\ \vec{v}_{j}' \cdot \vec{s}_{j}' &= \lambda_{j}v_{j} \Big\{ 1 + (\vec{u} \cdot \vec{v}_{j})\frac{1 - v_{j}^{2}}{v_{j}^{2}} \Big\}. \\ K'_{j} &= 1 - \frac{E'_{j}}{E'_{j} + m_{j}}(\vec{v}_{j}' \cdot \vec{s}_{j}') = 1 - \frac{E_{j}}{E_{j} + m_{j}}\lambda_{j}v_{j} \Big( 1 + \frac{\vec{u} \cdot \vec{v}_{j}}{v_{j}^{2}} \frac{m_{j}}{E_{j}} \Big). \end{split}$$

This gives

$$\frac{1}{E'_j}\vec{B'} = K'_j\vec{v}'_j - \frac{m_j}{E'_j}\vec{s}'_j = (1 - \lambda_j v_j) \left\{ \vec{u} - (1 - \vec{u} \cdot \vec{v}_j)\lambda_j \frac{\vec{v}_j}{v_j} \right\}.$$

### **Angular averaging**

Taking into account that  $\langle \vec{v}_j \rangle = 0$  and  $\langle \vec{s}_j \rangle = 0$ :

$$\langle E'_j \rangle = E_j , \qquad \langle \vec{q}' \rangle = \vec{u} E_j , \qquad \langle \vec{v}'_j \rangle = \left(1 - \frac{v_j^2}{3}\right) \vec{u} ,$$

$$\left\langle \frac{\vec{v}'_j}{v'_j} \right\rangle = \frac{2}{3} \frac{\vec{u}}{v_j} ,$$

$$\langle \vec{s}'_j \rangle = \lambda_j v_j \frac{2}{3} \frac{E_j}{E_j + m_j} \vec{u} , \qquad \left\langle \vec{v}'_j \cdot \vec{s}'_j \right\rangle = \lambda_j v_j = \left\langle \vec{v}_j \cdot \vec{s}_j \right\rangle .$$

This yields

$$\left\langle \frac{1}{E'_j} B^{0\prime} \right\rangle = 1 - \left\langle \vec{v}_j' \cdot \vec{s}_j' \right\rangle = 1 - \lambda_j v_j ,$$
$$\left\langle \frac{1}{E'_j} \vec{B'} \right\rangle = \left\langle K'_j \vec{v}_j' - \frac{m_j}{E'_j} \vec{s}_j' \right\rangle = \left( 1 - \frac{2}{3} \lambda_j v_j - \frac{v_j^2}{3} \right) \vec{u} .$$

With these relations, one can find the expressions for  $\mathcal{F}_j(\lambda_j)$ .
## **Unpolarized targets**

Averaging over the polarizations of the parent nuclei we find

$$\Big\langle \frac{1}{3} \sum_{\lambda} \frac{\varepsilon_{\mu}(\lambda) \varepsilon_{\nu}^{*}(\lambda) X^{\mu\nu}}{4E_{e}E_{j}^{\prime}} \Big\rangle = \frac{1}{2E_{e}} \Big\{ A^{0}(1-\lambda_{j}v_{j}) - \frac{1}{3} \Big(1-\frac{2}{3}\lambda_{j}v_{j} - \frac{v_{j}^{2}}{3}\Big) \vec{u} \cdot \vec{A} \Big\}.$$

If the spin state of the produced electron is not measured – sum over  $s_e$ :

$$\left\langle \frac{1}{3} \sum_{\lambda, s_e} \frac{\varepsilon_\mu(\lambda) \varepsilon_\nu^*(\lambda) X^{\mu\nu}}{4E_e E'_j} \right\rangle = 1 - \lambda_j v_j - \frac{1}{3} \left( 1 - \frac{2}{3} \lambda_j v_j - \frac{v_j^2}{3} \right) \vec{u} \cdot \vec{v}_e \,.$$

If the electron direction is not observed but its spin state is:

$$\int \frac{d\Omega_e}{4\pi} \Big\langle \frac{1}{3} \sum_{\lambda} \frac{\varepsilon_{\mu}(\lambda) \varepsilon_{\nu}^*(\lambda) X^{\mu\nu}}{4E_e E'_j} \Big\rangle = \frac{1}{2} \Big\{ 1 - \lambda_j v_j + \frac{1}{9} \Big( 1 + 2\frac{m_e}{E_e} \Big) \Big( 1 - \frac{2}{3} \lambda_j v_j - \frac{v_j^2}{3} \Big) \vec{u} \vec{s}_e \Big\}.$$