

#### - a (subjective) status report

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### Plan

- Idea of the no-boundary proposal (1981)
- Early explorations (1980s 2000s)
- Recent developments (last few years)

#### First appearance – September 1981

PONTIFICIAE ACADEMIAE SCIENTIARVM SCRIPTA VARIA

#### ASTROPHYSICAL COSMOLOGY

PROCEEDINGS OF THE STUDY WEEK ON COSMOLOGY AND FUNDAMENTAL PHYSICS

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EDITED BY H. A. BRÜCK, G. V. COYNE AND M. S. LONGAIR.



#### Allocution of Pope John Paul II

Toute hypothèse scientifique sur l'origine du monde, comme celle d'un atome primitif d'où dériverait l'ensemble de l'univers physique, laisse ouvert le problème concernant le commencement de l'univers. La science ne peut par ellemême résoudre une telle question: il y faut ce savoir de l'homme qui s'élève au-dessus de la physique et de l'astrophysique et que l'on appelle la métaphysique; il y faut surtout le savoir qui vient de la révélation de Dieu.

"Science by itself cannot resolve this question [of the beginning of the universe]... it requires the knowledge that comes from the revelation of God"

#### First appearance – September 1981

#### THE BOUNDARY CONDITIONS OF THE UNIVERSE

S.W. HAWKING University of Cambridge, Cambridge

This paper considers the questions of what are the boundary conditions of the universe and where should they be imposed. It is difficult to define boundary conditions at the initial singularity and, even if one could, they would be insufficient to determine the evolution of the universe. In order to overcome this problem it is suggested that one should adopt the Euclidean approach and evaluate the path integral for quantum gravity over positive definite metrics. If one took these metrics to be compact, one would avoid the need to specify any boundary conditions for the universe. This approach might explain why the apparent cosmological constant is zero, why the universe is spatially flat, and why it was in thermal equilibrium at early times.

#### [Pontif.Acad.Sci.Scr.Varia 48 (1982) 563-574]

- Going back in time to an earlier phase, one can always ask what came before
- This leads to an infinite regression...



- If the geometry is smoothly rounded off, then
  - The infinite regression is avoided
  - The initial singularity is avoided
  - We may not need to specify any boundary conditions



### Expectations

 Ideas spelled out in great detail by Hartle & Hawking [*Phys.Rev.D* 28 (1983) 2960-2975]



• The wave function of the universe should be calculated via a path integral defined by a sum over compact, regular, Euclidean metrics

$$\Psi(final) = \int_{no-boundary}^{final} e^{-S_E/\hbar}$$

## Expectations

- The w.f. should be real (because defined over real Euclidean metrics)
- The w.f. is expected/hoped to be unique
- The w.f. represents the ground state of the universe

$$e^{iEt} \stackrel{t\,=\,-i au}{
ightarrow} e^{E au}$$
 lowest E survives as  $au
ightarrow -\infty$ 

- The w.f. gives different probabilities for different initial conditions
- Topology change can be incorporated:  $\Psi(a=0) \neq 0$



#### Issues

- Conflict with uncertainty principle
- Friedmann equation (with k=+1):

$$\dot{a}^2 + 1 = a^2 \left(\frac{1}{6}\dot{\phi}^2 + \cdots\right)$$

- Compactness requires that at origin a=0
- Regularity requires a condition on expansion rate,  $\dot{a}^2 + 1 = 0$  at a = 0 (i.e.  $a_{,\tau} = \pm 1$ )
- Hence one is trying to fix both the field and the momentum

#### Issues

Conformal mode problem

$$ds^{2} = -N^{2}dt^{2} + a(t)^{2}d\mathbf{x}^{2}$$
$$S = \int dt N \left( -3a\frac{\dot{a}^{2}}{N^{2}} + \frac{1}{2}a^{3}\frac{\dot{\phi}^{2}}{N^{2}} + \cdots \right)$$

Opposite signs

• Action unbounded above and below, so how should the Euclidean path integral  $\int e^{-S_E/\hbar}$  be defined?

- Uniqueness?
- Early studies by Halliwell and Louko showed that a wide variety of boundary conditions and integration contours (especially for the lapse N) seemed possible in minisuperspace models

# Early explorations

# Early works

 Pragmatic approach: Focus on saddle points (= possibly complex solutions of classical equations of motion) of path integral, as they provide dominant contribution

[*recent Refs.:* De Alwis; Halliwell, Hartle & Hertog]

- Advantages:
  - Can specify both field values and momenta at nucleation
  - Do not need to worry about conformal mode problem
  - Uniqueness is only a problem if there exist multiple inequivalent saddle points



• A sequence of instantons with changing boundary data shows how the wave function evolves



• If the wave function is of WKB form, i.e. slowly varying amplitude, fast varying phase (as the universe expands),

$$\Psi \approx A e^{iP}$$
 with  $\partial P \gg \partial A$ 

then one can assign an approximately conserved probability  $|\Psi|^2 = A^2$  to the associated history

[Hartle, Hawking & Hertog; Vilenkin]

- Does the wave function automatically reach WKB form?
- No! need attractor mechanism
- Best understood example: inflation
- E.g.  $m^2\phi^2$  potential

$$\begin{split} \Psi(a_f,\phi_f,\phi_{SP}) &\approx e^{\frac{i}{\hbar}S_{on-shell}} \\ &\approx e^{\frac{12\pi^2}{\hbar V(\phi_{SP})} + i\frac{2\pi^2}{\hbar}\sqrt{\frac{2}{3}}m\phi_f a_f^3} \end{split}$$

Weighting almost constant for slow-roll

Phase grows as volme

• Only other mechanism known: ekpyrosis



- WKB reached as the universe contracts
- Weighting

$$\Psi \approx e^{\frac{1}{|V(\phi_{SP})|}}$$

 Connection to present day requires an understanding of the bounce phase

[Battarra & JLL]

- The no-boundary wave function explains the origin of classical realms, but *only* in regions of the potential allowing for inflation (positive, flat) or ekpyrosis (negative, steep)
- Low values of potential come out as preferred, i.e. short inflationary phases or long ekpyrotic phases
- This has led to speculations that one needs to use anthropic reasoning to argue for a long inflationary phase, and possibly eternal inflation
- In my view this is premature, as a rigorous understanding of probabilities, and of the possible scalar potentials in fundamental theory, are still lacking

• Consider perturbations around saddle geometry, e.g. tensor perturbations h (for spherical harmonic k)

$$\ddot{h} + 3\frac{\dot{a}}{a}\dot{h} + \frac{k(k+2)}{a^2}h = 0$$

- 2 solutions: one is regular at a=0, the other blows up
- Regular solution corresponds to Bunch-Davies vacuum (for de Sitter case)
- Thus the no-boundary proposal explains the initial state of perturbations

A subtlety



Effectively opposite Wick rotations

A subtlety



# Recent developments

### No-Boundary Proposal – Implementation?

• We will consider the simple system

$$\Psi = \int_{\mathcal{C}} \delta N \, \delta q \, e^{iS(N,q)/\hbar}$$

Can add ghosts and choose constant N gauge in a integral – see e.g. [Teitelboim]

with

$$S = \frac{1}{2} \int \mathrm{d}^4 x \sqrt{-g} \left( R - 2\Lambda \right)$$

Scale factor squared

• With the metric  $ds^2 = -\frac{N^2}{q(t)}dt^2 + q(t)d\Omega_3^2$ 

[Halliwell & Louko]

the action becomes quadratic

$$S = 2\pi^2 \int \mathrm{d}t \left( -\frac{3}{4N} \dot{q}^2 + 3N - N\Lambda q \right)$$

### **No-Boundary Proposal**

• Implementation?

 First impose boundary condition that universe starts at zero size q(t=0)=0

• This implements the idea that we are summing over compact metrics

# Ordinary integral for lapse

• We are left with an ordinary integral over the lapse function

$$\Psi = \int \frac{\mathrm{d}N}{N^{1/2}} e^{\frac{i}{\hbar} \left[ N^3 \frac{\Lambda^2}{36} + N \left(3 - \frac{\Lambda}{2} q_1\right) - \frac{3}{4N} q_1^2 \right]}_{\text{Integrate over real N}}$$

• There are 4 saddle points

$$N_s = \frac{3}{\Lambda} \left[ \pm \left(\frac{\Lambda}{3}q_1 - 1\right)^{1/2} \pm i \right]$$

- The saddle points are complex
- Now we can apply Picard-Lefschetz theory (framework for systematic saddle point approximation)

### Steepest descent of lapse integral

• Arrows: downwards flow



[Feldbrugge, JLL & Turok]

#### Steepest descent of lapse integral



[Feldbrugge, JLL & Turok]

### Remarks

Euclidean contour diverges, hence sum over Euclidean metrics *does not exist* 

the stable of th

Even if a stable saddle point is included, the steepest descent line flows to the unstable saddle point, hence instabilities will be present

Debate: [Diaz-Dorronsoro, Halliwell, Hartle, Hertog & Janssen; Feldbrugge, JLL & Turok]

# New definition

- Can impose regularity, rather than compactness, as a boundary condition on "initial"  $\frac{\dot{q}}{2N} = i$
- This effectively fixes Wick rotation, and eliminates unstable saddle points



Allows for a Lorentzian contour of integration

At the saddle points compactness is nevertheless achieved: a(0)=0

> [Louko; Di Tucci, JLL & Sberna]

## Correspondence with AdS



- Gravitational path integrals in AdS require the identical boundary condition in interior
- AdS/CFT implies  $\Psi = Bi \left[ \left( \frac{18\pi^2}{-\hbar\Lambda} \right)^{2/3} \left( 1 \frac{\Lambda}{3} q_f \right) \right]$

## Correspondence with AdS



For AdS: relevant saddle point is N<sub>-</sub>

Analytic continuation: 
$$Bi\left[\left(\frac{18\pi^2}{-\hbar\Lambda}\right)^{2/3}\left(1-\frac{\Lambda}{3}q_f\right)\right] = \sqrt{3}Ai\left[\left(\frac{18\pi^2}{\hbar\Lambda}\right)^{2/3}\left(1-\frac{\Lambda}{3}q_f\right)\right]$$

For dS: relevant saddle is also N<sub>-</sub>

## Correspondence with AdS



Implications:

- no contribution from topology changing (Hawking-Moss type) geometries at nucleation of the universe

- Stokes phenomenon describes emergence of time

# Simple no-boundary condition

• At the nucleation of the universe, wave function satisfies Wheeler-DeWitt equation in momentum space  $a \rightarrow i^{-1}$ 

$$\hat{H}_{(p)}\Psi = 0 \rightarrow (p^2 + 36\pi^4)\Psi + 12\pi^4\Lambda i\frac{\partial\Psi}{\partial p} = 0$$

- Regularity requires,  $p_0 = -6\pi^2 i$  so that we find that at the no-boundary point we have

$$i\frac{\partial}{\partial p}\Psi = \hat{q}\Psi = 0$$
 no-boundary condition

- Two interpretations:
  - No momentum flow into/out of universe at nucleation
  - Zero size condition imposed in momentum, rather than real, space

#### Some open questions

- Rigorous definition of probabilities
- Testable predictions?
- Going beyond minisuperspace
- Effective 4d vs full string theory
  - Robust to inclusion of Riemann<sup>^</sup>n terms [Jonas & JLL]

#### My collaborators on these projects:









#### Alice Di Tucci

#### Job Feldbrugge

#### Michal Heller



#### **Caroline** Jonas

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#### Neil Turok