A timeless history of time

Enrico Pajer

My brilliant collaborators:

Sadra Jazayeri (Postdoc)

David Stefanyszyn (Postdoc)

Scott Melville (Hawking Fellow)

Jakub Supeł (PhD student)

Harry Goodhew (PhD student)

Gordon Lee (PhD student)

All results are based on a series of recent papers

- 2004.09587: “On the symmetries of Cosmological Correlators”, with D. Green
- 2007.00027: “The Boostless Bootstrap: Amplitudes without Lorentz Boosts”, with J. Supel & D. Stefanyszyn
- 2009.02898: “The Cosmological Optical Theorem”, with H. Goodhew and S. Jazayeri
- 2010.12818: “A Boostless Bootstrap for the Bispectrum”
- 2103.08649: “From Locality and Unitarity to Cosmological Correlators”, with S. Jazayeri and D. Stefanyszyn
- 2103.09832: “Cosmological Cutting Rules”, with S. Melville

Motivations

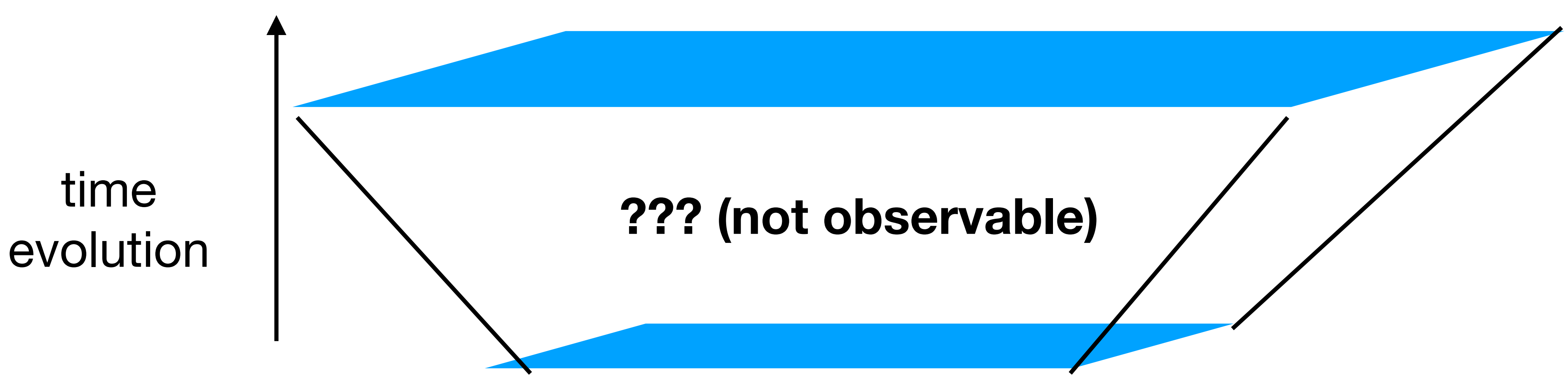
- Cosmological observations probe quantum field theory on curved spacetime and the perturbative regime of quantum gravity
- Studying the primordial universe we hope to learn about new degrees of freedom and interactions beyond the standard model and perhaps collect hints on the UV-completion of gravity
- In this talk I will describe recent progress in computing and understanding primordial observables using fundamental principles such as unitarity, locality and symmetries


Observables

The future boundary

- The goal of (primordial cosmology) is to compute Ψ at the end of inflation (on the reheating surface). We don't see the time dependence!
- Mathematically this is the space-like future conformal boundary of quasi de Sitter spacetime, t -> infinity (or η -> 0 in conformal time)

$$ds^2 = -dt^2 + a^2 dx^2 = a^2(-d\eta^2 + dx^2)$$



$$\boxed{\Psi[\phi; \eta \to 0]}$$

$$\Psi_{BD}[\phi; -\infty]$$

The wavefuncion

- The *wavefunction of the universe,* is a functional of the all fields in the theory (including the metric) at some time:

$$\Psi[\phi] = \exp\Big[- \sum_{n=2}^{\infty} \int \psi_n \phi(\mathbf{k}_1) \ldots \phi(\mathbf{k}_n) \Big] \,,$$

- All probabilities can be computed as in QM

$$\langle \mathcal{O} \rangle = \int d\phi \, \Psi^* \, \mathcal{O} \Psi$$

- It can be computed perturbatively from a path integral

$$\Psi[\phi(\mathbf{x}); \eta_0] = \int_{\text{vacuum}}^{\phi(\mathbf{x}, \eta_0)} [\mathcal{D}\phi] \, e^{iS[\phi]}$$

Cosmological correlators

- The Ψ_n coefficient are closely related to *cosmological correlators*, which determine the statistic of the Cosmic Microwave Background anisotropies and of Large Scale Structure inhomogeneities (e.g. galaxies, Dark Matter, …)

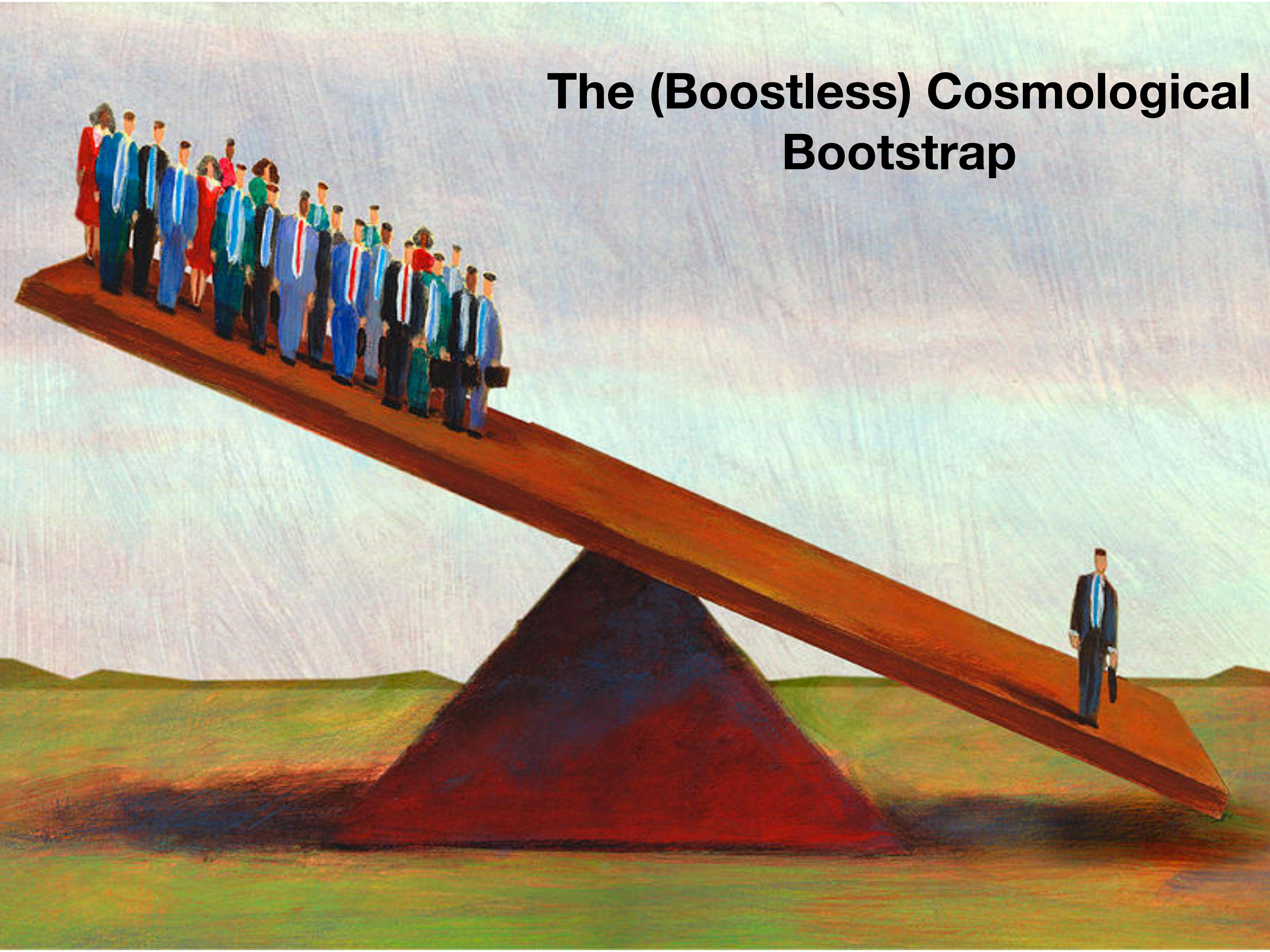
- For example

$$P(k) = \frac{1}{2\,\mathrm{Re}\,\psi_2(k)}$$

$$B_3 = -\frac{2}{\prod_a^3 \mathrm{Re}\,\psi_2(k_a)}\,\mathrm{Re}\,\psi_3$$

$$B_4 = -\frac{2}{\prod_a^4 \mathrm{Re}\,\psi_2(k_a)}\left[\mathrm{Re}\,\psi_4 - \sum_{s,t,u}\frac{\mathrm{Re}\,\psi_3\,\mathrm{Re}\,\psi_3}{\mathrm{Re}\,\psi_2}\right]$$

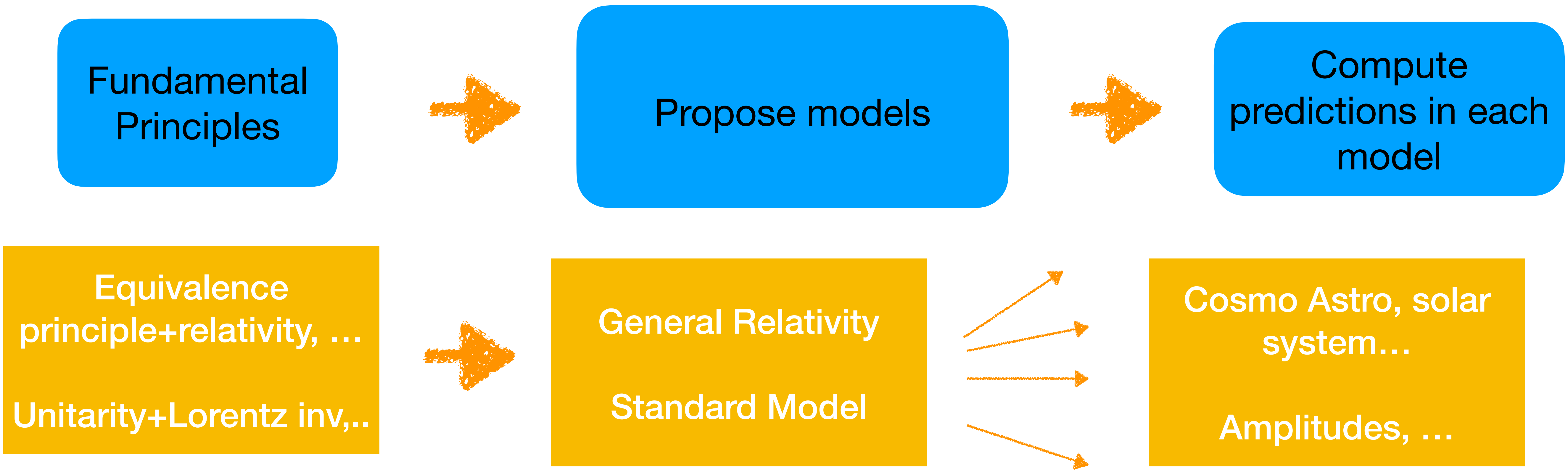
The (Boostless) Cosmological Bootstrap



Making predictions

- How do we make predictions?

- Option 1:

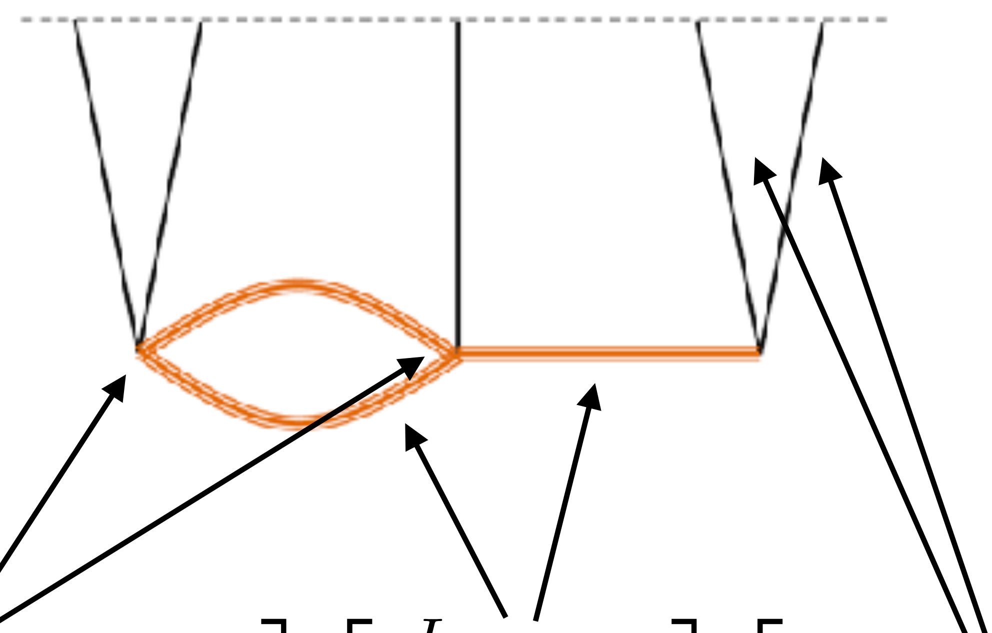


- Option 1 is great when we have *a well-established theory*, e.g. General relativity or the Standard Model, and *many* observables

Feynman Diagrams

- Given a model, e.g. GR + a scalar, we can compute the wavefunction with the following Feynman rules.

- For a given set of fields (i.e. of free propagators), the vertices F are computed from the model Lagrangian



$$\psi_n = \left[\prod_{A}^{V} \int d\eta_A \, F_A\right] \left[\prod_{m}^{I} G(p)\right] \left[\prod_{a}^{n} K(k_a)\right]$$

$$\begin{aligned}\psi_n &= \psi_n(\{k\}; \{p\}; \{\mathbf{k}\}) \\ &= \psi_n(\text{external energies}; \text{internal energies}; \text{contractions})\end{aligned}$$

“Bulk” computations

This approach is straightforward conceptually but has a few *drawbacks*:

1. The relation between Lagrangian and observables is many to one because of *field redefinitions and gauge symmetries*

2. Many different “Bulk Lagrangians” give the similar boundary observables

3. Even at tree-level there are V *nested integrals* (due to the lack of time-translation invariance; cf. amplitudes). Hard to compute!

4. Fundamental principles (e.g. unitarity & locality) are obscured: How do I know a given wavefunction comes from a unitary & local theory?

5. More ambitiously, to derive *cosmological positivity bounds and non-perturbative correlators* we need a more fundamental understanding

Making predictions: the Bootstrap approach

- option 2:

Fundamental Principles

~~Propose models~~

Compute predictions

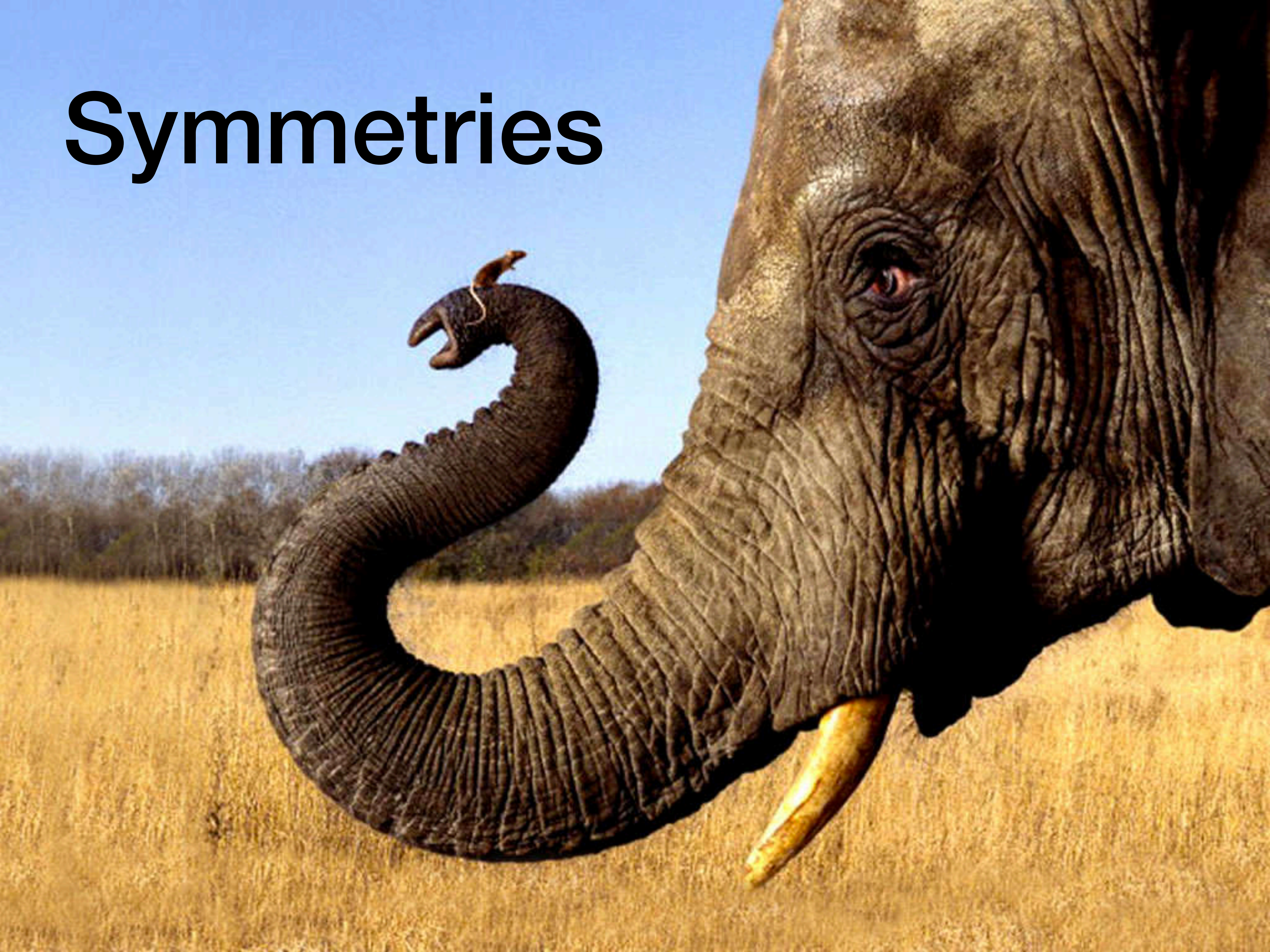
Unitarity, locality, symmetries

Too many theories of inflation, modified gravity Dark Energy, Dark Matter,

Cosmological corraltors

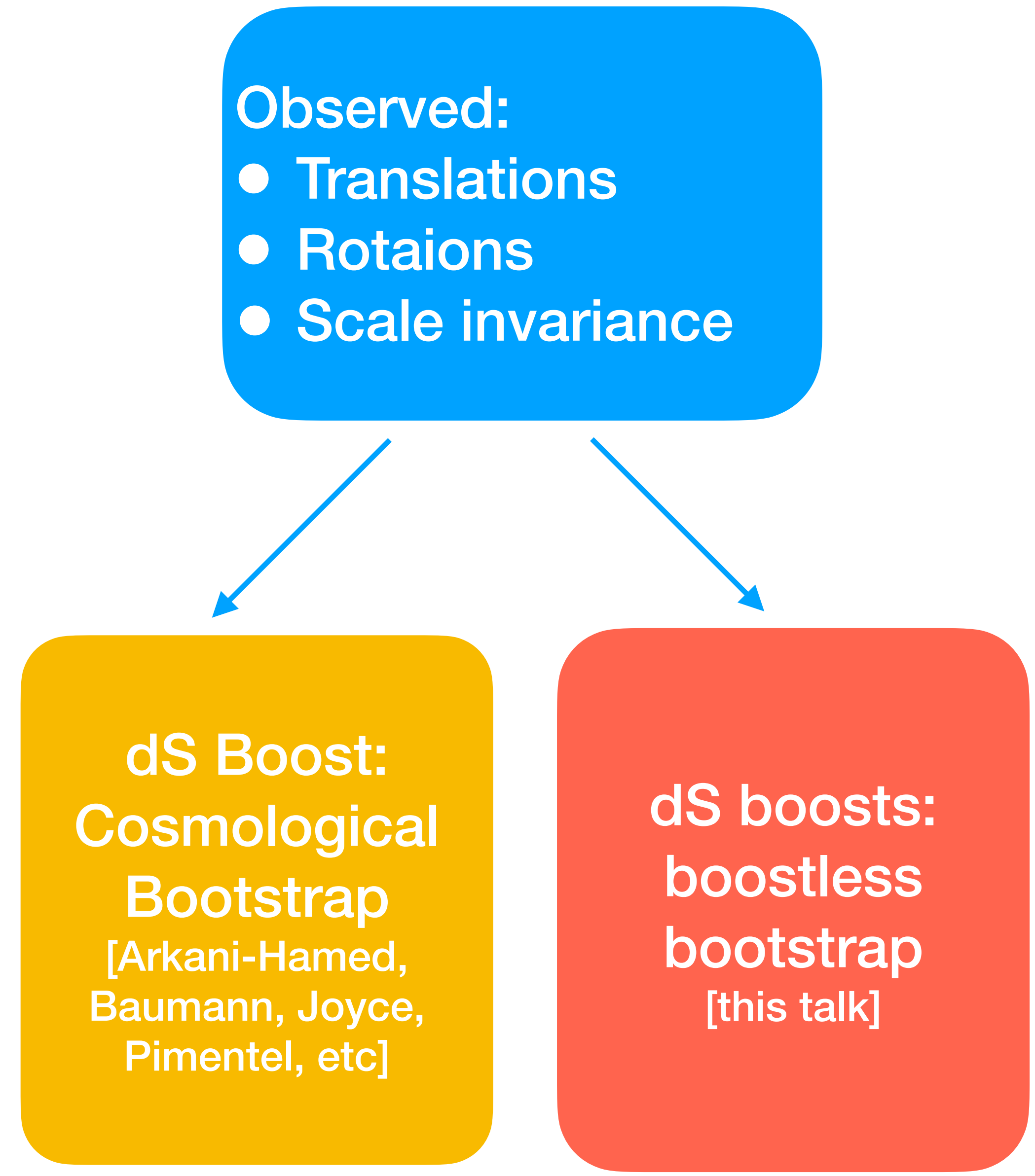
- Option 2 is great when we have *too many theories*, e.g. inflation or Beyond the Standard Model, *and* we compute always the same observables, e.g. cosmo correlators or amplitudes

Symmetries



Observed symmetries

- Cosmological perturbations are observed to be statistically *homogeneous* and *isotropic*
- Primordial perturbations are also observed to be approximately scale invariant
- Anything else?
  - With *de Sitter boost* we can derive general results and connect with Conformal Field Theory and holography. See beautiful progress by Maldacena, Pimentel, Arkani-Hamed, Baumann, Joyce, etc…
  - If we are instead more interested in phenomenology, we cannot assume Boost invariance. Here is why…



Three theorems [Green & EP ’20]

Assuming homogeneity, isotropy and scale invariance, I’ll prove:

Theorem 1: The correlators of curvature perturbations are uniquely characterized by their soft limits (no field redefinition ambiguities)

Theorem 2: The only theory of *curvature perturbations* with full de Sitter symmetries is the free theory

Theorem 3: de Sitter symmetries are the largest possible set of symmetries for any single scalar field

Th. 1 & 2 are valid *only in single-clock cosmology* (while Th. 3 is general)

There are no further assumptions about the particle content (any mass and spin) or the interactions

Theorem 3

- The space of symmetries is constrained by self-consistent dynamics, as in the Coleman-Mandula theorem.

- The only symmetries that can be linearly-realized on single scalar are

assumed observed symmetries

$$\sum_{a=1}^{n} \vec{k}_a \langle \phi(k_1) \ldots \phi(k_n) \rangle = 0$$ translations

$$\sum_{a=1}^{n} k_a^{[i} \partial_{k_a^{j]}} \langle \phi(k_1) \ldots \phi(k_n) \rangle = 0$$ rotations

$$\sum_{a=1}^{n} (3 - \Delta + k_a \partial_{k_a}) \langle \phi(k_1) \ldots \phi(k_n) \rangle = 0$$ dilations

$$\sum_{a=1}^{n} \left[2\vec{k} \cdot \vec{\partial} \partial_i - k_i \partial^2 + 2\,(3 - \Delta)\, \partial_i \right] \langle \phi(k_1) \ldots \phi(k_n) \rangle = 0$$ dS boosts

additional

Th. 2: Conformal = free

- De Sitter symmetries act on ζ as

Exact on in the decoupling slow-roll limit ε, η →0

$$\boxed{\begin{aligned} P_i &: \delta\zeta = -\partial_{x^i}\zeta\,, \\ M_{ij} &: \delta\zeta = 2x_{[i}\partial_{x^j]}\zeta\,, \\ D &: \delta\zeta = -\vec{x}\cdot\vec{\partial}\zeta\,, \end{aligned}}$$

$$\boxed{K^i : \delta\zeta = -2x^i\left(\vec{x}\cdot\vec{\partial}\zeta\right) + x^2\partial_{x^i}\zeta\,,}$$

translations

rotations

dilations

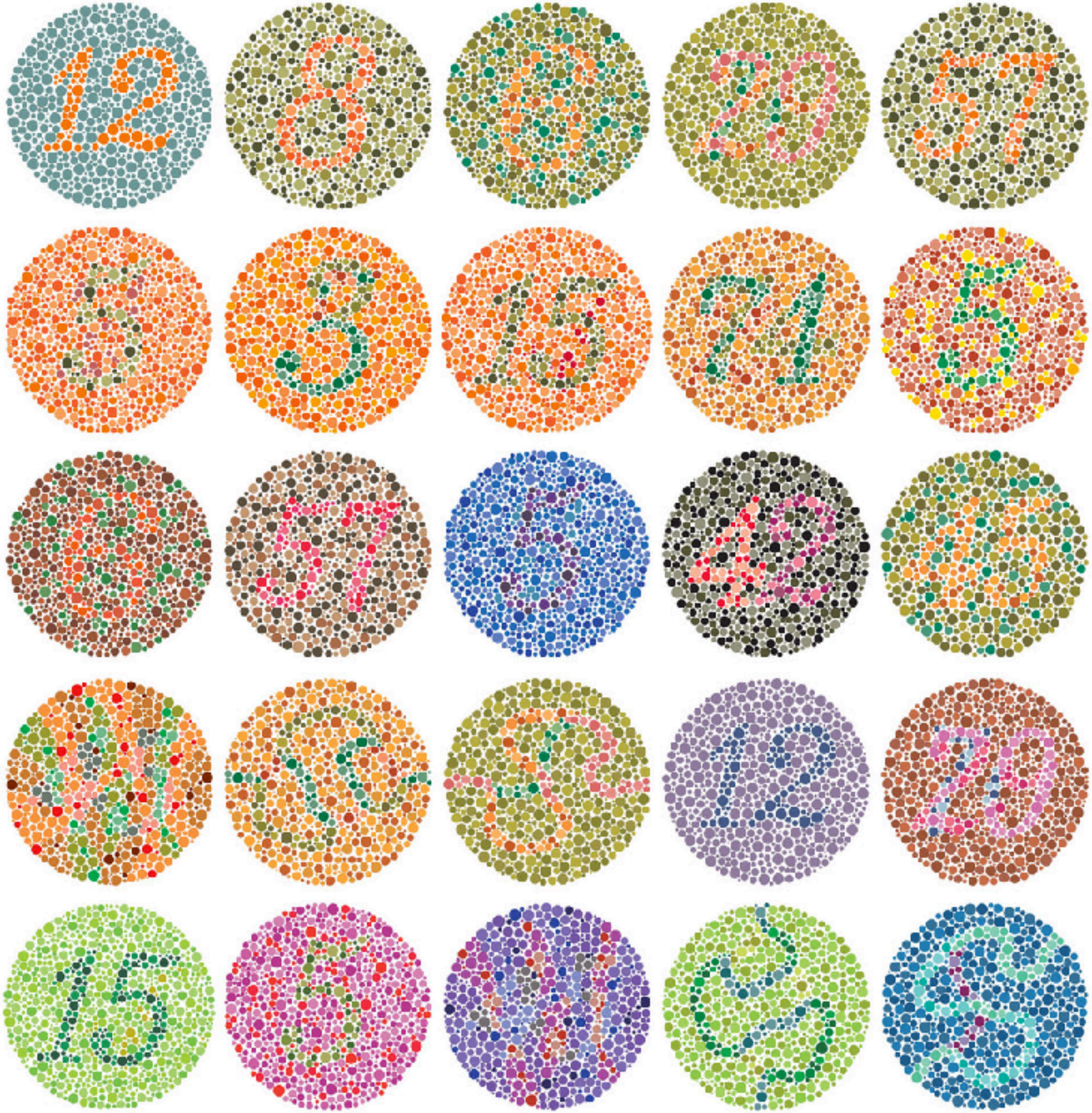
boosts

- Single-clock cosmology must also be invariant under

$$D_{\rm NL} : \delta\zeta = -1 - \vec{x}\cdot\vec{\partial}\zeta\,,$$

$$K^i_{\rm NL} : \delta\zeta = -2x^i - 2x^i\left(\vec{x}\cdot\vec{\partial}\zeta\right) + x^2\partial^{x^i}\zeta\,.$$

soft theorems



Locality

Locality

- Locality is colloquially the fact that what happens here cannot affect a system far away. Operators commute for space-like separation and correlators must factorise at large distances (*cluster decomposition*).

- A common sufficient condition for locality is *Manifest Locality*: Lagrangian interactions are products of operators *at the same spacetime point*. No inverse laplacians are allowed.

- Remarkably, we proved that the wavefunction must satisfy the very simple *Manifestly Local Test (MLT)* [Jazayeri, EP & Stefanyszyn '21]

$$\boxed{\frac{\partial}{\partial k_c}\psi_n(k_1, ..., k_n; \{p\}; \{\mathbf{k}\})\Big|_{k_c=0} = 0\,, \qquad \forall c = 1, \ldots, n\,,}$$

Derivation

There are two derivations: (*i*) a pure boundary derivation using unitarity and singularities (see paper) and (*ii*) a bulk derivation that uses the Feynman rules.

Notice that as k->0 there is no linear term in K

$$\lim_{k\to 0} K(k,\eta) = \lim_{k\to 0}(1-ik\eta)e^{ik\eta} = \left(1 + 0 \times k\eta + \frac{1}{2}k^2\eta^2 + \ldots\right)$$

which is also true for all time derivatives. The same is then true for the wavefunction coefficient

$$\psi_n = \left[\prod_A^V \int d\eta_A \, F_A\right] \left[\prod_m^I G(p)\right] \left[\prod_a^n K(k_a)\right]$$

Manifest locality

- This is true as long as there are *only positive powers of k* in the interactions, i.e. the theory is *manifestly local.*

- All large non-Gaussianities in single field inflation come from manifestly local interactions in the EFT of inflation, e.g.

$$\mathcal{L} \supset \dot{\phi}^3 + (\partial\phi)^2\dot{\phi} + \dot{\phi}^4 + \dots$$

- But gravity has not manifestly local interactions after we integrate out the non-dynamical lapse and shift (to which our MLT does not apply)

$$\mathcal{L}_{GR} \supset \dot{\zeta}^2\nabla^{-2}\dot{\zeta} + \dots$$

- Manifest locality -> locality but *not* vice versa. How does locality relate inverse Laplacians to massless spinning particles in the spectrum, e.g. solid inflation?

Amplitude limit

- The residue of the total-energy pole (kT=k1+…+kn=0) of (tree-level) correlators is fixed by the (UV-limit of the) amplitude [Maldacena & Pimentel ’11; Raju ’12; Arkani-Hamed et al ’17-‘18; Benincasa ’18]

- The precise relation is [Goodhew, Jazayeri & EP ’20]

$$\lim_{k_T \to 0} B_n = \frac{(-1)^n H^{p+n-1}(p-1)!}{2^{n-1}} \times \frac{\mathrm{Re}\left(i^{1+n+p} A_n\right)}{\left(\prod_{a=1}^{n} k_a\right)^2 k_T^p} \,,$$

- where p is fixed by dimensional analysis and scale invariance. For the bispectrum it’s simply the number of derivatives. More generally [EP ’20]

$$p = 1 + \sum_\alpha (D_\alpha - 4)$$

The MLT enforces the amplitude to be manifestly local, i.e. only have positive powers of momenta

Unitarity

Unitary time evolution

- In Quantum Mechanics we compute probabilities, which must be between 0 and 1 to make sense

- This requires the positive norm of states in the Hilbert space *and* Unitary time evolution, $UU^\dagger=1$. Colloquially this is the *conservation of probabilities*

- The consequences of unitarity for particle physics amplitudes were discover over *60 years ago*: the Optical theorem and Cutkosky Cutting Rules.

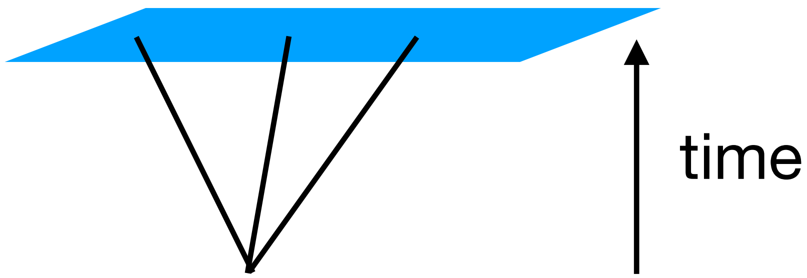
- In cosmology we don't see the time evolution, so how can we see it's unitary?!

The Cosmological Optical Theorem (COT) [Goodhew, Jazayeri, EP '20]

- From unitarity, $UU^{\dagger}=1$, we found infinitely many relations.

- The simplest applies to contact n-point functions

$$\psi_n(\{k\},\{\mathbf{k}\}) + \psi_n^*(-\{k\},-\{\mathbf{k}\}) = 0$$



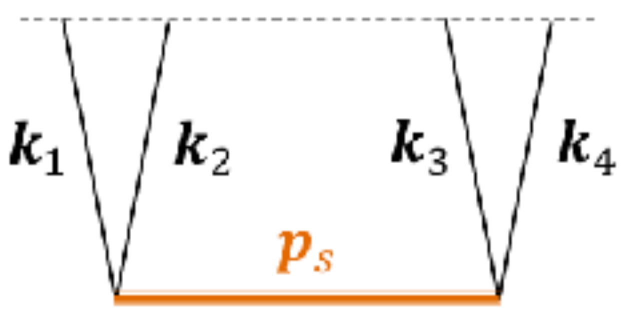
- It follows from unitarity time evolution, but the equation does not involve time! Time "emerges" at boundary as in holography…

- This is a Cosmological Optical Theorem (COT) and can be interpreted as fixing a "discontinuity"

$$\mathrm{Disc}\psi_n(\{k\},\{\mathbf{k}\}) \equiv \psi_n(\{k\},\{\mathbf{k}\}) + \psi_n^*(-\{k\},-\{\mathbf{k}\})$$

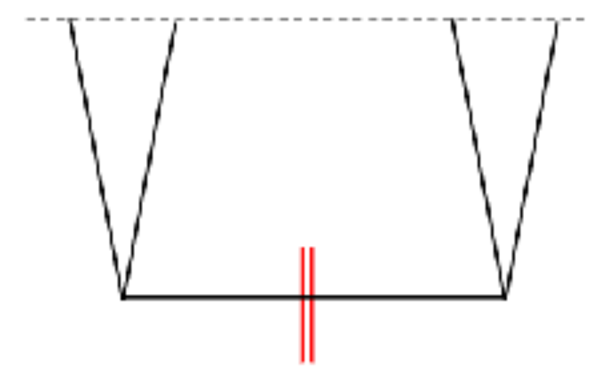
Exchange diagrams

- The next simplest case is a 4-particle exchange diagram (trispectrum). The Cosmo Optical Theorem (COT) is

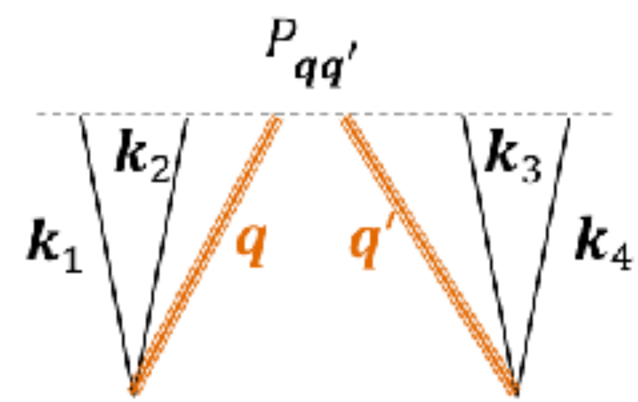


$i\,\mathrm{Disc}_{p_s}\left[i\psi^{(s)}_{k_1k_2k_3k_4}\right]$

$=$



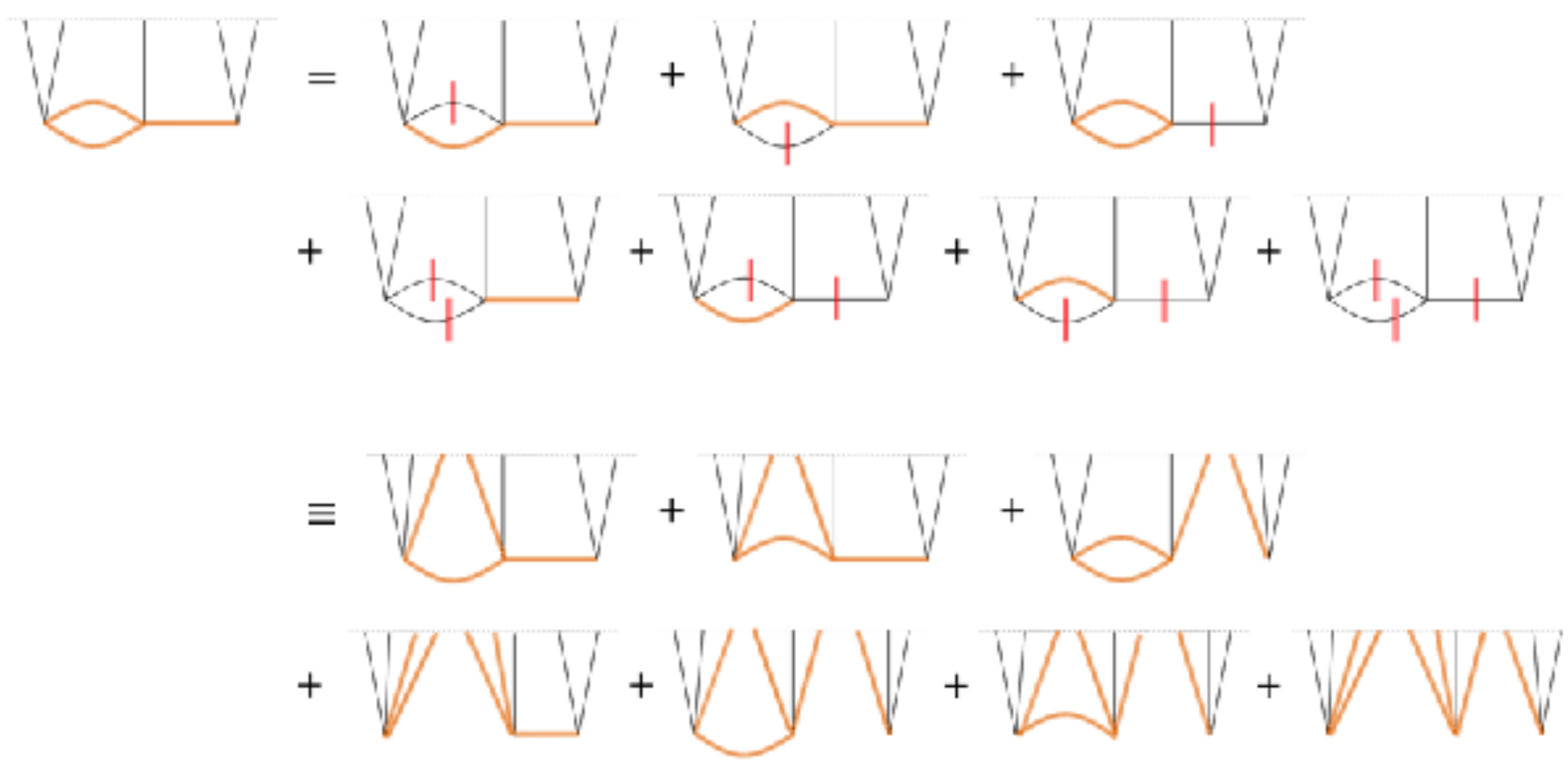
$\equiv$



$i\,\mathrm{Disc}_{q}\left[i\psi_{k_1k_2q}\right]\; P_{qq'}\; i\,\mathrm{Disc}_{q'}\left[i\psi_{q'k_3k_4}\right]$

General diagrams

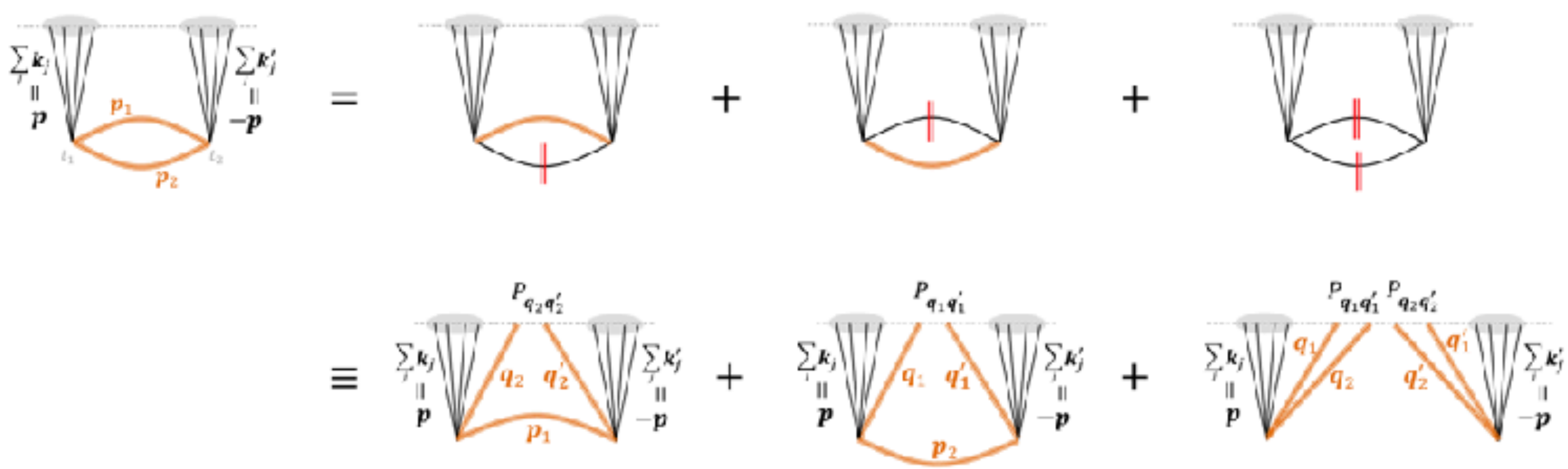
- These relations are valid to *all order in perturbation theory to any number of loops for fields of any mass and spin and arbitrary interactions (around any FLRW admitting a Bunch Davies initial condition)* [Goodhew, Jazayeri & EP ’21; Melville & EP ’21]



- These are Cosmological Cutting Rules. With a 60 year delay over particle physics, we finally understand unitarity in cosmology.

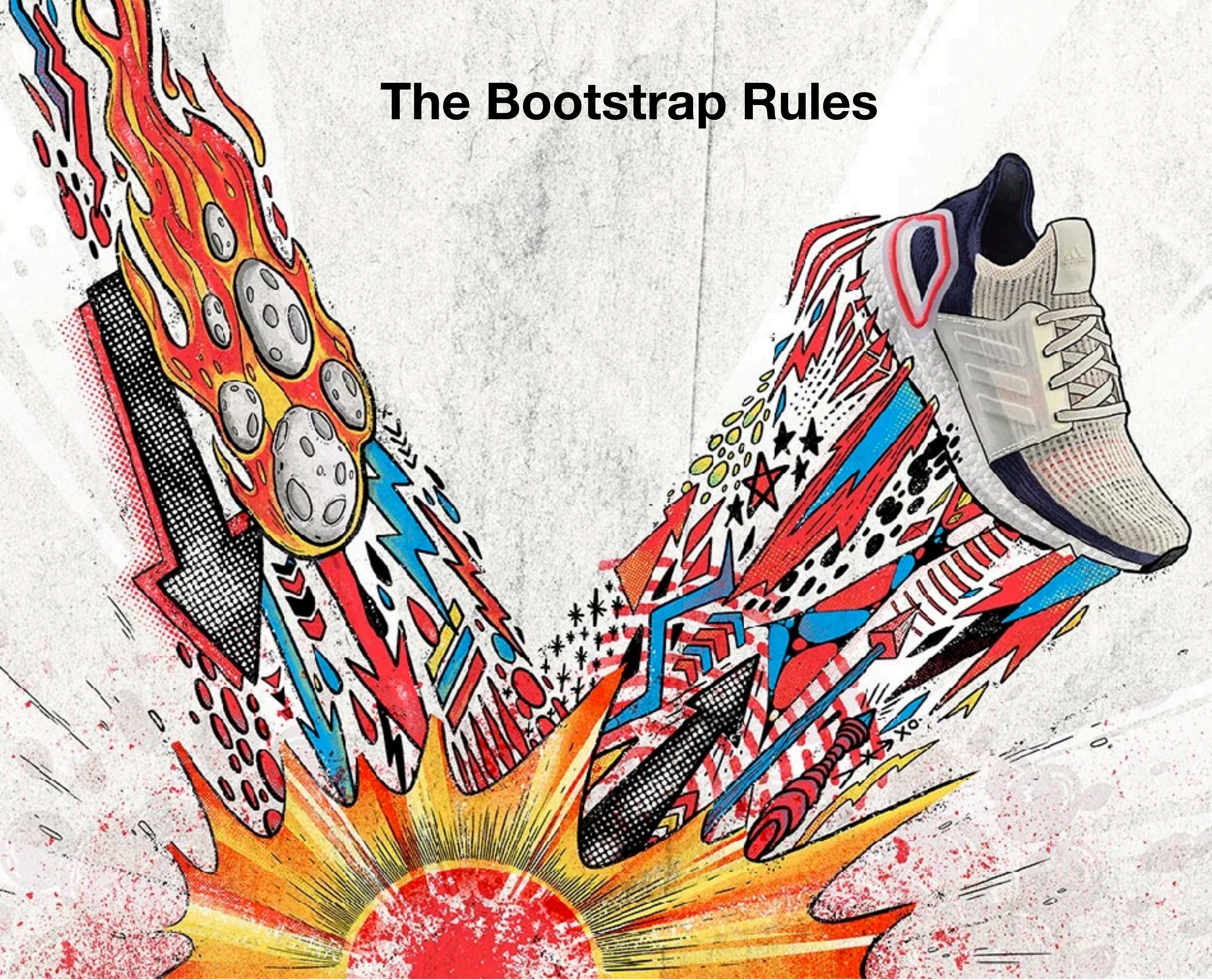
Loop corrections

- Unitarity gives us also *loop corrections*! For example we compute the leading 1-loop corrections for the power spectrum in the EFT of inflation, from tree-level results.



$$i\mathrm{Disc}\left[i\psi^{\text{1-loop}}_{\mathbf{k}_1\mathbf{k}_2}\right] = \frac{H^2}{f_\pi^4}\frac{ik^3}{480\pi}\frac{(1-c_s^2)^2}{c_s^4}\left[(4\tilde{c}_3+9+6c_s^2)^2+15^2\right]$$

The Bootstrap Rules



Bootstrap Rules

- To complete the derivation we need a set of Bootstrap Rules [EP '20]
- As an example, let's bootstrap the bispectrum (3-point function) of a scalar

Scale invariance

Bose symmetry

$$\psi_3 = \frac{\mathrm{Poly}_{3+p}(\overbrace{k_T, e_2, e_3})}{k_T^p}$$

$$k_T \equiv k_1 + k_2 + k_3$$
$$e_2 \equiv k_1 k_2 + k_2 k_3 + k_1 k_3$$
$$e_3 \equiv k_1 k_2 k_3$$

tree level in dS

Bunch Davies vacuum

The calculation

- The Bootstrap Rules reduced the problem to determining the numerical constants C_mn via the Manifestly Local Test

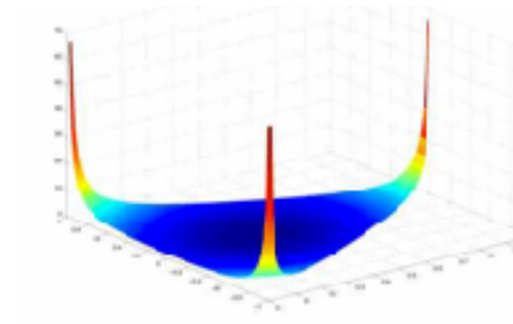
$$\psi_3^{(p)}(k_1, k_2, k_3) = \frac{1}{k_T^p} \sum_{n=0}^{\lfloor \frac{p+3}{3} \rfloor} \sum_{m=0}^{\lfloor \frac{p+3-3n}{2} \rfloor} C_{mn} k_T^{3+p-2m-3n} e_2^m e_3^n,$$

$$\partial_{k_1} \psi_3 \Big|_{k_1=0} = 0$$

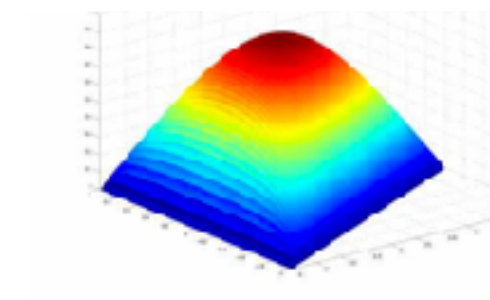
- This yields all manifestly local bispectra for a scalar to *any order in derivatives* in the EFT of inflation

- This gives order by order the shapes of non-Gaussianity that are constraint e.g. by the Cosmic Microwave Background, e.g. the Planck mission

Shapes of non-Gaussianity



$$\psi_3^{(0)} = A_0 \left[4e_3 - e_2k_T + (3e_3 - 3e_2k_T + k_T^3)\log(-k_T\eta/\mu)\right]$$
$$\psi_3^{(1)} = 0$$
$$\psi_3^{(2)} = A_2 \left[-k_T^3 + 3k_Te_2 - 11e_3 + \frac{4e_2^2}{k_T} + \frac{4e_2e_3}{k_T^2}\right]$$
$$\psi_3^{(3)} = A_3\frac{1}{k_T^3}(k_T^6 - 3k_T^4e_2 + 11k_T^3e_3 - 4k_T^2e_2^2 - 4k_Te_2e_3 + 12e_3^2) + A_3'\frac{e_3^2}{k_T^3}$$



- Ψ contains the famous *local non-Gaussianity,* while Ψ the so-called *equilateral and orthogonal non-Gaussianities*, the main tar gets of non-Gaussian searches in the CMB and galaxy surveys!

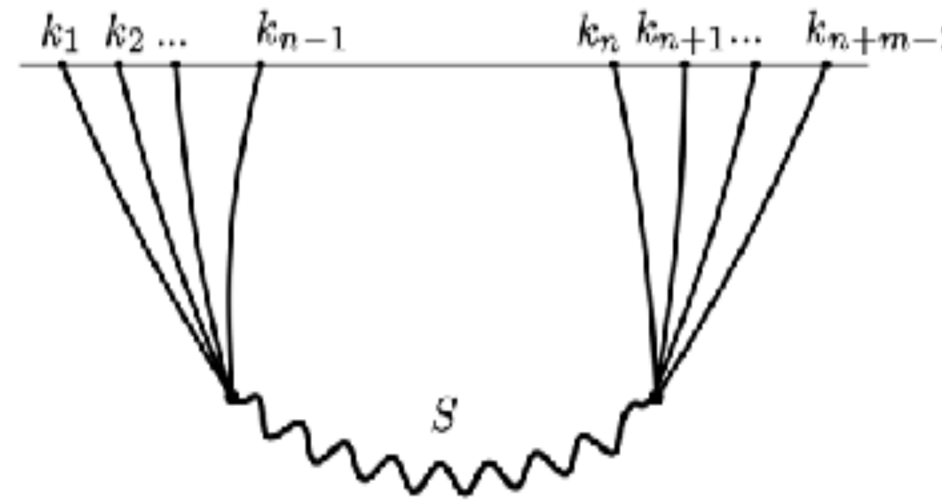
- In the standard approach the numerical coefficient come from time integrations, here they’re fixed algebraically

Comments and extensions

- A similar derivation gives all possible graviton non-Gaussianities! [EP ’20; Cabass, EP, Stefanyszyn & Supel to appear]
- The bootstrap derivation
  - is numerical much faster than performing the traditional (in-in) time integrals.
  - makes the role of fundamental principle transparent
  - can be extended to any number of field of any spin

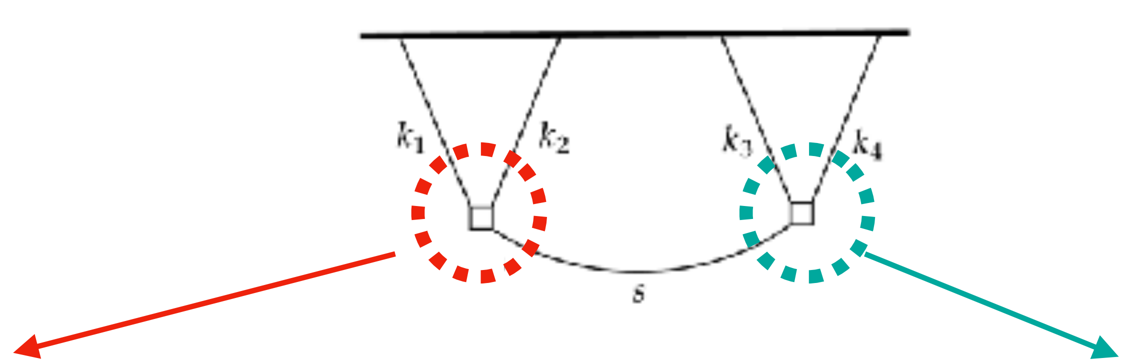
Exchange diagrams

- Also exchange diagrams can be computed using *partial energy recursion relations.* These use the Cosmo Optical Theorem to fix all residue of partial energy singularities.



Tree-level n-point function have only two types of singularities:

- *Total energy poles*, the residue is an amplitude: $$k_T = \sum_{a=1}^{n} k_a \to 0$$
- P*artial energy poles*, all residues are fixed by the COT



$$E_L = k_1 + k_2 + s, \qquad s = |\mathbf{k}_1 + \mathbf{k}_2| \qquad E_R \equiv k_3 + k_4 + s,$$

Partial Energy Recursion Relations

- The residues of all partial energy singularities of the exchange diagram are fixed by the Cosmo Optical Theorem
- Since the correlator is an analytic function, it is determined by these residues (plus a residue at infinity which is fixed by locality and unitarity up to a contact term)

$$\psi_4(E_L, E_R) = \oint \frac{dz}{z}\psi_4(E_L + z, E_R - z)$$
$$= \sum \mathrm{Res}\left[\mathrm{Cosmo\ Opt\ Th}\right] + \mathrm{Boundary}$$

Summary

- Cosmological observations test high energy physics, the perturbative regime of quantum gravity and discover new particles and forces.

- In the last two years we have made tremendous progress on understanding how fundamental principles such as unitarity and locality are encoded in observables (cosmo correlators).

- We have a *Cosmological Optical Theorem*, a *Manifestly Local Test* that enforce *unitarity* and *locality* of the observables without any reference to time.

- Fundamental principles are so powerful that observables are *bootstrapped* directly from them, without the need to write down explicit models and Lagrangians.

Connections

This bootstrap program connects with many topics discussed at this workshop:

- *de Sitter in String theory*: from the singularity of correlators and unitarity we can prove whether a UV-complete correlator/wavefunction exists or not and if it comes from a string theory

- *phenomenology, CMB & LSS*: the bootstrap makes predictions for all primordial observables compatible with the chosen fundamental principle. So we can use the data to extract model-independent information

- *Gravitational waves, scalar-tensor theories, modified gravity*: we can predict all graviton correlators for any modified theory of gravity that respects unitarity locality and the chosen symmetries

- *Quantum gravity in dS*: this time-less history of time gives us the rules that the hypothetical CFT/QFT holographic dual of dS must obey. What does the Cosmo Optical Theorem and Manifestly Local test mean for the QFT/CFT??