Holographic routes to de Sitter

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1704.05075 with C.Charmousis and E.Kiritsis
1711.08462, 1807.09794 with J.K.Ghosh, E.Kiritsis, L.Witkowski
1904.02727 with A.Amariti, C.Charmousis, D. Forcella and E.Kiritsis

Introduction

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- de Sitter cosmology was/is at some point or another a good approximation for the observed universe.

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However:

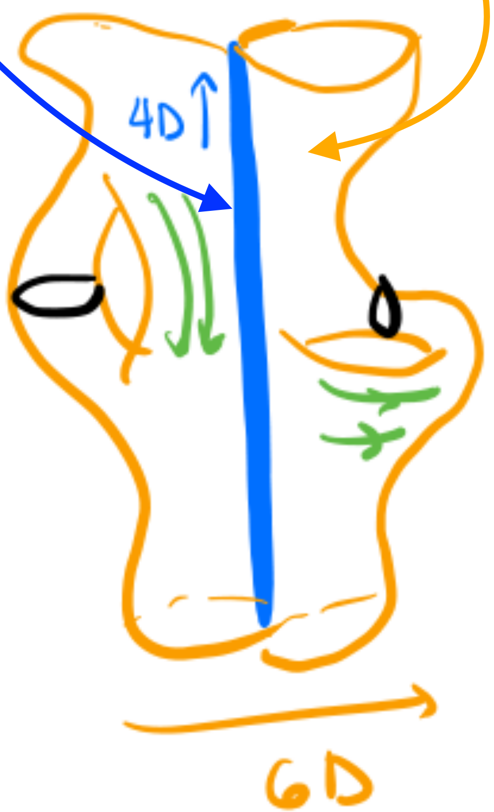
Empirical fact: 4D de Sitter spacetime is extremely difficult to construct within string theory (in a controllable and reliable way).

One of the most troubling clashes between fudamental theory and observation.

Constructing dS in String Theory

Typical 4D de Sitter constrution in string theory: warped compactification

$$ds_{10}^2 = e^{2A(y)} \underbrace{g_{\mu\nu}(x) dx^\mu dx^\nu}_{\text{4D}} + \underbrace{g_{mn}(y) dy^m dy^n}_{\text{6D}} \quad + \quad \text{Fluxes, D-branes, etc}$$



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1. Compact 6d manifold (finite 4d M_p) $\Rightarrow$ low-energy 4D EFT for $g_{\mu\nu}(x)$ + a bunch of scalars
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3. $V^{eff} > 0$ at the stabilized minimum;

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$\Rightarrow$ 4D de Sitter is a solution of effective 4D theory for $g_{\mu\nu}(x)$

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More rigourosly: solve the full 10D equations first, reduce later.

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TLDR: Obtain dS as a classical sollution of an effective 4D theory of gravity + other stuff

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- Tree-level, perturbative and non-perturbative effects all enter the effective potential
- Need to up-lift a negative V_{eff} to a (small) positive V_{eff} via susy-breaking elements (anti D-branes). Is this under control (huge debate)?
- Does dS survive when solving the full 10D theory?

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More generally: there are conjectures about possible *in principle* obstructions for this approach to work (*swampland* program).

This talk: alternative approach

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I will illustrate this in a simple bottom-up model based on holography.

- Gravity side: non-compact asymmetric braneworld
- Dual field theory: no *dynamical* 4d gravity in the UV;
- Emergent 4d gravity coupled to observed fields;
- No *local* 4d EFT description
- Positive curvature because of either:
  1. External sources turned on;
  2. Excited state above the vacuum;
- Mechanism to control the cosmological constant *(self-tuning)*

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Top down sting th.

KKLT-like

Outline

- Braneworld Setup
- Self-tuning Minkowski vacua.
- dS #1: Stabilized de Sitter brane in an RG flow geometry
- dS #2: de Sitter geometry on a moving brane.

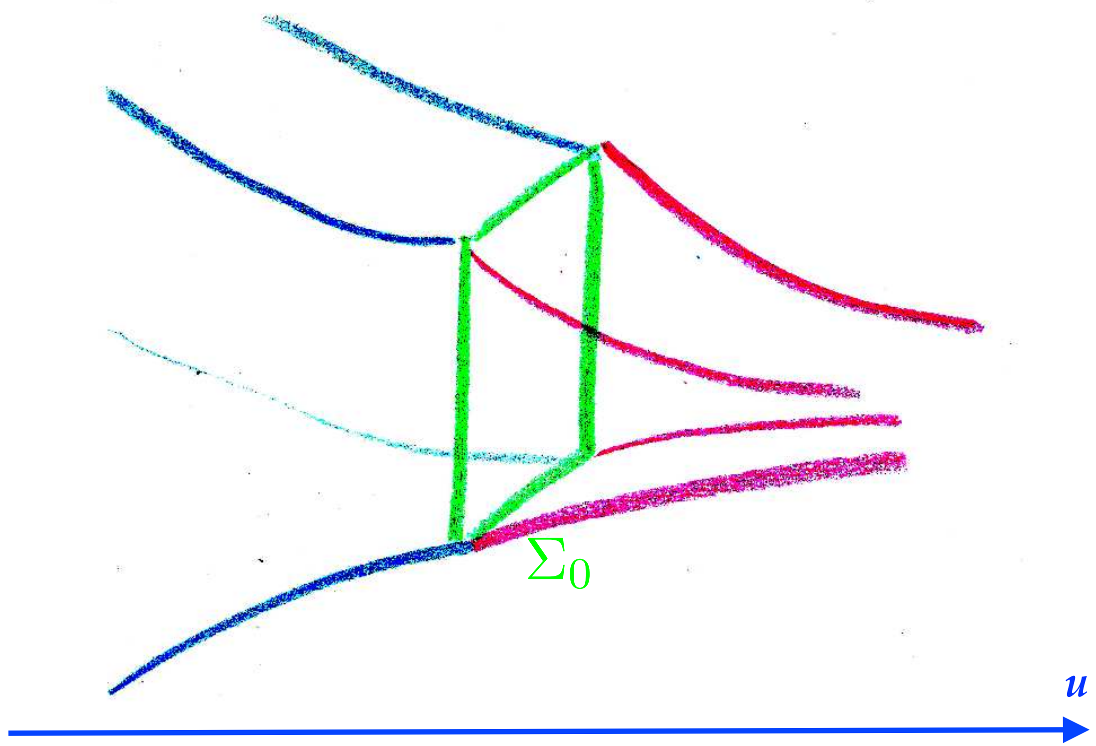
The Model

- 5D Einstein-Dilaton gravity $g_{ab}(x^\mu, u), \varphi(x^\mu, u)$
- 4D Defect (with SM on it)
- Integrating out SM fields gives 5D Einstein-Dilaton + localized effective action for on the defect for induced fields $\gamma_{\mu\nu}, \varphi$
- Holography (aka *gauge/gravity duality*): Dual interpretation as a purely 4D theory with SM + a strongly interacting QFT

The model

$$S = M^3 \int d^4x \int du \sqrt{-g} \left[R - \frac{1}{2} g^{ab} \partial_a \varphi \partial_b \varphi - V(\varphi) \right]$$

$$+ M^3 \int_{\Sigma_0} d^4\sigma \sqrt{-\gamma} \left[-W_B(\varphi) - \frac{1}{2} Z(\varphi) \gamma^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + U(\varphi) R^{(\gamma)} \right]$$



The model

5d Bulk action

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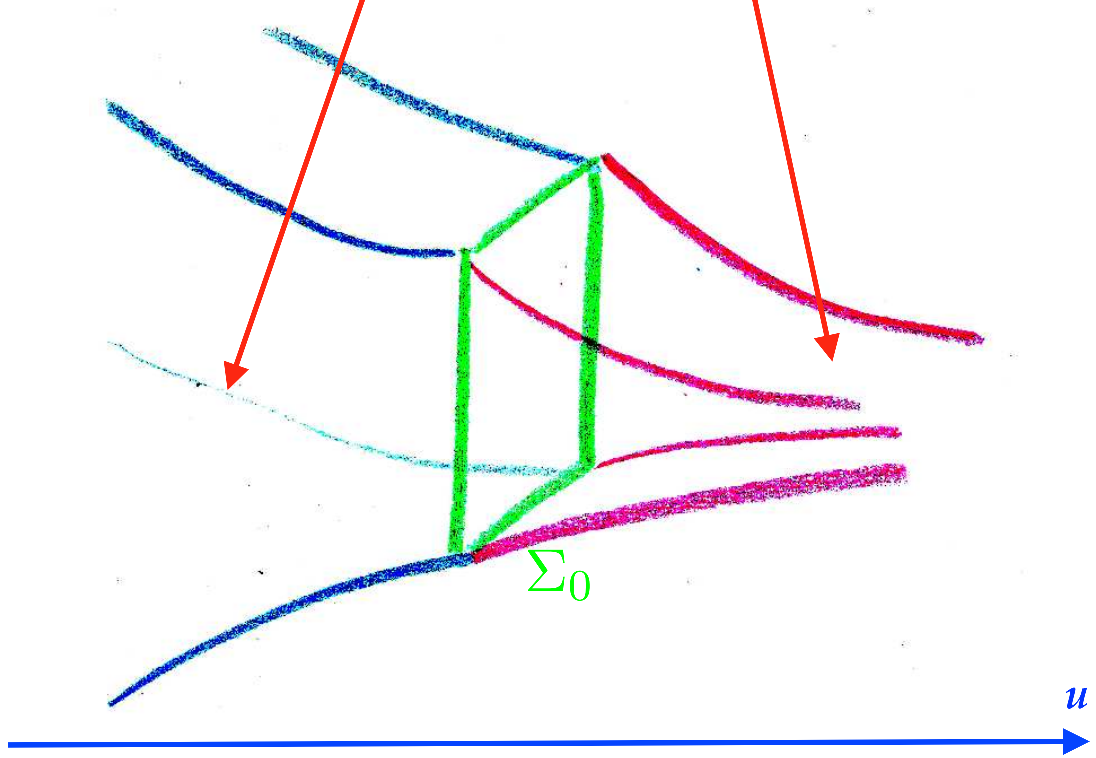

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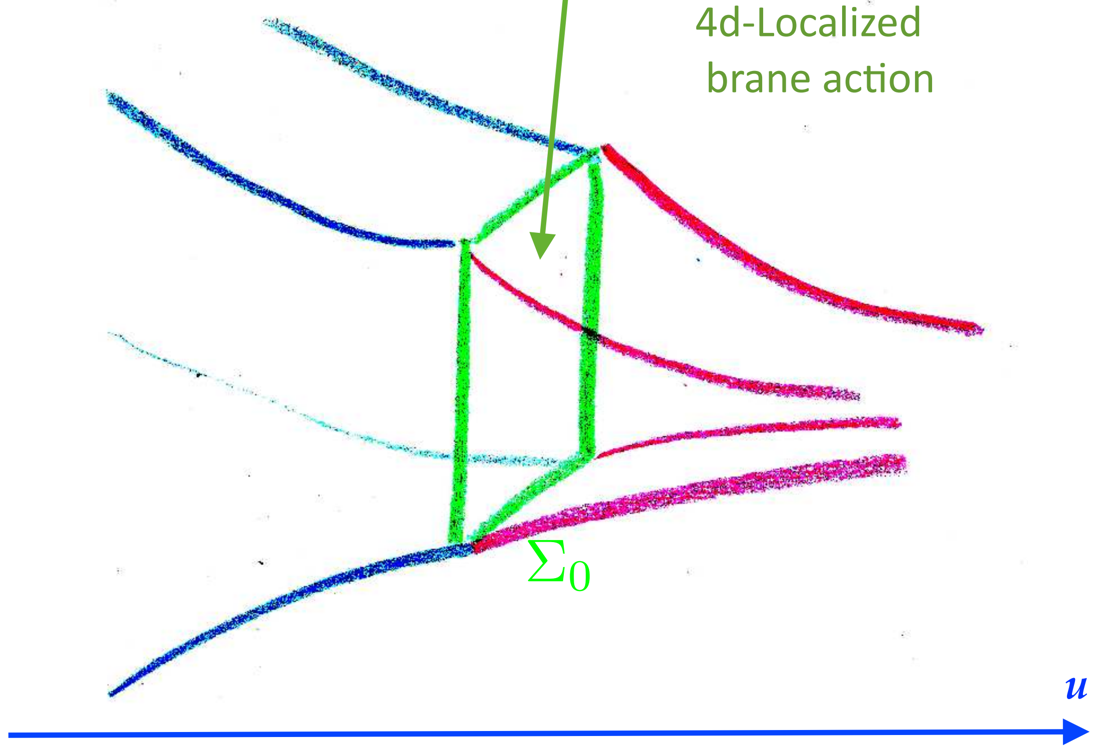
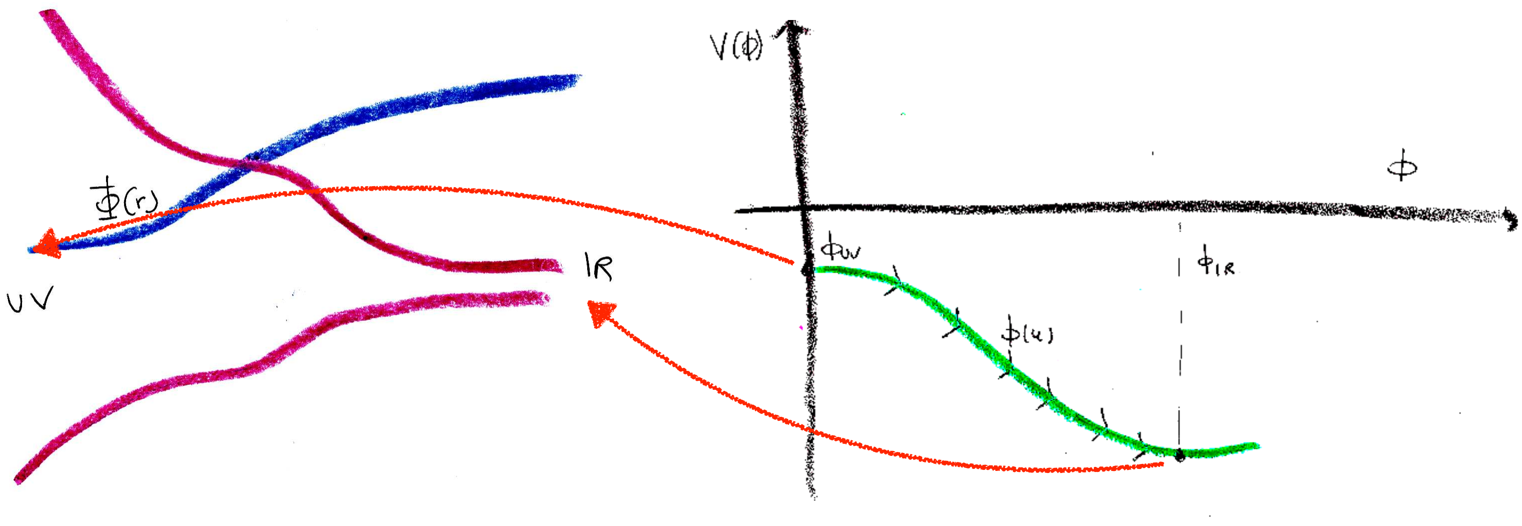
4d-Localized
brane action



Minkowski vacuum solution

Bulk geometry: holographic RG flow

$$ds^2 = du^2 + e^{2A(u)}\eta_{\mu\nu}dx^\mu dx^\nu, \qquad \varphi = \varphi(u)$$



Supported by negative bulk potential with one or more AdS extrema.

Minkowski vacuum solution

Two solutions joined at the brane:

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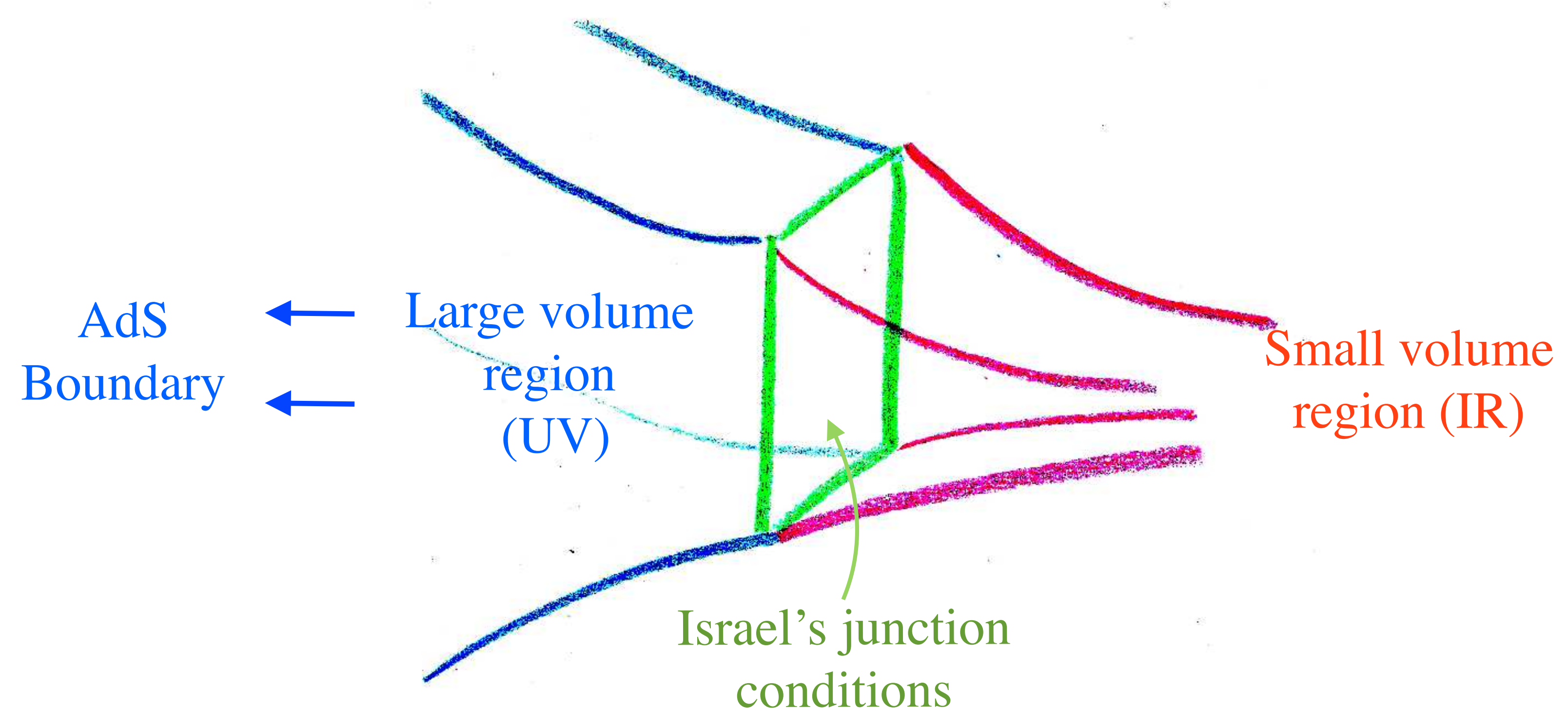

IR Regularity + junction conditions $\Rightarrow$ isolated solution(s) for generic brane vacuum energy (*self-tuning of CC*)

Emergent gravity on the brane

Do gravitational interactions between brane sources look 4d?

Emergent gravity on the brane

Do gravitational interactions between brane sources look 4d?

- Volume is infinite in the UV $\Rightarrow$ no low energy 4d gravity.
- Localized Einstein term $\Rightarrow$ existence of a 4d-like graviton resonance (Dvali,Gabadadze,Porrati, '00) at "short" distances.

$$S = M^3 \int du\, d^4x\, \sqrt{g} R_5 + \ldots + M^3 \int_{u=u_0} d^4x\, \sqrt{\gamma} U(\varphi_0) R_4$$

$$M_p \simeq M^3 U(\varphi_0)$$

- Localized EH term will be generated generically when SUSY is broken (that can be at a high scale, and generating also a CC is not an issue).

Where to find de Sitter

Generically, isolated stabilized 4d Minkowski-brane solutions exist.
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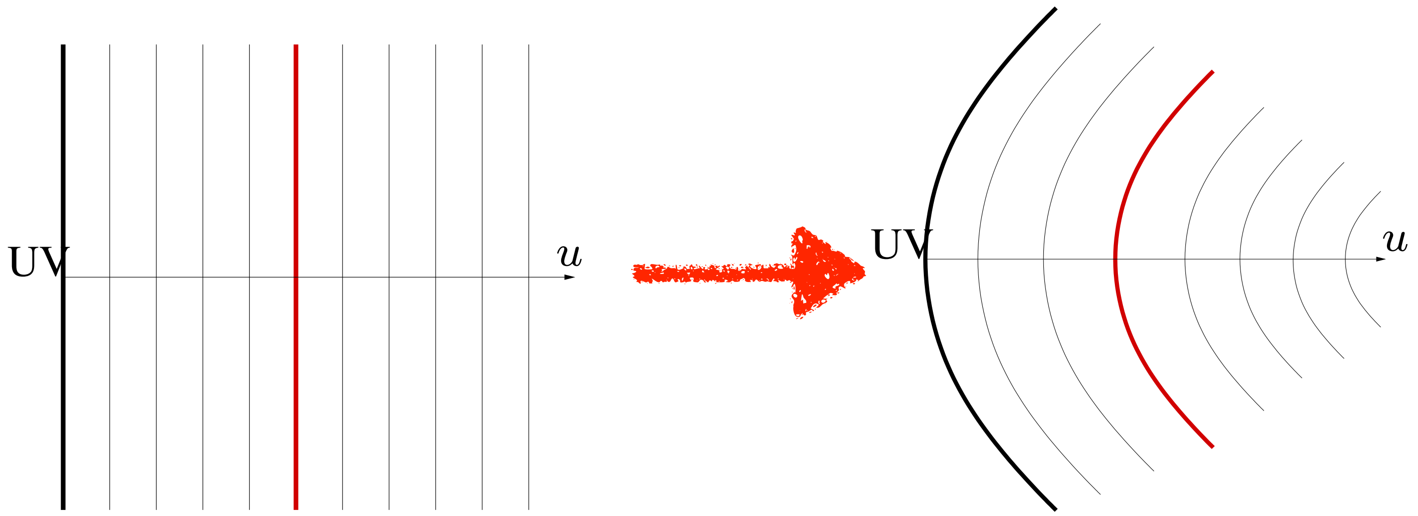
What about 4d curved-brane solutions?

- In gauge/gravity duality, the theory is specified by fixing the asymptotics of the boundary metric in the UV region.
- Generically, no stabilzed curved-brane with the same boundary conditions as the flat vacuum solution.
- Two options:
  1. Change the boundary theory and turn on metric source on the boundary *(Forced holography)*
  2. Depart from vacuum state and look at time-dependent excited states *(Brane cosmology)*

Option 1: Stabilized de Sitter 4d brane



Need two ingredients:

1. Bulk: Holographic RG flows of QFTs on curved spacetimes
2. Brane: Solve junction conditions for a curved brane

Holographic RG flows on curved manifolds

For the full bulk solution, take the ansatz:

$$ds^2 = du^2 + e^{2A(u)}\zeta_{\mu\nu}dx^\mu dx^\nu, \qquad \varphi = \varphi(u)$$

with $\zeta_{\mu\nu}$ an Einstein metric:

$$R^{(\zeta)}_{\mu\nu} = \frac{R}{4}\zeta_{\mu\nu} \quad R = \text{scalar curvature in the dual QFT in the UV}$$

- Geometry controlled by dimensionless parameter:

$$\mathcal{R} \equiv \frac{R}{\varphi_-^{2/\Delta_-}}$$

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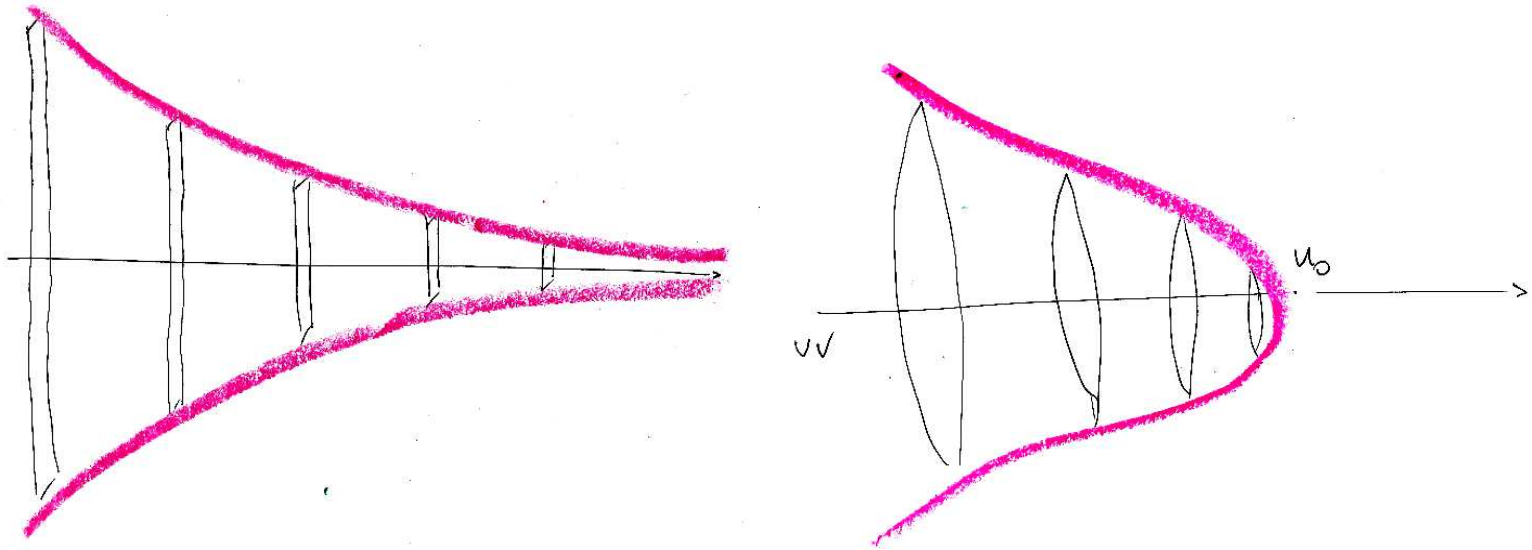
- φ_-: Relevant coupling driving the RG flow $=$ asymptotic boundary condition for $\varphi(u)$.

RG flows on $(d)S_4$



$R = 0$ $\qquad\qquad$ $R > 0$

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- $R \neq 0$: spacetimes ends at finite u_0, with $e^{A(u)} \sim (u_0 - u)$

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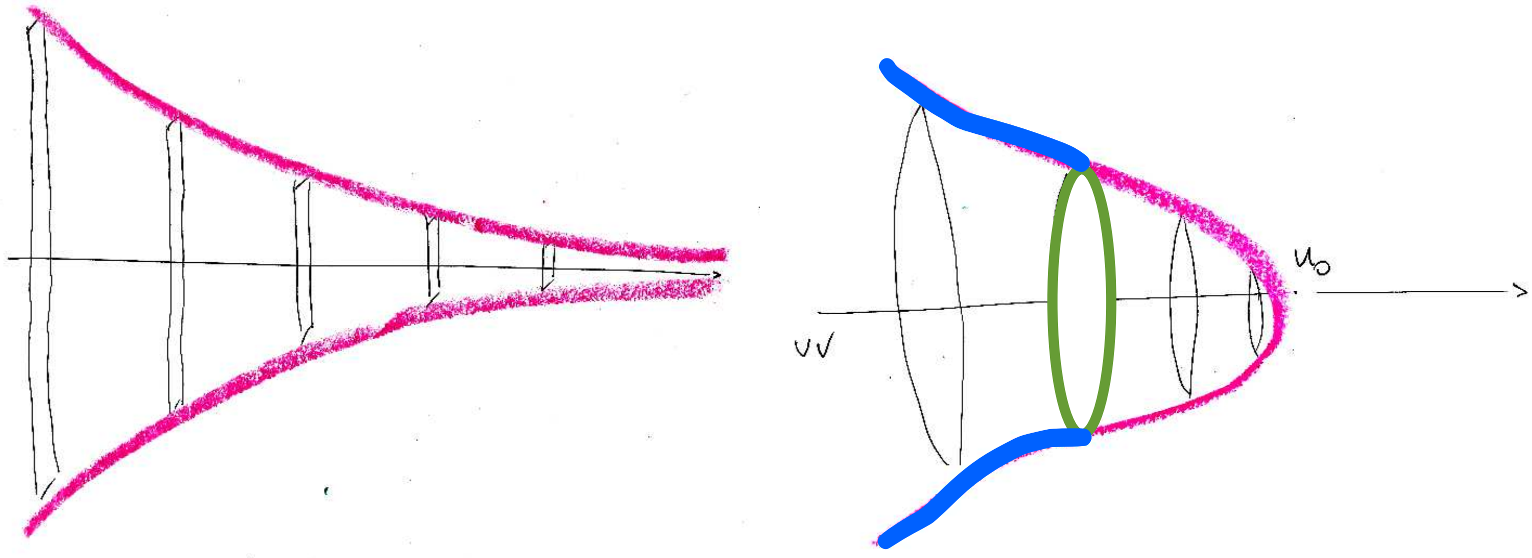

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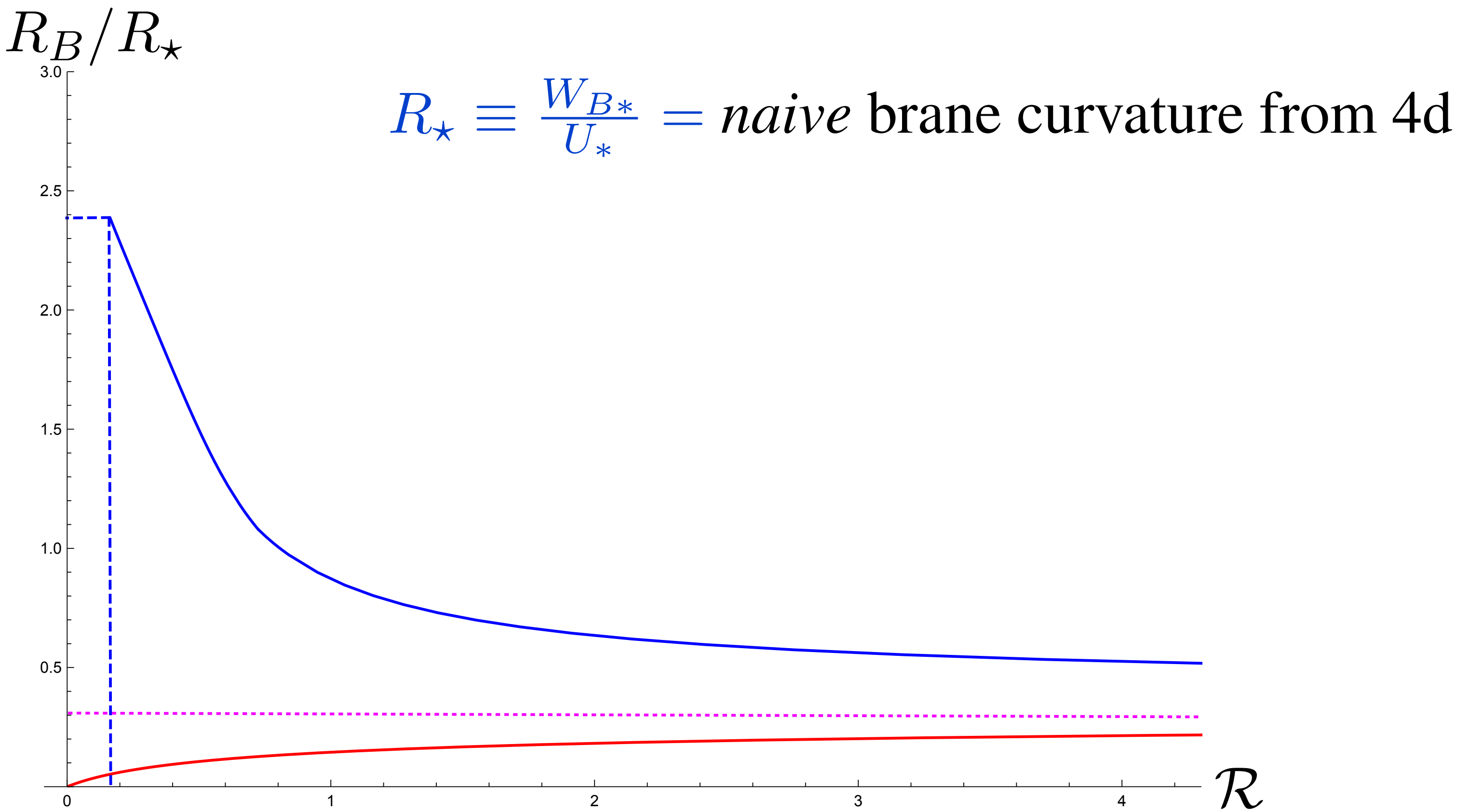
- Curvature effect subleading in the UV, but dominates in the IR
- $R \neq 0$: spacetimes ends at finite u_0, with $e^{A(u)} \sim (u_0 - u)$
- a stabilised de Sitter brane can be introduced in this setup. Position and 4d curvature determined *dynamically* by the bulk geometry + UV boundary condition + brane parameters.

Example

Take quartic bulk potential $V(\varphi)$ and exponential $W_B(\varphi)$ and $U(\varphi)$:



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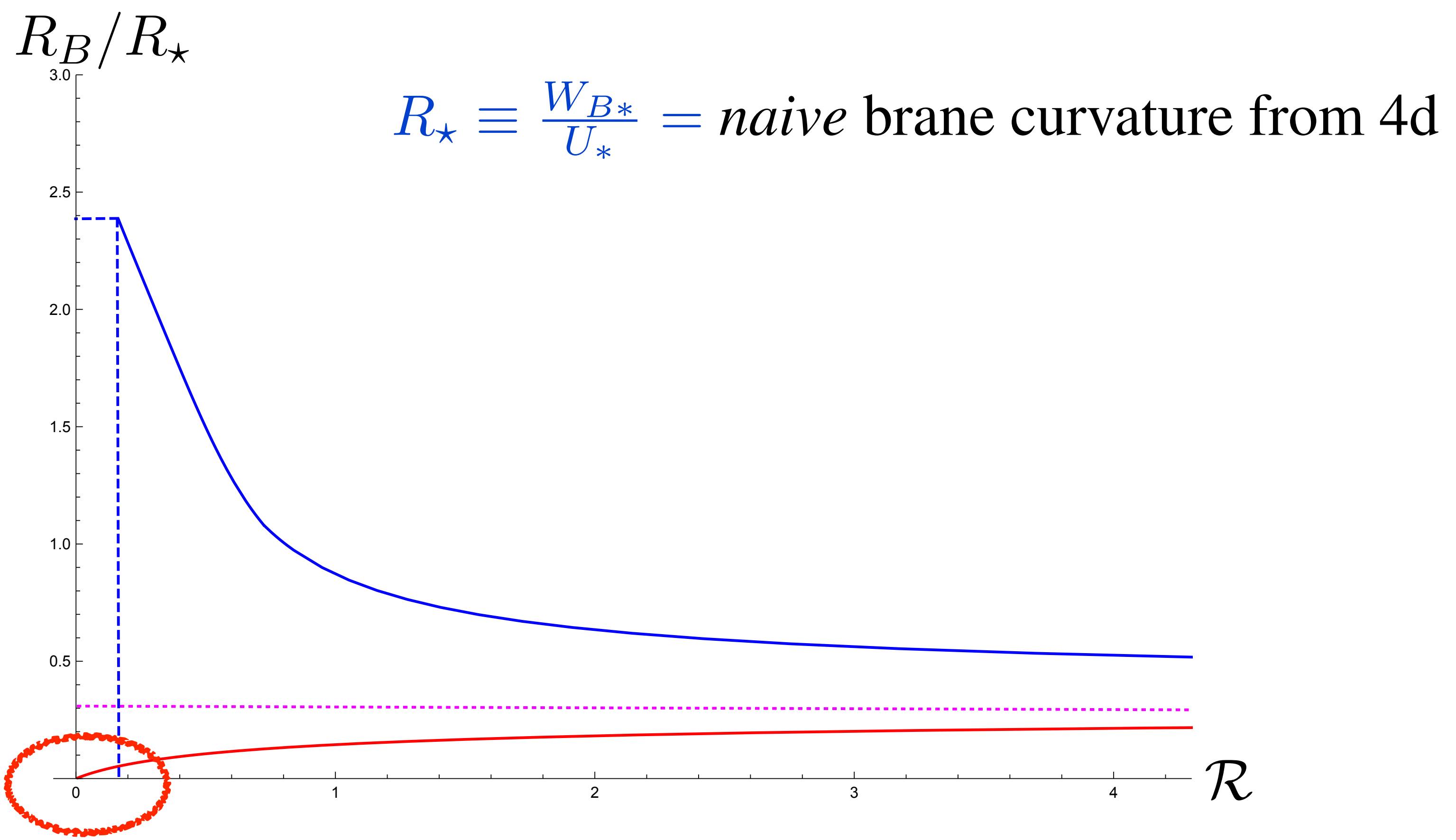

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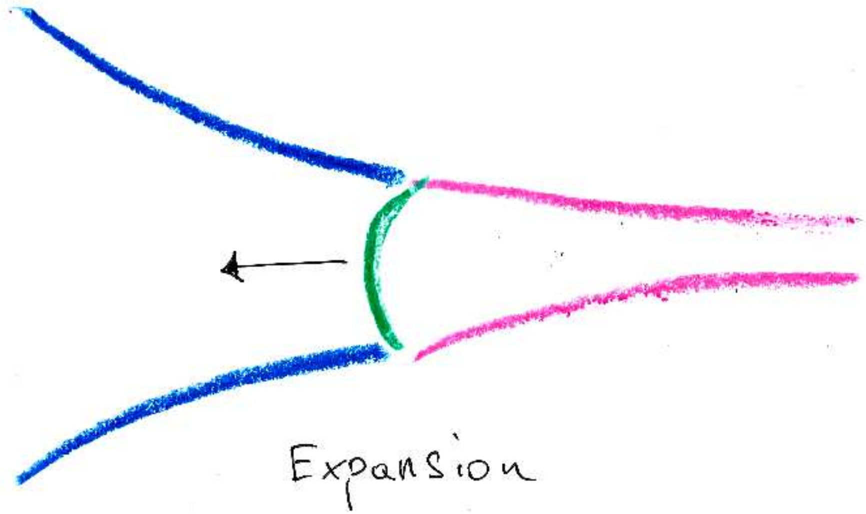
Option 2: Cosmological de Sitter brane

A brane moving with a non-zero velocity in warped geometry experiences a FRW induced metric (brane cosmology)



Can the 4d induced metric be de Sitter *without* sources for boundary metric?

Realize dS as an excited state in the same theory which admits the Minkowski vacuum

Amariti, Charmousis, Forcella, Kiritsis, FN '19

(I Can't Get No) Backreaction

Jagger,Richards 1965

To get a qualitative grip: look at the system in the probe limit:

- Bulk is the same as the (static) vacuum

$$ds^2 = du^2 + e^{2A(u)}\left(-dt^2 + d\vec{x}^2\right), \quad \varphi = \varphi(u),$$

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$$ds^2_{brane} = -(e^{2A} - \dot{u}^2)dt^2 + e^{2A(u(t))}d\vec{x}^2 \rightarrow -d\tau^2 + e^{2A(u(\tau))}d\vec{x}^2$$

All is needed is bulk scale factor $A(u)$ plus trajectory $u(\tau)$.

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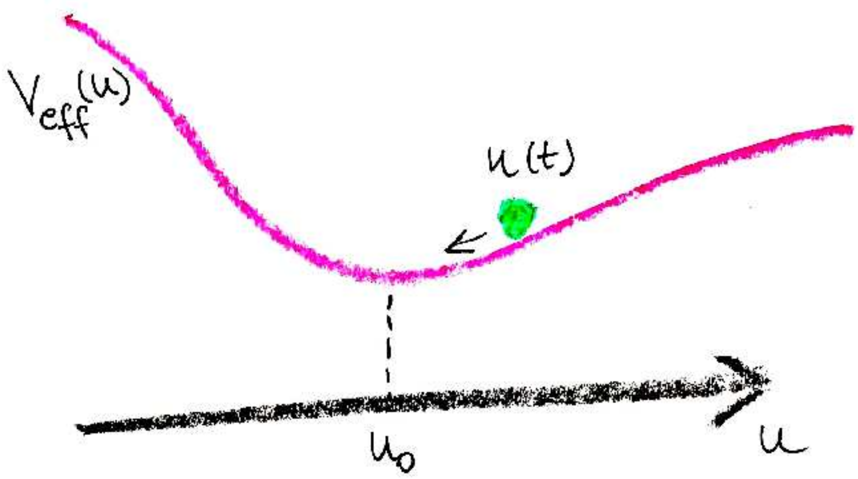
Con: Not generically applicable (probe condition may fail)

Pro: $u(\tau)$ exactly solvable after $A(u)$ is given

Recovering self-tuning

- Brane trajectory $u(t)$ described by a classical Lagrangian system with "energy" E an integral of the motion.
- Non-relativistic limit $\dot{u} \ll e^{2A}$ $\Rightarrow$ Point particle in a potential



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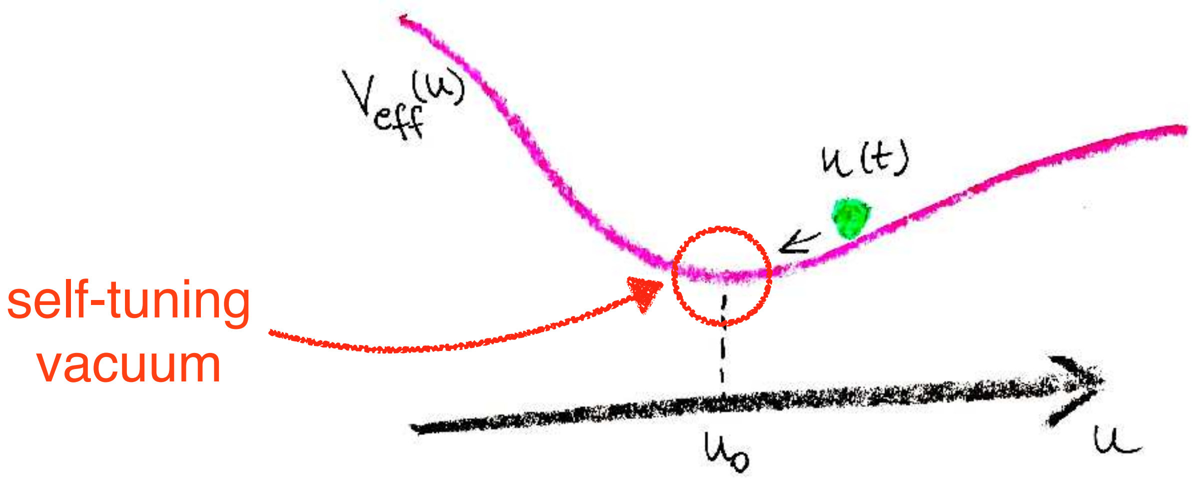

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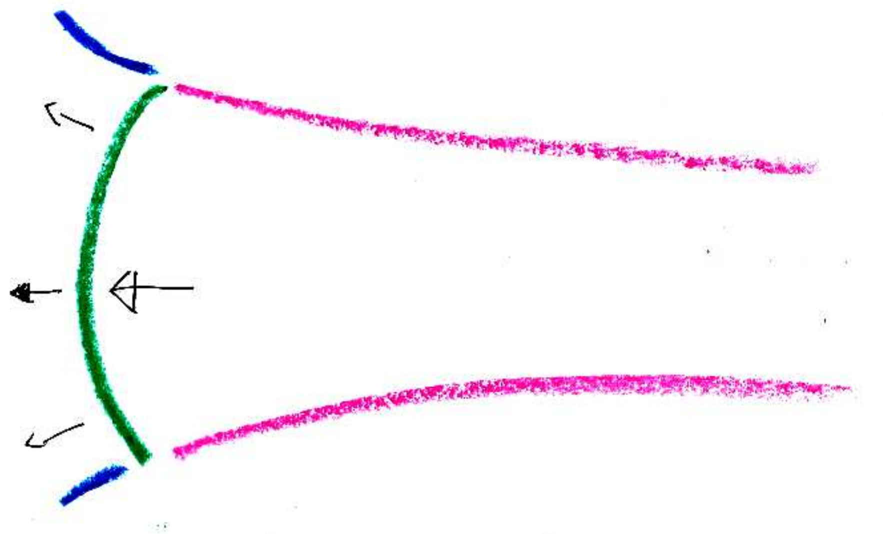
UV regime



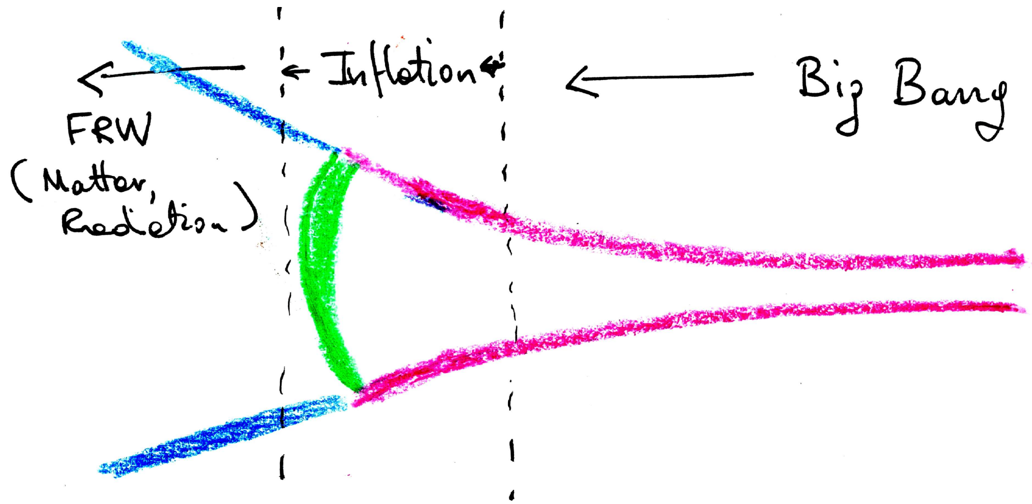
- Scalar approaching UV fixed point at $\varphi = 0$:

$$\varphi \simeq 0 \quad W, W_B, U_B, Z_B \;\to\; \text{constants.}$$

$$u(\tau) \simeq \tau \ell H_{eff} \quad \Rightarrow \quad a(\tau) \simeq \exp\left[-\tau H_{eff}\right]$$

- Solution approaches a de Sitter brane with $H_{eff} = \sqrt{\frac{W_B}{U_B}}\Big|_{\varphi=0}$.
- Same H as one would get from the 4d induced action alone

Intermediate inflation period



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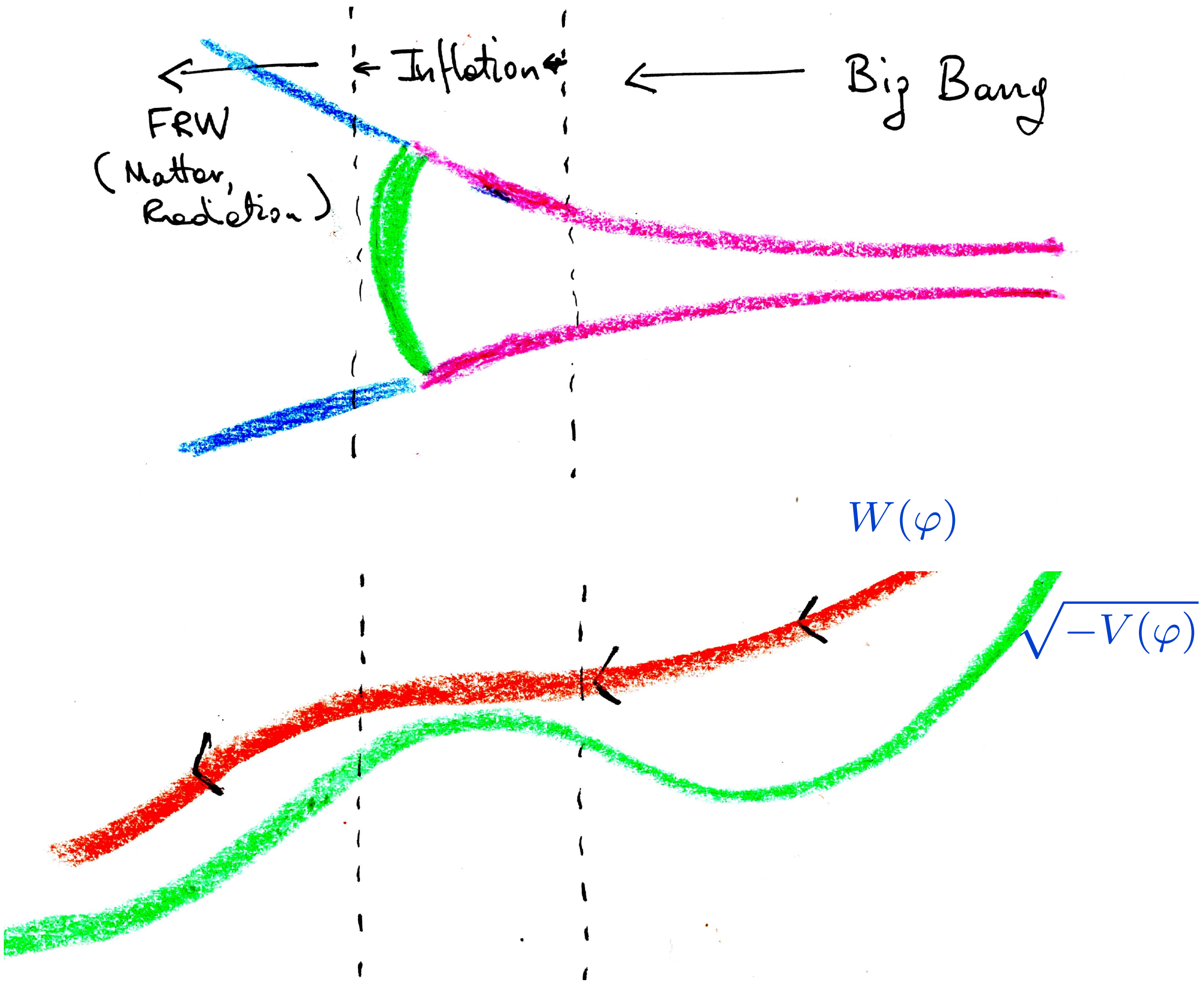


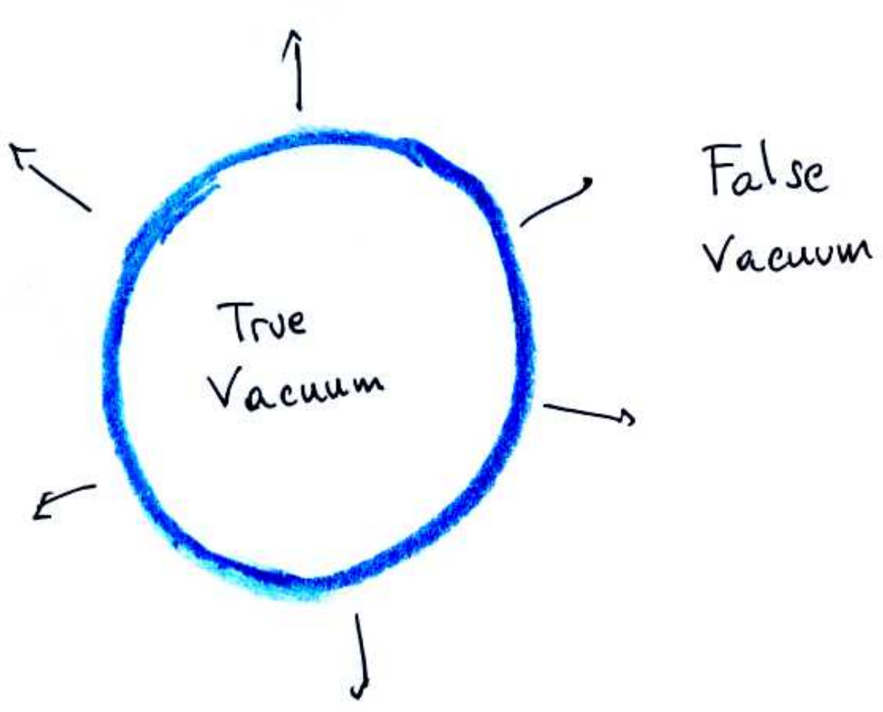
A period of inflation can be realized around intermediate extrema of the bulk potential.

Bubble-wall de Sitter

Related ideas by Danielsson *et al.* '18 -'19:

$dS_4 =$ wall on a vacuum bubble in $AdS_5 \to AdS_5$ vacuum decay.

- Vacuum decay by brane nucleation (infinitely thin, cannot be realized with scalars and a potential).
- Spatial sections are spheres.
- Universe starts *big*



Conclusion and outook

- Alternative realizations of dS which are not *vacua*
  - External sources
  - Excited state
- Can we realize any of this from top-down?
- Some constraints evaporate
  - No finite volume;
  - No worries about constant vacuum energy term;
  - Can one get scales right?

The Model

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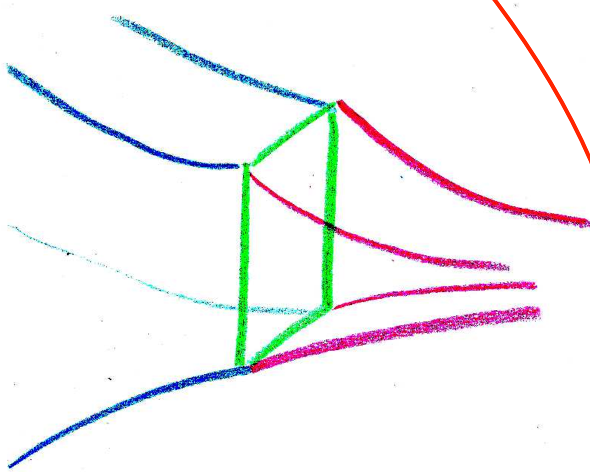


Solve bulk Einstein equations + Israel junction conditions

$$\left[\gamma_{\mu\nu}\right] = \left[\varphi\right] = 0; \quad \left[K_{\mu\nu} - \gamma_{\mu\nu} K\right] = \frac{1}{\sqrt{-\gamma}} \frac{\delta S_{loc}}{\delta \gamma^{\mu\nu}}; \quad \left[n^a \partial_a \varphi\right] = -\frac{1}{\sqrt{-\gamma}} \frac{\delta S_{loc}}{\delta \varphi}$$

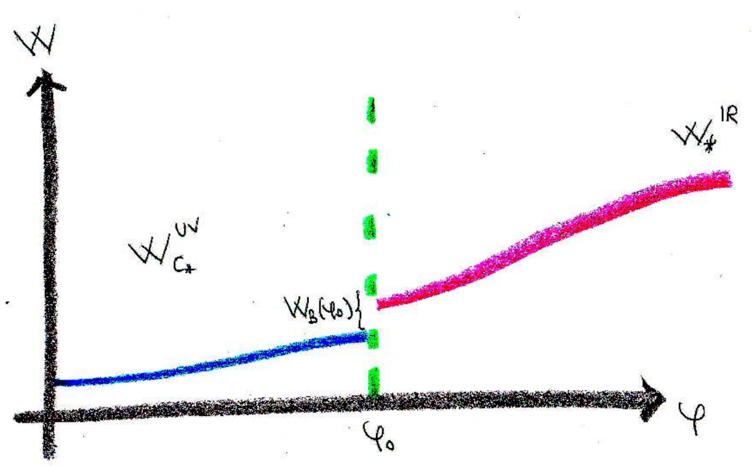
First order formalism

Solutions are conveniently characterized by scalar function $W(\varphi)$

$$W = -6\dot{A}, \quad W' = \dot{\varphi}, \quad -\frac{1}{3}W^2 + \frac{1}{2}\left(W'\right)^2 = V$$

Junction conditions in φ-space:

$$[W_{UV} - W_{IR}]_{\varphi_0} = W_B(\varphi_0), \qquad \left[W'_{UV} - W'_{IR}\right]_{\varphi_0} = W'_B(\varphi_0)$$

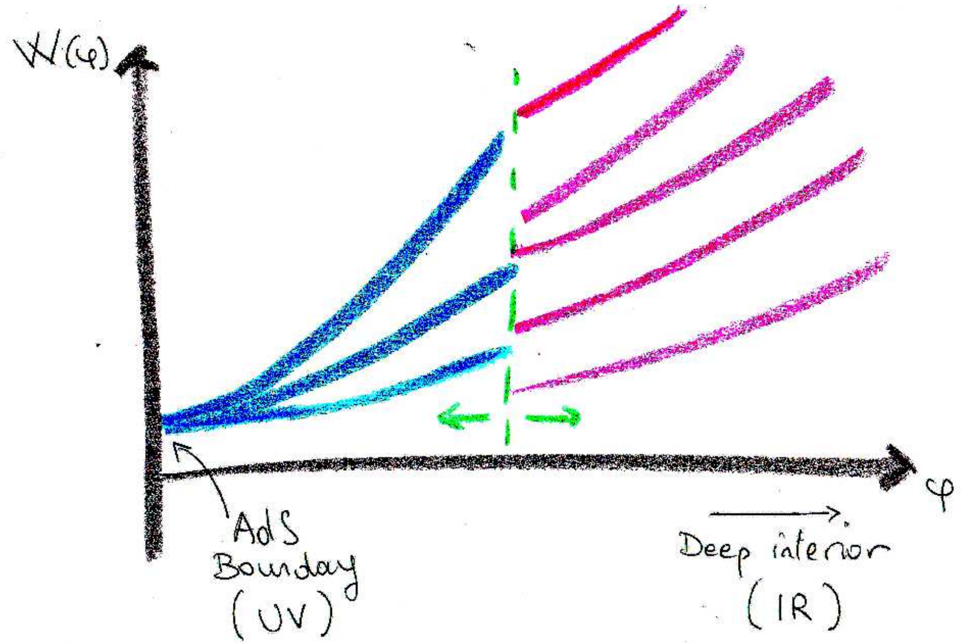


IR Regularity + junction conditions $\Rightarrow$ isolated solution(s) for generic brane vacuum energy (*self-tuning of CC*)

Self-tuning

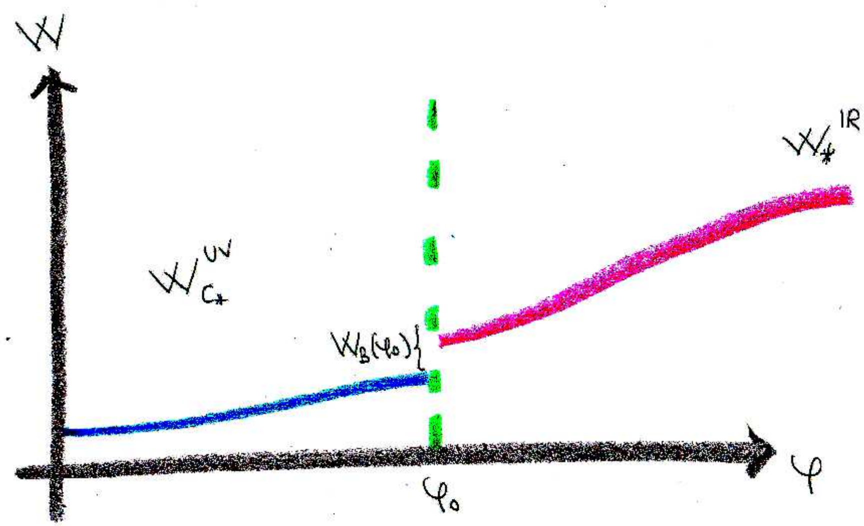
$$-\frac{1}{3}W^2 + \frac{1}{2}\left(W'\right)^2 = V$$

$$\left[W^{UV} - W^{IR}\right]_{\varphi_0} = W_B(\varphi_0), \qquad \left[\frac{dW^{UV}}{d\varphi} - \frac{dW^{IR}}{d\varphi}\right]_{\varphi_0} = \frac{dW_B}{d\varphi}(\varphi_0)$$



one-parameter family of solutions on each side.

Self-tuning

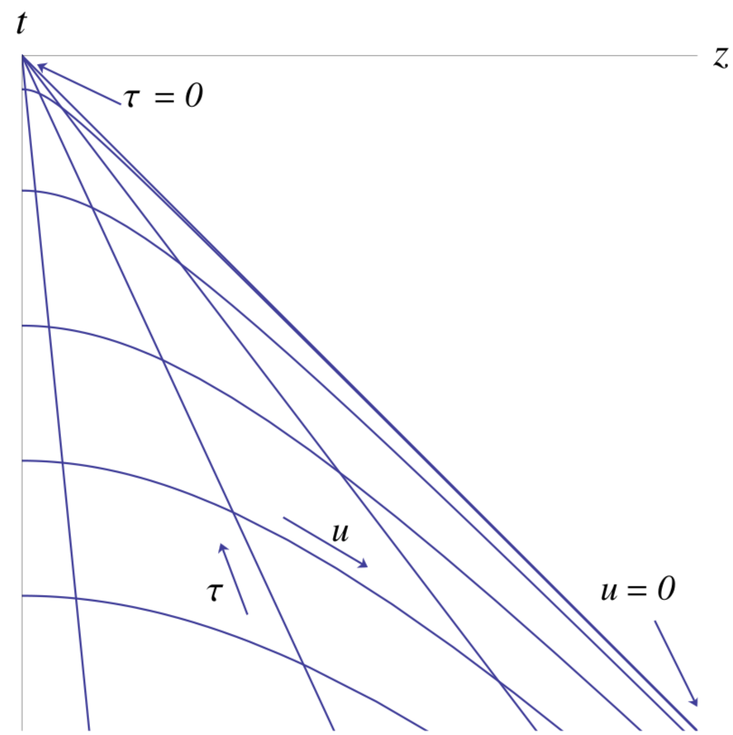


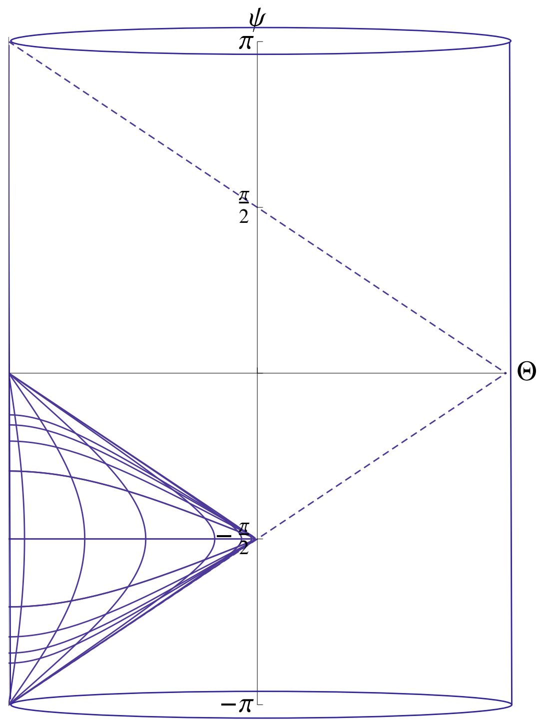
- Regularity fixes the IR solution
- Israel's junction conditions fix both UV solution and the brane position.
- For generic brane vacuum energy $\sim \Lambda^4$, UV geometry and brane position adjust so that the brane is flat and the UV glues to the regular IR (*self-tuning*).

Fixed-point solution

UV limit: solutions approaches $(d)S_d$ slicing of $(E)AdS_{d+1}$.

$$ds^2 = du^2 + \sinh^2(u_0 - u)d\Omega_4^2 \qquad R^{uv} = 4d(d-1)e^{-2u_0}$$





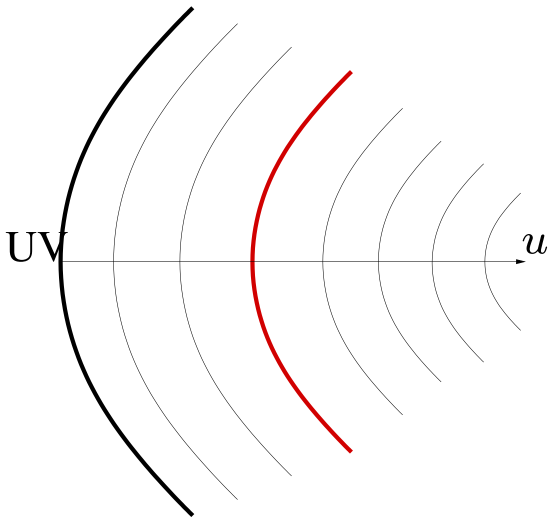
dS_d cosmological patch covers 1/4 Poincarè patch of AdS_{d+1}

Stabilized de Sitter brane

Ghosh, Kiritsis, FN, Witkowski, 1807.09794

Introduce 3 superpotentials $W(\varphi), S(\varphi), T(\varphi)$

$$W = -2(d-1)\dot{A}, \quad S = \dot{\varphi}, \quad T = e^{-2A}R$$



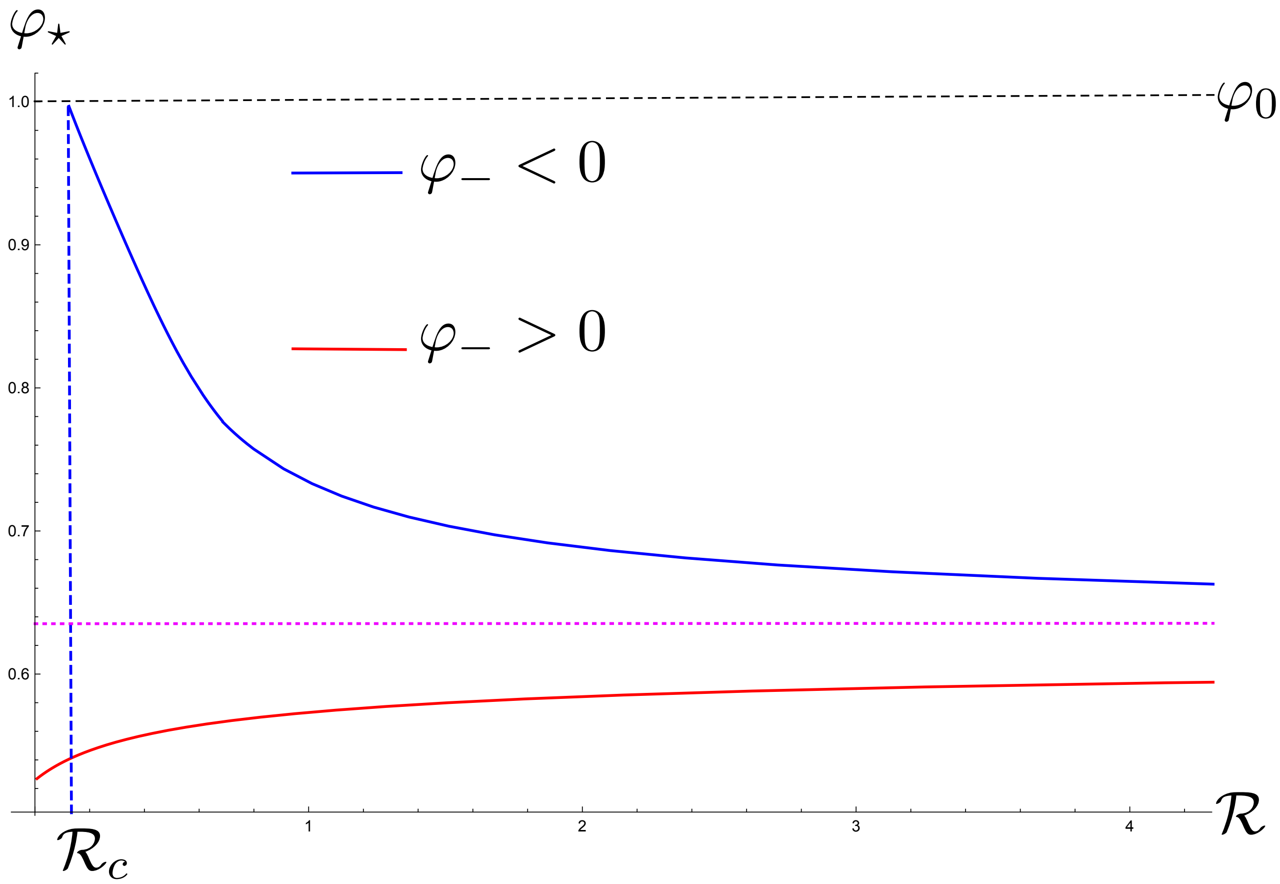
$$[W_{UV} - W_{IR}]_{\varphi_*} = [W_B + U\,T/2]_{\varphi_*} \,,$$

$$[S_{UV} - S_{IR}]_{\varphi_*} = \left[W_B' - U_B' T\right]_{\varphi_*}$$

- IR Regularity + Junction eqs $\Rightarrow$ Stabilized de Sitter brane at φ_* ($\neq$ Minkowski value φ_0)
- Equivalently: use flat boundary metric but turn on time-dependent scalar field source $\varphi_-(t) \sim t^{-\Delta_-}$.

Example

Take quartic bulk potential $V(\varphi)$ and exponential $W_B(\varphi)$ and $U(\varphi)$:



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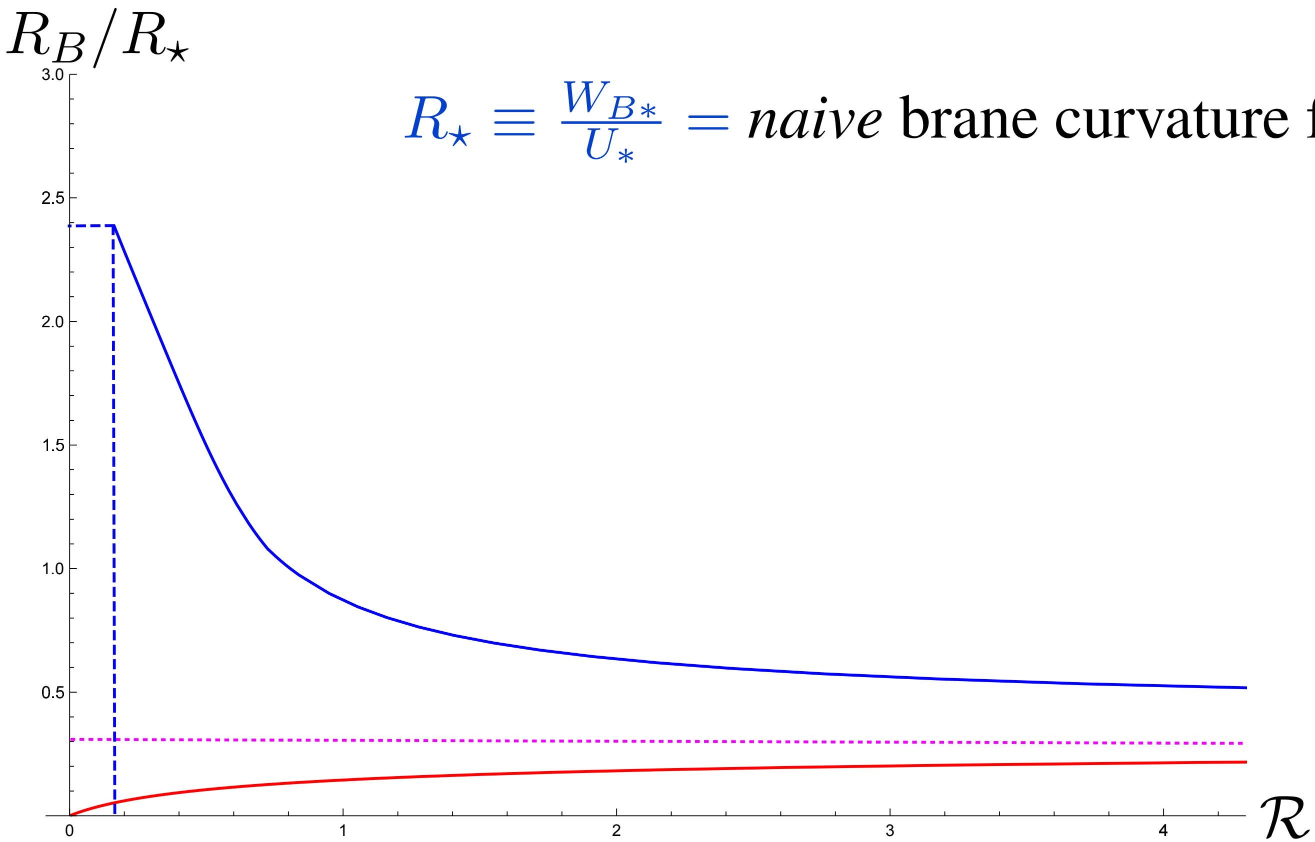

$R_\star \equiv \frac{W_{B*}}{U_*} =$ *naive* brane curvature from 4d

$$\mathcal{R} = R/|\varphi_-|^{2/\Delta_-}$$