A classification of Scalar-Tensor theories: Applications to Cosmology and Astrophysics

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Plan of the talk

1. Higher-Order Scalar-Tensor Theories in a nutshell : a very brief “history”
2. The DHOST Classification
3. Cosmology, Astrophysics and Constraints

1. Higher-Order Scalar-Tensor Theories in a nutshell

A BRIEF INTRODUCTION and MOTIVATIONS

General Relativity is a beautiful theory...

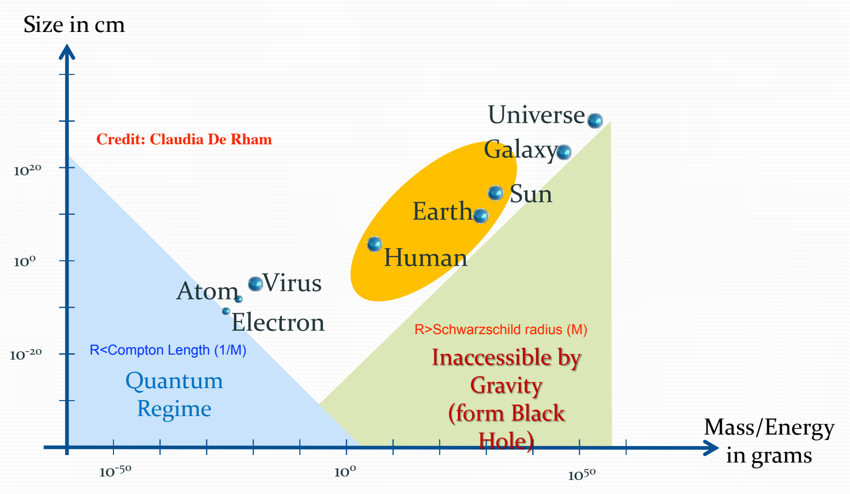
Space-time is described in terms of Lorentz geometry in total agreements with today observations...

... But it has limits

- Planck scale : UV completion of General Relativity.
- Very large (cosmological) scales : dark energy ?
  - Accelerated expansion of the universe leads to troubles

$\Longrightarrow$ Going beyond General Relativity : Modifications of GR to test the gravitational interaction at these different scales and to propose deviations that we can constrain...

Narrow window of tests of General Relativity



Uniqueness of General Relativity with a cosmological constant :

- Hypothesis 1 : Space-time is of dimension 4 (+ symmetries)
- Hypothesis 2 : Gravity is described by a metric (spin 2) only
- Hypothesis 3 : Euler-Lagrange equations are second order

$\Longrightarrow$ Lovelock theorem (1971) : Einstein gravity + Cosmological constant

$$S[g_{\mu\nu}] = \frac{c^4}{16\pi G_N} \int d^4x \sqrt{-g} \, (R - 2\Lambda)$$

No much room available for alternative theories...

Relax some of the hypothesis of Lovelock Theorem

- Gravity comes with a scalar field ϕ : a fifth force which is expected to be responsible for dark energy $\Longrightarrow$ Scalar-Tensor theories
- Equations of motion are not necessarily second order PDE

Motivations

- Adding a scalar is the simplest possibility, but there exist other interesting scenarii : massive gravity, bi-gravity, vectors, extra-dimensions... $\rightarrow$ Most of them contain a scalar mode
- Higher order equations because the dynamics of gravity is governed by an action with second order derivatives ($\partial_\mu \partial_\nu g_{\rho\sigma} \rightarrow \partial_\mu \partial_\nu \phi$) : very rich phenomenology !

⋆ The landscape of Scalar-Tensor theories has evolved a lot in the last 20 years : from Brans-Dicke to DHOST theories...

First order Scalar-Tensor theories : from Brans-Dicke to K-essence

The metric comes with a scalar field with kinetic energy $X \equiv g^{\mu\nu}\phi_\mu\phi_\nu$

- Brans-Dicke like theories : $S_{\rm BD}[g_{\mu\nu},\phi] = \int d^4x\sqrt{-g}\,[F(\phi)R - Z(\phi)X - U(\phi)]$

- K-essence theories : $S_{\rm K}[g_{\mu\nu},\phi] = \int d^4x\sqrt{-g}\,[F(\phi)R + G_2(\phi,X)]$

Possibility of non-minimal coupling to external matter fields

$$S_{\rm mat}[g_{\mu\nu},\phi;\psi] = \int d^4x\sqrt{-g}\,A(\phi)\,g^{\mu\nu}\partial_\mu\psi\,\partial_\nu\psi$$

$\Longrightarrow$ Violation of the equivalence principle : severe constraints of non-minimal couplings

- Very interesting phenomenology for Inflation and Dark energy

Brans, Dicke (1961) - Armendariz-Picon, Damour, Mukhanov (1999) - Armendariz-Picon, Mukhanov, Steinhardt (2000) - Damour, Esposito-Farese (1992)

The Dvali-Gabadadze-Porrati model : a breakthrough

- Decoupling limit of the 5D DGP model leads to the cubic Galileon

$$S_{\mathrm{cubic}}[g_{\mu\nu},\phi] = \int d^4x\sqrt{-g}\,[F(\phi)R + G_2(\phi,X) + G_3(\phi,X)\Box\phi]$$

$\Longrightarrow$ Novelty : higher order Lagrangians but still second order eom.

Very interesting (Late) cosmological phenomenolgy

- $\Diamond$ Consider the example Lagrangian : $L[g_{\mu\nu},\phi] = F(\phi)R + X - \frac{1}{2\Lambda^3}X\Box\phi$
- Self-accelerating solutions : $\Lambda \sim (M_{\mathrm{P}}H_0^2)^{1/3}$ where H_0 : Hubble constant
- Vainshtein screening : $r_V \sim (\mathcal{M}/(M_{\mathrm{P}}\Lambda^3))^{1/3}$ (If $\mathcal{M} = M_\odot$ then $r_V \sim 100pc$)

Dvali, Gabadadze, Porrati (2000) - Kobayaski (2019)

Horndeski theories : Second order equations of motion

Generalization of DGP model

$$S_{\mathrm{H}}[g_{\mu\nu}, \phi] = \int d^4x\sqrt{-g}\left(G_2 + G_3\Box\phi + G_4 R + 2G_{4X}\left[\phi_{\mu\nu}\phi^{\mu\nu} - (\Box\phi)^2\right] + \cdots\right)$$

- Each “coefficient” is a function $G_A(\phi, X)$ with $X \equiv g^{\mu\nu}\phi_\mu\phi_\nu$

Why Horndeski theories are so interesting ?

- Self-accelerating solutions without cosmological constant, screening, etc.
- General belief : most general relevant Scalar-Tensor theories.

$\Longrightarrow$ It provides a parametrization of consistant modifications of GR (EFT) : a large and interesting framework to test GR at large scales but not only (Inflation, BH).

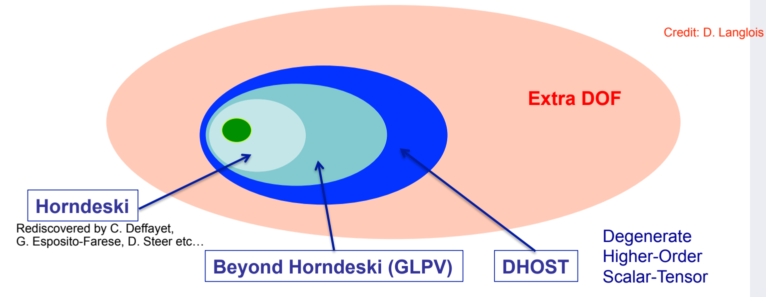
$\longrightarrow$ But this is not the end of the story as many “viable” theories do not belong to the Horndeski class : GLPV, mimetic, cuscuton etc. $\Longrightarrow$ Need of a classification...

Horndeski (1974) - Nicolis, Rattazzi, Trincherini (2008) - Deffayet, Esposito-Farese (2009) - Deffayet, Deser, Esposito-Farese (2009) - Deffayet, Gao, Steer, Zahariade (2011) - Kobayashi, Yamaguchi, Yokoyama (2011) - Chamseddine, Mukhanov (2013) - Gleyzes, Langlois, Piazza, Vernizzi (2015)

2. The DHOST Classification

Classification of Higher-Order Theories

Most general “viable” Scalar-Tensor theories with Lagrangians that contains (up to) second order derivatives. The eom are not necessary second order but (at most) one scalar propagates in addition to the two gravitons.



Langlois, Noui (2016) - Crisostomi, Koyama, Tasinato (2016) - Ben Achour, Langlois, Noui (2016)

Toy-model : Higher order particle and the Ostrogradsky ghost

Dynamics of a higher order point like particle $\phi(t)$

Action : $S[\phi(t)] = \frac{1}{2}\int dt\,\left(\dot{\phi}^2 - \omega^2\phi^2 + \alpha\ddot{\phi}^2\right)$, EoM : $\ddot{\phi} + \omega^2\phi - \alpha\,\ddot{\ddot{\phi}} = 0$.

Degrees of Freedom

- Need of 4 initial conditions : $\phi(0)$, $\dot{\phi}(0)$, $\ddot{\phi}(0)$ and $\dddot{\phi}(0)$ $\Longrightarrow$ 2 particles propagate !

Unbounded (from above and below) energy

- An equivalent Lagrangian for the two degrees of freedom

$$L[\phi,\psi] = \frac{1}{2}\left[\dot{\phi}^2 - \omega^2\phi^2 + \alpha(2\dot{\phi}\dot{\psi} - \psi^2)\right] = \frac{1}{2}\left[(\dot{\phi}+\alpha\dot{\psi})^2 - \alpha^2\dot{\psi}^2 - \omega^2\phi^2 - \alpha\psi^2\right]$$

$\Longrightarrow$ The extra DoF is a ghost : there is an instability. This is the Ostrogradsky ghost.

Ostrogradski (1850)

Coupling two particles $\phi(t)$ and $q(t)$

$$S[\phi, q] = \frac{1}{2} \int dt \left(\dot{\phi}^2 - \omega^2 \phi^2 + \alpha \, \ddot{\phi}^2 + \dot{q}^2 - \omega^2 q^2 + 2\alpha \ddot{\phi} \dot{q} \right)$$

<u>EoM</u> : $\ddot{\phi} + \omega^2 \phi - \alpha \ddddot{\phi} - \alpha \dddot{q} = 0$ and $\ddot{q} + \omega^2 q + \alpha \dddot{\phi} = 0$.

How many Degrees of Freedom ?

Not easy to guess... In general, such a theory propagates 3 DOF : ϕ, q and the ghost !

Evading Ostrogradski instability

- Here, the theory is DEGENERATE $\Longrightarrow$ NO GHOST !

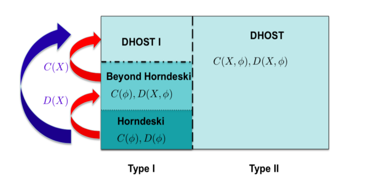
$$S[\phi, q] = \frac{1}{2} \int dt \left(\dot{Q}^2 + \dot{\phi}^2 - \omega^2 \phi^2 - \omega^2 X^2 \right) , \quad Q = q + \alpha \dot{\phi}$$

Langlois, Noui (2016) - Motohashi, Noui, Suyama, Yamaguchi, Langlois (2016) - de Rham, Matas (2016) - Crisostomi, Klein, Roest (2017)

The (quadratic) DHOST Lagrangian

$$G_2 + G_3\Box\phi + G_4 R + A_1\phi_{\mu\nu}\phi^{\mu\nu} + A_2\Box\phi^2 + A_3\Box\phi\phi^\mu\phi^\nu\phi_{\mu\nu} + A_4(\phi_{\mu\nu}\phi^\nu)^2 + A_5(\phi_{\mu\nu}\phi^\mu\phi^\nu)^2$$

- 3 relations between G_4 and $A_I \Longrightarrow$ 3 free functions in the quadratic part of action



Disformal transformations of the metric

$$g_{\mu\nu} \mapsto \tilde{g}_{\mu\nu} = C(\phi, X)g_{\mu\nu} + D(\phi, X)\phi_\mu\phi_\nu\,, \qquad S[g_{\mu\nu}, \phi] = \tilde{S}[\tilde{g}_{\mu\nu}, \phi]$$

Degeneracy is preserved by disformal transformations : one identifies disformal classes.

3. Cosmology, Astrophysics and Constraints

Cosmology and Linear Perturbations

Quadratic action for the perturbations ζ (scalar) and γ_{ij} (tensors)

- One considers a cosmological background $a(t)$ with self-acceleration

$$S = \int dt\, d^3x\, a^3 \left\{ \frac{M^2}{2} A \left[\dot{\zeta}^2 - c_s^2 \frac{(\partial\zeta)^2}{a^2} \right] + \frac{M^2}{8} \left[\dot{\gamma}_{ij}^2 - c_T^2 \frac{(\partial\gamma_{ij})^2}{a^2} \right] \right\}$$

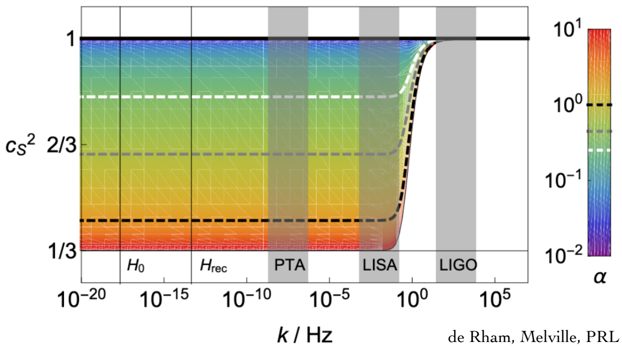
- Tensor modes do not propagate at speed of light in general : $c_T^2 = \frac{G_4}{G_4 - XA_1}$
- GW feel the fifth force and propagate in a medium

DHOST After GW170817 : $|c_T - 1| < 3.10^{-15}$

- Severe constraints on DHOST actions if taken strictly : $A_1 = 0$ etc.
- But rainbow argument : limit of validity of DHOST at GW scale

Langlois, Mancarella, Noui, Vernizzi (2017) - Creminelli, Vernizzi (2017) - Ezquiaga, Zumalacarregui (2017) - Sakstein, Jain (2017) - de Rham, Melville (2018) - Creminelli, Lewandowski, Tambalo, Vernizzi (2018)

DHOST Theories as Effective Field Theories : $\Lambda \sim 300Hz$



de Rham, Melville, PRL (2018)

Gravitational Rainbow by de Rham and Melville (2018)

GW170817 probes DHOST Theories at its limit of validity : $\Lambda = (H_0^2 M_P)^{1/3} \sim 300Hz$

Quasi-static approximation : $Hr \ll 1$

$$ds^2 = -(1+2\Phi)dt^2 - (1-2\Psi)dx^2\,, \quad \phi = \phi_c(t) + \chi(r)$$

Gravitational laws

$$\mathcal{M}(r) \equiv 4\pi \int_0^r \bar{r}^2 \rho(\bar{r}) \mathrm{d}\bar{r}$$

$$\frac{d\Phi}{dr} = \frac{G_{\rm N}\,\mathcal{M}(r)}{r^2} + \Xi_1\, G_{\rm N}\,\mathcal{M}''(r)\,,$$

$$\frac{d\Psi}{dr} = \frac{G_{\rm N}\,\mathcal{M}(r)}{r^2} + \Xi_2 \frac{G_{\rm N}\,\mathcal{M}'(r)}{r} + \Xi_3\, G_{\rm N}\,\mathcal{M}''(r)$$

with $(8\pi G_{\rm N})^{-1} \equiv 2f\,(1+\Xi_0)$

Modifications of Newton laws can be constrained

Theoretical bound : $\Xi_1 > -1/6$ $\star$ With non-relativistic stars : $-1/12 < \Xi_1 < 0.2$

Langlois, Saito, Yamauchi, Noui, Vernizzi (2018) - Crisostomi, Koyama (2018), Dima, Vernizzi (2018)

Conclusion

Summary and Perspectives

Systematic study of large class of modified gravity theories

- Full classification of DHOST theories with NO GHOST
- Very interesting applications to (late time) cosmology

Applications to Black Holes : very rich physics

- Parametrization of consistent deviations from GR
- Background solutions $\Longrightarrow$ Imaging of the solution (see Meudon)
- Perturbations about these solutions : subtle entanglement between the scalar and the polar gravitational mode
- New techniques for computation of QNM (H. Roussille, D. Langlois and KN)