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Fundamental interactions
Medical applications
New Nuclear systems

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Particle Matter Interactions
For medical applications
Outline

Introduction
   Aim, Units, particle, energy, cross section

Charged Particles
   Protons, Heavy ions
   Bethe formula, corrections
   Range, Bragg peak
   Straggling

   Bremsstrahlung, Cerenkov

   Electrons/Positrons
Neutral Particles
  Beer Lambert Law

Photons
  Photo Electric
  Compton
  Pair production
dominant effect vs energy and Z

Neutrons
  Diffusion
  Absorption
  Nuclear reaction
The goal of these lectures is to give an overall knowledge of particle matter interaction in the medical energies ranges.

Medical energies ranges?

Ranges define by application

Medical
Nuclear energy
Nuclear physics

Particles

Photons, neutrons, ions
Electrons, positrons

No HEP particles

Forces

EM, Strong
Weak, Gravitation
Some formulae and units

• Velocity \( \text{m/s, cm/ns, light?} \)
• Length \( \text{m, \(\mu\text{m, fm}\)} \)
• Energy \( \text{J, eV} \)
  
  \text{Rx 100 keV-Few MeV, PET 511 keV,}
  \text{Proton Therapy 200 MeV, Carbon therapy 400 MeV/A}
  \text{Fission 200 MeV, thermal neutron 0,025 eV…}

• Mass \( \text{Kg, g, amu} \)
• Energy per nucleon \( \text{E/A} \)
  
  \text{Same energy per nucleon} \rightarrow \text{same velocity}

• Photon Energy/momentum

  \( E(\text{keV}) = \frac{12,394}{\lambda}(\text{Å}) \)
Solid angle $d\Omega$, sr

$$d\Omega = \sin \varphi \, d\varphi \, d\theta \rightarrow S/R^2$$

Whole space $4\pi$ sr

Solid angle is fundamental for detection as well as radiation safety

Cross section

Considered a flux of particle type A hitting a medium made of particle B
How many particle A are scattered in a solid angle $d\Omega$?
The number of scattered A particles in a thickness $dx$ is

$$N_s = F S N \delta x \frac{d\sigma}{d\Omega}$$

Where $F$ is the flux (particle per unit area)
$S$ is the surface covered by the beam
$N$ the density of B particle
$\delta x$ the thickness

By dimensionnal analysis $d\sigma/d\Omega$ is a surface
It is called the differential cross section the unit is cm$^2$ or barn = $10^{-24}$cm$^2$
It can be interpreted as a probability of interaction

$d\sigma/d\Omega$ can be dependant of angle, energy, spin....

The total cross section is given by

$$\int d\sigma/d\Omega$$ over all parameters

The total number of interaction is

$$N_s = FS N_\sigma \, dx$$
Cross Section

Cross section $\sigma$ is the interaction probability
If the incident particle and the target particle are in the same projected area (along the incident particle velocity) they will interact

The physical size of the particles does not relate to the cross section
the cross section depends on what force it acting on the particle

It has the dimension of a surface m$^2$, cm$^2$ ... or usually barn
1 barn = 10$^{-24}$ cm$^2$

Reaction rate is directly related to cross section

$N_{\text{reactions}} = N_{\text{projectiles}} \times N_{\text{target}} \times \text{cross section}$
example

Incident neutron data / ENDF/B-VII.1 / U235 // Cross section
Mean free path

The cross section is the interaction probability we can define a quantity $\lambda$ where a particle $A$ does not interact with the $B$ particles.

What is the probability for a particle to survive up to $x$

$P(x)$ probability of non-interaction up to $x$
$\omega dx$ probability to interact in $dx$

$$P(x+dx) = P(x) (1-\omega dx)$$

$$P(x)+dP/dx \ dx = P(x)-P(x) \ \omega dx$$

$$dP=-P \omega dx$$

$P=C \exp(-\omega x)$ $C=1$ as $P(0)=1$

The mean distance without an interaction is

$$\lambda = \int xP(x)dx / \int p(x)dx = 1/\omega$$

$\omega$? we saw that $N \sigma$ is the probability to interact within $dx \Rightarrow \lambda = 1/N \sigma$
Surface density, thickness

Thickness \( t \) homogeneous with a length

For a given material of mass density \( \rho \) we can define \( \rho t \) which is in g/cm\(^2\) and called Surface density.

This unit is related to the density of interacting centers.

We will see that equivalent surface density make the same effects on an incoming beam.
IONIZING RADIATION

Two types of particles

Neutral particles (photons) and charged particles (electron positron, ions)

The common fact between these two types of particles is that when they interact with matter

Electric charge is produced by ionization

Either directly (charged particle) or indirectly (neutral particle)

The two types of particle will exhibit different behavior
CP always interact with all electrons (coulomb interaction) therefore the cross section is very high and actually non measurable the particle will always interact and produce charges

→ Radiation effect =100% Detection Efficiency =100%

Range
CP lose energy along their path

Neutral very rare interaction → crosssection (interaction probability)
Sometimes no interaction or very little
→ Radiation effect ≠100% Detection Efficiency ≠ 100%

Mean free path
NP number will be attenuated along their path
Energy loss. Bethe formula

Ion travelling along x with a velocity \( v \)

\( F_c \) Coulomb force between the electron and the ion
Only the perpendicular component of the force $F_c$ will modify $P$ by $\Delta P$

$$F_{c\perp} = F_c \frac{b}{\sqrt{x^2 + b^2}}$$

Then the change in momentum is given by

$$\Delta p = \int_{-\infty}^{\infty} F_{c\perp} \frac{dx}{v} = \frac{ze^2b}{v} \int_{-\infty}^{\infty} \frac{dx}{(x^2 + b^2)^{3/2}} = \frac{2ze^2}{vb}$$

The electron gains an energy of

$$\Delta E = \frac{(\Delta p)^2}{2m_e} = \frac{2z^2e^4}{m_e v^2 b^2}$$

Which is also the energy loss of the particle
This is the energy loss due to a single electron.
We have to sum up all the contributions for all impact parameters \( b \).

The number of electrons between \( b \) and \( b + db \) is given by:

\[
Z N 2\pi \ b \ db \ dx
\]

Where \( Z \) is the atomic number of the medium and \( N \) number of atoms per \( \text{cm}^3 \).
The energy loss induced by these electrons is then:

\[
dE = 2\pi \ b \ db \ dx \ Z N \frac{2z^2e^4}{m_e v^2 b^2}
\]

We have now to integrate over all the impact parameters the following expression, finally the energy loss for a thickness \( dx \) is:

\[
- \frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} \ N Z \ln \frac{b_{\text{max}}}{b_{\text{min}}}
\]
We have to define the to integration limits bmin and bmax

The natural choice will be 0 and infinity
But the physics meaning of zero and infinity is ??
The other point is that the integral will not be finite...

We have to find physics arguments to choose bmin and bmax

bmin is the closest the ion and electron will be, this is also the impact parameter where the electron will gain the maximum energy

The electron change in momentum can be at most

\[ 2mev \]

Which correspond to back scattering then b is derived as

\[ \frac{2ze^2}{vb_{\text{min}}} = 2mev \]

For b max even though the coulomb interaction is infinite screening will affect its range
bmax is related to the minimum energy change the ion can induce on the electron. We will choose this minimum as the electron ionization potential I. It means that the minimum energy transfer has to be I, the ionization potential.

\[ b_{\text{max}} = \frac{Z e^2}{v} \left( \frac{2}{m_e I} \right)^{1/2} \]

A common formula for I is
\[ I = 11.5 \times Z \text{ (eV)} \]

Finally, the energy loss through a thickness dx is

\[ - \frac{dE}{dx} = \frac{4\pi Z^2 e^4}{m_e v^2} N Z \ln \left[ \frac{2m_e v^2}{I} \right]^{1/2} \]

The previous calculation is an a classical approximation and a full quantum mechanics calculation gives

\[ - \frac{dE}{dx} = \frac{4\pi Z^2 e^4}{m_e v^2} N Z \left[ \ln \frac{2m_e v^2}{I} - \ln \left( 1 - \beta^2 \right) - \beta^2 - \frac{C_k}{Z} \right] \]

Bethe Bloch
in CGS
erg cm\(^{-1}\)

It differs from the classical approximation by the square root and there are some corrections. Relativistic and electron shells (K)
Energy loss rate for ions- Bethe Bloch formula

In CGS unit

\[ \frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} NZ \left[ \ln \frac{2m_e v^2}{I} - \ln \left( 1 - \beta^2 \right) - \frac{\beta^2}{2} - \frac{C_k}{Z} \right] \]

- Linear stopping power
- \( Z \): target atomic number
- \( N \): target density cm\(^{-3}\)
- \( z \): projectile atomic number \( z = 1 \) for proton
- \( e \): unit charge \((1,6 \times 10^{-19} \text{C})\)
- \( m_e \): electron mass \(9.1 \times 10^{-31} \text{kg}\)
- \( v \): ion velocity
- \( I \): ionization potential of the target material

Relativistic corrections

Atomic Shell correction
Units

The formula is in CGS units to have it in J cm\(^{-1}\)
we have to multiply it by \(1/(4\pi\varepsilon_0)^2\)
Then from J to MeV divide by 1.6 \(10^{-13}\)

The energy loss should be in MeV/cm but it sometimes appear in MeV cm\(^2/g\)

\[
\frac{dE}{dx} \text{ (MeV/mg/cm}^2) = \frac{1}{\rho} \frac{dE}{dx} \text{ (MeV/cm)}
\]

Application
Do we have to calculate the Bethe formula for a given ion and material??

Tables, data bases
https://www.nist.gov/pml/stopping-power-range-tables-electrons-protons-and-helium-ions
Software , SRIM
http://www.srim.org/
Is the energy loss by a single electron very small so we can consider the ion trajectory as straight?

The maximum transfer is (Backscattering) given by the previous arguments. Maximum change of momentum is at most $2m_e v$ a thousand times less than the ion momentum. We can consider the ion trajectory as straight.

At the end of its path, the ion will have very little energy and this statement will not be true anymore.

For electron multiple scattering will occur and the trajectory will be « chaotic »

Due to their mass and that the electron diffuse on electron the Bethe formula is different. For electrons

$$
\frac{dE}{dx} = \frac{2 \pi e^4 N Z}{m_e \gamma^2} \left[ \ln \frac{m_e v^2 E}{2 I^2 (1 - \beta^2)} - \ln^2 \left( 2 \sqrt{1 - \beta^2} - 1 + \beta^2 \right) + (1 - \beta^2) + \frac{1}{8} \left( 1 - \sqrt{1 - \beta^2} \right)^2 \right]
$$

$$g = \frac{\pi r^2}{\sqrt{1 - b^2}}$$
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\]

\[
\gamma = \frac{1}{\sqrt{1 - \beta^2}} \quad \beta = \frac{v}{c}
\]

$v$ the electron velocity, $c$ speed of light.
1 μm marker

Number of electrons: 2000
Number backscattered: 139
10keV electrons

Element 6, C
Atomic weight: 12.01
Density: 2.34

Backscattering Coefficient: 6%

ELECTRONS
The Bethe Bloch Formula

\[ \frac{dE}{dx} = \frac{4\pi z^2e^4}{m_en^2} N Z \left[ \ln \frac{2m_e v^2}{I} - \ln \left(1 - \beta^2\right) - \beta^2 - \frac{C_k}{Z} \right] \]

The stopping power is dependant on

- the incident particle charge \( z^2 \)
- The target atomic number \( Z \)

Energy dependance three zones

For high energy the relativistic term \( \ln (1-\beta^2) \) dominates and as \( v \) increase \( dE/dx \) increase

For lower energy the first term \( \ln (2m_e v^2/I) \) dominates

We know that for \( E=0 \) \( dE/dx=0 \) note that the ion will neutralized at the end of its path
Bethe Bloch Formula

\[ \frac{dE}{dx} = \frac{4\pi z^2\epsilon^4}{m_e\nu^2} NZ \left[ \ln \frac{2m_e\nu^2}{I} - \ln \left(1 - \beta^2\right) - \beta^2 \frac{C_k}{Z} \right] \]
The domains’ limit are

3 x incident particle mass, $3 m_p c^2$ for proton 3 GeV

500 x the ionization potential $I$, if $I \sim 10 \text{ eV}$ and $Z = 25 \rightarrow 125 \text{ keV}$

It means that considering the energy range we are interested in only the zone 2 and 3 will be of interest for our applications

Proton therapy 150 Mev

Hadron Therapy Carbon ion 400 MeV/A (limit)

Fission product 100 MeV $\rightarrow \sim 1\text{ MeV/A}$ very slow

For the medium energy

$$\frac{dE}{dx} = k (\ln k' v^2)/v^2$$

$\rightarrow$ Stopping power decreases as the energy increases
Incident particle dependance
\( \frac{dE}{dx} \) varies with \( z \) square

Same \( z \) different mass

Same energy
\( \frac{dE}{dx} \) is proportional to \( z^2 m_p \lambda(E) \) for a same charge (isotope) the heavier one is stopped faster

Deuteron and proton of same energy ...

Same velocity
\( \frac{dV}{dx} \) is proportional to \( z^2 / m_p \lambda(v) \) for a same charge (isotope) the lightest one is stopped faster

Deuteron and proton of same velocity...
Target dependance

dE/dx is proportional to NZ and I
Heavy material will have a larger stopping power

In units of energy per path unit it will have a broad range of variation

If dE/dx is express in mev/g/cm² it is more or less constant

dE/dx \propto \frac{1}{r}
dE/dx \propto \frac{NZ}{NAm_n} \propto \frac{Z}{A}

which is more or less constant over the entire table of elements

For the same mass the dE/dX is independant of the element
<table>
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<th>ENERGY FOR A=1</th>
<th>ENERGY PER MASS UNIT</th>
<th>ELECTRONIC STOPPING POWER IN UNITS OF MEV/IMG/SQ CM</th>
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Orders of magnitude

Heavy charged particle loss their energy very rapidly, and they are highly ionizing

10 MeV protons in Al ($\rho=2.7 \text{ g/cm}^3$)

0.034 MeV/mg/cm²

Range?

10 MeV p $dE/dx \sim 90 \text{ MeV/cm}$ $\Rightarrow$ in about 1 mm the protons have lost all their energy
5 MeV Alphas' range is about 5 cm in air
250 Mev protons in Tissue $\Rightarrow \sim 35$ cm

Definition of range

$$R = \int_0^V \left( \frac{dv}{dx} \right)^{-1} \, dx$$

$$R = \int_0^E \left( \frac{dE}{dx} \right)^{-1} \, dx \quad .... \quad \text{Tables, software}$$

http://www.srim.org/
<table>
<thead>
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<th>ENERGY PER MASS UNIT</th>
<th>RANGE IN UNITS OF MG/SQ CM</th>
<th>ENERGY FUN A+1</th>
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<td>12.0000</td>
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Bragg curve

Ionization as a function of range, penetration in matter

Nb of ionization

X
Bragg curve

Ionization as a function of range

Nb of ionization
Energy Loss of Alphas of 5.49 MeV in Air
(Stopping Power of Air for Alphas of 5.49 MeV)

Stopping Power [MeV/cm]

Path Length [cm]
Other energy loss processes

Nuclear/Atomic

At the end of the path the ion is neutralized or almost neutralized. It can hit a nucleus and sometimes displace it.

dE/dx = (dE/dx)_{elec} + (dE/dx)_{nuc}

This process is very small but can have consequences on detector (local defect).

Nuclear reactions can be responsible of particle loss as well.
Bremmsthralung/ braking radiation

Any accelerated charge particle emits photons
Acceleration = change in velocity (vector)

This phenomenon will be large for light particle (electrons)

\[ m \times \text{acceleration} = \sum \text{forces} \rightarrow \text{very easy to accelerate electrons} \]

For the coulomb force it scales as \( z^2/m \)

For electron the ratio between ionization loss and radiation loss is given by

\[ \frac{(dE/dx)_{\text{bremsstr}}}{(dE/dx)_{\text{bethe}}} \approx \frac{E \,(\text{MeV}) \, Z}{700} \]

For lead \( Z=82 \) 9 MeV electrons loss equally by ionization and by radiations

Application X ray Tubes, synchrotron light sources
Heated filament emits electrons by thermionic emission. Electrons are accelerated by a high voltage. X-rays produced when high speed electrons hit the metal target.

\[ I \propto \frac{1}{E^2} \]
Cerenkov radiation

Can only happen when the charged particle travels faster than the speed of light in the material \((c/n)\) in the index.

\[ \cos \theta = \frac{1}{\beta n} \]

With the angle \(\theta\) we can deduce the velocity.

Nuclear reactor
Particle with velocity greater than speed of light in the medium.
The light is in the visible to the UV domain

\[ \frac{d^2N_{ph}}{dL \, d\lambda} = 2\pi \alpha Z^2 \sin^2 \theta \frac{1}{\lambda^2} \]

Loss around 1keV/cm
1% of ionization losses

Univ Minn.
Others
Noise radiations/scintillations see gammas
A few words again about electrons ...

Due to their light mass they lose their energy with a chaotic behavior

As a rule of thumb

For all solid materials

~1MeV/mm

The heavier shorter is the range

They are very often subject to Bremsstrahlung and Cerenkov
What about Positron $\beta^+$ ??

They have also a modified bethe formula,,,

They lose their energy at about the same rate as electrons

There is a big difference at the end of their path

Since they are anti matter they cannot stay in our world an annihilates with an electron

La radioactivite.com
Free Range

Fluorine-18

\[ \gamma (511 \, \text{keV}) \]

Average range

<table>
<thead>
<tr>
<th>Element</th>
<th>Range</th>
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</thead>
<tbody>
<tr>
<td>Carbon-11</td>
<td>4.1 mm</td>
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<tr>
<td>Nitrogen-13</td>
<td>5.4 mm</td>
</tr>
<tr>
<td>Oxygen-15</td>
<td>8.2 mm</td>
</tr>
<tr>
<td>Fluorine-18</td>
<td>2.6 mm</td>
</tr>
</tbody>
</table>

\[ E = 2 \, m_e c^2 \]
Application nuclear medicine  PET imaging
Positron emission tomography
IRM

PET
Straggling

To straggle definition (dictionary.com)
1. to stray from the road, course, or line of march.
2. to wander about in a scattered fashion; ramble.
3. to spread or be spread in a scattered fashion or at irregular intervals: The trees straggle over the countryside.

Because of the large interaction number and probabilistic processes (cross section) each particle history will have some statistical fluctuations

The mean will be in accordance with the Bethe formula or other formulas

There will be fluctuation in energy, angle, range....
Energy straggling

As particles crossed a thin target their mean energy will be given by

\[ E = E_0 - \frac{dE}{dx} \Delta x \]

Even if the energy spread was null before the target it will broadened by straggling and the energy spread (sigma) will be given by

\[
\frac{N(E) dE}{N} = \frac{1}{\alpha \pi^{1/2}} \exp \left[ -\frac{(E_0 - E)^2}{\alpha^2} \right]
\]

\[
\sigma^2 = 4 \pi Z^2 e^4 N Z x_0 \left[ 1 + \frac{kI}{m_e V^2} \ln \frac{2m_e V^2}{I} \right]
\]

The broadening is larger as the particle is lighter (more subject to straggling) electrons.
Straggling of 3 MeV Protons Through a 3.3 mg cm-2 gold foil
Angular Straggling

The same is true for the angular divergence of a beam

Even if the mean direction is the same before and after the thin target there is some angular opening in the beam.

\[ \theta_{\text{rad}} = \left[ 8.91 \frac{NZ(z+1)z^2}{E^2} \ln \frac{3.52 \times 10^{10} N(z+1)}{Z \left( Z^{2/3} + z^{2/3} \right)} \right]^{9/2} \]

7 MeV protons going through a 55 mg/cm² thick aluminium foil are slowed down to 4 MeV. The energy straggling is 70 keV and the angular straggling is 5°.
Any beam target interaction will modify the beam properties

Broadening in energy, direction, charge

These effects can limit the performances of detection systems.

Example: telescope or ΔE E method

The ΔE E method is the only method to identify simultaneously the energy, the charge, and the mass of an ion.
Photons

As neutral particle, photons will interact very rarely with matter. When they interact they interact individually with one particle

The attenuation law (Beer Lambert)
Is given by

\[ I(x) = I_0 e^{-\mu x} \]

\( I_0 \) is the initial intensity, \( I(x) \) is the intensity after \( x \), \( \mu \) is the linear attenuation coefficient
\( \mu = N \sigma \)
\( N \) the number of target per volume unit, \( \sigma \) the total cross section
Total cross section is the sum of all the cross section, ie all the interaction processes

\[ \sigma = \sum_i \sigma_i \]

\(\sigma_i\) are the corresponding cross section for the \(i\) interaction.

We will cover different interaction:

- Elastic scattering
- Photo electric
- Compton
- Pair Production

Of course there are many others.
Photo electric effect

The photon is giving all its energy to a bound electron

The electron energy $E_{e-}$ after the photo electric effect is given by

$$E_{e-} = E_0 - B_i$$

$E_0$ is the initial photon energy
$B_i$ is the electron binding energy

The photo electric effect has a threshold
Historically unexplained phenomenon ~1900

Experiment: Electric current when a metal plate is illuminated by light

http://www.relativitycalculator.com/photoelectric_effect.shtml
Experimentally what can be vary

The light intensity
The light frequency
The stopping voltage
The cathode material

In the classical explanation
Intensity= EM Field intensity
→ How does it explain that the electrons have more energy???
What links the electron energy and the frequency???
Different behavior with different metals?

Another unexplained experimental fact is the instantaneous current (no time needed to gain energy).
Quantum explanation
Albert Einstein 1905, using Planck's idea of quantum particle (Photons)
quanta = photons = $h\nu$
No more EM wave

$$E_e = h\nu - \phi$$
Scaling Laws

\[ \sigma_{ph} \propto \frac{Z^5}{E^{3.5}} \]

Outside the shell discontinuities

Varies strongly with Z and with E

The denser the material the higher the absorption
The higher the energy the more transparent is the absorber

One can represent
\[ \frac{\mu}{\rho} \]
with \( \mu = N\sigma \) and \( \rho \) the specific mass in g/cm\(^3\)
Compton effect

Incoherent diffusion on an electron
Avant la collision :

\[ E = \hbar \nu \]
\[ p = \hbar \kappa \]

Après la collision :

\[ \sum p_{xf} = \sum p_{xi} \]

\[ \Rightarrow \quad p_2 \cos(\theta) + p_e \cos(\phi) = p_1 \]

\[ \Rightarrow \quad p_e \cos(\phi) = p_1 - p_2 \cos(\theta) \]

\[ \Rightarrow \quad p_e^2 \cos^2(\phi) = p_1^2 - 2p_1p_2 \cos(\theta) + p_2^2 \cos^2(\theta) \]
Momentum

\[ \sum p_{y_f} = \sum p_{y_i} \]

\[ \Rightarrow \quad p_2 \sin(\theta) + p_e \sin(\phi) = 0 \]

\[ \Rightarrow \quad p_e \sin(\phi) = -p_2 \sin(\theta) \]

\[ \Rightarrow \quad p_e^2 \sin^2(\phi) = p_2^2 \sin^2(\theta) \]

\[ \begin{align*}
[ p_e^2 \cos^2(\phi)] + [ p_e^2 \sin^2(\phi)] &= [ p_1^2 - 2p_1p_2 \cos(\theta) + p_2^2 \cos^2(\theta)] + [ p_2^2 \sin^2(\theta)] \\
\Rightarrow \quad p_e^2 \left( \cos^2(\phi) + \sin^2(\phi) \right) &= p_1^2 - 2p_1p_2 \cos(\theta) + p_2^2 \left( \cos^2(\theta) + \sin^2(\theta) \right) \\
\Rightarrow \quad p_e^2 &= p_1^2 - 2p_1p_2 \cos(\theta) + p_2^2 \quad (\Gamma)
\end{align*} \]
Energy

\[ \sum E_f = \sum E_i \]

\[ \Rightarrow \quad E_{\gamma_2} + E_{ef} = E_{\gamma_1} + E_{ei} \]

\[ \Rightarrow \quad p_2c + E_{ef} = p_1c + E_{ei} \]

\[ \Rightarrow \quad p_2c + \sqrt{p_{ef}^2 c^2 + m_e^2 c^4} = p_1c + \sqrt{p_{ei}^2 c^2 + m_e^2 c^4} \]

\[ \Rightarrow \quad p_2c + \sqrt{p_{e}^2 c^2 + m_e^2 c^4} = p_1c + m_e c^2 \]

\[ \Rightarrow \quad \sqrt{p_{e}^2 c^2 + m_e^2 c^4} = p_1c - p_2c + m_e c^2 \]

\[ \Rightarrow \quad \sqrt{p_{e}^2 c^2 + m_e^2 c^4} = c(p_1 - p_2) + m_e c^2 \]

\[ \Rightarrow \quad p_{e}^2 c^2 + m_e^2 c^4 = (c(p_1 - p_2) + m_e c^2)^2 \]

\[ \Rightarrow \quad p_{e}^2 c^2 + m_e^2 c^4 = c^2(p_1 - p_2)^2 + 2m_e c(p_1 - p_2) + m_e^2 c^4 \]

\[ \Rightarrow \quad p_{e}^2 = (p_1 - p_2)^2 + 2m_e c(p_1 - p_2) \quad (4) \]
\[
\begin{align*}
[p_e^2] - [p_e^2] &= [p_1^2 - 2p_1p_2 \cos(\theta) + p_2^2] - [(p_1 - p_2)^2 + 2m_e c(p_1 - p_2)] \\
\Rightarrow &\quad 0 = p_1^2 - 2p_1p_2 \cos(\theta) + p_2^2 - (p_1 - p_2)^2 - 2m_e c(p_1 - p_2) \\
\Rightarrow &\quad 0 = p_1^2 - 2p_1p_2 \cos(\theta) + p_2^2 - (p_1^2 - 2p_1p_2 + p_2^2) - 2m_e c(p_1 - p_2) \\
\Rightarrow &\quad 0 = -2p_1p_2 \cos(\theta) + 2p_1p_2 - 2m_e c(p_1 - p_2) \\
\Rightarrow &\quad 0 = 2p_1p_2(1 - \cos(\theta)) - 2m_e c(p_1 - p_2) \\
\Rightarrow &\quad 2m_e c(p_1 - p_2) = 2p_1p_2(1 - \cos(\theta)) \\
\Rightarrow &\quad p_1 - p_2 = \frac{p_1p_2}{m_e c}(1 - \cos(\theta)) \\
\Rightarrow &\quad \frac{h}{\lambda_1} - \frac{h}{\lambda_2} = \frac{(h/\lambda_1)(h/\lambda_2)}{m_e c}(1 - \cos(\theta)) \\
\Rightarrow &\quad \frac{1}{\lambda_1} - \frac{1}{\lambda_2} = \frac{h}{m_e c \lambda_1 \lambda_2}(1 - \cos(\theta)) \\
\Rightarrow &\quad \lambda_2 - \lambda_1 = \frac{h}{m_e c}(1 - \cos(\theta)) \\
\Rightarrow &\quad \lambda_f - \lambda_i = \frac{h}{m_e c}(1 - \cos(\theta)) \quad \blacksquare
\end{align*}
\]
And

\[
h' = \frac{\hbar}{1 + \frac{\hbar}{mc^2(1 - \cos \theta)}}
\]

\(h'\) energy of the scattered photon
\(h\) energy of the initial photon
\(mc^2\) electron energy mass
\(\theta\) scattering angle

\[
E_e = h - h'
\]

Electron energy

REMEMBER the CP ionize the matter not the NP
The cross section is given by the Klein Nishina formula

\[
\frac{d\sigma}{d\Omega} = \frac{1}{2r_0^2} \frac{1 + \cos^2 \theta}{\left[1 + 2\varepsilon \sin^2 \frac{\theta}{2}\right]^2} \left\{1 + \frac{4\varepsilon^2 \sin^4 \frac{\theta}{2}}{\left[1 + \cos^2 \theta\right]\left[1 + 2\varepsilon \sin^2 \frac{\theta}{2}\right]}\right\}
\]

With \(r_0\) the classical radius of the electron

\(e^2/mc^2 \sim 2,8 \times 10^{-13} \text{ cm}\) and \(\varepsilon\) the ratio between the energy and the rest mass \(mc^2\)
By integrating over all angles the total cross section is given by

\[ \sigma = 2\pi \cdot r_0^2 \left\{ \frac{1 + \varepsilon}{\varepsilon} \left[ \frac{2 + 2\varepsilon}{1 + 2\varepsilon} - \frac{\ln(1 + 2\varepsilon)}{\varepsilon} \right] + \frac{\ln(1 + 2\varepsilon)}{2\varepsilon} - \frac{1 + 3\varepsilon}{(1 + 2\varepsilon)^2} \right\} \]

\varepsilon = \frac{E}{m_\mu c^2}
Pair Production from sea of “negative” mass
Dominance domains

Cross sections and attenuation coefficient:

Photons and medical applications

**X Ray radiography**
Contrast obtain by different absorption ($\mu_{\text{bone}} - \mu_{\text{tissue}}$)
Low energy X rays = absorption = dose

**Nuclear medecine**
Single photon
Radio isotope with a biological molecule
Isotope concentration is imaged
Common isotope is 99Tcm (140 keV rather high)
High transmission
Low detection efficiency $\rightarrow$ dose

PET Positron+ 2 511 keV gammas
CP interaction (Blur) $\rightarrow$ annihilation $\rightarrow$ Gammas (detection efficiency)

**Radio therapy**
Few MeV photons
No bragg peak continous attenuation still the main solution using radiations for cancer treatment
Neutrons matter interactions

Reminder

Mn=939 MeV/c^2
Q=0
S=1/2
T_{1/2} = 12 minutes

Neutrons are named with their energy

Cold, thermal, fast and so on

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<th>Neutron energy</th>
<th>Energy range</th>
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<td>0.025 eV</td>
<td>Thermal neutrons</td>
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<td>0.025 eV-0.4 eV</td>
<td>Epithermal neutrons</td>
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<td>0.4 eV-0.6 eV</td>
<td>Cadmium neutrons</td>
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<td>EpiCadmium neutrons</td>
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<td>1 eV-10 eV</td>
<td>Slow neutrons</td>
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<td>Resonance neutrons</td>
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<td>Intermediate neutrons</td>
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<td>1 MeV-20 MeV</td>
<td>Fast neutrons</td>
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<td>&gt; 20 MeV</td>
<td>Relativistic neutrons</td>
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## How to produce neutrons

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<th>Price</th>
<th>Pros</th>
<th>Cons</th>
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<td>Reactor</td>
<td>Up to 10¹⁴ n/cm²/s</td>
<td>Extremely high 10M€</td>
<td>High flux At any energy</td>
<td>Expensive Not easy to use safety</td>
</tr>
<tr>
<td>DT generator</td>
<td>Up to 10⁹ n/s</td>
<td>100 k€</td>
<td>Moderate flux Easy to use cost</td>
<td>cost</td>
</tr>
<tr>
<td>Isotopique source</td>
<td>Up to 10⁶ n/s</td>
<td>Few k€</td>
<td>Cost Easy to use</td>
<td>Low flux</td>
</tr>
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</table>
Absorption

The neutron interaction will depend only on the Nuclear Structure of the nucleus.
Elastic Scattering

A neutron interacts with a nucleus and lose some energy

\[ E_n = E_0 \frac{A^2 + 1 + 2A \cos \theta}{(A+1)^2} \]

The recoil nucleus gains \( E_0 - E_n \)

This the process to slow down neutrons
It is called moderation

The medium where the moderation takes place is called a moderator

Kinematically the lower the A the more efficient could be the moderation

Example a few cm of water is enough to slow down fast neutron to thermal energies.
More or less constant in our energy domain
Hydrogen, Iron, Uranium elastic cross section
more or less constant except in the resonances region
Radiative capture \((n, \gamma)\)

The neutron is captured by the nucleus \((A)\), the \((A+1)\) nucleus emits a gamma.

This is the process called activation and this is how material in a neutron environment gets activated.

The cross section varies as \(1/V\).
Nuclear reactions

Charge particles

\[ ^{3}\text{He} + n \rightarrow ^{3}\text{H} + p + 765 \text{ KeV} \quad (19) \]

\[ ^{10}\text{B} + n \rightarrow ^{7}\text{Li} + \alpha + 2,78 \text{ MeV} \quad (20) \]

\[ ^{6}\text{Li} + n \rightarrow ^{3}\text{H} + \alpha + 4,78 \text{ MeV} \quad (21) \]

\[ ^{235}\text{U} + n \rightarrow \text{fission products} + \sim 160 \text{ MeV} \quad (22) \]

The reaction energy is released as kinetic energy.

Cross section is varying like 1/V.

The slower the neutrons the higher the reaction rate.
Resonance

Resonance occurs when the neutron energy is exactly the difference between the energy level of the initial nucleus $A$ plus the neutron binding energy $S_n$ and one of the energy levels of the $A+1$ nuclei.
Glenn F Knoll. Wiley

WR Leo
Springer

Marmier Sheldon
Academic press