ORBITAL PROPERTIES AND GRAVITATIONAL WAVES SIGNATURES OF STRANGE CRYSTAL PLANETS

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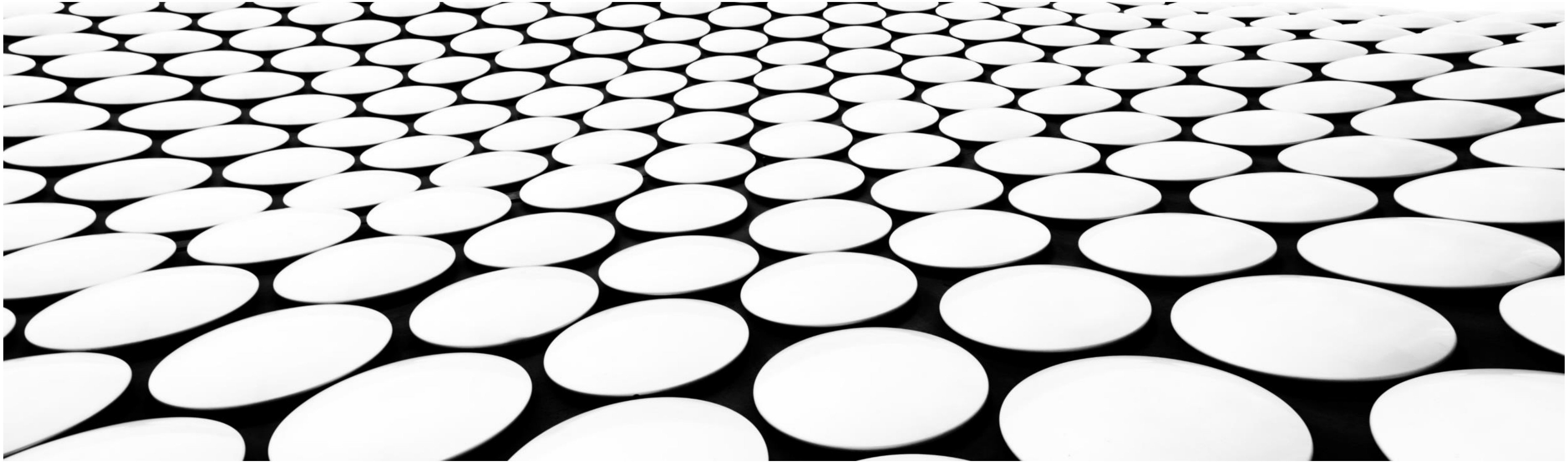
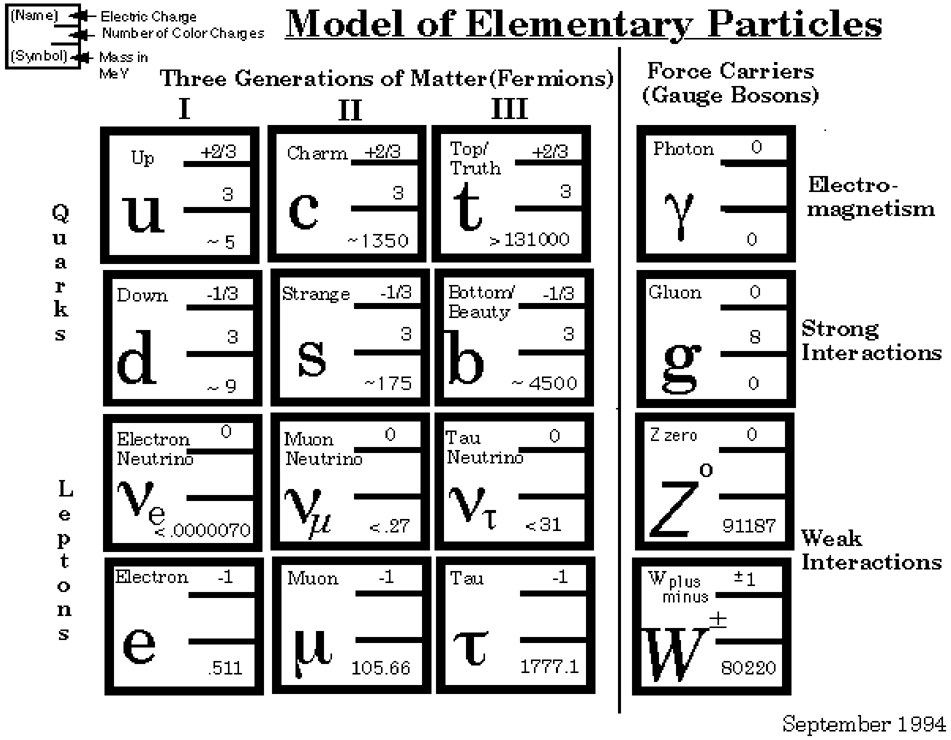
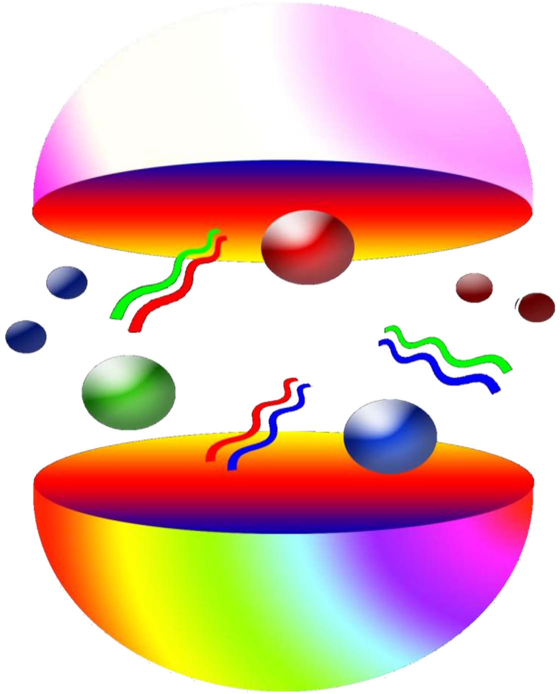


QUARKS…

- Building blocks of baryonic matter





BODMER-WITTEN-TERAZAWA'S STRANGE MATTER HYPOTHESIS

- The strange matter hypothesis surmises that the true and absolute ground state of the trong interaction is a deconfined state of quark matter, consisting of up, down and strange quarks (in aproximately the same proportion).
- The confined state of quarks that we know would be merely a very long-lived state, but not absolutely stable,
- There is no strong evidence either to support or oppose such hypothesis. There is, in fact, many works that show that such hypothesis is perfectly compatible with our current understanding of the universe.
- If the confined state of quarks is indeed a very long lived (but not absolutely stable) state, then only in very long time scale (such that of stellar evolution) it would be possible for hadrons to transform in strange matter.
- Compact stars being the end result of a star's life cycle would be ideal candidates to search for such matter.

SIMPLE ENERGY CONSIDERATIONS FOR THE STRANGE MATTER HYPOTHESIS

- Considering a gas of two-flavor (*u*,*d*) quark matter

$$\text{energy per baryon} \sim 1100\ MeV$$

- For three-flavor quark-matter *(u,d,s)*

$$\text{energy per baryon} \lesssim 930\ \text{MeV}$$

- Recalling that for ^{56}Fe

$$\text{energy per baryon} \sim 930\ \text{MeV}$$

- Whether or not strange quark matter is absolutely stable will depend on (currently not completely understood) properties of the strong interaction (particularly QCD confinement and the strange quark mass).

THE MIT BAG MODEL

- First description on a obscure french article by *P. N. Bogoliubov: “Sur un modele à quarks quase-indépendants” (1967)*
- Later improved (without knowledge of Bogoliubov work) by five MIT scientists: *Chodos, Jaffe, Johnson, Thorn and Weisskopf (1974)*.
- The MIT version fixed the major problem of Bogoliubov’s description (energy-momentum conservation and causality).
- This was done by introducing a phenomenological “confining” pressure (BAG) to account for confinement.
- Evidently there are shortcomings in the MIT bag model, most prominent of which is violation of Chiral Symmetry
- First applications to compact stars by Farhi, Alcock e Olinto (1986)

THE MIT BAG MODEL – FUNDAMENTALS

- Quarks are described by a fermi gas.
- up and down quarks are massless.
- Strange quark is massive.
- Confinement is included as a scalar field representing the “confining” pressure (the “BAG”)
- One can also include one gluon exchange (1st order) corrections
- Pairing effects can also be included (although we do not consider them here)

THE MIT BAG MODEL – FUNDAMENTALS

Thermodynamic Potentials

$$\Omega_u = -\frac{\mu_u^4}{4\pi^2}\left(1-\frac{2\alpha_c}{\pi}\right),$$

$$\Omega_d = -\frac{\mu_d^4}{4\pi^2}\left(1-\frac{2\alpha_c}{\pi}\right),$$

$$\Omega_e = -\frac{\mu_e^4}{12\pi^2},$$

$$\begin{aligned}\Omega_s = &-\frac{1}{4\pi^2}\{\mu_s\sqrt{\mu_s^2-m_s^2}(\mu_s^2-\frac{5}{2}m_s^2)+\frac{3}{2}m_s^4\ln(\frac{\mu_s+\sqrt{\mu_s^2-m_s^2}}{m_s})\\ &-\frac{2\alpha_c}{\pi}[3(\mu_s\sqrt{\mu_s^2-m_s^2}-m_s^2\ln(\frac{\mu_s+\sqrt{\mu_s^2-m_s^2}}{m_s}))^2-2(\mu_s^2-m_s^2)^2\\ &+3m_s^4\ln^2\frac{m_s}{\mu_s}+6\ln\frac{\sigma}{\mu_s}(\mu_s m_s^2\sqrt{\mu_s^2-m_s^2}-m_s^4\ln(\frac{\mu_s+\sqrt{\mu_s^2-m_s^2}}{m_s}))]\}.\end{aligned}$$

Chemical equilibrium and charge neutrality

$$\mu_d = \mu_s \equiv \mu,$$

$$\mu_u + \mu_e \equiv \mu,$$

$$\rho_i = -\frac{\partial\Omega_i}{\partial\mu_i},$$

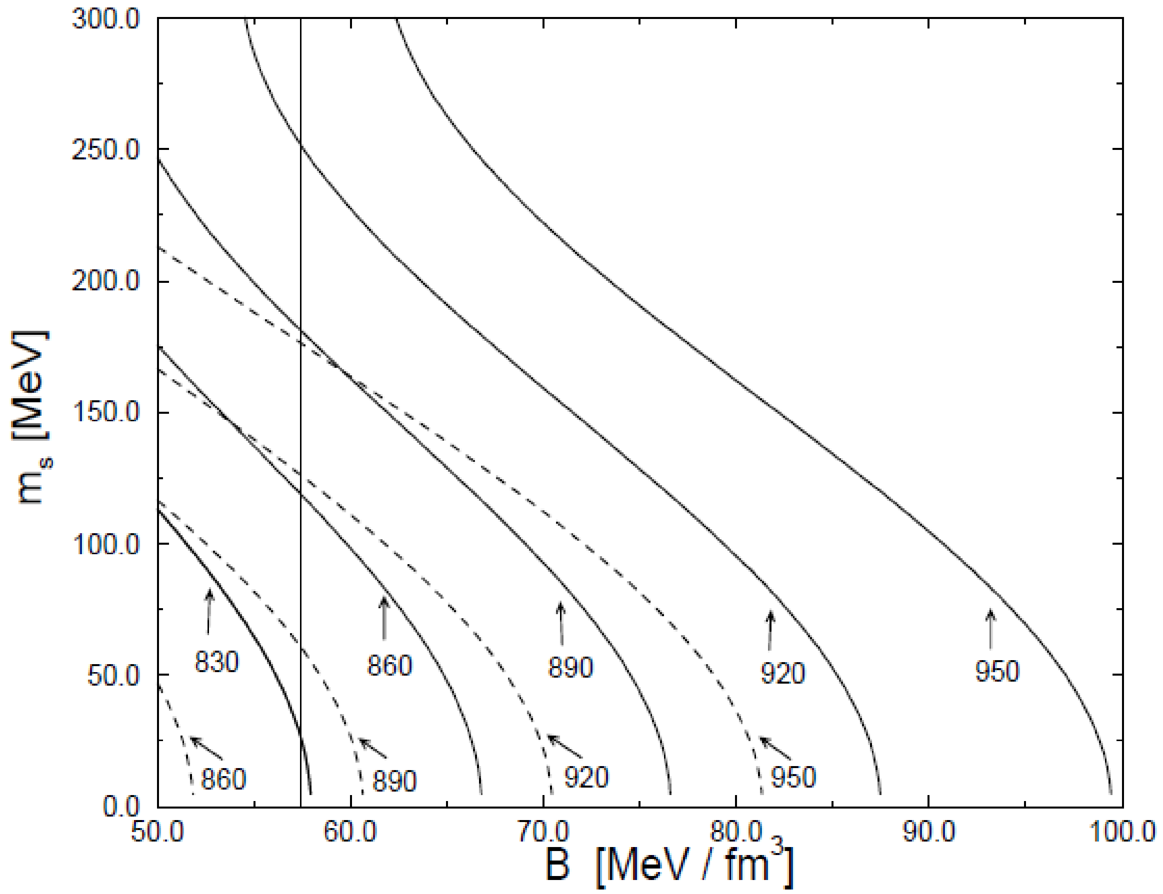
$$\sum_{i=u,d,s,e} q_i\rho_i = 0 = \frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e.$$

Equation of State

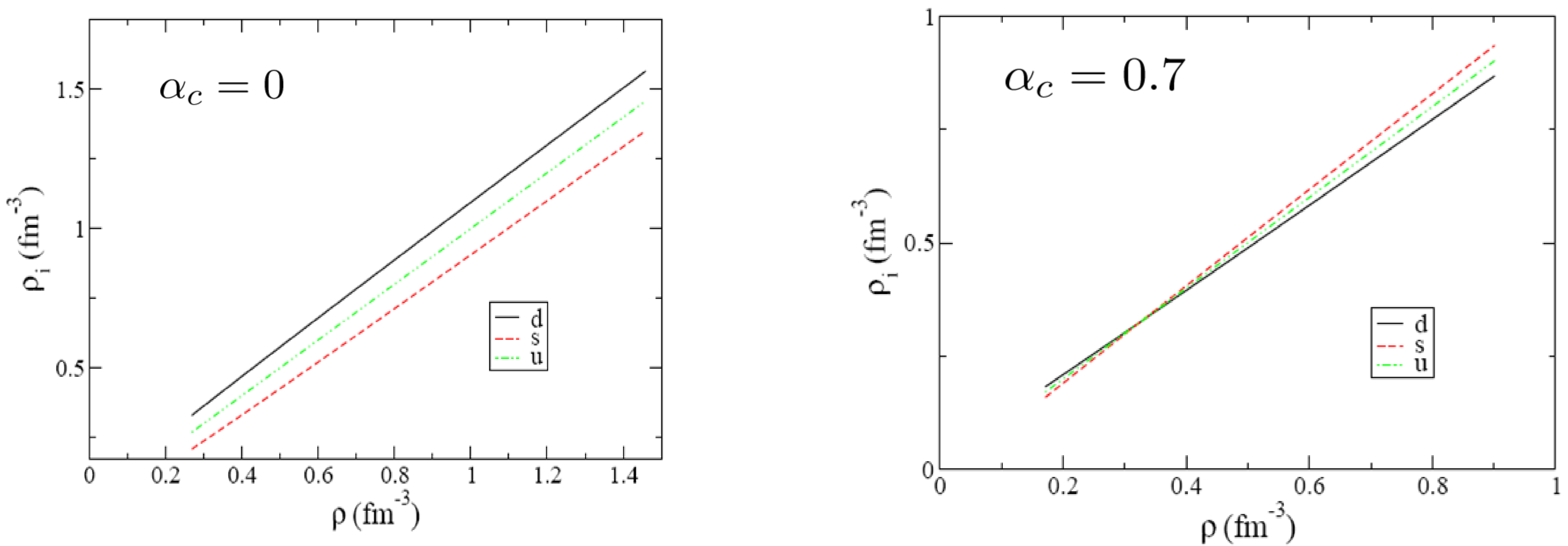
$$\epsilon = \sum_{i=u,d,s,e}(\Omega_i + \mu_i\rho_i) + B.$$

$$p = -B - \sum_{i=u,d,s,e}\Omega_i.$$

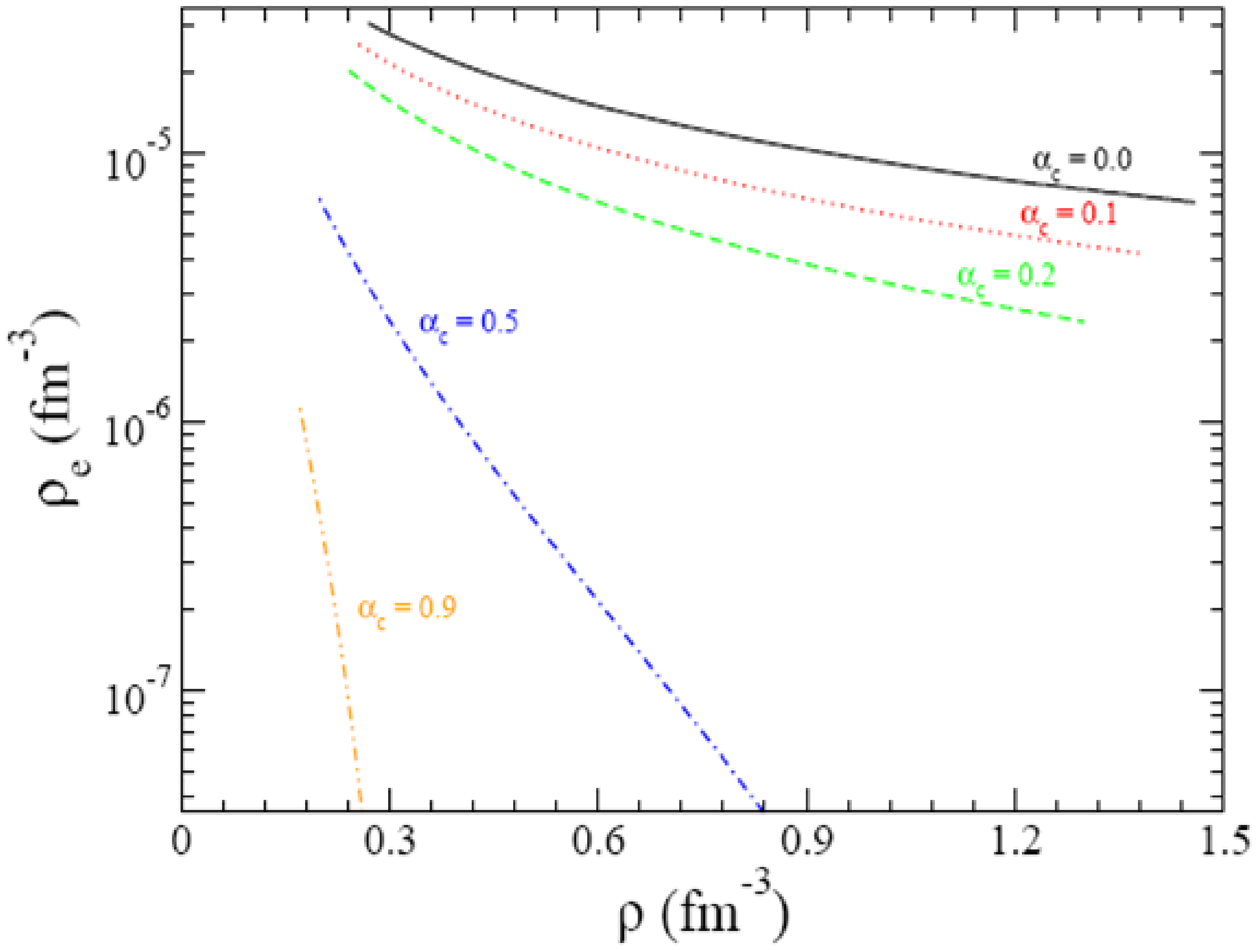
THE MIT BAG MODEL – STABILITY ANALYSIS



THE MIT BAG MODEL – QUARK COMPOSITION



THE MIT BAG MODEL – ELECTRON COMPOSITION



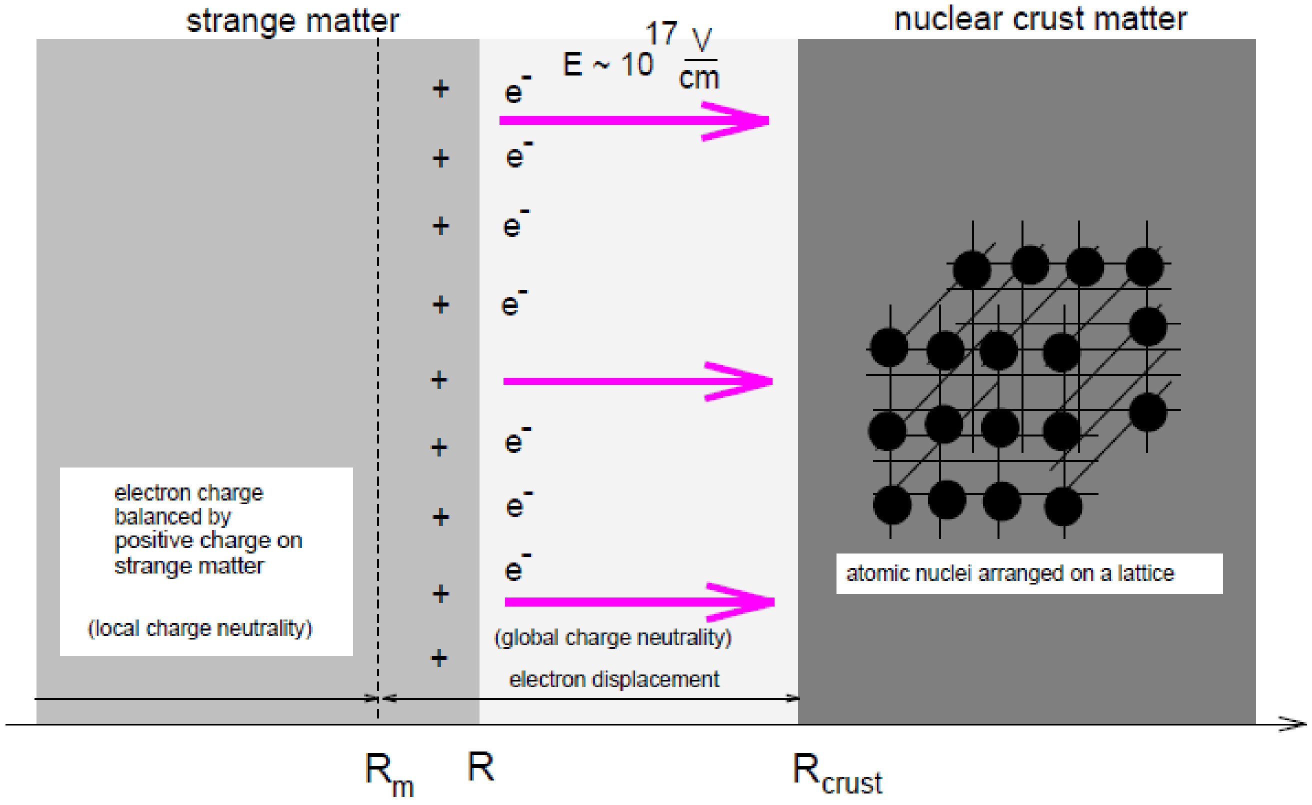
THE NUCLEAR CRUST

Quark Matter has a finite density at zero pressure ($\varepsilon \sim 3B$)

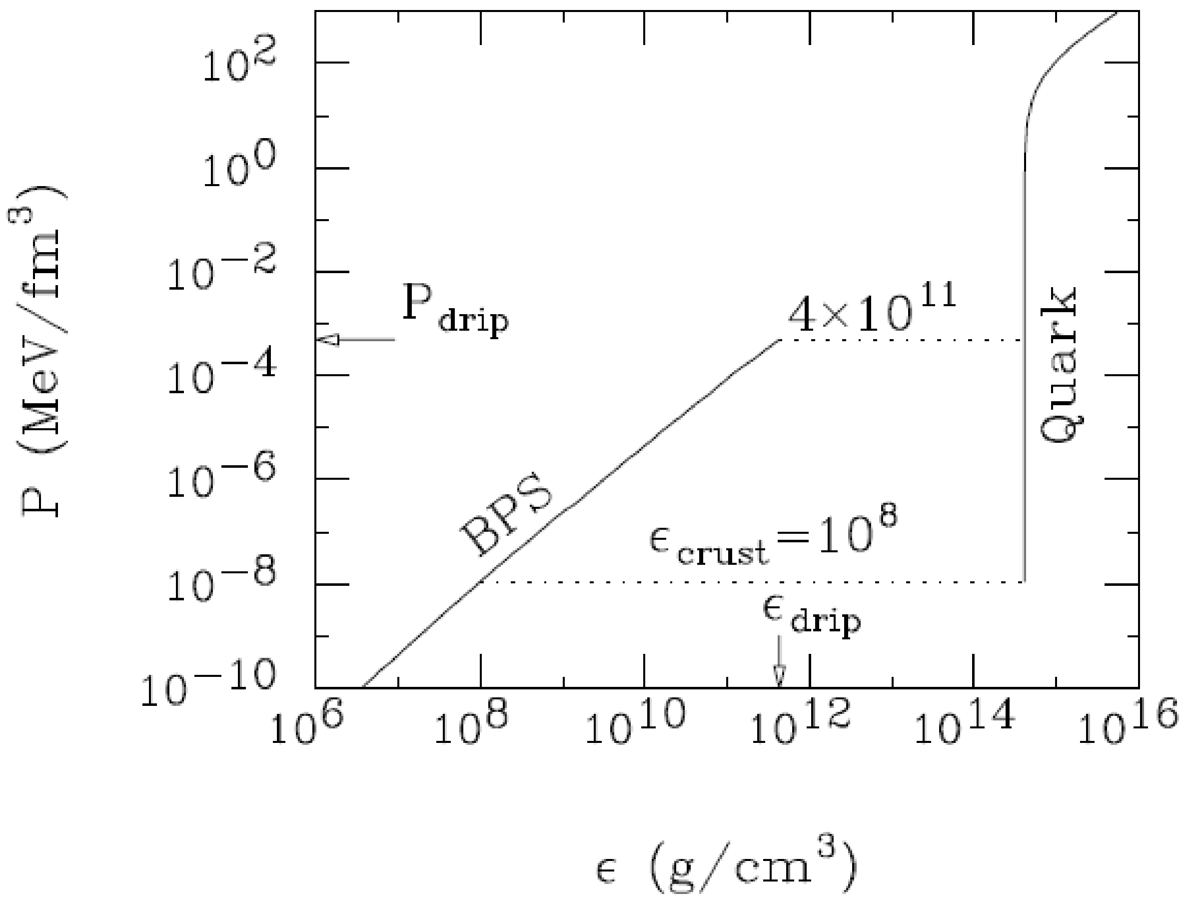
Electrons are not bound by the strong interaction, but rather by the electromagnetic force.

Electron screening give rise to an ultra high electric field.

Such field may support a nuclear matter crust (below drip density)



THE EQUATION OF STATE



MACROSCOPIC STRUCTURE

- Must resort to General Relativity

$$ds^2 = e^{\nu(r)}c^2dt^2 - e^{\lambda(r)}dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2) \longrightarrow \boxed{\text{Metric}}$$

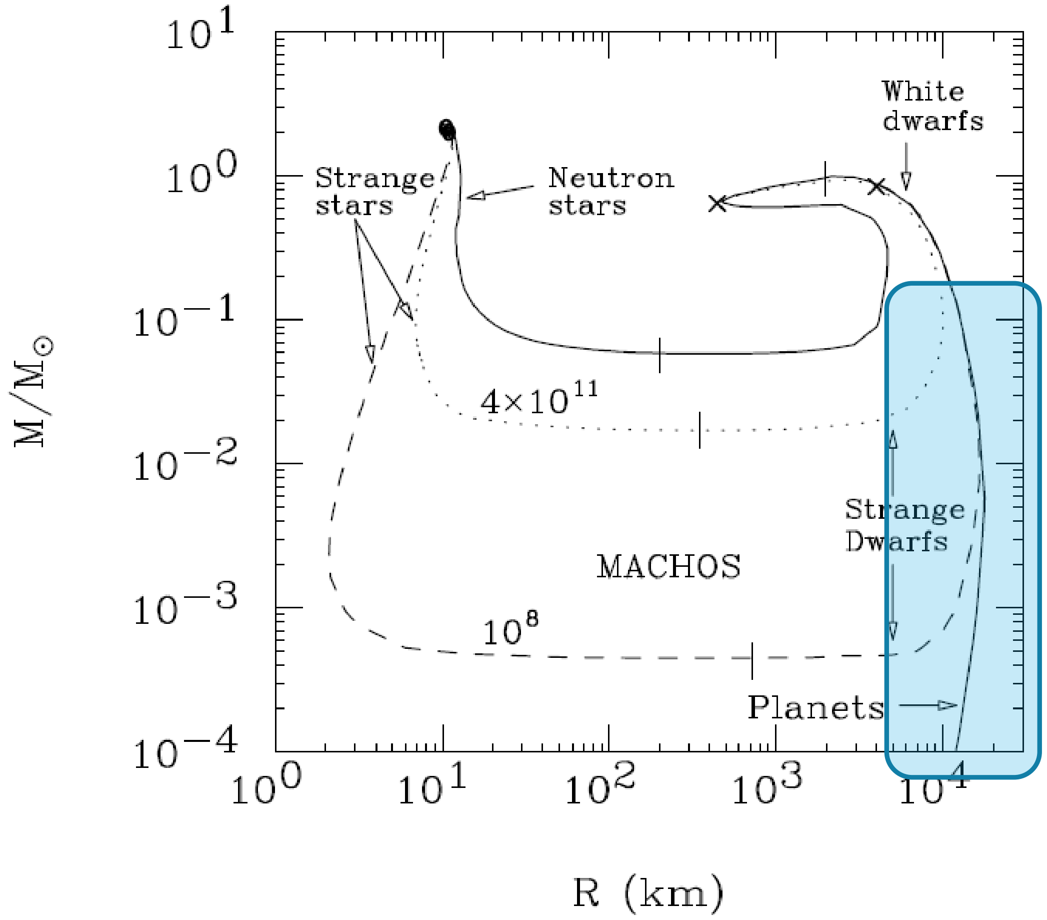
$$T^\mu_\nu = (p + \epsilon)u^\mu u_\nu + p\delta^\mu_\nu \longrightarrow \boxed{\text{Energy-Momentum Tensor (EOS)}}$$

$$G_{\nu\mu} = 8\pi T_{\nu\mu} \longrightarrow \boxed{\text{Einstein's Equation}}$$

$$\frac{dp}{dr} = -\frac{[p(r) + \epsilon(r)][m(r) + 4\pi r^3 p(r)]}{r(r - 2m(r))} \longrightarrow \boxed{\text{TOV equation (General Relativistic Hydrostatic equilibrium)}}$$

$$m(r) = 4\pi \int_0^r dr r^2 \epsilon(r) \longrightarrow \boxed{\text{Stellar mass}}$$

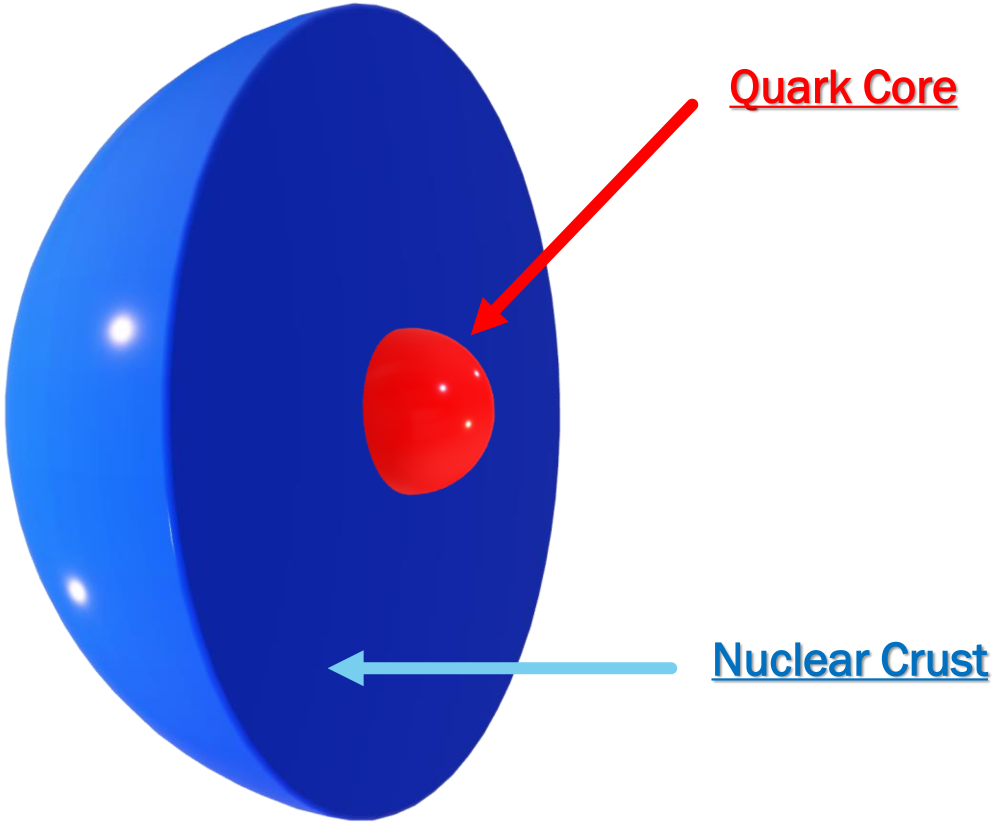
STELLAR SEQUENCE



Planet-like objects

STRANGE PLANETS

- Small density strange stars.
- Very small (macroscopic) quark core.
- Very extense nuclear crust



QUARK MATTER IS SELF BOUND!

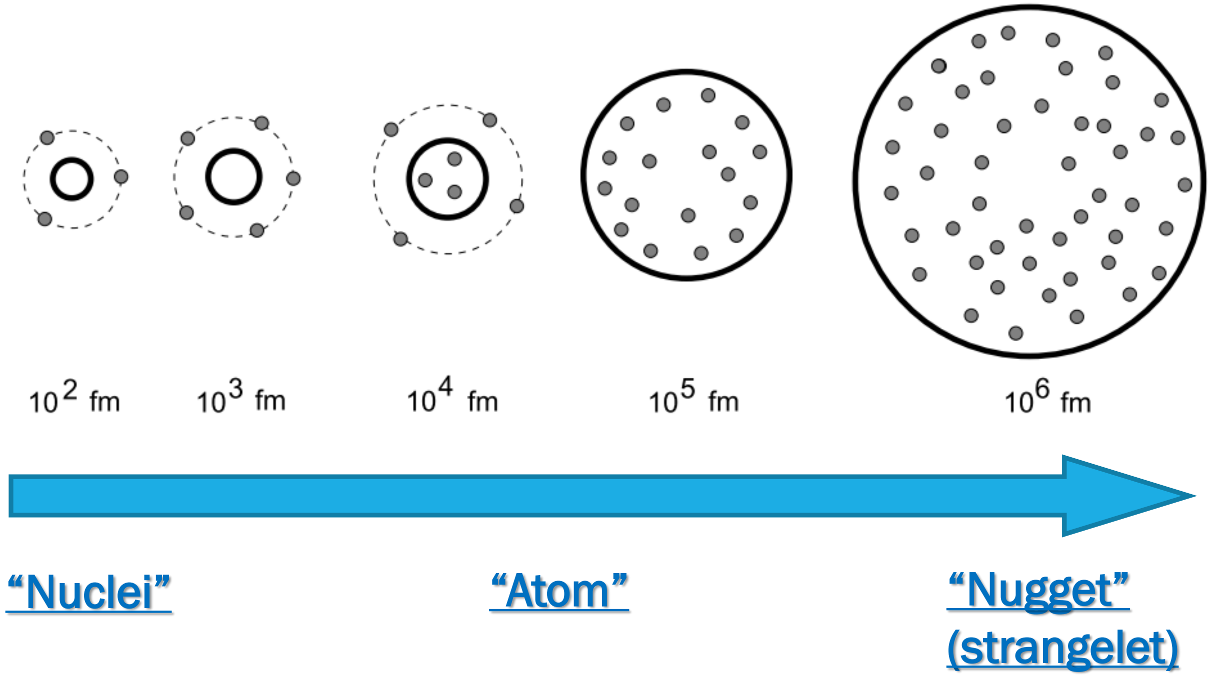
Neutron Stars are not giant nuclei!

They are gravitationally bound!

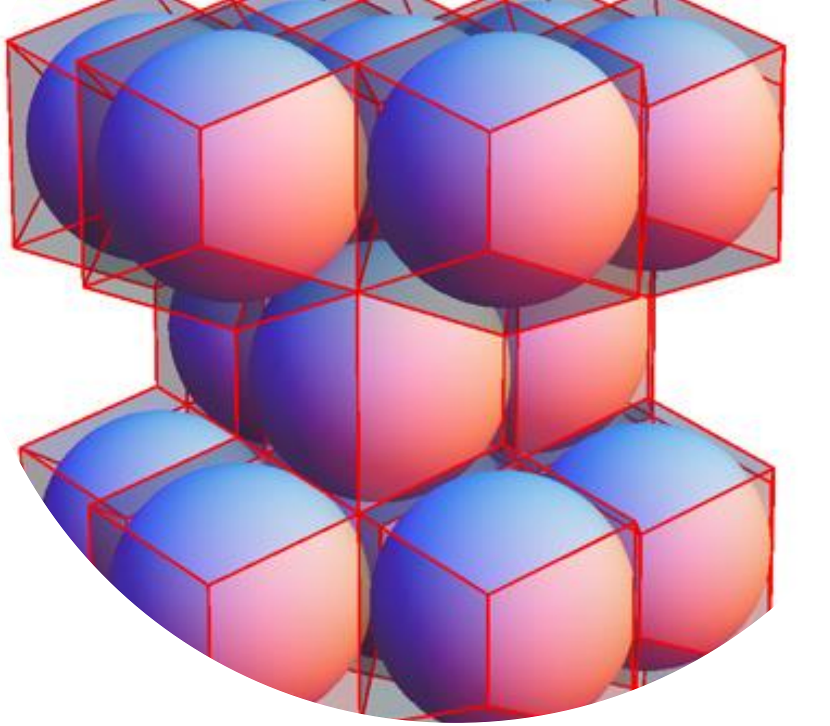
Quark matter (under Witten's hypothesis) is absolutely stable.

Quark stars are then self-bound!

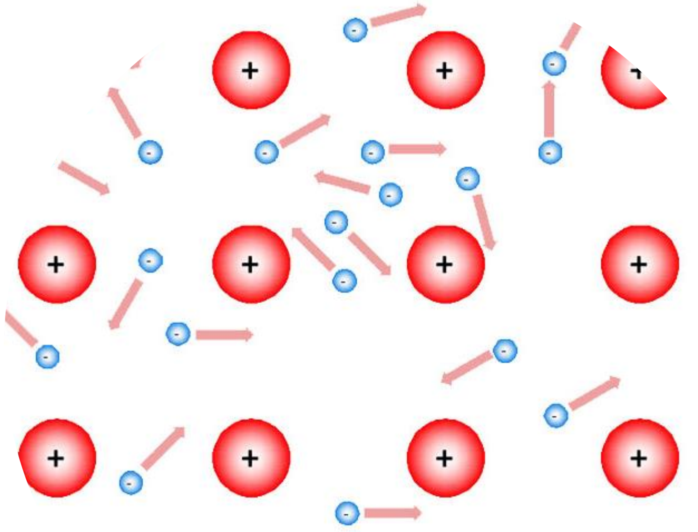
They can have very different sizes.



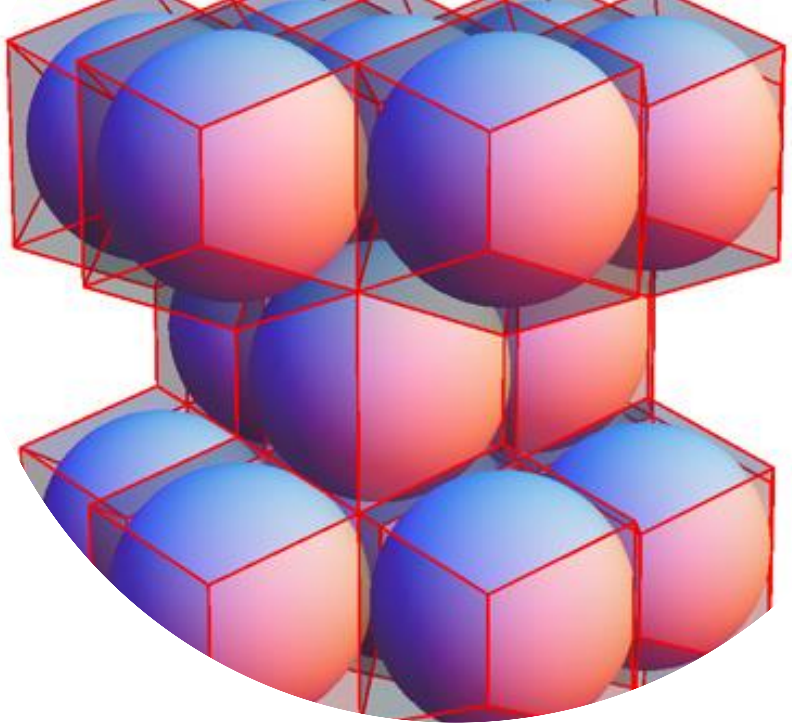
INTERMISSION: WHITE DWARF STRUCTURE



- Composed of Ions immersed in a sea of degenerate electrons.
- Ions are organized in a crystalline structure.
- Electrons form a degenerate gas.
- Pressure comes mostly from electrons.
- Energy density comes from Ions



INTERMISSION: WHITE DWARF STRUCTURE



- Given a nuclear species A

$$A = \frac{4}{3}\pi R^3 \rho \,,$$

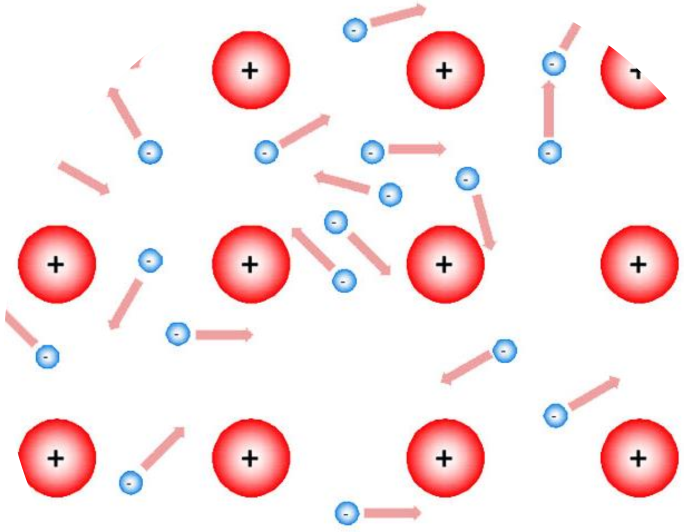
- The electrons form a degenerate gas.

$$k_e = (3\pi^2 \rho_e)^{1/3} = \left(3\pi^2 \frac{Z}{A}\rho\right)^{1/3}$$

- The equation of state is then given by

$$\epsilon = E_{\text{total}}/V \quad \Longrightarrow \quad \epsilon(\rho) = \frac{\rho}{A}\left(M(A,Z) - Zm_e - \frac{9}{10}\frac{(Ze)^2}{R}\right) + \epsilon_e(k_e)\,,$$

$$p(\rho) = p_e(k_e) - \frac{3}{10}\left(\frac{4\pi}{3}\right)^{1/3} Z^{2/3} e^2 \rho_e^{4/3}\,.$$



STRANGELET CRYSTALS

- For lower densities it maybe more energetically favorable for quark matter to fragmente into smaller (strangelet) pieces.
- Such strangelets, much like in a White dwarf, will form a lattice as to minimize energy.
- Whether or not such crystals can be formed will depend on the value of the quark matter surface energy.
- We assume that the surface energy is such that strangelet formation is allowed.

STRANGELET CRYSTALS

We consider a wide range of strangelets.

We also investigate the effect of different surface tensions.

We find that strangelets have a *Z/A* ratio is smaller when compared to ordinary nuclei

Label	A	Z	E (10^6 MeV)		
			$\sigma = 0.2$	$\sigma = 0.6$	$\sigma = 1.0$
Stra_1	5×10^3	581	4.5189	4.5189	4.5201
Stra_2	1×10^4	793	9.0258	9.0277	9.0297
Stra_3	5×10^4	1527	45.055	45.060	45.066
Stra_4	1×10^5	1986	90.073	90.082	90.091

STRANGELET CRYSTAL - EOS

Label	A	Z	E (10^6 MeV)		
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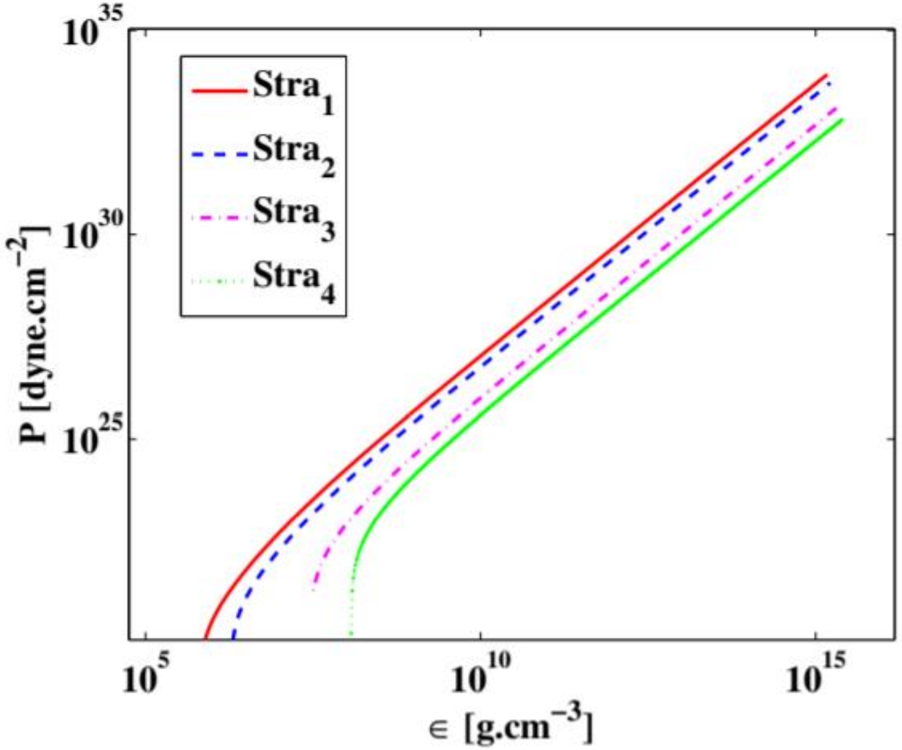
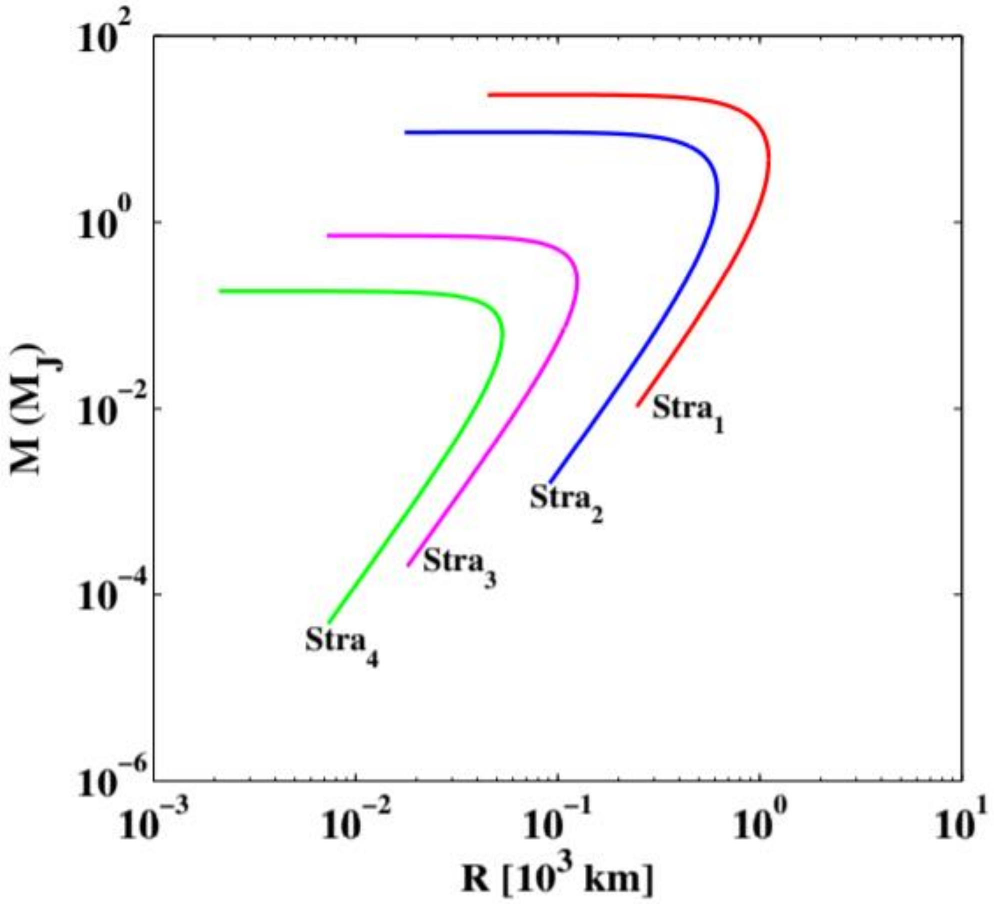


STRANGELET CRYSTAL PLANET - SEQUENCE

- Planetary Properties can be obtained by solving the structure equations.
- We found objects with masses comparable to that of observed planets.
- The strangelet planets are much smaller however.

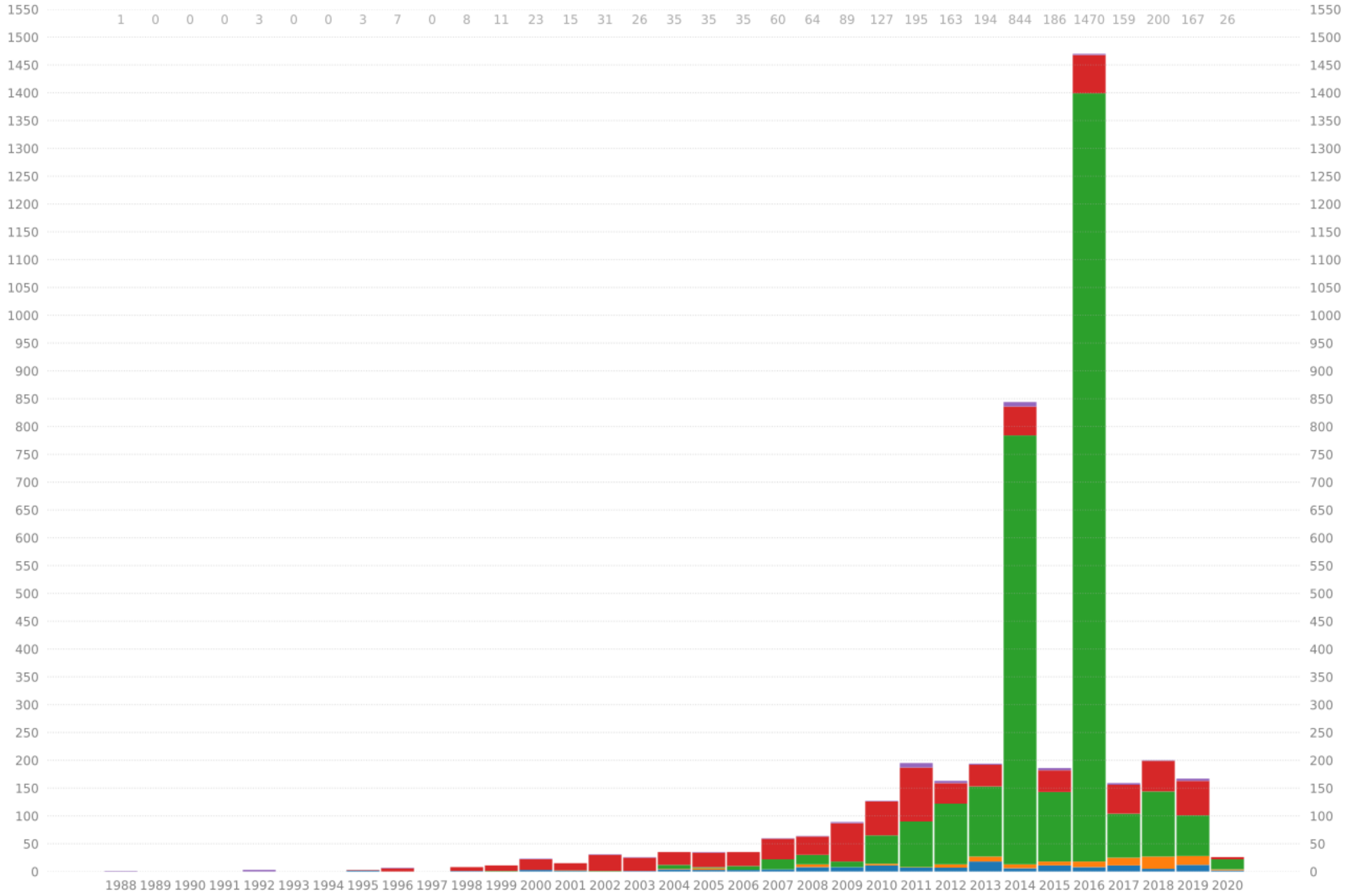


INTERMISSION II: EXO-PLANETS

Extrasolar planets (planets outside of the solar system)

Recent technological advances have allowed to a rapid increase in the observation of such objects (over 4000 as of now).

Observations indicate a wide range of mass (from a few Luna masses to over 30 Jupiters)

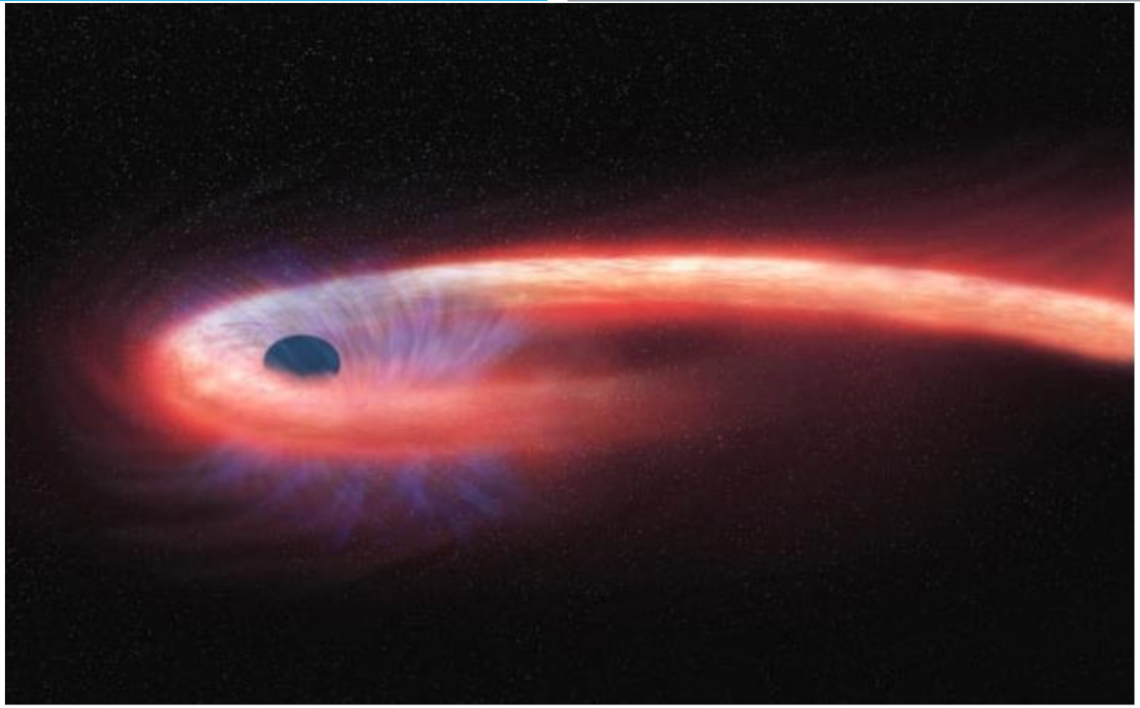


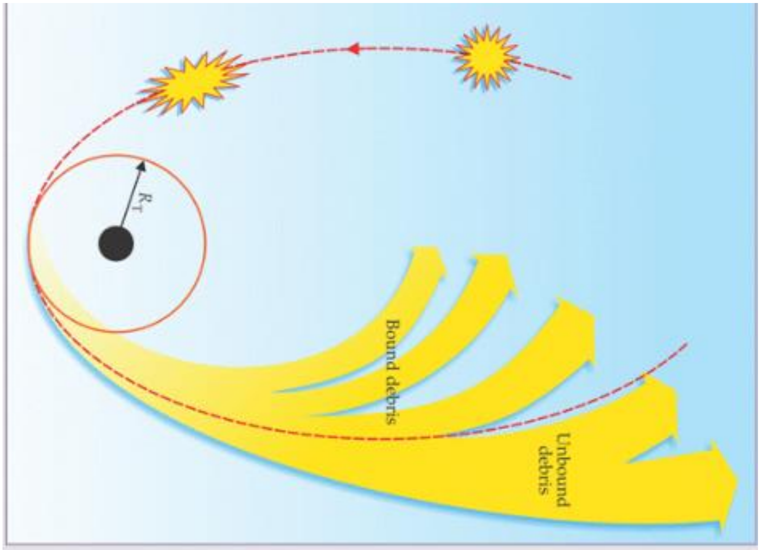
TIDAL DISRUPTION RADIUS

Orbit at which tidal forces would lead to the destruction of the orbiting object: TDE – Tidal Distruction Even

$$r_{td} \approx 1.5 \times 10^6 \left(\frac{M_\star}{1.4 M_\odot}\right)^{1/3} \left(\frac{\bar{\epsilon}}{4 \times 10^{14}}\right)^{-1/3}$$

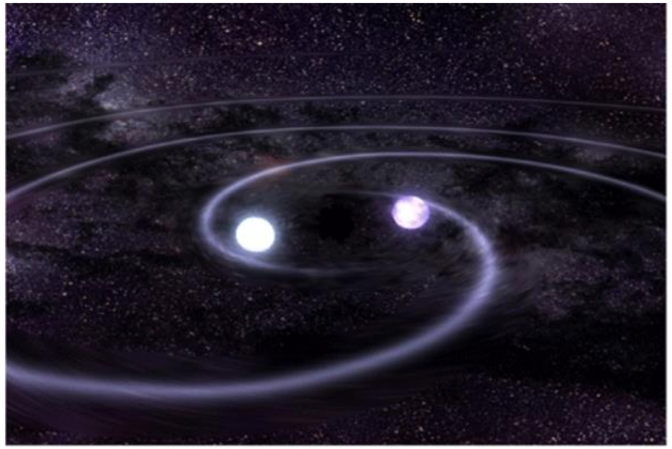
$$\frac{r^3}{P_{orb}^2} \approx \frac{G M_\star}{4\pi^2}.$$

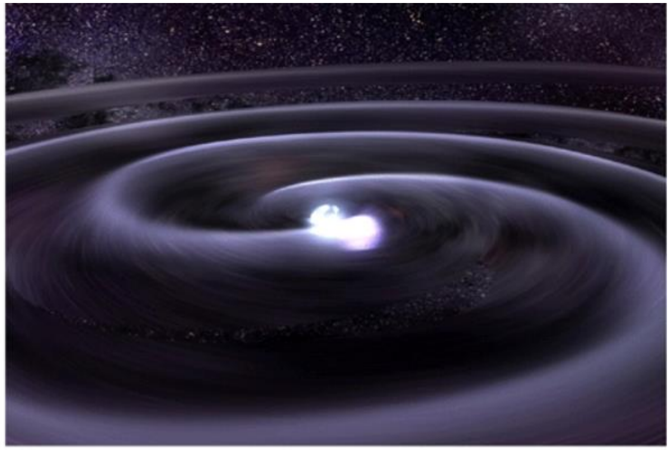


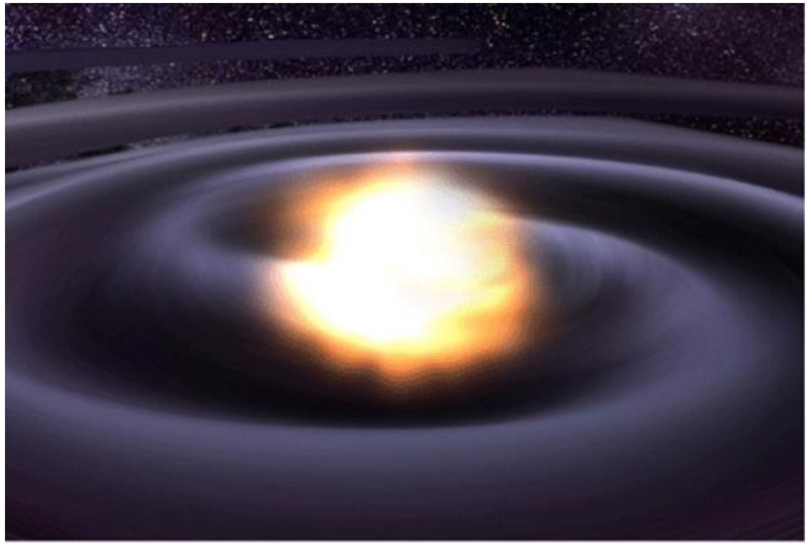


GRAVITATIONAL WAVE AMPLITUDE

Due to the high compactness of strangelet planets, their orbit may give rise to possibily detectable gravitational waves.







$$h = l\left(\frac{M_\star}{1.4M_\odot}\right)^{2/3}\left(\frac{\bar{\epsilon}}{4\times10^{14}}\right)^{4/3}\left(\frac{R}{10^4}\right)^{3}\left(\frac{d}{10}\right)^{-1}$$

STRANGELET PLANETS OBSERVABLES

For the strangelet Crystal planets investigated we have found

Stra	$M(M_J)$	$R(km)$	$\bar{\epsilon}$ (g/cm^3)	r_{td} (cm)	P_{orb} (ms)
1	23.3	96	2.5×10^{10}	3.8×10^7	107
2	9.2	53	5.9×10^{10}	2.8×10^7	69.9
3	7.2	8.0	1.3×10^{13}	4.7×10^6	4.6
4	0.181	3.65	3.5×10^{12}	7.3×10^6	9.0

$Stra$	$M(M_J)$	$R(km)$	$\bar{\epsilon}$ (g/cm^3)	h
1	23.3	96	2.51×10^{10}	3.08×10^{-21}
2	9.2	53	5.87×10^{10}	1.61×10^{-21}
3	7.2	8.0	1.33×10^{10}	7.68×10^{-21}
4	0.181	3.65	3.53×10^{12}	1.24×10^{-22}

Object Planets	$\bar{\epsilon}$ (g/cm^3)	r_{td} (cm)	P_{orb} (s)	h
Ordinary Planets				
Low density	10	5.1×10^{10}	~ 5263	4.9×10^{-29}
High density	30	5.6×10^{10}	~ 6100	7.1×10^{-26}
Strangelet Crystal Planets	$\sim 10^{10} - 10^{12}$	$\sim 4 \times 10^6 - 3 \times 10^7$	$\sim 0.009 - 0.107$	$\sim 10^{-22} - 10^{-21}$
Strange Planets	$\sim 4.0 \times 10^{14}$	$\sim 1.5 \times 10^6$	$\sim 8.45 \times 10^{-4}$	$\sim 10^{-23} - 10^{-21}$

SUMMARY

- The hypothesis of absolutely strange matter is still an open question, thus any possible mechanism for detecting it should be considered.
- Compact stars are prime subjects for such search. The presence of a nuclear crust, however, may mask the results.
- Technological advances have allowed us to increase the rate at which we can observe exoplanets.
- The wide range of properties exhibited by exoplanets make these objects good candidates for the search of quark matter.
- We have shown that strange crystal planets can orbit much closer to their host stars, as well as much faster.
- Furthermore future GW observatories, with higher sensibilities may allow us to search for GW signatures of such systems.