

LOW- Q^2 PARAMETRIZATIONS OF THE $\gamma^* N \rightarrow N^*$ TRANSITION AMPLITUDES

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Status of the problem [1]

- The electromagnetic structure of the N^* resonances can be parametrized in terms of $\gamma^* N \rightarrow N^*$ helicity amplitudes ($\alpha \simeq 1/137$, $K = \frac{M_R^2 - M_N^2}{2M_R}$) defined at N^* rest frame

$$\begin{aligned} A_{3/2} &= \sqrt{\frac{2\pi\alpha}{K}} \langle N^*, S'_z = +\frac{3}{2} | \epsilon_+ \cdot J | N, S_z = +\frac{1}{2} \rangle \\ A_{1/2} &= \sqrt{\frac{2\pi\alpha}{K}} \langle N^*, S'_z = +\frac{1}{2} | \epsilon_+ \cdot J | N, S_z = -\frac{1}{2} \rangle \\ S_{1/2} &= \sqrt{\frac{2\pi\alpha}{K}} \langle N^*, S'_z = +\frac{1}{2} | \epsilon_0 \cdot J | N, S_z = +\frac{1}{2} \rangle \end{aligned}$$

ϵ_m photon polarization; $|\mathbf{q}|$ photon 3-momentum

- There are correlations between the helicity amplitudes near the pseudothreshold (PT): when N and N^* are both at rest $q = (\omega, \mathbf{0})$, $\omega = M_R - M_N$, $|\mathbf{q}| = 0$, and $Q^2 = -(M_R - M_N)^2 < 0$

$\frac{1}{2}^+$	$A_{1/2} \propto \mathbf{q} $, $S_{1/2} \propto \mathbf{q} ^2$
$\frac{1}{2}^-$	$A_{1/2} \propto 1$, $S_{1/2} \propto \mathbf{q} $ $S_{1/2} \propto A_{1/2} \mathbf{q} $
$\frac{3}{2}^+$	$A_{1/2} \propto \mathbf{q} $, $S_{1/2} \propto \mathbf{q} ^2$ $S_{1/2} \propto (A_{1/2} - \frac{1}{\sqrt{3}}A_{3/2}) \mathbf{q} $
$\frac{3}{2}^-$	$A_{3/2} \propto \mathbf{q} $,
	$A_{1/2} \propto 1$, $S_{1/2} \propto \mathbf{q} $ $S_{1/2} \propto (A_{1/2} + \sqrt{3}A_{3/2}) \mathbf{q} $
	$A_{3/2} \propto 1$, $(A_{1/2} - \frac{1}{\sqrt{3}}A_{3/2}) \propto \mathbf{q} ^2$

[Consequence of the gauge-invariant structure of the transition current J^μ]

- Most popular effect: Siegert's theorem: $S_{1/2} \propto E|\mathbf{q}|$
Scalar amplitude \propto (electric amplitude) $|\mathbf{q}|$ [2, 3, 4, 5] See $\frac{1}{2}^-$
- These correlations cannot be ignored at low- Q^2 ...
- Most empirical parametrizations of the data ignore the PT constraints
Example: Jefferson Lab parametrizations [6]

Methodology [1]

- Use a known parametrization of the data for $Q^2 \geq Q_P^2$

$$A(Q^2) = A^{(0)} + A^{(1)}(Q^2 - Q_P^2) + \frac{A^{(2)}}{2!}(Q^2 - Q_P^2)^2 + \frac{A^{(3)}}{3!}(Q^2 - Q_P^2)^3 + \dots,$$

- Consider analytical continuation for $-(M_R - M_N)^2 \leq Q^2 \leq Q_P^2$

$$A = |\mathbf{q}|^n (\alpha_0 + \alpha_1 |\mathbf{q}|^2 + \alpha_2 |\mathbf{q}|^4 + \alpha_3 |\mathbf{q}|^6),$$

- Coefficients α_l determined by:
PT constraints \oplus continuity of A , A' , A'' [A''' in some cases]
- Study sensibility of the paramet. w/ the data $Q_P^2 = 0.1, 0.3, 0.5 \text{ GeV}^2$

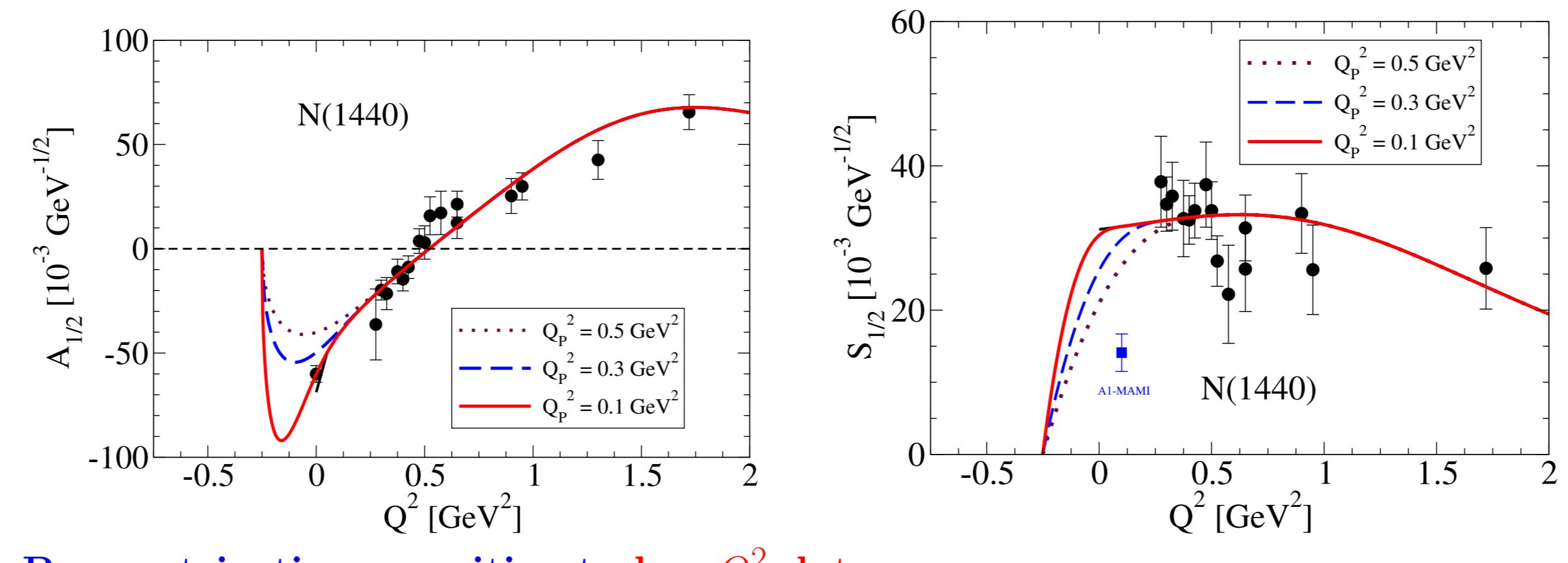
Example $\frac{1}{2}^-$

$$\eta = \frac{M_R - M_N}{M_R + M_N}, b = \sqrt{\frac{(M_R + M_N)^2 + Q^2}{8M_N M_R (M_R - M_N)}} [7]$$

$$\begin{aligned} \text{Current: } J^\mu &= F_1 \left(\gamma^\mu - \frac{q q^\mu}{q^2} \right) + F_2 \frac{i \sigma^{\mu\nu} q_\nu}{M_R + M_N} \quad \text{Kinematic-Singularity-Free FF: } F_i \\ A_{1/2} &= 2b(F_1 + \eta F_2), \quad S_{1/2} = -\sqrt{2}b(M_R - M_N) \frac{|\mathbf{q}|}{Q^2} (F_1 + \eta F_2) + \mathcal{O}(|\mathbf{q}|^3). \\ \Rightarrow \sqrt{2}(M_R - M_N) S_{1/2} &= A_{1/2} |\mathbf{q}| \quad \text{Siegert's theorem [3, 4, 7]} \end{aligned}$$

$N(1440)\frac{1}{2}^+$

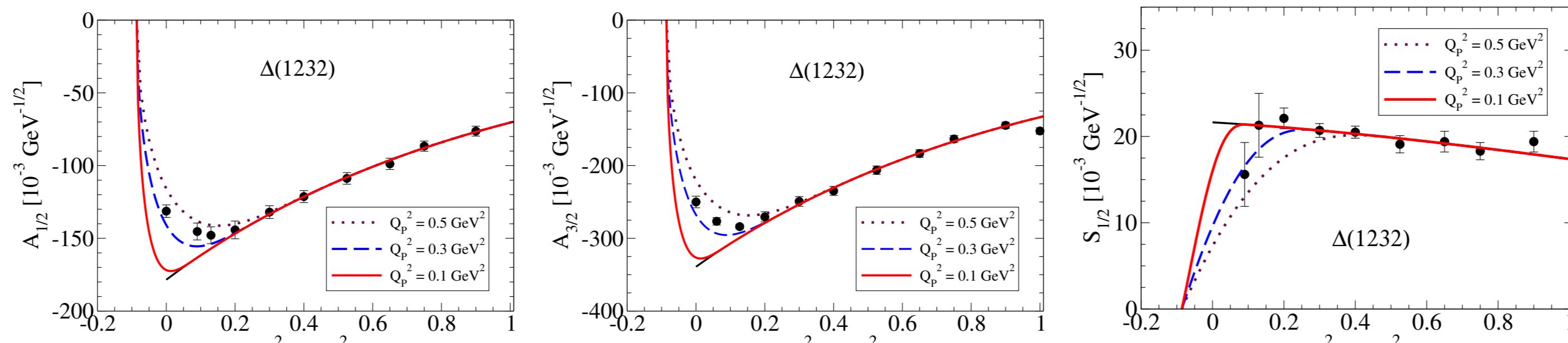
$$A_{1/2} \propto |\mathbf{q}|, S_{1/2} \propto |\mathbf{q}|^2 \quad \text{Data from [8]}$$



Parametrizations sensitive to low- Q^2 data.

$\Delta(1232)\frac{3}{2}^+$

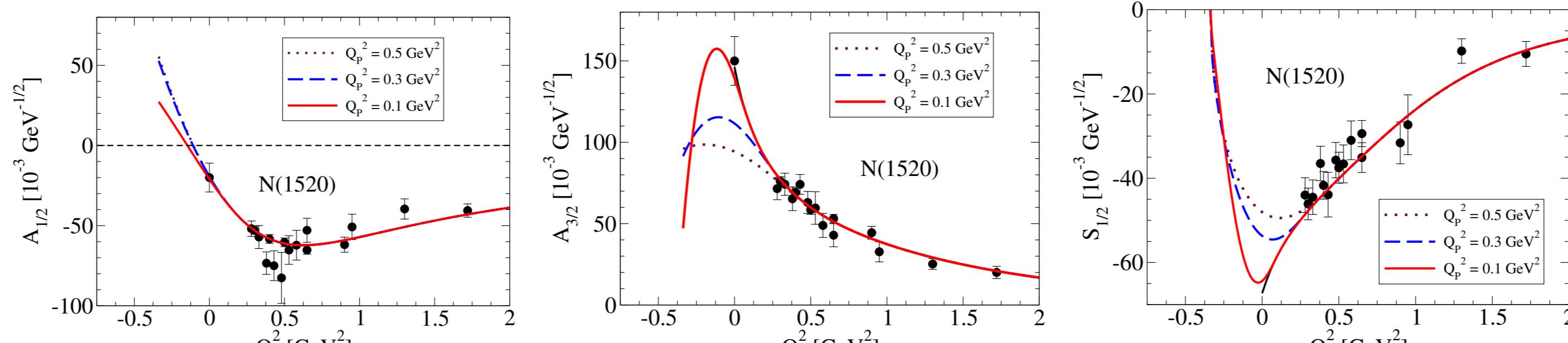
$$S_{1/2} \propto (A_{1/2} - \frac{1}{\sqrt{3}}A_{3/2})|\mathbf{q}| [9, 10, 11, 12]$$



Evidence of PT constraints (turning points); Best parametrization $Q_P^2 = 0.3 \text{ GeV}^2$

$N(1520)\frac{3}{2}^-$

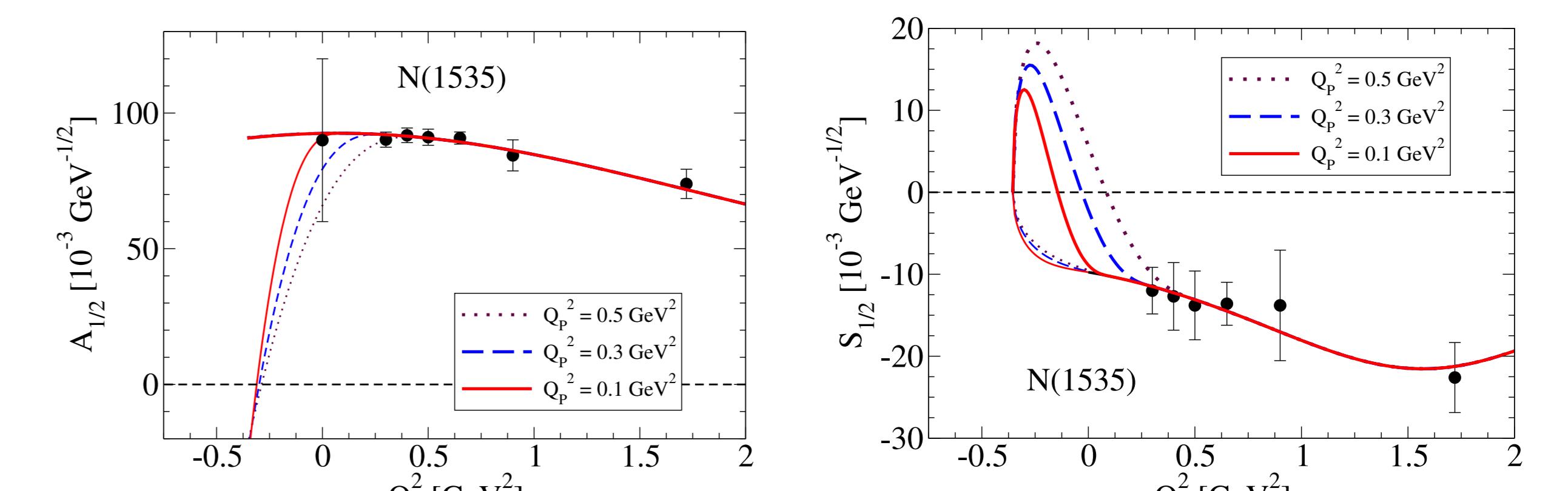
$$S_{1/2} \propto (A_{1/2} + \sqrt{3}A_{3/2})|\mathbf{q}|, (A_{1/2} - \frac{1}{\sqrt{3}}A_{3/2}) \propto |\mathbf{q}|^2 [9]$$



Some evidence of PT constraints; Best parametrization $Q_P^2 = 0.1 \text{ GeV}^2$.

$N(1535)\frac{1}{2}^-$

$$S_{1/2} \propto A_{1/2}|\mathbf{q}| \quad \text{Large uncertainty for } A_{1/2}(0)$$



(1) Thick lines: smooth $A_{1/2}$ (small $\frac{d^3}{dQ^6} A_{1/2}$) – large $\frac{d^3}{dQ^6} S_{1/2}$) – overlap of lines $A_{1/2}$

(2) Thin lines: smooth $S_{1/2}$ (large $\frac{d^3}{dQ^6} A_{1/2}$) – small $\frac{d^3}{dQ^6} S_{1/2}$)

Only new data in the range $Q^2 = 0.0-0.3 \text{ GeV}^2$ can decide which extension is the best

Discussion and Conclusions

- We derive analytic continuations of the Jefferson Lab parametrizations [6] for $Q^2 < Q_P^2$
Parametrizations compatible with large Q^2 data [8]
- Method can be extended to other parametrizations
- Evidences of PT constraints $\Delta(1232)$, $N(1535)$, $N(1520)$
- New parametrizations compatible with PT conditions and low- Q^2 data – choose Q_P^2
- New low- Q^2 data ($Q^2 < 0.3 \text{ GeV}^2$) fundamental to establish the shape of the $N(1535)$ amplitudes

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