Dynamical gluon mass generation and instability of the Gribov-Zwanziger theory:
an effective potential calculation

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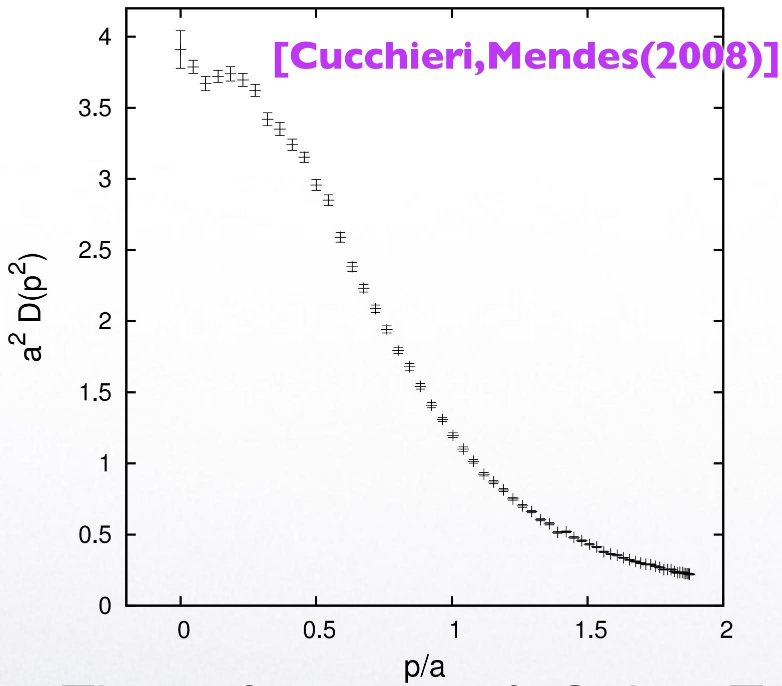
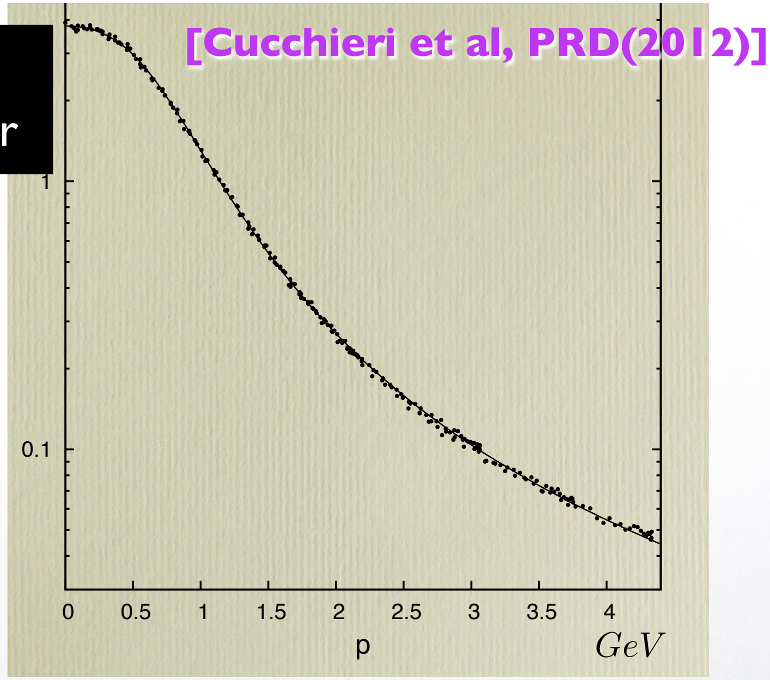
Outline

- Motivation: ***dynamical gluon mass generation => ubiquitous in nonperturbative analyses, but no quantitative prediction in RGZ yet!***
- the Gribov(-Zwanziger) approach to quantize Yang-Mills theories beyond PT
- Status of Refined-GZ framework
- The one-loop effective potential of GZ in the presence of dimension-two condensates
- Numerical results: instability of GZ theory and condensates
- Conclusions and perspectives

- **Finite infrared gluon propagator in Landau gauge:**
  - early predictions in Dyson-Schwinger studies [Aguilar, Natale (2004); Frasca (2007)]
  - High-precision lattice YM results for large systems [Cucchieri, Mendes (2008)]



Gluon Propagator



- The refinement of Gribov-Zwanziger theory was exactly proposed to reconcile with these results via the generation of dim. 2 condensates.

- However, there is still no self-consistent prediction of this theory for the values of the condensates.

[Gribov (1978)]

The Gribov problem:

- In the Landau gauge, for instance, the theory assumes the form

$$\int \mathcal{D}A\mathcal{D}\bar{c}\mathcal{D}c\mathcal{D}b\, e^{-S_{YM}+S_{gf}}$$

$$S_{gf} = b^a \partial_\mu A^a_\mu - \bar{c}^a \mathcal{M}^{ab} c^b \,, \qquad \mathcal{M}^{ab} = -\partial_\mu \left(\delta^{ab}\partial_\mu + g f^{abc} A^c_\mu \right)$$

- Gribov copies $\rightarrow$ zero eigenvalues of the Faddeev-Popov operator $\mathcal{M}^{ab}$.
- Copies cannot be reached by small fluctuations around $A = 0$ (perturbative vacuum) $\rightarrow$ pertubation theory works.
- Once large enough gauge field amplitudes have to be considered (non-perturbative domain) the copies will show up enforcing the effective breakdown of the Faddeev-Popov procedure.

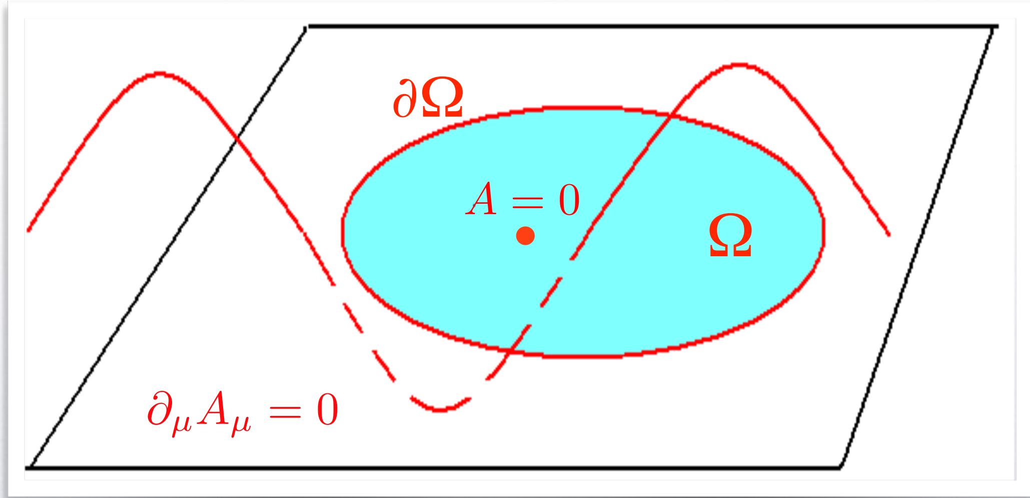
- Gribov proposed a way to eliminate (infinitesimal) Gribov copies from the integration measure over gauge fields: **the restriction to the (first) Gribov region Ω**

$$\int [DA]\delta(\partial A)\det(\mathcal{M})\mathrm{e}^{-S_{\mathrm{YM}}} \longrightarrow \int_\Omega [DA]\delta(\partial A)\det(\mathcal{M})\mathrm{e}^{-S_{\mathrm{YM}}} \qquad S_{\mathrm{YM}} = \frac{1}{4}\int_x F^2$$

with $\Omega = \{A^a_\mu \;;\; \partial A^a = 0, \mathcal{M}^{ab} > 0\}$

$$\boxed{\mathcal{M}^{ab} = -\partial_\mu\left(\delta^{ab}\partial_\mu + f^{abc}A^c_\mu\right) = -\partial_\mu D^a_\mu}$$

(Faddeev-Popov operator)



The Gribov-Zwanziger action

- The **restriction** can be implemented as a **gap equation** for the vacuum energy obtained as: **[Zwanziger (1989,...)]**

$$Z = e^{-V\mathcal{E}(\gamma)} = \int \mathcal{D}A\,\delta(\partial A)\;\det\mathcal{M}\;e^{-\left(S_{YM}\underbrace{+\gamma^4 H(A)-\gamma^4 V D(N^2-1)}_{=:\,+\gamma^4\mathcal{H}}\right)}$$

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Gap equation:

$$\frac{\partial \mathcal{E}(\gamma)}{\partial \gamma} = 0 \Rightarrow \langle H(A) \rangle_{1PI} = V D(N^2-1)$$

$$H(A) = \int_p \int_q A^a_\mu(-p) \left(\mathcal{M}^{ab}\right)^{-1} A^b_\mu(q)$$

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$$H(A) = \int_p \int_q A^a_\mu(-p) \left(\mathcal{M}^{ab}\right)^{-1} A^b_\mu(q)$$

- Using auxiliary fields, this can be cast in a *local* form: $Z = \int [\mathcal{D}\Phi]\ \delta(\partial A)\ \det \mathcal{M}\ e^{-S_{\mathrm{GZ}}}$

- The GZ theory is unstable against the formation of certain dimension 2 condensates, giving rise to a refinement of the effective IR action:

[Dudal et al (2008)]

$$S_{\mathrm{YM}} \xrightarrow[\text{Gribov restriction(UV}\rightarrow\text{IR)}]{} S_{\mathrm{GZ}} = S_{\mathrm{YM}} + \gamma^4 \mathcal{H}$$

Dynamical generation of dim.2 condensates

$$\boxed{S_{\mathrm{RGZ}} = S_{\mathrm{YM}} + \gamma^4 \mathcal{H} + \frac{m^2}{2} AA - M^2 \left(\overline{\varphi}\varphi - \overline{\omega}\omega\right)}$$

The Refined Gribov-Zwanziger action

- The GZ theory is unstable against the formation of certain dimension 2 condensates, giving rise to a refinement of the effective IR action:

[Dudal et al (2008)]

$$S_{\rm YM} \xrightarrow[\text{restriction(UV}\to\text{IR)}]{\text{Gribov}} S_{\rm GZ} = S_{\rm YM} + \gamma^4 \mathcal{H}$$

Dynamical generation of dim.2 condensates

$$\boxed{S_{\rm RGZ} = S_{\rm YM} + \gamma^4 \mathcal{H} + \frac{m^2}{2} AA - M^2 \left(\overline{\varphi}\varphi - \overline{\omega}\omega\right)}$$

Gap equation for the Gribov param.:

$$\frac{\partial \mathcal{E}(\gamma)}{\partial \gamma} = 0 \Rightarrow \langle H(A) \rangle_{1PI} = V D (N^2 - 1)$$

The parameters *M* and *m* are obtained via minimization of an effective potential for:

$$\langle \overline{\varphi}\varphi - \overline{\omega}\omega \rangle \neq 0 \qquad \langle A^2 \rangle \neq 0$$

- Non-perturbative effects included: $(\gamma, M, m) \propto \mathrm{e}^{-\frac{1}{g^2}}$

A checklist for RGZ

✓ *(can be cast in a) local and renormalizable action*

✓ *reduces to QCD (pure gauge) at high energies*

✓ *consistent with gluon 'confinement': confining propagator (no physical propagation; violation of reflection positivity)*

✓ *consistent with lattice IR results ?*

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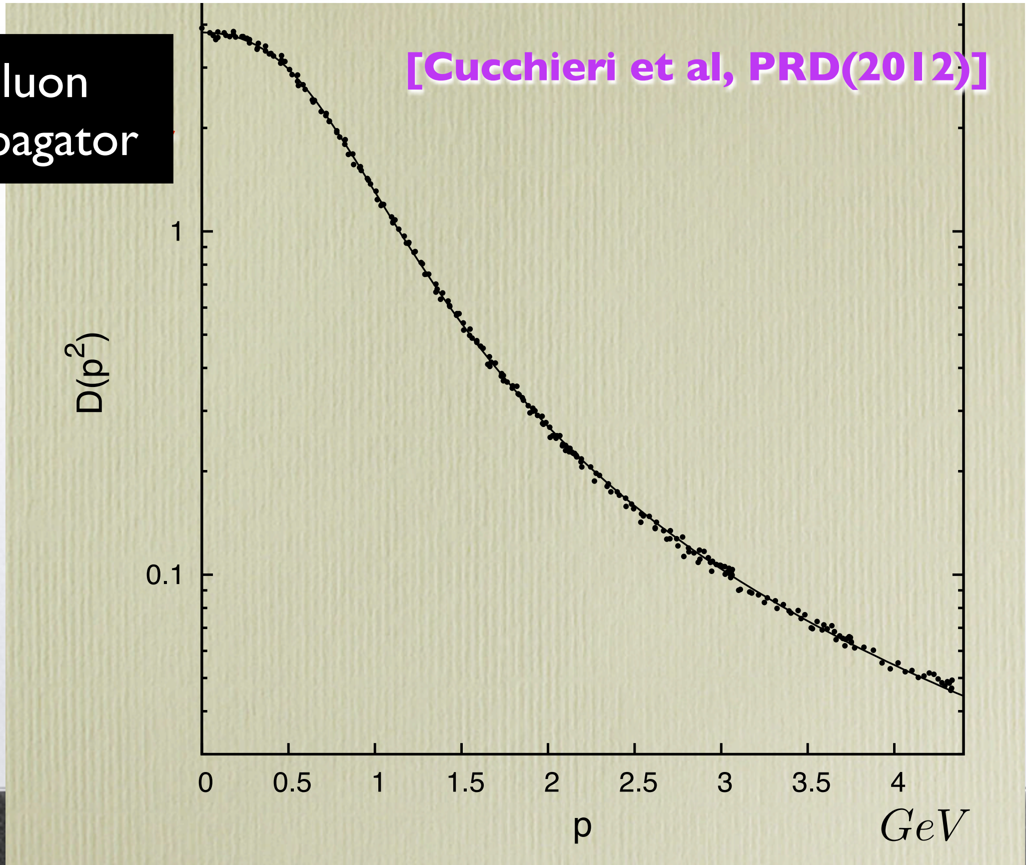

$$\langle A^a_\mu A^b_\nu \rangle_p = \delta^{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) D(p^2)$$

$$D_{\rm fit}(p^2) = C \frac{p^2 + s}{p^4 + u^2 p^2 + t^2}$$

$$C = 0.56(0.01)\,,\; u = 0.53(0.04)\,\mathrm{GeV}\,,$$

$$t = 0.62(0.01)\,\mathrm{GeV}^2\,,\; u = 2.6(0.2)\,\mathrm{GeV}^2$$

poles: $m^2_\pm \;=\; (0.352 \pm 0.522 i)\mathrm{GeV}^2$

$$D_{\rm RGZ}(p^2) = \frac{p^2 + M^2}{p^4 + (M^2 + m^2) p^2 + 2 g^2 N \gamma^4}$$

✓ ***consistent with lattice IR results***

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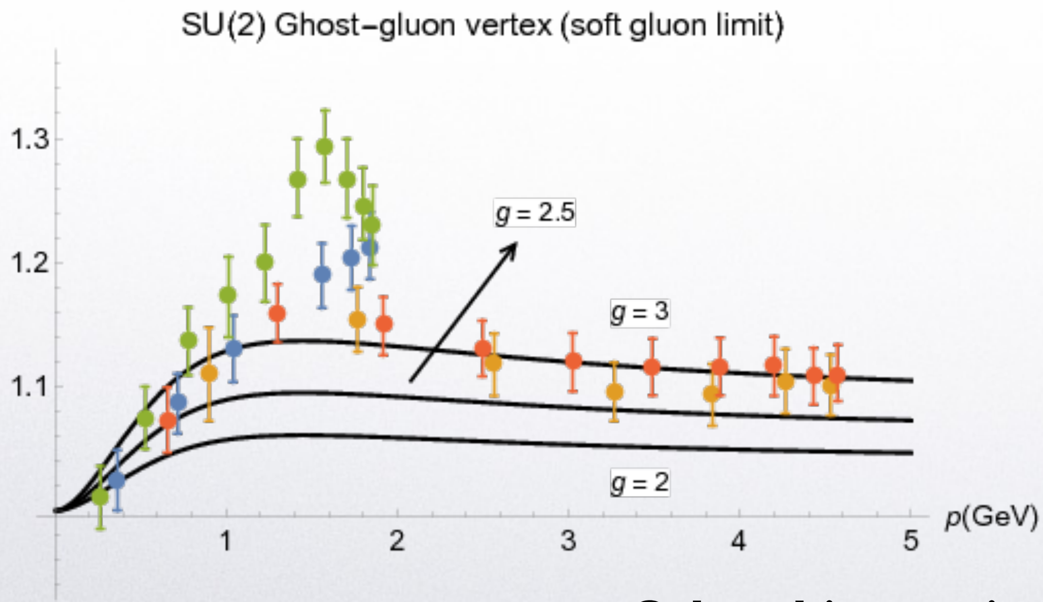
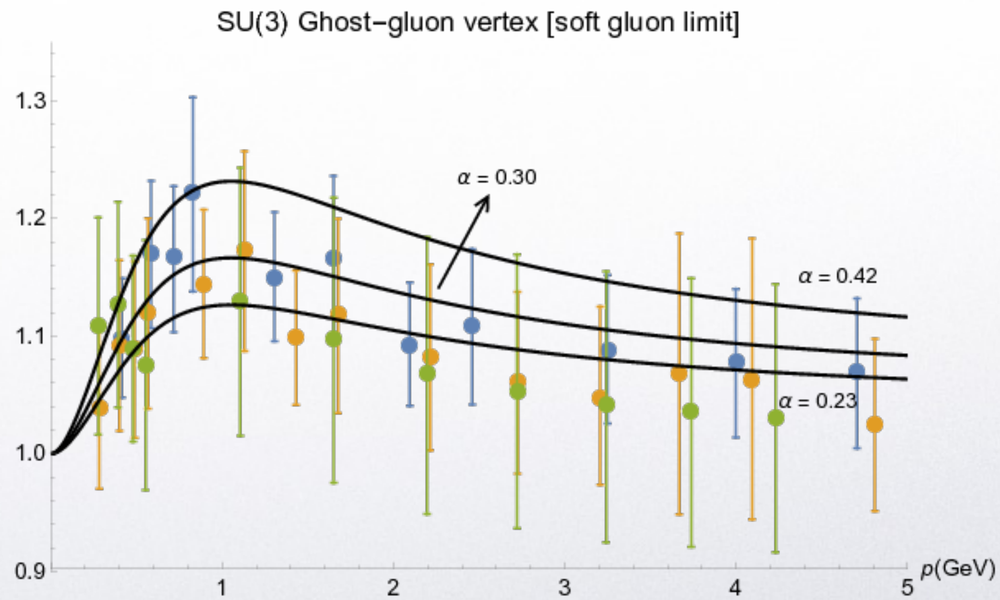
[Mintz, LFP, Sorella, Pereira (2018)]





Other kinematic regions: **[Guimaraes, Mintz, LFP, Pelaez, Sorella; *in prep.* (2021)]**

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✓ *physical spectrum of bound states ??*

Glueball masses are obtained by computing two-point correlation functions of composite operators with the appropriate quantum numbers and casting them in the form of a Källén-Lehmann spectral representation. A lot of caveats of course!

J^{PC}	confining gluon propagator
0^{++}	2.27
2^{++}	2.34
0^{-+}	2.51
2^{-+}	2.64

[Dudal,Guimaraes,Sorella, PRL(2011), PLB(2014)]

-Lattice: (1) Y. Chen *et al.* PRD **73**, 014516 (2006)
-Flux tube model: M. Iwasaki *et al.* PRD **68**, 074007 (2003).
-Hamiltonian QCD: A. P. Szczepaniak and E. S. Swanson, PLB **577**, 61 (2003).
-AdS/CFT: K. Ghoroku, K. Kubo, T. Taminato and F. Toyoda, arXiv:1111.7032.
-AdS/CFT: H. Boschi-Filho, N. R. F. Braga JHEP 0305, 009 (2003)

J^{PC}	Lattice	Flux tube model	Hamiltonian QCD	ADS/CFT
0^{++}	1.71	1.68	1.98	1.21
2^{++}	2.39	2.69	2.42	2.18
0^{-+}	2.56	2.57	2.22	3.05
2^{-+}	3.04	–	–	–

RGZ: Correct hierarchy of masses

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✓ *consistent with lattice IR results (propagators, ghost-gluon vertex)*

✓ *physical spectrum of bound states? Glueballs w/ masses compatible w/ lattice*

✓ *other applications…* **[Canfora *et al*, Sobreiro *et al*, …]**

✓ ***Exact BRST invariance***

- A gauge-invariant gluon field: [Dell'Antonio & Zwanziger ('89), van Ball ('92), Lavelle & McMullan ('96)]

$$f_A[u] \equiv \min_{\{u\}} \mathrm{Tr} \int d^d x \, A^u_\mu A^u_\mu \qquad A^h_\mu = \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2}\right) \phi_\nu$$

$$A^u_\mu = u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u \qquad \phi_\nu = A_\nu - ig\left[\frac{1}{\partial^2}\partial A, A_\nu\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\nu \frac{1}{\partial^2}\partial A\right] + \mathcal{O}(A^3)\,.$$

- Localization is possible through the introduction of a Stueckelberg field ξ^a:

$$A^h_\mu = (A^h)^a_\mu T^a = h^\dagger A^a_\mu T^a h + \frac{i}{g} h^\dagger \partial_\mu h\,, \qquad h = e^{ig\,\xi^a T^a}$$

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- <u>The BRST-invariant Gribov region and condensates:</u> **[Capri *et al* (2016,2017)]**

$$\Omega = \{A^a_\mu;\ \partial_\mu A^a_\mu = i\alpha b^a, \qquad \mathcal{M}^{ab}(A^h) = -\partial_\mu D^{ab}_\mu(A^h) > 0\}$$ (ex. in Linear Cov. Gauges)

$$\langle A^a_\mu(p) A^b_\nu(-p)\rangle = \frac{p^2 + M^2}{p^4 + (M^2 + m^2)p^2 + M^2 m^2 + \lambda^4}\mathcal{P}_{\mu\nu}(p)\delta^{ab} + \frac{\alpha_g}{p^2} L_{\mu\nu}\delta^{ab}$$

$\langle \bar\varphi^{ab}_\mu \varphi^{ab}_\mu \rangle$ $\qquad$ $\langle A^{h,a}_\mu A^{h,a}_\mu \rangle$

$$\boxed{\begin{aligned}
&sA^a_\mu = -D^{ab}_\mu c^b\,,\\
&sc^a = \frac{g}{2} f^{abc} c^b c^c\,, \qquad s\bar c^a = ib^a\\
&sb^a = 0\,,\\
&s\varphi^{ab}_\mu = 0\,, \qquad s\omega^{ab}_\mu = 0\,,\\
&s\bar\omega^{ab}_\mu = 0\,, \qquad s\bar\varphi^{ab}_\mu = 0\,,\\
&s\varepsilon^a = 0\,, \qquad s(A^h)^a_\mu = 0\,,\\
&sh^{ij} = -igc^a (T^a)^{ik} h^{kj}\,.
\end{aligned}}$$

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✓ *Exact BRST invariance*

✗ *no quantitative prediction without fitting lattice data for propagators*

✗ *no general definition of physical operators, unitarity*

✗ *confinement properties: linear potential, etc…*

✗ …

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✗ *…*

Constructing the effective potential of GZ theory

- To explicitly calculate the values of the condensates in RGZ, one should construct an effective potential for the composite operators:

$$\Sigma[\cdots,\tau,Q] = S + \tau \underbrace{\boxed{\frac{1}{2}A^{h,a}_{\mu}A^{h,a}_{\mu}}}_{O_A} + Q \underbrace{\boxed{\bar{\varphi}^{ac}_{\mu}\varphi^{ac}_{\mu}}}_{O_\varphi} \xRightarrow{\text{Legendre transf.}} \Gamma[O_A, O_\varphi] \xRightarrow{\text{Minimize}} \langle O_I \rangle$$

- For composite operators (mass dimension 2 or higher) a lot of complications appear!

- In the non-BRST-invariant formulation of RGZ, there could be many more condensates and the full effective potential calculation was never achieved.

[cf. Dudal, Sorella & Vandersickel (2011)]

The LCO effective potential of GZ theory

- For composite operators (dim 2 or higher), this is not so straightforward… [Verschelde ('95)]

$$\boxed{\Sigma = S + S_{A^2} + S_{\varphi\bar{\varphi}} + S_{\text{vac}}} \quad \begin{cases} S_{A^2} = \int d^dx Z_A (Z_{\tau\tau}\tau + Z_{\tau Q} Q) \frac{1}{2} A^{h,a}_\mu A^{h,a}_\mu \,, \\ S_{\bar{\varphi}\varphi} = \int d^dx Z_{QQ} Z_\varphi Q \bar{\varphi}^{ac}_\mu \varphi^{ac}_\mu \,, \\ S_{\text{vac}} = -\int d^dx \left(\frac{Z_\zeta \zeta}{2} \tau^2 + Z_\alpha \alpha Q^2 + Z_\chi \chi Q \tau \right) \end{cases}$$

- Nonlinear terms in the currents necessary to cancel divergences + Mixing
- The usual Legendre transform does not work, but one can use Hubbard-Stratonovich transformations to eliminate these nonlinear terms in the currents and construct an effective potential that can be properly minimized.
- Finite parts of the LCO parameters $\zeta,\ \alpha,\ \xi$ have to be computed separately, requiring that the effective potential obeys the usual RG equation.

needed (n+1) loops for n-loop results [cf. Dudal, Sorella & Vandersickel (2011)]

- **In this talk:** [Dudal, Felix, LFP, Rondeau, Vercauteren, EPJC (2019)]
  - BRST-invariance allows us to work with Landau gauge and "only" two condensates (still 4-dim. parameter space, with the Gribov parameter and renormalization scale)
  - More convenient Hubbard-Stratonovich transformation that eliminates the necessity of n+1-loop calculations. Similar technique first used in [Lemes, Sarandy, Sorella (2003)]

$$1 = \int [\mathcal{D}\sigma_1]\, e^{-\frac{1}{2Z_\zeta}\int d^dx\left(\sigma_1 + \frac{\bar{a}}{2}A^2 + \bar{b}Q + \bar{c}\tau\right)^2},$$

$$1 = \int [\mathcal{D}\sigma_2]\, e^{+\frac{1}{2Z_\alpha}\int d^dx\left(\sigma_2 + \bar{d}\bar{\varphi}\varphi + \bar{e}Q + \frac{\bar{f}}{2}A^2\right)^2},$$

(auxiliary fields σ_1, σ_2 play the role of the composite fields)

$$\begin{aligned}
\bar{a} &= \frac{Z_A Z_{\tau\tau}}{\sqrt{\zeta}}\mu^{\epsilon/2} \\
\bar{b} &= -\frac{Z_\chi \chi}{\sqrt{\zeta}}\mu^{-\epsilon/2}, \\
\bar{c} &= -Z_\zeta\sqrt{\zeta}\mu^{-\epsilon/2}, \\
\bar{d} &= \frac{Z_\varphi Z_{QQ}}{\sqrt{-2\alpha + \frac{Z_\chi^2\chi^2}{Z_\alpha Z_\zeta \zeta}}}\mu^{\epsilon/2}, \\
\bar{e} &= Z_\alpha\sqrt{-2\alpha + \frac{Z_\chi^2\chi^2}{Z_\alpha Z_\zeta\zeta}}\ \mu^{-\epsilon/2}, \\
\bar{f} &= -\frac{\frac{Z_A Z_{\tau\tau} Z_\chi \chi}{Z_\zeta \zeta} + Z_A Z_{\tau Q}}{\sqrt{-2\alpha + \frac{Z_\chi^2\chi^2}{Z_\alpha Z_\zeta\zeta}}}\mu^{\epsilon/2}.
\end{aligned}$$

coefficients chosen to eliminate nonlinear terms in the currents

The LCO effective potential of BRST-inv. GZ theory (cont.)

- After introducing the HS identities, current terms are now linear:

$$e^{-\Gamma(Q,\tau)} = \int [\mathcal{D}\Phi][\mathcal{D}\sigma_1 \mathcal{D}\sigma_2'] \exp\Bigg[-S_{GZ} - \int d^dx \left(\frac{\sigma_1^2}{2Z_\zeta}\left(1 - \frac{\bar{b}^2}{\bar{e}^2}\frac{Z_\alpha}{Z_\zeta}\right) - \frac{\sigma_2'^2}{2Z_\alpha} - \frac{\bar{b}}{\bar{e}}\frac{\sigma_1\sigma_2'}{Z_\zeta}\right.$$
$$+ \left(\frac{1}{2Z_\zeta}\left(\bar{a} - \frac{\bar{f}\bar{b}}{\bar{e}}\right)\sigma_1 - \frac{\bar{f}}{2Z_\alpha}\sigma_2'\right)A^2 - \left(\frac{\bar{b}\bar{d}}{\bar{e}}\frac{1}{Z_\zeta}\sigma_1 + \frac{\bar{d}}{Z_\alpha}\sigma_2'\right)\overline{\varphi}\varphi$$
$$\left. + \frac{\bar{a}^2}{8Z_\zeta}(A^2)^2 - \frac{1}{2Z_\alpha}\left(\frac{\bar{f}}{2}A^2 + \bar{d}\overline{\varphi}\varphi\right)^2 + \underline{\frac{\bar{c}}{Z_\zeta}\sigma_1\tau - \frac{\bar{e}}{Z_\alpha}\sigma_2' Q}\right)\Bigg]$$

- The condensates are directly related to the sigma fields:

$$\langle A_\mu^{h,a} A_\mu^{h,a}\rangle \;\longleftrightarrow\; m^2 = \sqrt{\frac{13Ng^2}{9(N^2-1)}}\langle\sigma_1\rangle$$

$$\langle \bar{\varphi}_\mu^{ab}\varphi_\mu^{ab}\rangle \;\longleftrightarrow\; M^2 = \sqrt{\frac{35Ng^2}{48(N^2-1)^2}}\langle\sigma_2'\rangle$$

One-loop effective potential

- The one-loop effective potential will only involve the quadratic terms in the fluctuations around the condensates. A standard calculation (Tr log of quadratic operators) gives the final analytic result (MSbar scheme):

$$\Gamma(m^2, M^2, \lambda^4) = -\frac{2(N^2-1)}{Ng^2}\lambda^4\left(1 - \frac{3}{8}\frac{Ng^2}{16\pi^2}\right) + \frac{9(N^2-1)}{13Ng^2}\frac{m^4}{2} - \frac{48(N^2-1)^2}{35Ng^2}\frac{M^4}{2} + \frac{(N^2-1)^2}{8\pi^2}M^4\left(-1 + \ln\frac{M^2}{\bar{\mu}^2}\right)$$
$$+ \frac{3(N^2-1)}{64\pi^2}\left(-\frac{5}{6}(m^4 - 2\lambda^4) + \frac{m^4 + M^4 - 2\lambda^4}{2}\ln\frac{m^2M^2 + \lambda^4}{\bar{\mu}^4}\right.$$
$$\left. -(m^2 + M^2)\sqrt{4\lambda^4 - (m^2 - M^2)^2}\arctan\frac{\sqrt{4\lambda^4 - (m^2 - M^2)^2}}{m^2 + M^2} - M^4\ln\frac{M^2}{\bar{\mu}^2}\right).$$

<u>To determine the condensates, one needs to:</u>

1. compute the Gribov parameter lambda through the gap equation: $\frac{\partial \Gamma}{\partial \lambda^4} = 0$
2. minimize the effective potential as a function of the condensates

Numerics: minimizing the one-loop effective potential of GZ

- The coupling constant and the renormalization scale (related by the RGE) must also be chosen in order to guarantee:

  (i) a valid perturbative approx. (above the nonperturbative background);
  (ii) valid solutions of the multi-dimensional extremization problem… **NOT EASY…**

- We were only able to find solutions meeting these criteria by considering a generic renormalization scheme, changing the first term in the effective potential to:

$$-\frac{2(N^2-1)}{Ng^2}\lambda^4\left(1-\left(\frac{3}{8}-b_0\right)\frac{Ng^2}{16\pi^2}\right)$$

- Now we have acceptable solutions, but the parameter b0 is NOT self-consistently determined. Applying the Principle of Minimal Sensitivity also does not work…

- A robust result is the instability of the zero-condensate case, meaning that GZ (scaling solution) is not even an acceptable phase of the theory [d=4].

[Dudal, Felix, LFP, Rondeau, Vercauteren, EPJC (2019)]

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- We were able to show that for both SU(3) and SU(2) the renormalization scheme (i.e. b0) can be chosen to give proper minima of the effective potential describing the available lattice data:

SU(3)

$$\frac{g^2N}{16\pi^2} = 0.40\,, \qquad \bar{\mu} = 1.41\,\Lambda_{\overline{\rm MS}} = 0.31\ \mathrm{GeV}\,,$$
$$\Gamma = -24\,\Lambda^4_{\overline{\rm MS}} = -0.059\ \mathrm{GeV}^4\,, \qquad \lambda^4 = 28\,\Lambda^4_{\overline{\rm MS}} = 0.071\ \mathrm{GeV}^4\,,$$
$$m^2 = 2.6\,\Lambda^2_{\overline{\rm MS}} = 0.13\ \mathrm{GeV}^2\,, \qquad M^2 = 7.8\,\Lambda^2_{\overline{\rm MS}} = 0.39\ \mathrm{GeV}^2\,.$$

SU(3) Lattice data: [Cucchieri,Dudal,Mendes,Vanderscikel (2012)]

SU(2)

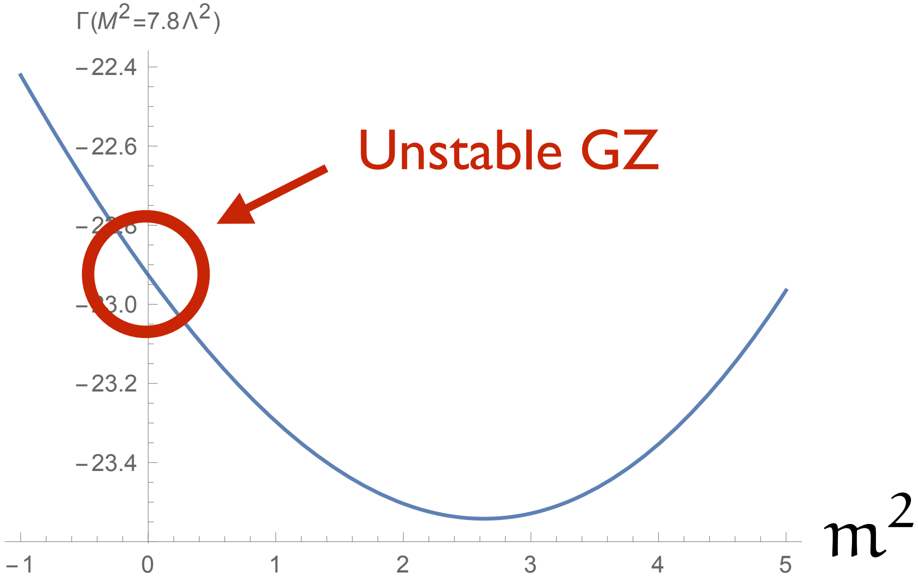
$$\frac{g^2N}{16\pi^2} = 1.24\,, \qquad \bar{\mu} = 1.12\,\Lambda_{\overline{\rm MS}} = 0.37\ \mathrm{GeV}\,,$$
$$\Gamma = -0.38\,\Lambda^4_{\overline{\rm MS}} = -0.0046\ \mathrm{GeV}^4\,, \qquad \lambda^4 = 9.1\,\Lambda^4_{\overline{\rm MS}} = 0.109\ \mathrm{GeV}^4$$
$$m^2 = 2.3\,\Lambda^2_{\overline{\rm MS}} = 0.25\ \mathrm{GeV}^2\,, \qquad M^2 = 2.9\,\Lambda^2_{\overline{\rm MS}} = 0.32\ \mathrm{GeV}^2\,.$$

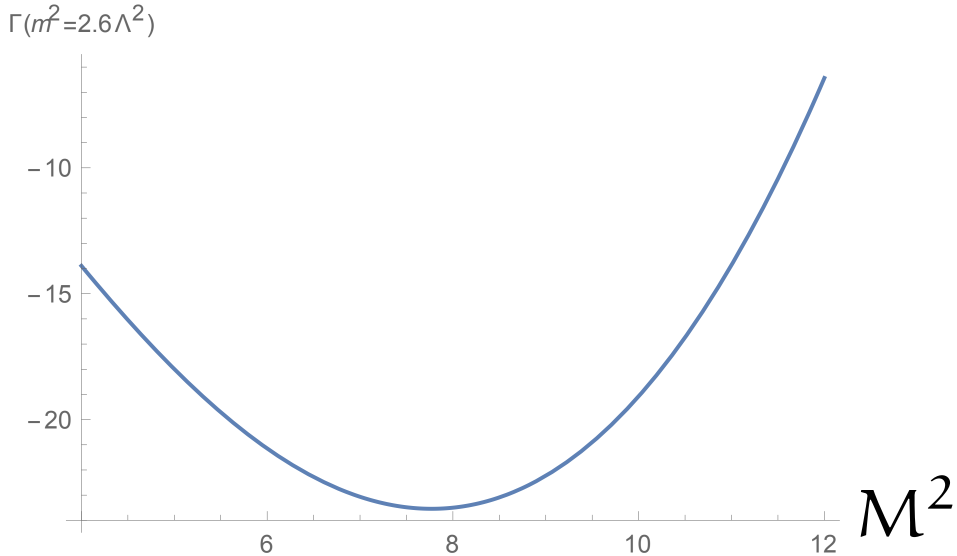
SU(2) Lattice data: [Dudal,Oliveira,Silva (2018)]

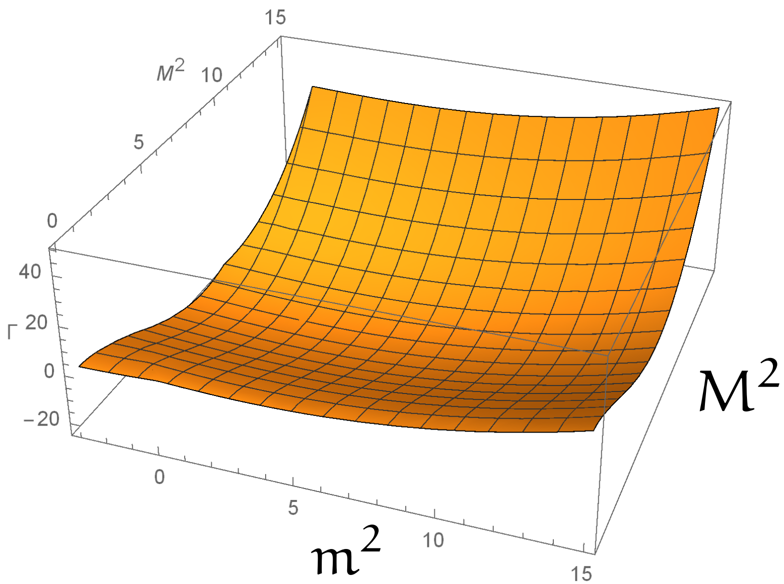
Unstable GZ and (stable) RGZ minimum

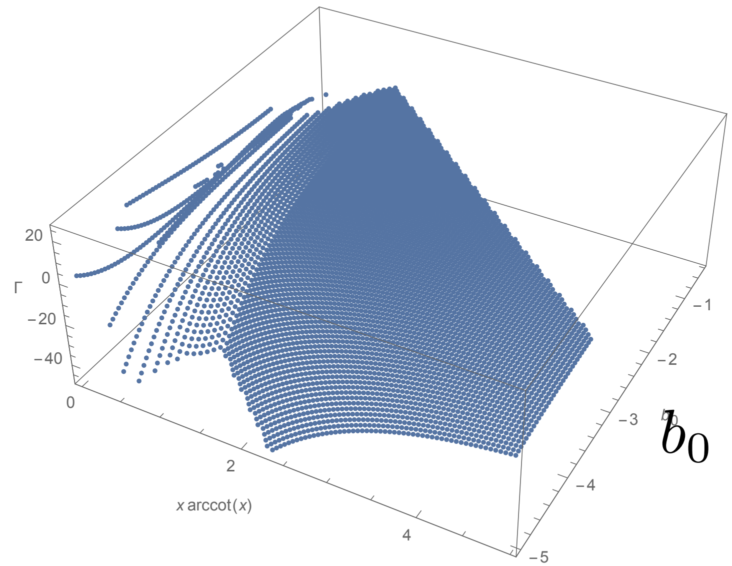
Effective Potential Plots [SU(3)]:

[Dudal, Felix, LFP, Rondeau, Vercauteren, EPJC (2019)]









Final comments

- ***Dynamical gluon mass generation should occur in IR YM theories.***
- The ***Gribov problem*** is present and should profoundly affect the IR regime of gauge-fixed non-Abelian gauge theories.
- The ***RGZ framework*** represents a consistent scenario to study the non-perturbative IR physics and has provided ***interesting results for the gluon sector fitting lattice propagators***.
- A fully self-consistent determination of the condensates in BRST-invariant RGZ is not yet available, but we have gone an important step forward by computing and minimizing the one-loop effective potential explicitly.
- We have shown that the GZ vacuum is unstable in the absence of condensates, whose generation properly stabilizes it. Condensate results are compatible with lattice data for SU(3) and SU(2).
- Strong dependence on the renormalization scheme probably indicates that the one-loop approximation is not a good one, calling for higher-loop computations improved RGEs, etc

Thank you for your attention!