

The gap equation in QCD and the origin of constituent quarks



Bruno El-Bennich

Laboratório de Física Teórica e Computacional
Universidade Cidade de São Paulo

Work in collaboration with

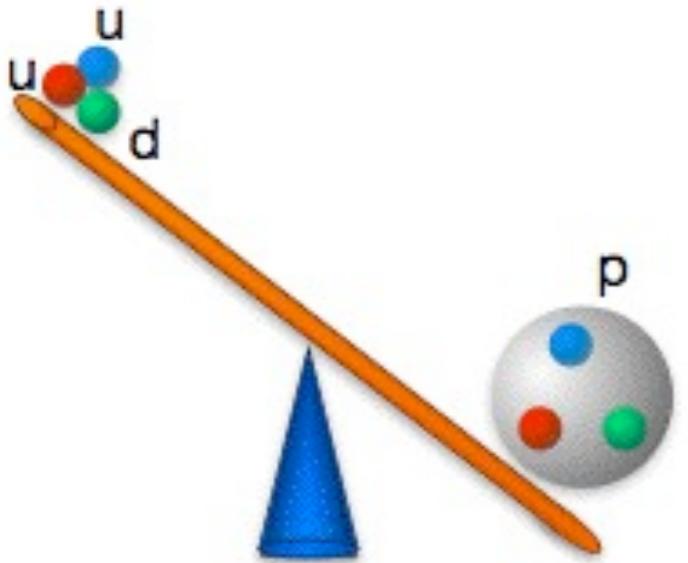
- Luis Albino, Instituto de Física Teórica, Unesp, Brazil
- Adnan Bashir, Universidad de Michoacán, Mexico
- José Roberto Lessa, Instituto Tecnológico de Aeronáutica, Brazil
- Orlando Oliveira, Universidade de Coimbra, Portugal
- Eduardo Rojas, Universidad de Nariño, Colombia
- Fernando Serna, Universidade Cidade de São Paulo, Brazil
- Roberto Correa da Silveira, Universidade Cidade de São Paulo, Brazil



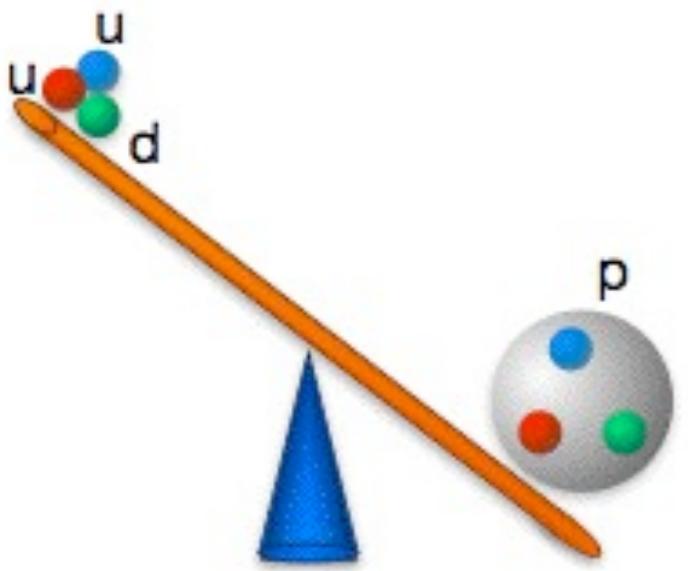
"WE COLLABORATE. I'M AN EXPERT, BUT NOT AN AUTHORITY, AND DR. GELPIS IS AN AUTHORITY, BUT NOT AN EXPERT."

Quantum ChromoDynamics

- >We strive for a description of interactions between quarks and gluons which form hadrons as observed in *Nature*.
- The key issue is: while the Brout-Englert-Higgs mechanism has been established as the essential explicit source of elementary particle's masses, the same cannot be said of the atoms and their nuclei.
- The lightest Nambu-Goldstone mode of QCD, the pion, is more than an order of magnitude heavier than the sum of two light current quarks
- The formation of hadronic and nuclear bound states via its fundamental constituents is an inherently *nonperturbative* problem.
- It involves precise knowledge of the infrared (long distance) regime of QCD and the dynamical generation of a constituent quark mass.



So where do the Hadron's masses
come from after all?! The Higgs
boson isn't doing the job alone!



Hint: the gluons interact with each other and have infinite ways to interact with the quark and “dress it”.

Dyson-Schwinger equation in QCD

The propagator can be obtained from QCD's gap equation: the Dyson-Schwinger equation (DSE) for the dressed-fermion self-energy, which involves the set of infinitely many coupled equations.

$$[\begin{array}{c} \rightarrow \\ p \end{array}]^{-1} = [\begin{array}{c} \rightarrow \\ p \end{array}]^{-1} + \text{Diagram}$$

$q = p - k$

$$\begin{aligned} S^{-1}(p) &= Z_2(i\gamma \cdot p + m^{\text{bm}}) + \Sigma(p) := i\gamma \cdot p A(p^2) + B(p^2) \\ \Sigma(p) &= Z_1 \int^\Lambda \frac{d^4 q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \Gamma_\nu^a(q, p) \end{aligned}$$

with the *running* mass function $M(p^2) = B(p^2)/A(p^2)$.

- $D_{\mu\nu}$: dressed-gluon propagator
 - $\Gamma_\nu^a(q, p)$: dressed quark-gluon vertex
 - Z_2 : quark wave function renormalization constant
 - Z_1 : quark-gluon vertex renormalization constant
- Each satisfies its own DSE !

Dyson-Schwinger equation in QCD

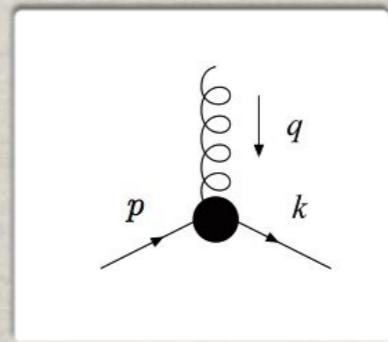
The propagator can be obtained from QCD's gap equation: the Dyson-Schwinger equation (DSE) for the dressed-fermion self-energy, which involves the set of infinitely many coupled equations.

$$[\begin{array}{c} \rightarrow \\ p \end{array}]^{-1} = [\begin{array}{c} \rightarrow \\ p \end{array}]^{-1} + \text{Diagram}$$

$$S^{-1}(p) = Z_2(i\gamma \cdot p + m^{\text{bm}}) + \Sigma(p) := i\gamma \cdot p A(p^2) + B(p^2)$$

$$\Sigma(p) = Z_1 \int^\Lambda \frac{d^4 q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \Gamma_\nu^a(q, p)$$

with the *running mass function* $M(p^2) = B(p^2)/A(p^2)$.



- $D_{\mu\nu}$: dressed-gluon propagator
 - $\Gamma_\nu^a(q, p)$: dressed quark-gluon vertex
 - Z_2 : quark wave function renormalization constant
 - Z_1 : quark-gluon vertex renormalization constant
- Each satisfies its own DSE !

Dyson-Schwinger equation in QCD

The propagator can be obtained from QCD's gap equation: the Dyson-Schwinger equation (DSE) for the dressed-field coupled equations.

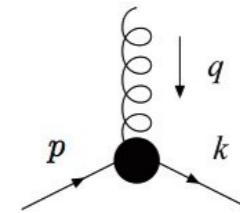
Running Quark Mass

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

$$S^{-1}(p) = Z_2(i\gamma \cdot p + m^{\text{bm}}) + \Sigma(p) := i\gamma \cdot p A(p^2) + B(p^2)$$

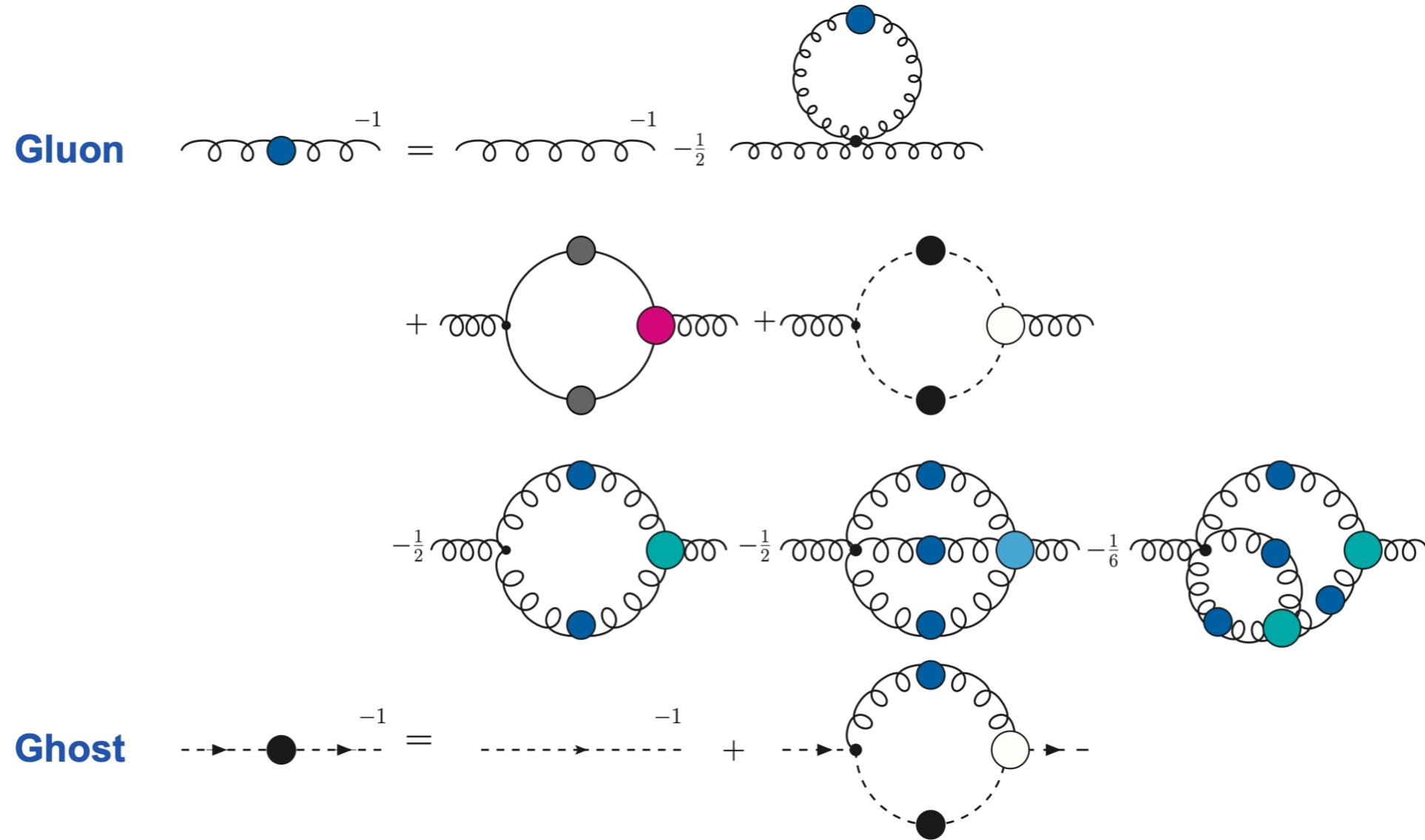
$$\Sigma(p) = Z_1 \int^\Lambda \frac{d^4 q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \Gamma_\nu^a(q, p)$$

with the *running* mass function $M(p^2) = B(p^2)/A(p^2)$.



- $D_{\mu\nu}$: dressed-gluon propagator
 - $\Gamma_\nu^a(q, p)$: dressed quark-gluon vertex
 - Z_2 : quark wave function renormalization constant
 - Z_1 : quark-gluon vertex renormalization constant
- Each satisfies its own DSE !

More DSE in **QCD**: Gluon and Ghost Propagators



However, in this work we employ dressed gluon and ghost propagators from lattice QCD.



Rainbow Truncation

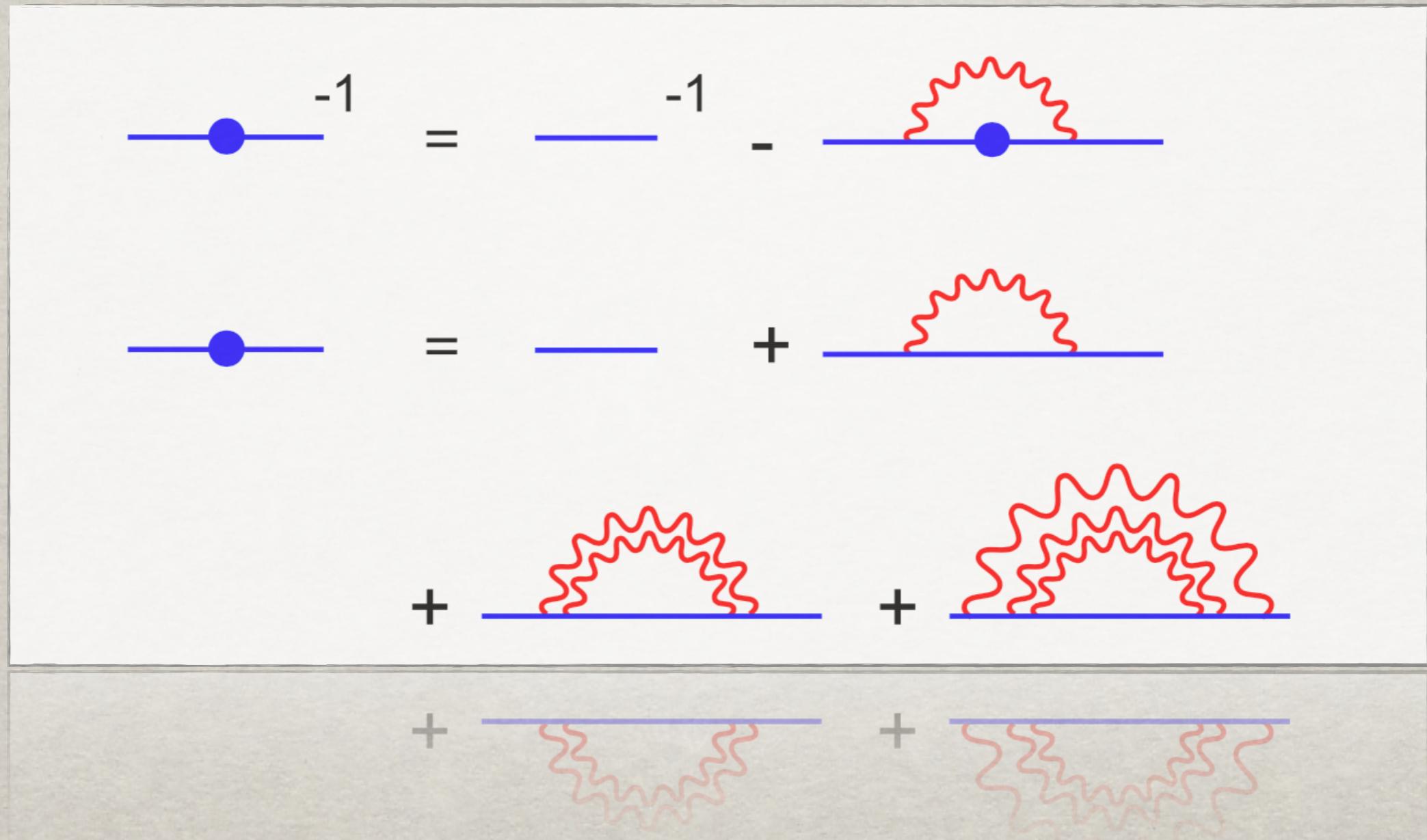
Since the Dyson-Schwinger Equation for QCD imply an infinite tower of non-linear integral equations, a symmetry preserving truncation scheme must be employed. The leading term in such a scheme is the **Rainbow-Ladder** (RL) truncation (Abelian approach).

$$\Gamma_\nu \rightarrow \gamma_\nu$$

RL truncation satisfies flavor non-singlet axial-vector Ward-Takahashi identities (chiral symmetry!) but has bad gauge dependence.

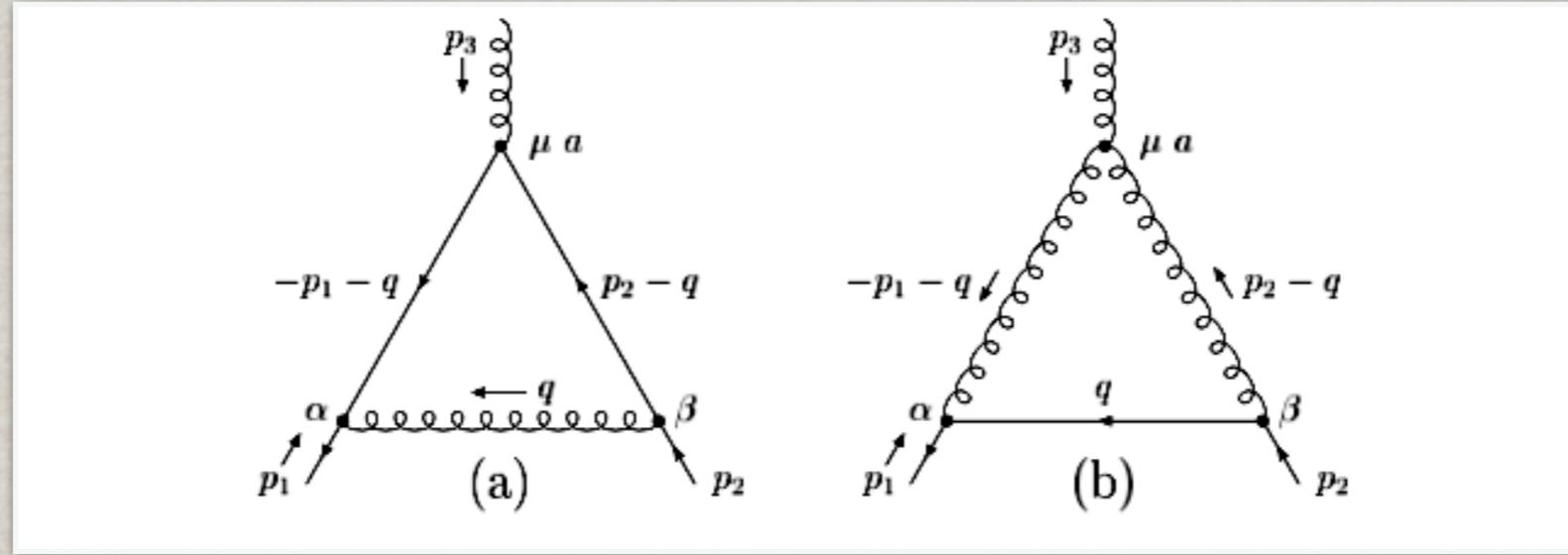
⇒ Landau gauge!

Rainbow Truncation



Here the bare gauge-boson propagator is used, can also be dressed.

The Quark-Gluon Vertex in QCD



- (a) *Abelian correction at one loop*
- (b) *Non-Abelian correction at one loop*

- The quark-gluon vertex in a tree-order is just $i \frac{\lambda_i}{2} \gamma_\mu$.
- However, already at one loop the Dirac-tensor structure is very complex.

Davydychev, Osland and Saks (2000)

Nonperturbative quark-gluon vertex: *restrictions*

- Must satisfy Slavnov-Taylor identities.
- $\Gamma_\mu(k, p)$ must be free of kinematic singularities for $k^2 \rightarrow p^2$.
- Must transform as bare vertex γ_μ under C, P and T transformations.
- Correct weak-coupling limit: must reduce to the perturbative limit when coupling is small.
- Vertex ansatz should lead to gauge independent physical observables, i.e. condensates, hadron masses, decay constants, form factors etc.



Nonperturbative quark-gluon vertex: tensor structure

The fermion-gauge-boson vertex can be decomposed into “longitudinal” and transverse components: $\Gamma_\mu(k, p) = \Gamma_\mu^L(k, p) + \Gamma_\mu^T(k, p)$



$$\Gamma_\mu^L(k, p) = \sum_{i=1}^4 \lambda_i(k^2, p^2) L_\mu^i(k, p)$$
$$\Gamma_\mu^T(k, p) = \sum_{i=1}^8 \tau_i(k^2, p^2) T_\mu^i(k, p)$$

$$\Gamma_\mu(k, p) \Big|_{k^2=p^2=q^2=\mu^2} = \gamma_\mu$$
$$q \cdot \Gamma_\mu^T(k, p) = 0$$

Nonperturbative quark-gluon vertex: tensor structure

Which independent tensor structures to specify the longitudinal and transverse vertex?
 Following Ball and Chiu (1980), one can write:

$$L_\mu^1(k, p) = \gamma_\mu$$

RL approximation

$$L_\mu^2(k, p) = \frac{1}{2}(k + p)_\mu \gamma \cdot (k + p)$$

$$L_\mu^3(k, p) = -i(k + p)_\mu$$

$$L_\mu^4(k, p) = -\sigma_{\mu\nu} (k + p)_\mu$$

$$T_\mu^1(k, p) = i [p_\mu(k \cdot q) - k_\mu(p \cdot q)]$$

$$T_\mu^2(k, p) = [p_\mu(k \cdot q) - k_\mu(p \cdot q)] \gamma \cdot t$$

$$T_\mu^3(k, p) = q^2 \gamma_\mu - q_\mu \gamma \cdot q$$

$$T_\mu^4(k, p) = -[p_\mu(k \cdot q) - k_\mu(p \cdot q)] p^\nu k^\rho \sigma_{\nu\rho}$$

$$T_\mu^5(k, p) = \sigma^{\mu\nu} q_\nu$$

$$T_\mu^6(k, p) = -\gamma_\mu (k^2 - p^2) + t_\mu \gamma \cdot q$$

$$T_\mu^7(k, p) = \frac{i}{2}(k^2 - p^2) [\gamma_\mu \gamma \cdot t - t_\mu] + t_\mu p^\nu k^\rho \sigma_{\nu\rho}$$

$$T_\mu^8(k, p) = -i\gamma_\mu p^\nu k^\rho \sigma_{\nu\rho} - p_\mu \gamma \cdot k + k_\mu \gamma \cdot p$$

Nonperturbative quark-gluon vertex: symmetries

Which steps to remedy a gauge dependence? Clearly must be beyond rainbow truncation as bare vertex violates gauge variance:

$$q_\mu i\Gamma_\mu(k, p) = S^{-1}(k) - S^{-1}(p)$$

Best “prepared” with Landau gauge to minimize dependence.

First step is the proposal for the longitudinal vertex by Ball and Chiu:

$$\Gamma_\mu^L(k, p) = \sum_{i=1}^4 \lambda_i(k^2, p^2) L_\mu^i(k, p)$$
$$\lambda_1(k^2, p^2) = \frac{1}{2} [A(k^2) + A(p^2)] \quad \lambda_2(k^2, p^2) = \frac{A(k^2) - A(p^2)}{k^2 - p^2}$$
$$\lambda_3(k^2, p^2) = \frac{B(k^2) - B(p^2)}{k^2 - p^2} \quad \lambda_4(k^2, p^2) = 0$$

Widely employed in phenomenology though transverse part remains undetermined.

What about **gauge covariance**, does it satisfy Landau-Khalatnikov-Fradkin transformations?

What about **multiplicative renormalizability**?

Nonperturbative quark-gluon vertex: symmetries

Which steps to remedy a gauge dependence? Clearly must be beyond rainbow truncation as bare vertex violates gauge variance:

$$iq^\mu \gamma_\mu \neq i\gamma \cdot k A(k^2) + B(k^2) - i\gamma \cdot p A(p^2) - B(p^2)$$

Best “prepared” with Landau gauge to minimize dependence.

First step is the proposal for the longitudinal vertex by Ball and Chiu:

$$\Gamma_\mu^L(k, p) = \sum_{i=1}^4 \lambda_i(k^2, p^2) L_\mu^i(k, p)$$
$$\lambda_1(k^2, p^2) = \frac{1}{2} [A(k^2) + A(p^2)] \quad \lambda_2(k^2, p^2) = \frac{A(k^2) - A(p^2)}{k^2 - p^2}$$
$$\lambda_3(k^2, p^2) = \frac{B(k^2) - B(p^2)}{k^2 - p^2} \quad \lambda_4(k^2, p^2) = 0$$

Widely employed in phenomenology though transverse part remains undetermined.

What about **gauge covariance**, does it satisfy Landau-Khalatnikov-Fradkin transformations?

What about **multiplicative renormalizability**?

Abelian Ward-Takahashi identities: divergence and curl

Ward-Takahashi identity:

$$q_\mu i\Gamma_\mu(k, p) = S^{-1}(k) - S^{-1}(p)$$

Transverse Ward-Takahashi identities:

$$\begin{aligned} q_\mu \Gamma_\nu(k, p) - q_\nu \Gamma_\mu(k, p) &= S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu}S^{-1}(k) \\ &+ 2im\Gamma_{\mu\nu}(k, p) + t_\lambda \varepsilon_{\lambda\mu\nu\rho} \Gamma_\rho^A(k, p) + A_{\mu\nu}^V(k, p) \end{aligned}$$

$$\begin{aligned} q_\mu \Gamma_\nu^A(k, p) - q_\nu \Gamma_\mu^A(k, p) &= S^{-1}(p)\sigma_{\mu\nu}^5 - \sigma_{\mu\nu}^5 S^{-1}(k) \\ &+ t_\lambda \varepsilon_{\lambda\mu\nu\rho} \Gamma_\rho(k, p) + V_{\mu\nu}^A(k, p) \end{aligned}$$

What is the origin of these transverse identities?

What is the origin of these transverse identities?

$$\psi(x) \longrightarrow \psi'(x) = \psi(x) + ig\alpha(x)\psi(x), \quad \bar{\psi}(x) \longrightarrow \bar{\psi}'(x) = \bar{\psi}(x) - ig\alpha(x)\bar{\psi}(x),$$



$$q_\mu \Gamma_V^\mu(p_1, p_2) = S_F^{-1}(p_1) - S_F^{-1}(p_2),$$

What is the origin of these transverse identities?

$$\psi(x) \longrightarrow \psi'(x) = \psi(x) + ig\alpha(x)\psi(x), \quad \bar{\psi}(x) \longrightarrow \bar{\psi}'(x) = \bar{\psi}(x) - ig\alpha(x)\bar{\psi}(x),$$



$$q_\mu \Gamma_V^\mu(p_1, p_2) = S_F^{-1}(p_1) - S_F^{-1}(p_2),$$

$$\delta_T \psi(x) = \frac{1}{4}g\alpha(x)\epsilon^{\mu\nu}\sigma_{\mu\nu}\psi(x), \quad \delta_T \bar{\psi}(x) = \frac{1}{4}g\alpha(x)\epsilon^{\mu\nu}\bar{\psi}(x)\sigma_{\mu\nu},$$

Infinitesimal Lorentz transformation

H.-x. He Phys.Rev. D80 (2009)

$$\begin{aligned} & \int D[\psi, \bar{\psi}, A] e^{i \int d^4x L_{\text{QED}}[\psi, \bar{\psi}, A]} \psi(x_1) \bar{\psi}(x_2) \\ &= \int D[\psi', \bar{\psi}', A'] e^{i \int d^4x L_{\text{QED}}[\psi', \bar{\psi}', A']} \psi'(x_1) \bar{\psi}'(x_2). \end{aligned}$$

What is the origin of these transverse identities?

$$\psi(x) \longrightarrow \psi'(x) = \psi(x) + ig\alpha(x)\psi(x), \quad \bar{\psi}(x) \longrightarrow \bar{\psi}'(x) = \bar{\psi}(x) - ig\alpha(x)\bar{\psi}(x),$$



$$q_\mu \Gamma_V^\mu(p_1, p_2) = S_F^{-1}(p_1) - S_F^{-1}(p_2),$$

$$\delta_T \psi(x) = \frac{1}{4}g\alpha(x)\epsilon^{\mu\nu}\sigma_{\mu\nu}\psi(x), \quad \delta_T \bar{\psi}(x) = \frac{1}{4}g\alpha(x)\epsilon^{\mu\nu}\bar{\psi}(x)\sigma_{\mu\nu},$$

Infinitesimal Lorentz transformation

H.-x. He Phys.Rev. D80 (2009)

$$\begin{aligned} & \int D[\psi, \bar{\psi}, A] e^{i \int d^4x L_{\text{QED}}[\psi, \bar{\psi}, A]} \psi(x_1) \bar{\psi}(x_2) \\ &= \int D[\psi', \bar{\psi}', A'] e^{i \int d^4x L_{\text{QED}}[\psi', \bar{\psi}', A']} \psi'(x_1) \bar{\psi}'(x_2). \end{aligned}$$

$$\begin{aligned} & iq^\mu \Gamma_V^\nu(p_1, p_2) - iq^\nu \Gamma_V^\mu(p_1, p_2) \\ &= S_F^{-1}(p_1)\sigma^{\mu\nu} + \sigma^{\mu\nu}S_F^{-1}(p_2) + 2m\Gamma_T^{\mu\nu}(p_1, p_2) \\ &+ (p_{1\lambda} + p_{2\lambda})\epsilon^{\lambda\mu\nu\rho}\Gamma_{A\rho}(p_1, p_2) - \int \frac{d^4k}{(2\pi)^4} 2k_\lambda \epsilon^{\lambda\mu\nu\rho}\Gamma_{A\rho}(p_1, p_2; k), \end{aligned}$$

Non-Abelian Ward-Takahashi identities: *divergence and curl*

Slavnov-Taylor identity:

$$i q \cdot \Gamma^a(k, p) = G(q^2) \left[S^{-1}(-k) H^a(k, p) - \bar{H}^a(p, k) S^{-1}(p) \right]$$

Transverse Slavnov-Taylor identities:

H.-X. He, PRD 80, 016004 (2009)

$$\begin{aligned} q_\mu \Gamma_\nu^a(k, p) - q_\nu \Gamma_\mu^a(k, p) &= G(q^2) [S^{-1}(p) \sigma_{\mu\nu} H^a(k, p) + \bar{H}^a(p, k) \sigma_{\mu\nu} S^{-1}(k)] \\ &+ 2im \Gamma_{\mu\nu}^a(k, p) + t_\alpha \epsilon_{\alpha\mu\nu\beta} \Gamma_\beta^{5a}(k, p) + A_{\mu\nu}^a(k, p) \end{aligned}$$

$$\begin{aligned} q_\mu \Gamma_\nu^{5a}(k, p) - q_\nu \Gamma_\mu^{5a}(k, p) &= G(q^2) [S^{-1}(p) \sigma_{\mu\nu}^5 H^a(k, p) - \bar{H}^a(p, k) \sigma_{\mu\nu}^5 S^{-1}(k)] \\ &+ t_\alpha \epsilon_{\alpha\mu\nu\beta} \Gamma_\beta^a(k, p) + V_{\mu\nu}^a(k, p) \end{aligned}$$

Vector and axialvector vertices must be decoupled !

Slavnov-Taylor Identity

One can relate the longitudinal form factors λ_i to the quark propagator's scalar and vector pieces, $B(p^2)$ and $A(p^2)$ via an ST

$$i q \cdot \Gamma^a(k, p) = G(q^2) \left[S^{-1}(-k) H^a(k, p) - \bar{H}^a(p, k) S^{-1}(p) \right]$$

Ghost dressing function

Quark-ghost scattering kernel

Decomposition of $H(k, p)$ and its conjugate in terms of Lorentz covariants:

$$H(p_1, p_2, p_3) = X_0 \mathbb{I}_D + i X_1 \gamma \cdot p_1 + i X_2 \gamma \cdot p_2 + i X_3 \sigma_{\alpha\beta} p_1^\alpha p_2^\beta$$

$$\bar{H}(p_2, p_1, p_3) = \bar{X}_0 \mathbb{I}_D - i \bar{X}_2 \gamma \cdot p_1 - i \bar{X}_1 \gamma \cdot p_2 + i \bar{X}_3 \sigma_{\alpha\beta} p_1^\alpha p_2^\beta$$

$$X_i \equiv X_i(p_1, p_2, p_3)$$

$$X_i(p, k, q) = \bar{X}_i(k, p, q)$$

Davydychev, Osland & Saks (2001)

A .C. Aguilar and J. Papavassiliou (2011)

A. C. Aguilar, J. C. Cardona, M. N. Ferreira and J.~Papavassiliou (2016, 2018)

Decoupling the transverse STIs

$$\begin{aligned}
 q_\mu \Gamma_\nu^a(k, p) - q_\nu \Gamma_\mu^a(k, p) &= G(q^2) [S^{-1}(p) \sigma_{\mu\nu} H^a(k, p) + \bar{H}^a(p, k) \sigma_{\mu\nu} S^{-1}(k)] \\
 &\quad + 2im \Gamma_{\mu\nu}^a(k, p) + t_\alpha \epsilon_{\alpha\mu\nu\beta} \Gamma_\beta^{5a}(k, p) + A_{\mu\nu}^a(k, p) \\
 q_\mu \Gamma_\nu^{5a}(k, p) - q_\nu \Gamma_\mu^{5a}(k, p) &= G(q^2) [S^{-1}(p) \sigma_{\mu\nu}^5 H^a(k, p) - \bar{H}^a(p, k) \sigma_{\mu\nu}^5 S^{-1}(k)] \\
 &\quad + t_\alpha \epsilon_{\alpha\mu\nu\beta} \Gamma_\beta^a(k, p) + V_{\mu\nu}^a(k, p)
 \end{aligned}$$

- The decoupling of the vector and axialvector vertices can be achieved by appropriate projections with two tensors which lead to **two** independent equations for each vertex ! S.-x. Qin, L. Chang, Y.-x. Liu, C.D. Roberts & S. Schmidt (2013)
- Using the two identities for the vector vertex, we can use another set of projections to isolate the **8** tensor structures of the transverse vertex as functions of the *quark propagator, the ghost dressing function, the quark-ghost scattering form factors and an hitherto undetermined nonlocal tensor structure.*

The unknown ingredient ...

The cumbersome nonlocal tensor structure originates in the Fourier transform of a 4-point function with a vector-vertex insertion and a Wilson line.

$$V_{\mu\nu} = \int \frac{d^4 k}{(2\pi)^4} 2k_\lambda \varepsilon^{\lambda\mu\nu\rho} \Gamma_{A\rho}(p_1, p_2; k)$$

H.-X. He, PRD 80, 016004 (2009)

$$\begin{aligned} & \int d^4x d^4x' d^4x_1 d^4x_2 e^{i(p_1 \cdot x_1 - p_2 \cdot x_2 + (p_2 - k) \cdot x - (p_1 - k) \cdot x')} \\ & \times \langle 0 | T \bar{\psi}(x') \gamma_\rho \gamma_5 U_P(x', x) \psi(x) \psi(x_1) \bar{\psi}(x_2) | 0 \rangle \\ & = (2\pi)^4 \delta^4(p_1 - p_2 - q) iS_F(p_1) \Gamma_{A\rho}(p_1, p_2; k) iS_F(p_2) \end{aligned}$$

The unknown ingredient ...

The cumbersome nonlocal tensor structure originates in the Fourier transform of a 4-point function with a vector-vertex insertion and a Wilson line.

$$V_{\mu\nu} = \int \frac{d^4 k}{(2\pi)^4} 2k_\lambda \varepsilon^{\lambda\mu\nu\rho} \Gamma_{A\rho}(p_1, p_2; k)$$

H.-X. He, PRD 80, 016004 (2009)

$$\begin{aligned} & \int d^4x d^4x' d^4x_1 d^4x_2 e^{i(p_1 \cdot x_1 - p_2 \cdot x_2 + (p_2 - k) \cdot x - (p_1 - k) \cdot x')} \\ & \times \langle 0 | T\bar{\psi}(x') \gamma_\rho \gamma_5 U_P(x', x) \psi(x) \psi(x_1) \bar{\psi}(x_2) | 0 \rangle \\ & = (2\pi)^4 \delta^4(p_1 - p_2 - q) iS_F(p_1) \Gamma_{A\rho}(p_1, p_2; k) iS_F(p_2) \end{aligned}$$

The unfamiliar, complicated components in these identities can be decomposed:

$$i T_{\mu\nu}^1 V_{\mu\nu} = Y_1(k, p) \mathbf{I}_D + Y_2(k, p) \gamma \cdot q + Y_3(k, p) \gamma \cdot t + Y_4(k, p) [\gamma \cdot q, \gamma \cdot t]$$

$$i T_{\mu\nu}^2 V_{\mu\nu} = Y_5(k, p) \mathbf{I}_D + Y_6(k, p) \gamma \cdot q + Y_7(k, p) \gamma \cdot t + Y_8(k, p) [\gamma \cdot q, \gamma \cdot t]$$

The unknown ingredient ...

We constrain the Y_i form factors with a known ansatz for transverse vertex based on perturbation theory, symmetry considerations and multiplicative renormalizability in a given limit $k^2 \gg p^2$.

Bashir, Bermudez, Chang & Roberts (2012)

$$\begin{aligned}\tau_1(k^2, p^2) &= \frac{a_1 \Delta_B(k^2, p^2)}{(k^2 + p^2)} \\ \tau_2(k^2, p^2) &= \frac{a_2 \Delta_A(k^2, p^2)}{(k^2 + p^2)} \\ \tau_3(k^2, p^2) &= a_3 \Delta_A(k^2, p^2) \\ \tau_4(k^2, p^2) &= \frac{a_4 \Delta_B(k^2, p^2)}{[k^2 + M^2(k^2)[p^2 + M^2(p^2)]]} \\ \tau_5(k^2, p^2) &= a_5 \Delta_B(k^2, p^2) \\ \tau_6(k^2, p^2) &= \frac{a_6(k^2 + p^2) \Delta_A(k^2, p^2)}{[(k^2 - p^2)^2 + (M^2(k^2) + M^2(p^2))^2]} \\ \tau_7(k^2, p^2) &= \frac{a_7 \Delta_B(k^2, p^2)}{(k^2 + p^2)} \\ \tau_8(k^2, p^2) &= a_8 \Delta_A(k^2, p^2)\end{aligned}$$

Gluon and ghost dressing functions

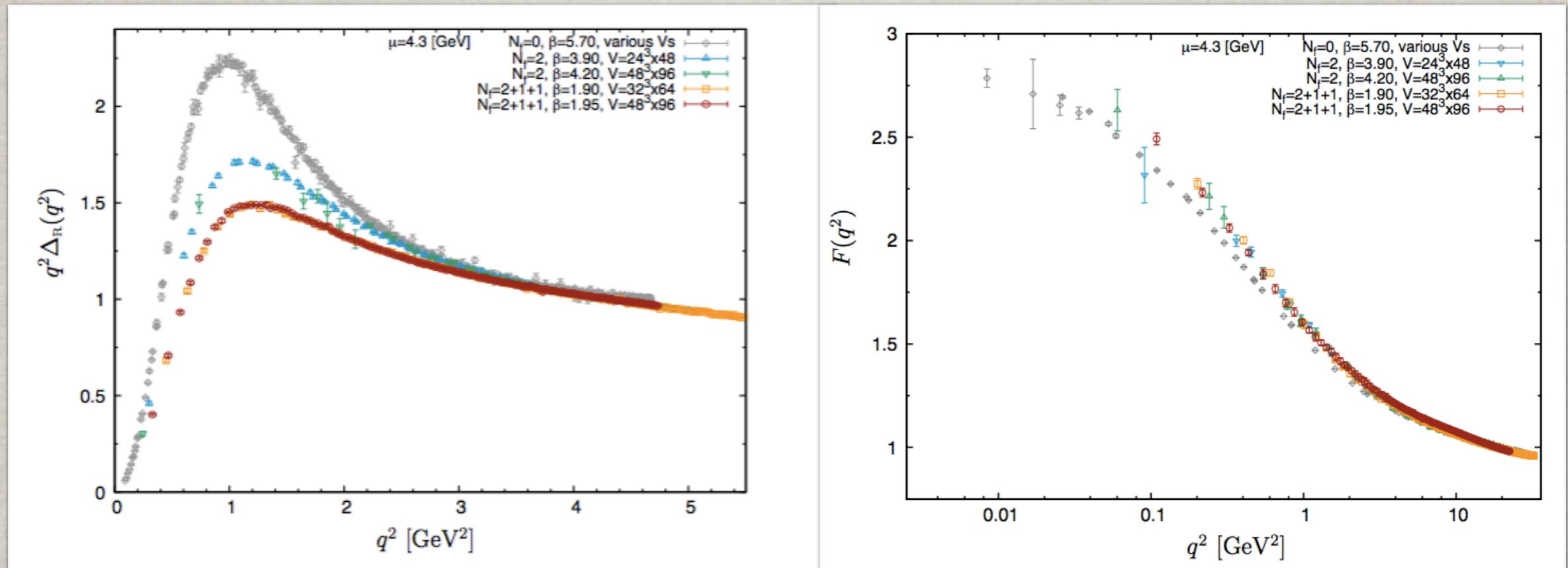
The gluon propagator in Landau gauge is:

$$\Delta_{\mu\nu}^{ab}(q) = \delta^{ab} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \Delta(q^2) \quad \Delta(q^2) \xrightarrow{q^2 \rightarrow \infty} \frac{1}{q^2}$$

The ghost propagator is:

$$D^{ab}(q^2) = -\delta^{ab} \frac{G(q^2)}{q^2} \quad G(q^2) \xrightarrow{q^2 \rightarrow \infty} 1$$

Gluon and ghost dressing functions

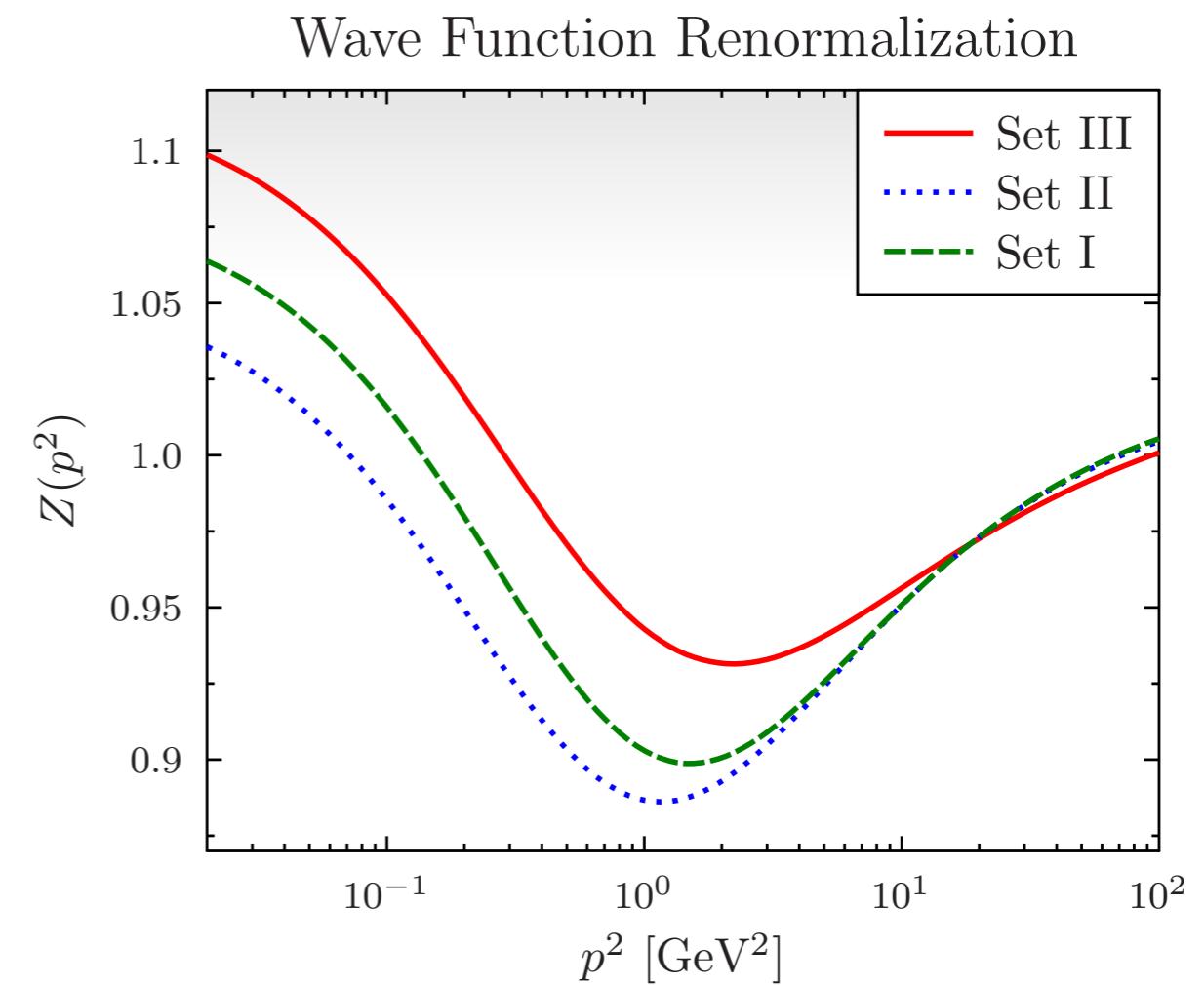
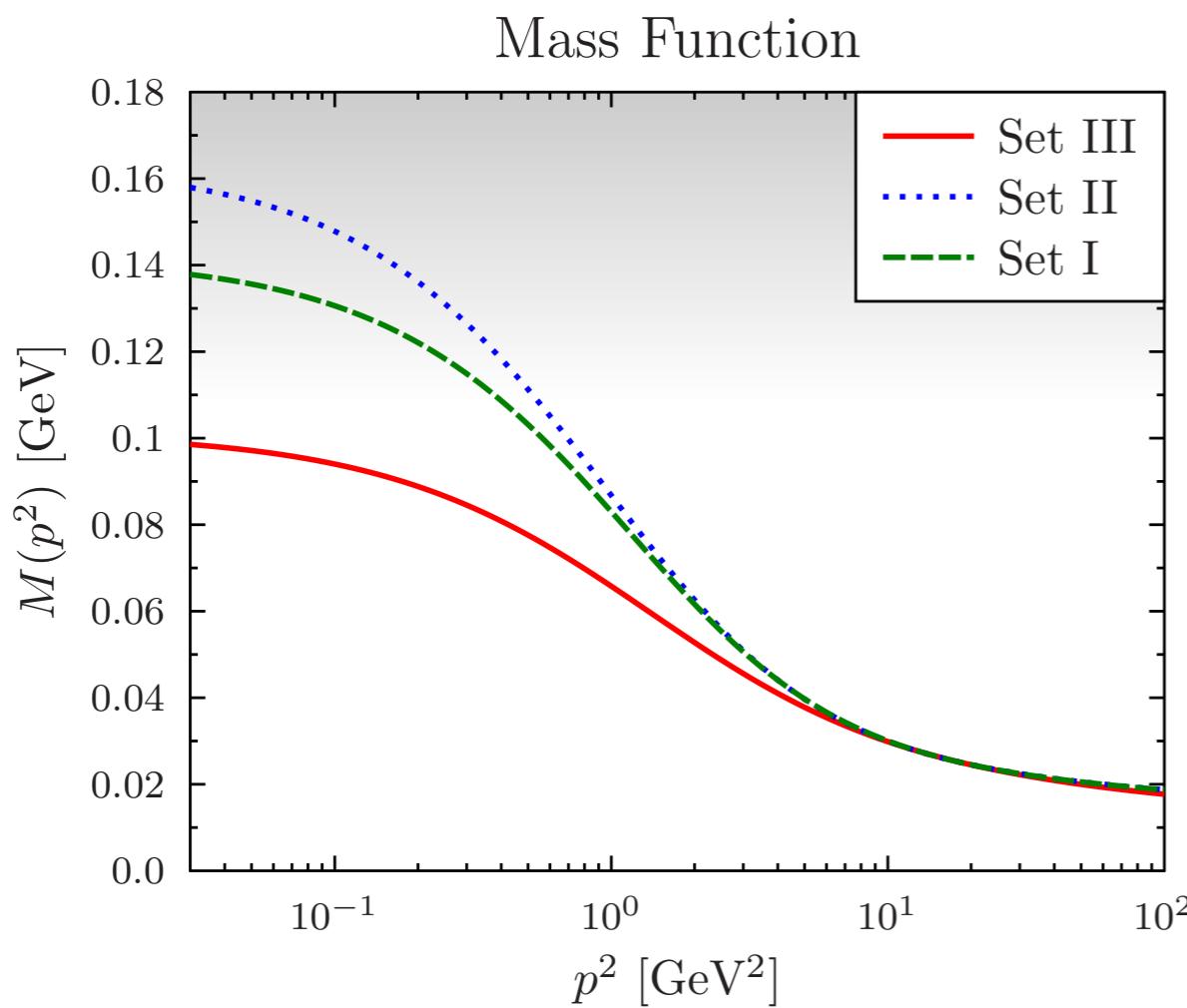


We study three sets of propagators from different collaborations:

- Set I: Bogolubsky *et al.*, Phys. Lett. B 676, 69 (2009)
- Set II: Dudal *et al.*, Annals Phys. 397, 351-364 (2018)
Duarte *et al.*, Phys. Rev. D 94 (2016)
- Set III: A. Ayala *et al.*, Phys. Rev. D 86, 074512 (2012)

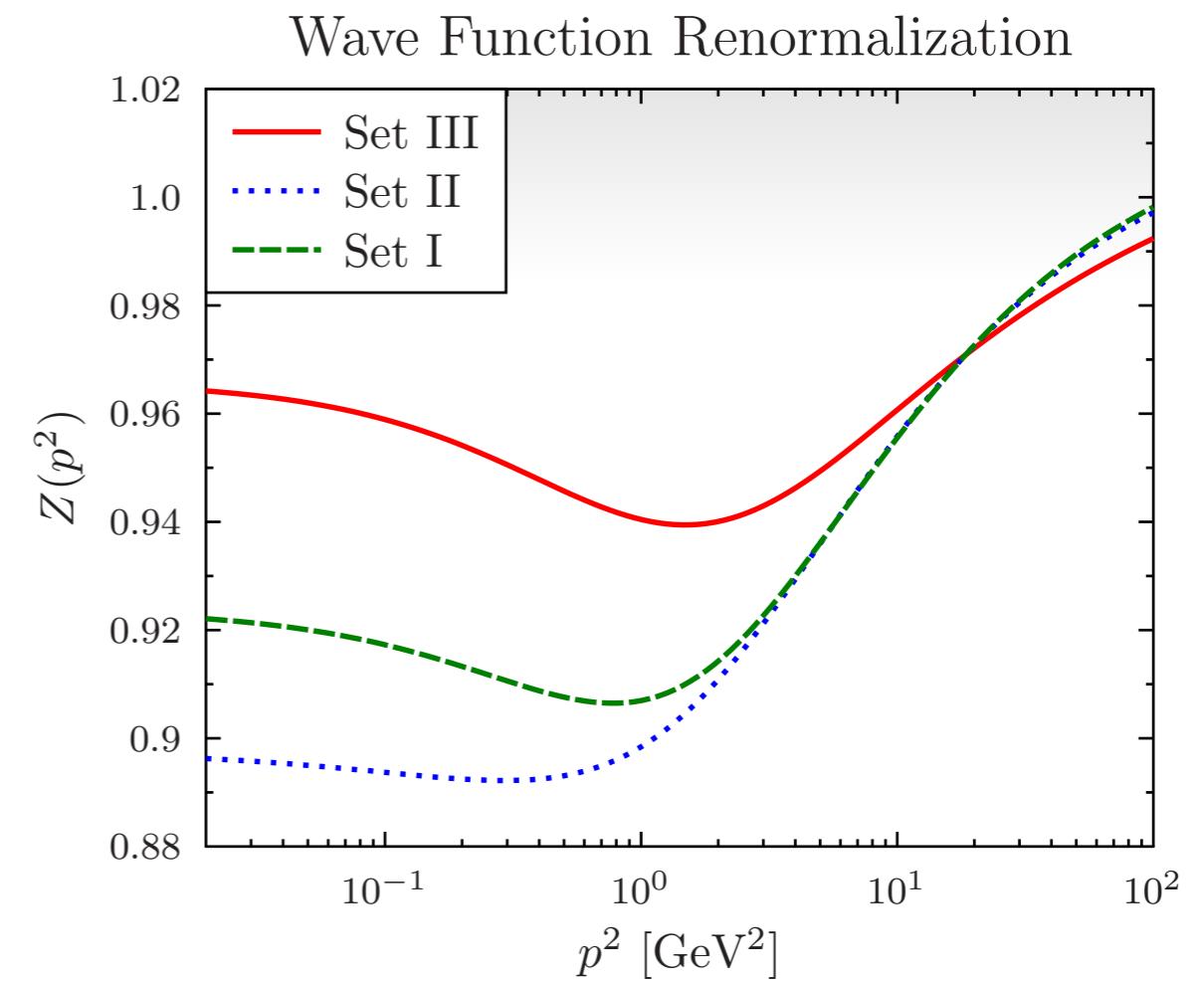
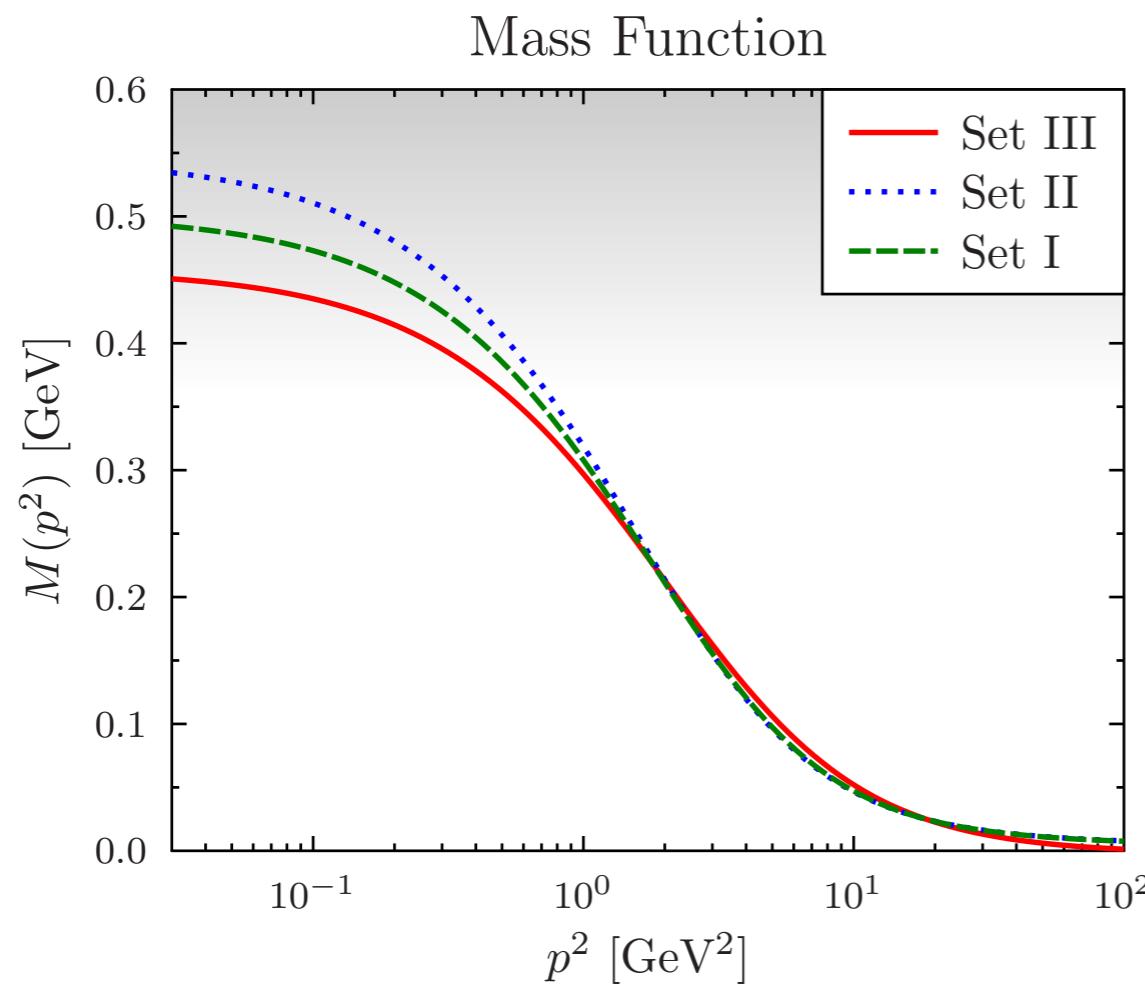
DSE Solutions with non-transverse vertex

$$\Gamma_\mu^L(k, p) = \sum_{i=1}^4 \lambda_i(k^2, p^2) L_\mu^i(k, p)$$

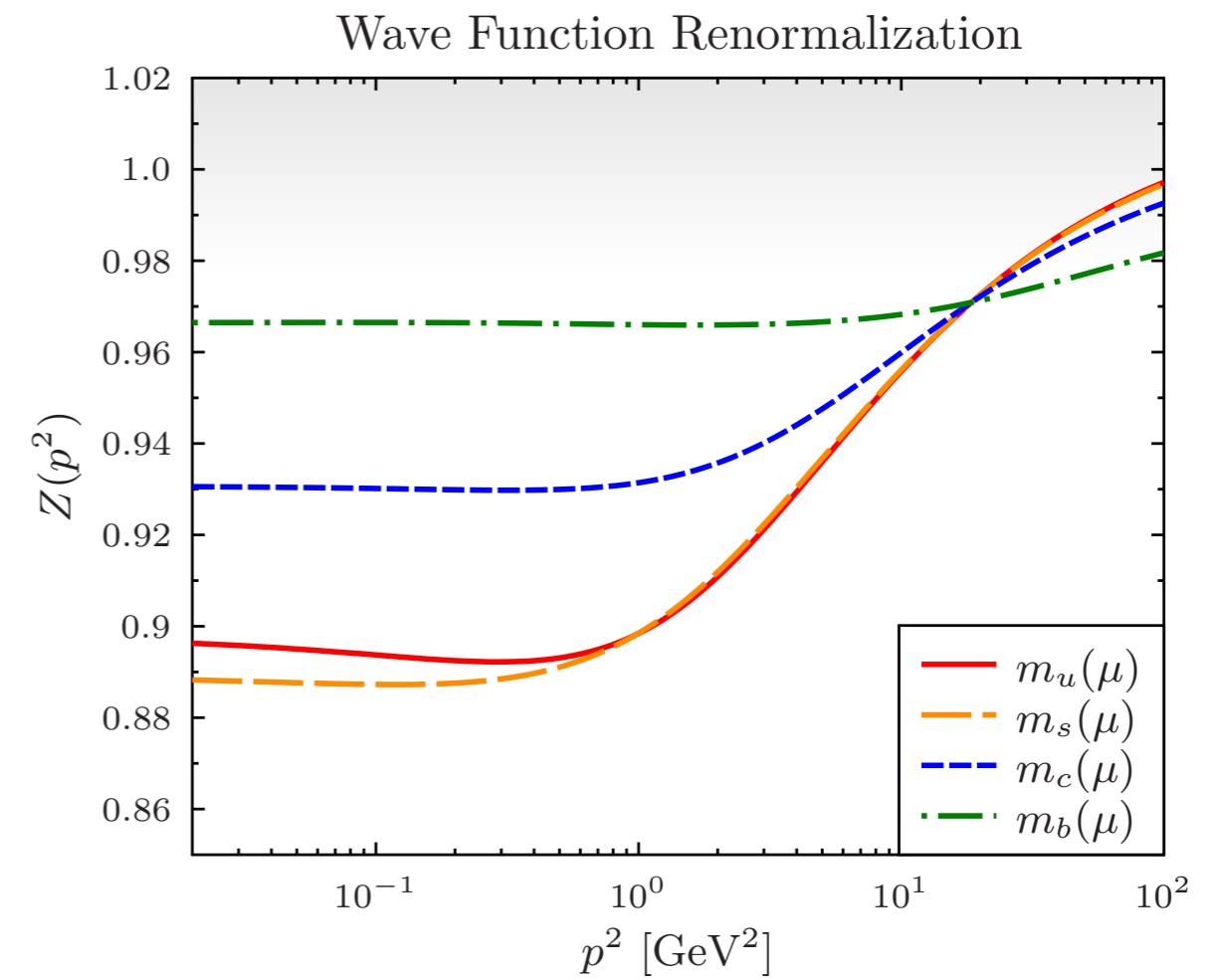
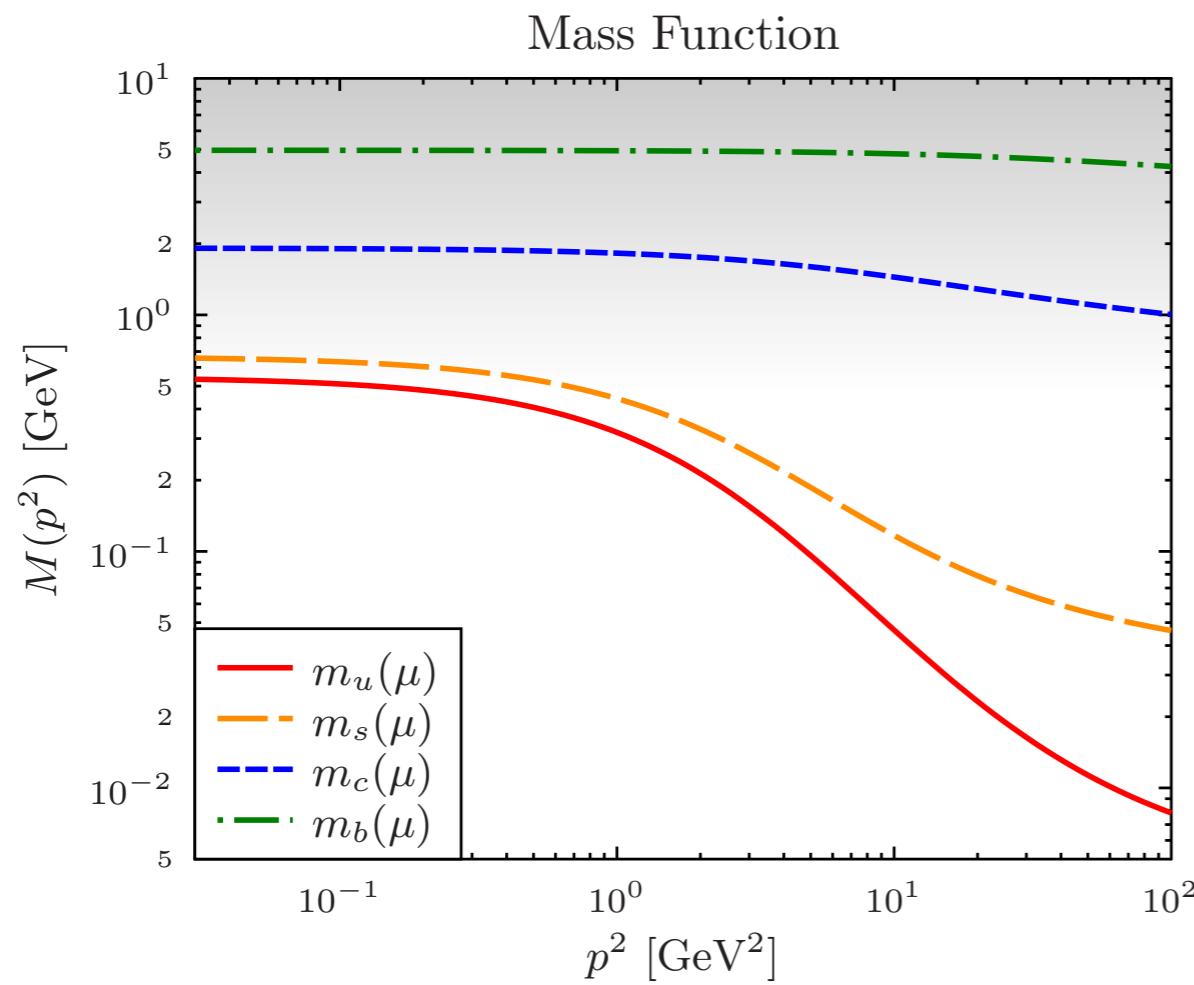


DSE solutions with full quark-gluon vertex

$$\Gamma_\mu(k, p) = \Gamma_\mu^L(k, p) + \Gamma_\mu^T(k, p)$$



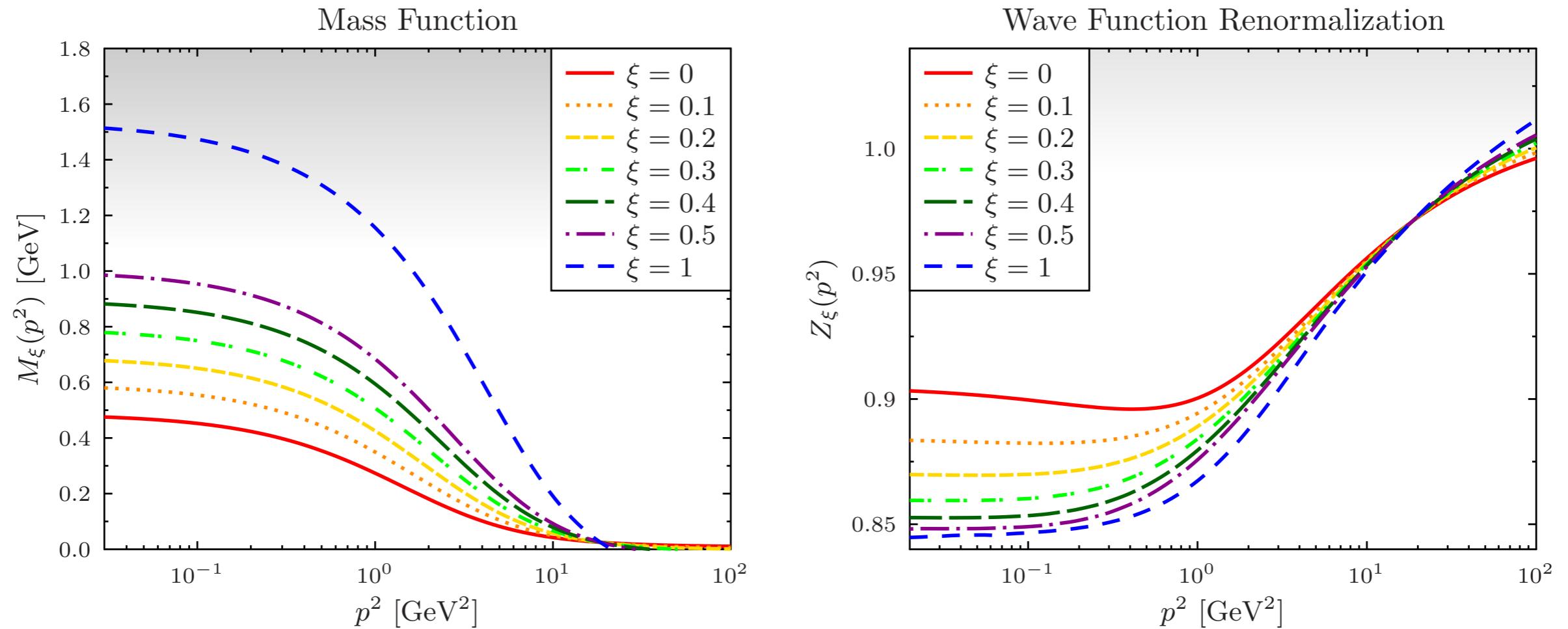
Flavor dependence of DSE solutions



DSE with gluon propagators in R_ξ gauge

Lattice QCD input for gluon: P. Bicudo, D. Binosi, N. Cardoso, O. Oliveira and P. J. Silva, Phys. Rev. D 92, 114514 (2015)

Extrapolation to Feynman Gauge using Padé parametrization of lattice gluon and ghost dressing function.

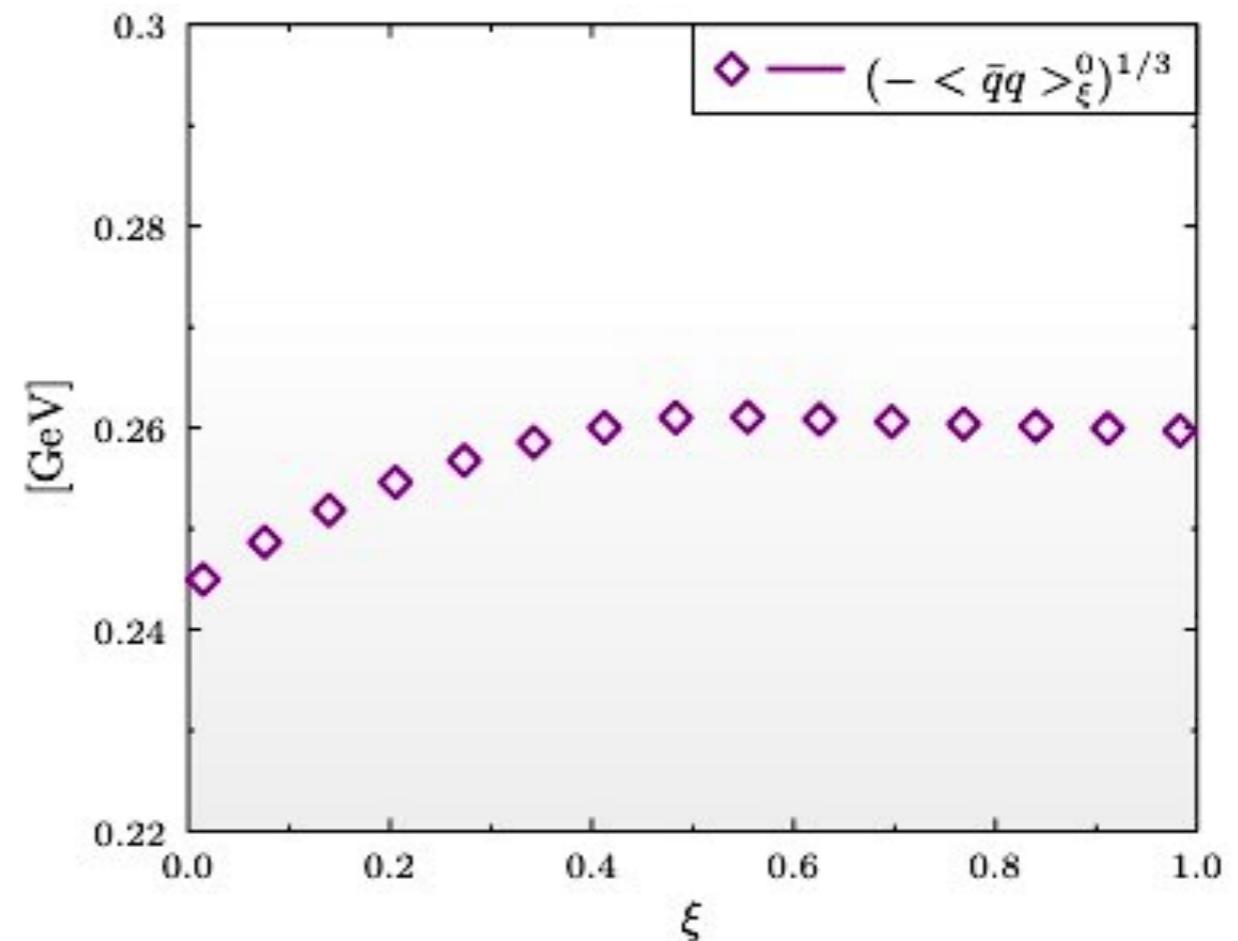
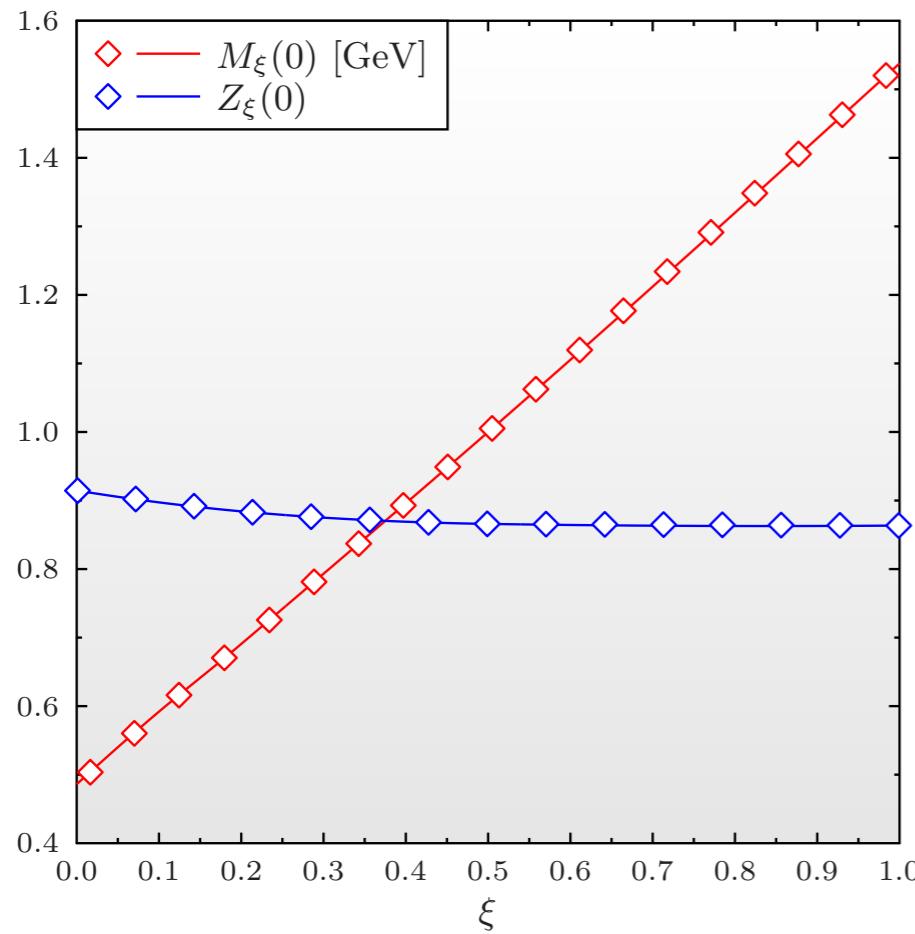


$$\alpha_s^\xi = 0.29 + 0.098\xi - 0.064\xi^2$$

A. C. Aguilar, D. Binosi and J. Papavassiliou, Phys. Rev. D95, 034017 (2017)

DSE with gluon propagators in R_ξ gauge: constituent mass and quark condensate

Lattice QCD input for gluon: P. Bicudo, D. Binosi, N. Cardoso, O. Oliveira and P. J. Silva, Phys. Rev. D 92, 114514 (2015)

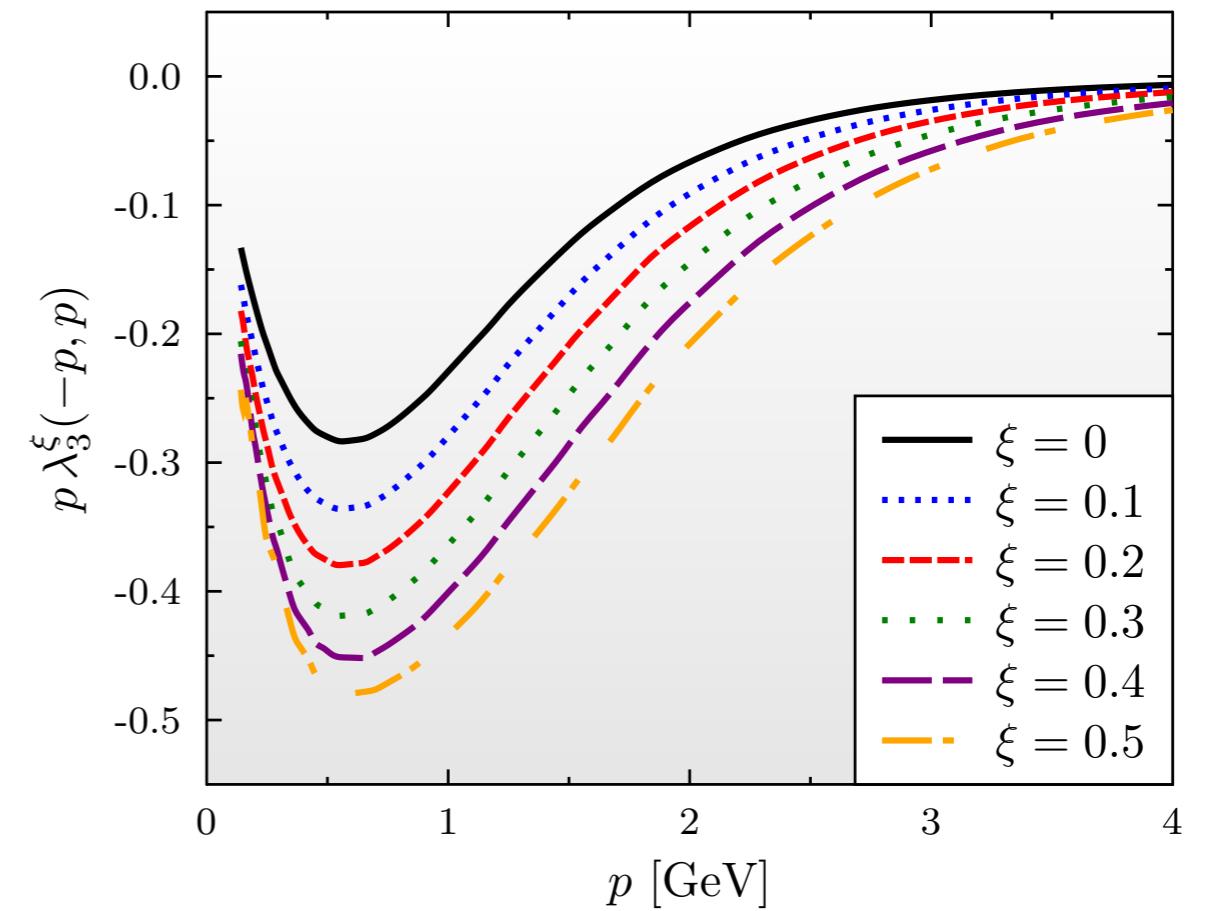
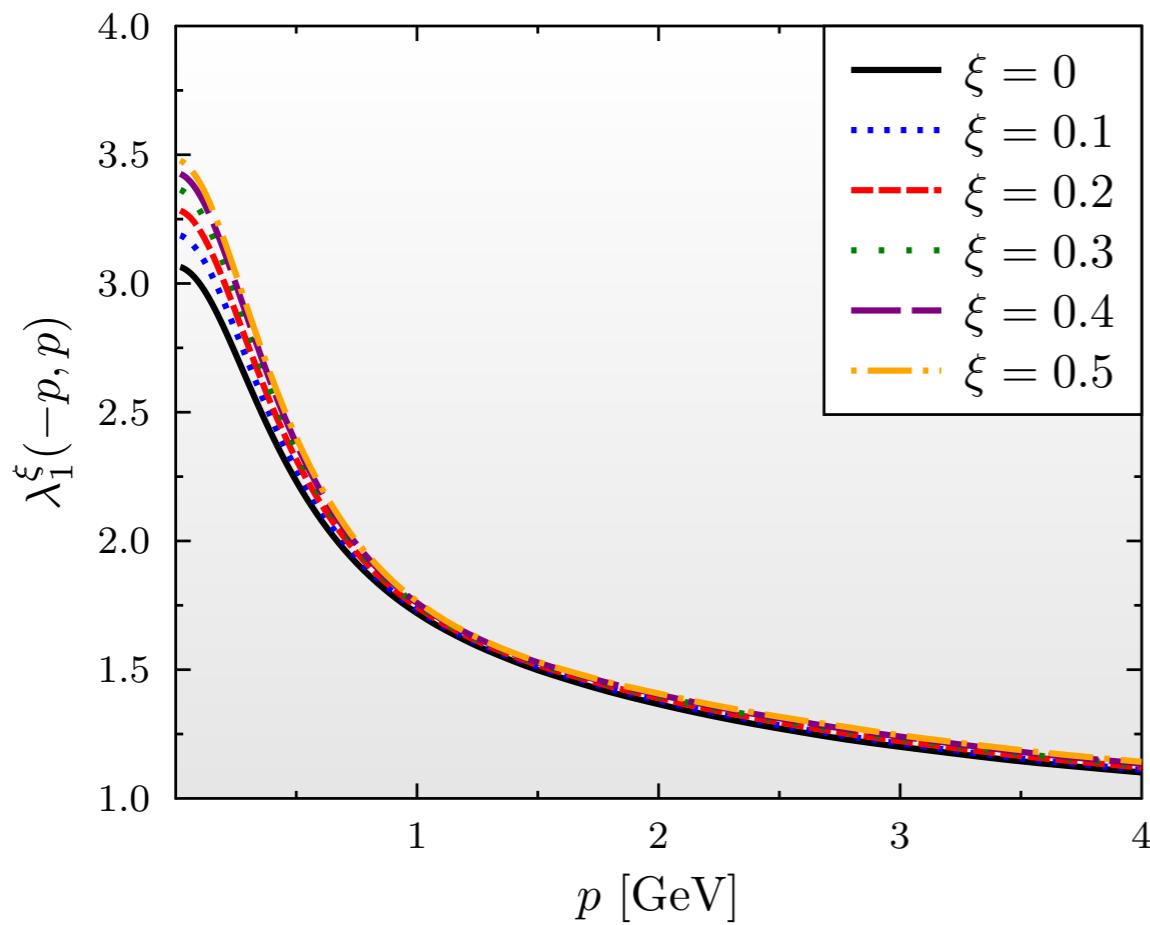


$$\alpha_s^\xi = 0.29 + 0.098\xi - 0.064\xi^2$$

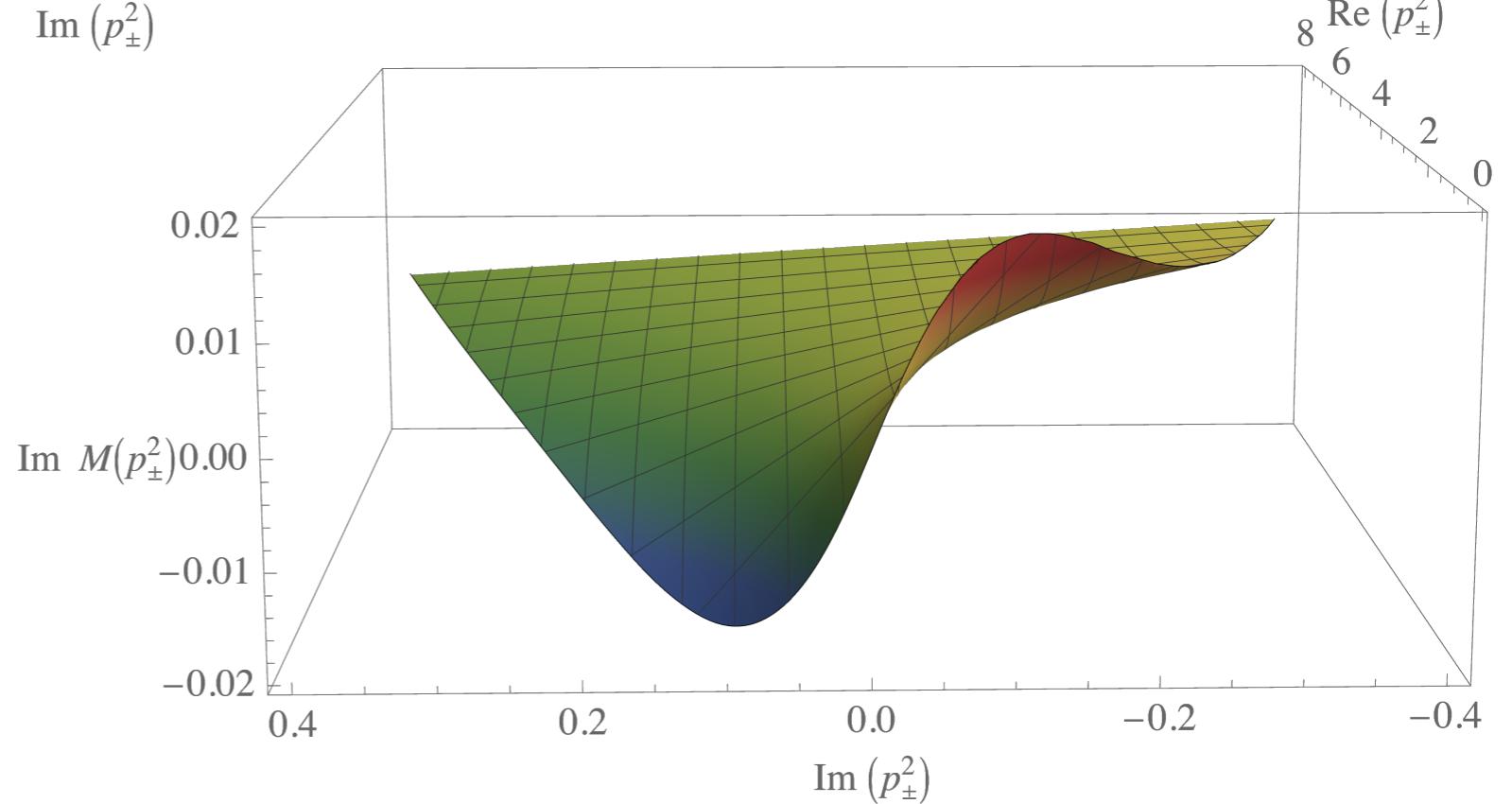
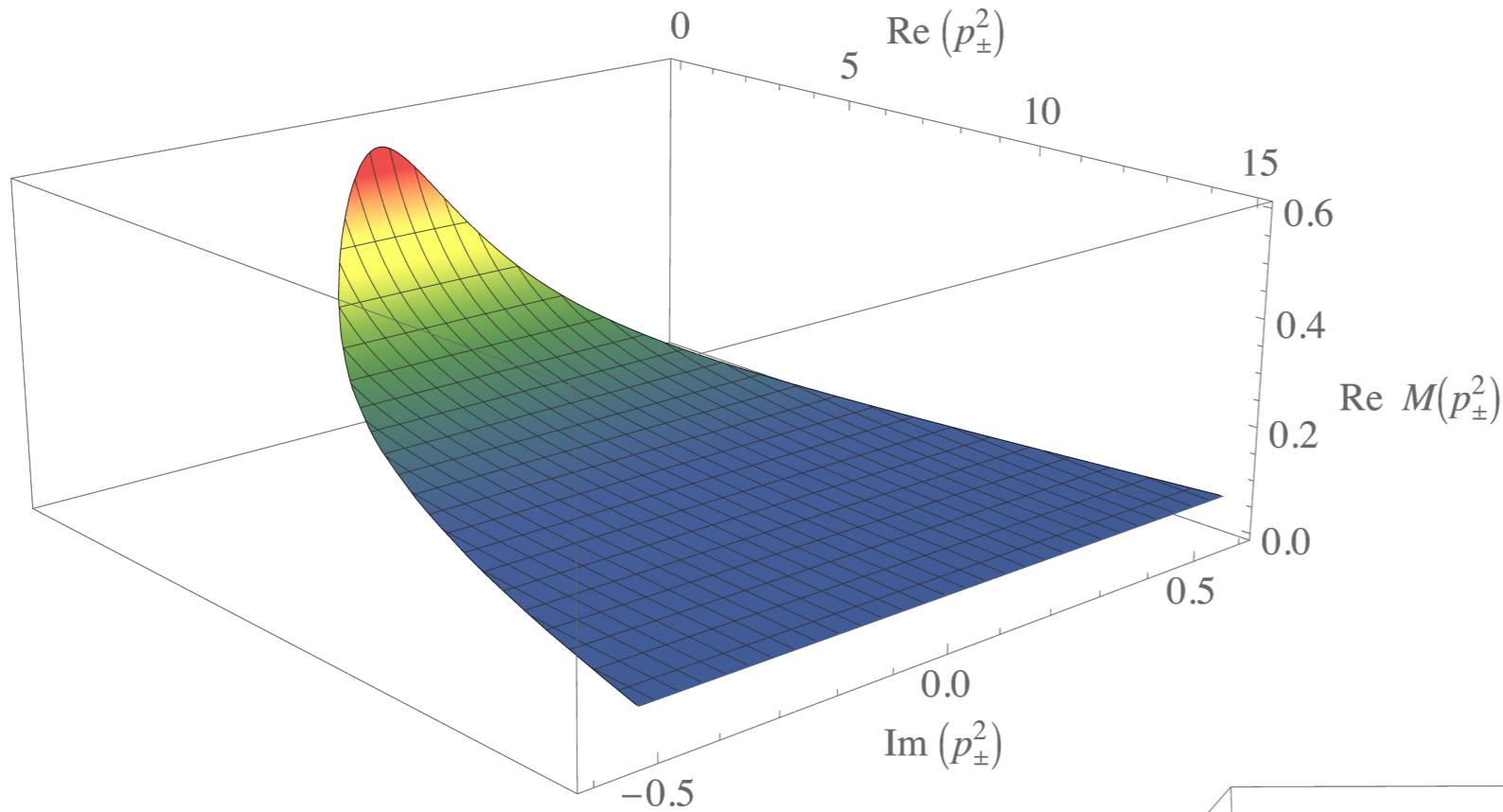
A. C. Aguilar, D. Binosi and J. Papavassiliou, Phys. Rev. D95, 034017 (2017)

DSE with gluon propagators in R_ξ gauge: quark-gluon vertex

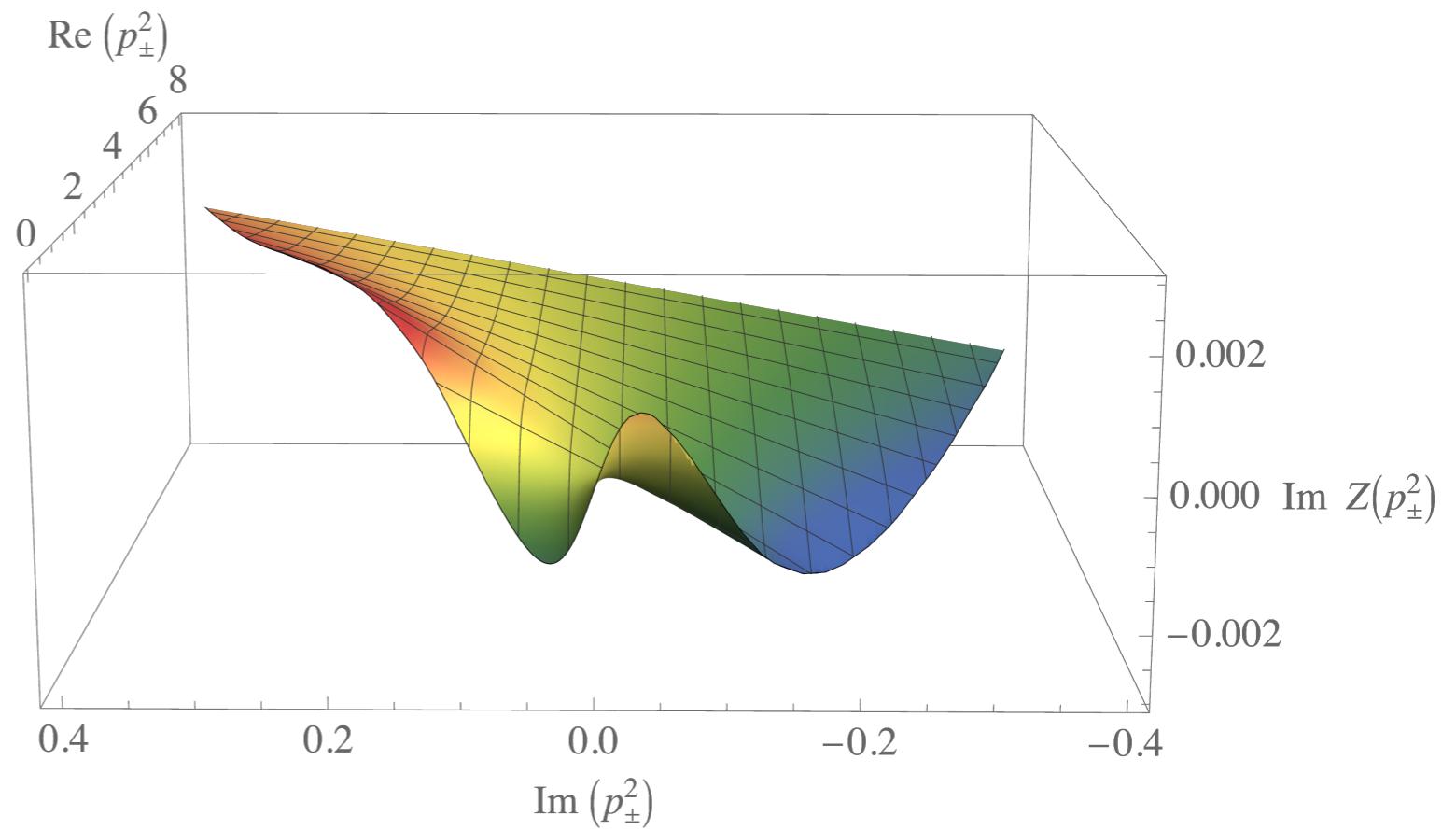
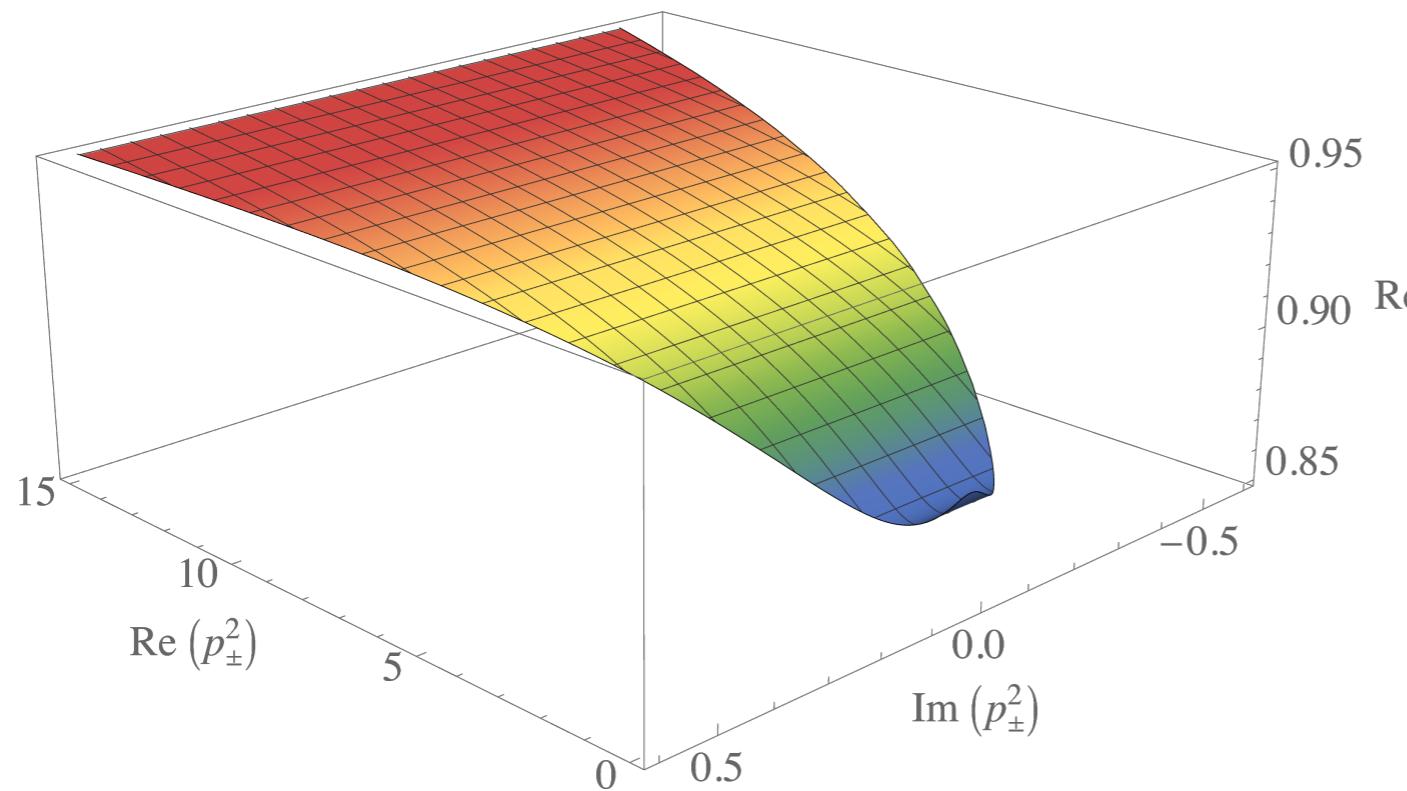
Lattice QCD input for gluon: P. Bicudo, D. Binosi, N. Cardoso,
O. Oliveira and P. J. Silva, Phys. Rev. D 92, 114514 (2015)



DSE on complex plane for light quarks: *mass function*



DSE on complex plane for light quarks: *wave renormalization*



Conclusions & Progress

- We derived a quark-gluon vertex from symmetries (gauge + Lorentz), that is we don't solve the inhomogeneous BSE for the quark-gluon vertex.
- The self-consistent solutions employ as ingredients gluon and ghost propagators from lattice QCD.
- Current status of DCSB still unsatisfying when only known terms are kept and multiplicative renormalizability is not satisfied.
- Next step: use Y_i form factors from nonlocal tensors (line integral).
- Adding these terms yields a mass function and DCSB known from phenomenological model, but still has parameters.
- A full self-consistent with additional coupled integral equations is underway.

Backup Slides

Transverse form factors of the quark-gluon vertex from transverse STIs

$$\begin{aligned}
\tau_3^{\text{QCD}} &= \frac{1}{2} G(q^2) X_0(q^2) \left[\frac{A(k^2) - A(p^2)}{k^2 - p^2} \right] + \frac{Y_2}{4\nabla(k, p)} - \frac{(k+p)^2(Y_3 - Y_5)}{8(k^2 - p^2)\nabla(k, p)} \\
\tau_5^{\text{QCD}} &= -G(q^2) X_0(q^2) \left[\frac{B(k^2) - B(p^2)}{k^2 - p^2} \right] - \frac{2Y_4 + Y_6^A}{2(k^2 - p^2)} \\
\tau_8^{\text{QCD}} &= -G(q^2) X_0(q^2) \left[\frac{A(k^2) - A(p^2)}{k^2 - p^2} \right] - \frac{2Y_8^A}{k^2 - p^2}
\end{aligned}$$

$$\begin{aligned}
\tau_1^{\text{QCD}} &= -\frac{Y_1}{2(k^2 - p^2)(k^2 p^2 - (k \cdot p)^2)} \\
\tau_2^{QCD} &= -\frac{Y_5 - 3Y_3}{4(k^2 - p^2)(k^2 p^2 - (k \cdot p)^2)}, \\
\tau_4^{QCD} &= \frac{Y_1 - (6Y_4 + Y_6)(k^2 - p^2) - Y_7(k + p)^2}{2(k^2 - p^2)^2(k^2 p^2 - (k \cdot p)^2)}, \\
\tau_6^{QCD} &= \frac{2Y_2(k - p)^2 - (Y_3 - Y_5)(k^2 - p^2)}{8(k^2 - p^2)(k^2 p^2 - (k \cdot p)^2)}, \\
\tau_7^{QCD} &= \frac{Y_1(k - p)^2 - 4Y_7(k^2 p^2 - (k \cdot p)^2)}{4(k^2 - p^2)(k^2 p^2 - (k \cdot p)^2)}
\end{aligned}$$

Transverse form factors of the quark-gluon vertex from transverse STIs

$$\tau_3^{\text{QCD}} = \frac{1}{2} G(q^2) X_0(q^2) \left[\frac{A(k^2) - A(p^2)}{k^2 - p^2} \right] + \frac{Y_2}{4\nabla(k, p)} - \frac{(k+p)^2(Y_3 - Y_5)}{8(k^2 - p^2)\nabla(k, p)}$$

$$\tau_5^{\text{QCD}} = -G(q^2) X_0(q^2) \left[\frac{B(k^2) - B(p^2)}{k^2 - p^2} \right] - \frac{2Y_4 + Y_6^A}{2(k^2 - p^2)}$$

$$\tau_8^{\text{QCD}} = -G(q^2) X_0(q^2) \left[\frac{A(k^2) - A(p^2)}{k^2 - p^2} \right] - \frac{2Y_8^A}{k^2 - p^2}$$

No dependence on $A(p^2)$ and $B(p^2)$ functions !

$$\begin{aligned}\tau_1^{\text{QCD}} &= -\frac{Y_1}{2(k^2 - p^2)(k^2 p^2 - (k \cdot p)^2)}, \\ \tau_2^{\text{QCD}} &= -\frac{Y_5 - 3Y_3}{4(k^2 - p^2)(k^2 p^2 - (k \cdot p)^2)}, \\ \tau_4^{\text{QCD}} &= \frac{Y_1 - (6Y_4 + Y_6)(k^2 - p^2) - Y_7(k + p)^2}{2(k^2 - p^2)^2(k^2 p^2 - (k \cdot p)^2)}, \\ \tau_6^{\text{QCD}} &= \frac{2Y_2(k - p)^2 - (Y_3 - Y_5)(k^2 - p^2)}{8(k^2 - p^2)(k^2 p^2 - (k \cdot p)^2)}, \\ \tau_7^{\text{QCD}} &= \frac{Y_1(k - p)^2 - 4Y_7(k^2 p^2 - (k \cdot p)^2)}{4(k^2 - p^2)(k^2 p^2 - (k \cdot p)^2)}\end{aligned}$$

Use an ansatz based on perturbation theory, symmetry considerations and multiplicative renormalizability

Guided by perturbation theory; draws on comparison with structural dependence of the Ball-Chiu vertex on $\mathbf{A}(p^2)$ and $\mathbf{B}(p^2)$. The perturbative limit of the transverse vertex conforms with its one loop expansion in the asymptotic limit of $k^2 \gg p^2$.

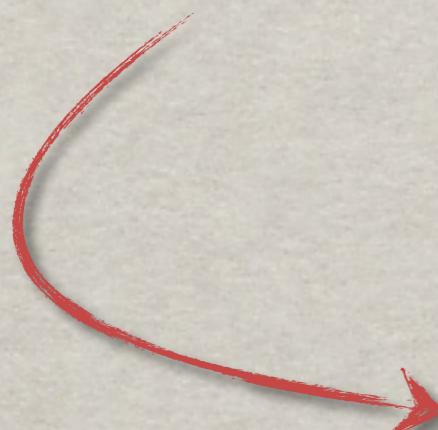
Bashir, Bermúdez, Chang & Roberts (2012)

$$\begin{aligned}
 \tau_1(k^2, p^2) &= \frac{a_1 \Delta_B(k^2, p^2)}{(k^2 + p^2)} \\
 \tau_2(k^2, p^2) &= \frac{a_2 \Delta_A(k^2, p^2)}{(k^2 + p^2)} \\
 \tau_3(k^2, p^2) &= a_3 \Delta_A(k^2, p^2) \\
 \tau_4(k^2, p^2) &= \frac{a_4 \Delta_B(k^2, p^2)}{[k^2 + M^2(k^2)][p^2 + M^2(p^2)]} \\
 \tau_5(k^2, p^2) &= a_5 \Delta_B(k^2, p^2) \\
 \tau_6(k^2, p^2) &= \frac{a_6(k^2 + p^2) \Delta_A(k^2, p^2)}{[(k^2 - p^2)^2 + (M^2(k^2) + M^2(p^2))^2]} \\
 \tau_7(k^2, p^2) &= \frac{a_7 \Delta_B(k^2, p^2)}{(k^2 + p^2)} \\
 \tau_8(k^2, p^2) &= a_8 \Delta_A(k^2, p^2)
 \end{aligned}$$

Use an ansatz based on perturbation theory, symmetry considerations and multiplicative renormalizability

Guided by perturbation theory; draws on comparison with structural dependence of the Ball-Chiu vertex on $\mathbf{A}(p^2)$ and $\mathbf{B}(p^2)$. The perturbative limit of the transverse vertex conforms with its one loop expansion in the asymptotic limit of $k^2 \gg p^2$.

Bashir, Bermúdez, Chang & Roberts (2012)



$$\begin{aligned}
 \tau_1(k^2, p^2) &= \frac{a_1 \Delta_B(k^2, p^2)}{(k^2 + p^2)} \\
 \tau_2(k^2, p^2) &= \frac{a_2 \Delta_A(k^2, p^2)}{(k^2 + p^2)} \\
 \tau_3(k^2, p^2) &= a_3 \Delta_A(k^2, p^2) \\
 \tau_4(k^2, p^2) &= \frac{a_4 \Delta_B(k^2, p^2)}{[k^2 + M^2(k^2)][p^2 + M^2(p^2)]} \\
 \tau_5(k^2, p^2) &= a_5 \Delta_B(k^2, p^2) \\
 \tau_6(k^2, p^2) &= \frac{a_6(k^2 + p^2) \Delta_A(k^2, p^2)}{[(k^2 - p^2)^2 + (M^2(k^2) + M^2(p^2))^2]} \\
 \tau_7(k^2, p^2) &= \frac{a_7 \Delta_B(k^2, p^2)}{(k^2 + p^2)} \\
 \tau_8(k^2, p^2) &= a_8 \Delta_A(k^2, p^2)
 \end{aligned}$$

*Can also be done using the full pQCD vertex derived
by Bermudez et al., Phys. Rev. D95 (2017).*

Comparing the transverse vertex derived from the STIs with this ansatz

$$\begin{aligned}
\tau_1(k^2, p^2) &= \frac{a_1 \Delta_B(k^2, p^2)}{(k^2 + p^2)} \\
\tau_2(k^2, p^2) &= \frac{a_2 \Delta_A(k^2, p^2)}{(k^2 + p^2)} \\
\tau_3(k^2, p^2) &= a_3 \Delta_A(k^2, p^2) \\
\tau_4(k^2, p^2) &= \frac{a_4 \Delta_B(k^2, p^2)}{[k^2 + M^2(k^2)[p^2 + M^2(p^2)]]} \\
\tau_5(k^2, p^2) &= a_5 \Delta_B(k^2, p^2) \\
\tau_6(k^2, p^2) &= \frac{a_6(k^2 + p^2) \Delta_A(k^2, p^2)}{[(k^2 - p^2)^2 + (M^2(k^2) + M^2(p^2))^2]} \\
\tau_7(k^2, p^2) &= \frac{a_7 \Delta_B(k^2, p^2)}{(k^2 + p^2)} \\
\tau_8(k^2, p^2) &= a_8 \Delta_A(k^2, p^2)
\end{aligned}$$



$$\begin{aligned}
\tau_1(k^2, p^2) &= -\frac{Y_1}{2(k^2 - p^2)\nabla(k, p)} \\
\tau_2(k^2, p^2) &= -\frac{Y_5 - 3Y_3}{4(k^2 - p^2)\nabla(k, p)} \\
\tau_3(k^2, p^2) &= \frac{1}{2} G(q^2) X_0(q^2) \left[\frac{A(k^2) - A(p^2)}{k^2 - p^2} \right] \\
&\quad + \frac{Y_2}{4\nabla(k, p)} - \frac{(k + p)^2(Y_3 - Y_5)}{8(k^2 - p^2)\nabla(k, p)} \\
\tau_4(k^2, p^2) &= -\frac{6Y_4 + Y_6^A}{8\nabla(k, p)} - \frac{(k + p)^2 Y_7^S}{8(k^2 - p^2)\nabla(k, p)} \\
\tau_5(k^2, p^2) &= -G(q^2) X_0(q^2) \left[\frac{B(k^2) - B(p^2)}{k^2 - p^2} \right] \\
&\quad - \frac{2Y_4 + Y_6^A}{2(k^2 - p^2)} \\
\tau_6(k^2, p^2) &= \frac{(k - p)^2 Y_2}{4(k^2 - p^2)\nabla(k, p)} - \frac{Y_3 - Y_5}{8\nabla(k, p)} \\
\tau_7(k^2, p^2) &= \frac{q^2(6Y_4 + Y_6^A)}{4(k^2 - p^2)\nabla(k, p)} + \frac{Y_7^S}{4\nabla(k, p)} \\
\tau_8(k^2, p^2) &= -G(q^2) X_0(q^2) \left[\frac{A(k^2) - A(p^2)}{k^2 - p^2} \right] - \frac{2Y_8^A}{k^2 - p^2}
\end{aligned}$$

Comparing the transverse vertex derived from the STIs with this ansatz

With this ansatz for the Y_i functions we come closer to a gauge covariant vertex and include additional mass terms that enhance DCSB.

The parameters a_i are constrained by multiplicative renormalizability and only certain combinations are allowed.

$$\begin{aligned}
 Y_1(k^2, p^2) &= -2a_1 [B(k^2) - B(p^2)] \frac{\Delta(k, p)}{k^2 + p^2} \\
 Y_2(k^2, p^2) &= \frac{1}{2} [A(k^2) - A(p^2)] \\
 &\times \left\{ (k^2 - p^2) (G(q^2)X_0(q^2) - 2a_3) \right. \\
 &\left. - 2 \left(\frac{k^2 + p^2}{k^2 - p^2} \right) (k + p)^2 a_6 \right\} \\
 Y_3(k^2, p^2) &= \frac{1}{2} [A(k^2) - A(p^2)] \\
 &\times \left\{ -(k - p)^2 (G(q^2)X_0(q^2) - 2a_3) \right. \\
 &\left. + 4 \frac{\Delta(k, p)}{k^2 + p^2} a_2 + 2(k^2 + p^2) a_6 \right\} \\
 Y_4(k^2, p^2) &= -\frac{B(k^2) - B(p^2)}{4k^2 p^2 (k^2 + p^2)} \left\{ 2(k^2 + p^2) \Delta(k, p) a_4 \right. \\
 &+ 2k^2 p^2 (k^2 + p^2) [a_5 - G(q^2)X_0] \\
 &\left. + k^2 p^2 (k + p)^2 a_7 \right\} \\
 Y_5(k^2, p^2) &= \frac{3}{2} [A(k^2) - A(p^2)] \\
 &\times \left\{ -(k - p)^2 [G(q^2)X_0 - 2a_3] \right. \\
 &\left. + \frac{4}{3} \frac{\Delta(k, p)}{k^2 + p^2} a_2 + 2(k^2 + p^2) a_6 \right\} \\
 Y_6^A(k^2, p^2) &= \frac{B(k^2) - B(p^2)}{2k^2 p^2 (k^2 + p^2)} \left\{ 2(k^2 + p^2) \Delta(k, p) a_4 \right. \\
 &+ 6k^2 p^2 (k^2 + p^2) (a_5 - G(q^2)X_0) \\
 &\left. + k^2 p^2 (k + p)^2 a_7 \right\} \\
 Y_7^S(k^2, p^2) &= a_7 [B(k^2) - B(p^2)] \frac{k^2 - p^2}{k^2 + p^2} \\
 Y_8^A(k^2, p^2) &= -\frac{1}{2} [A(k^2) - A(p^2)] (a_8 + G(q^2)X_0)
 \end{aligned}$$