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Diffusion of conserved charges in relativistic heavy ion collisions

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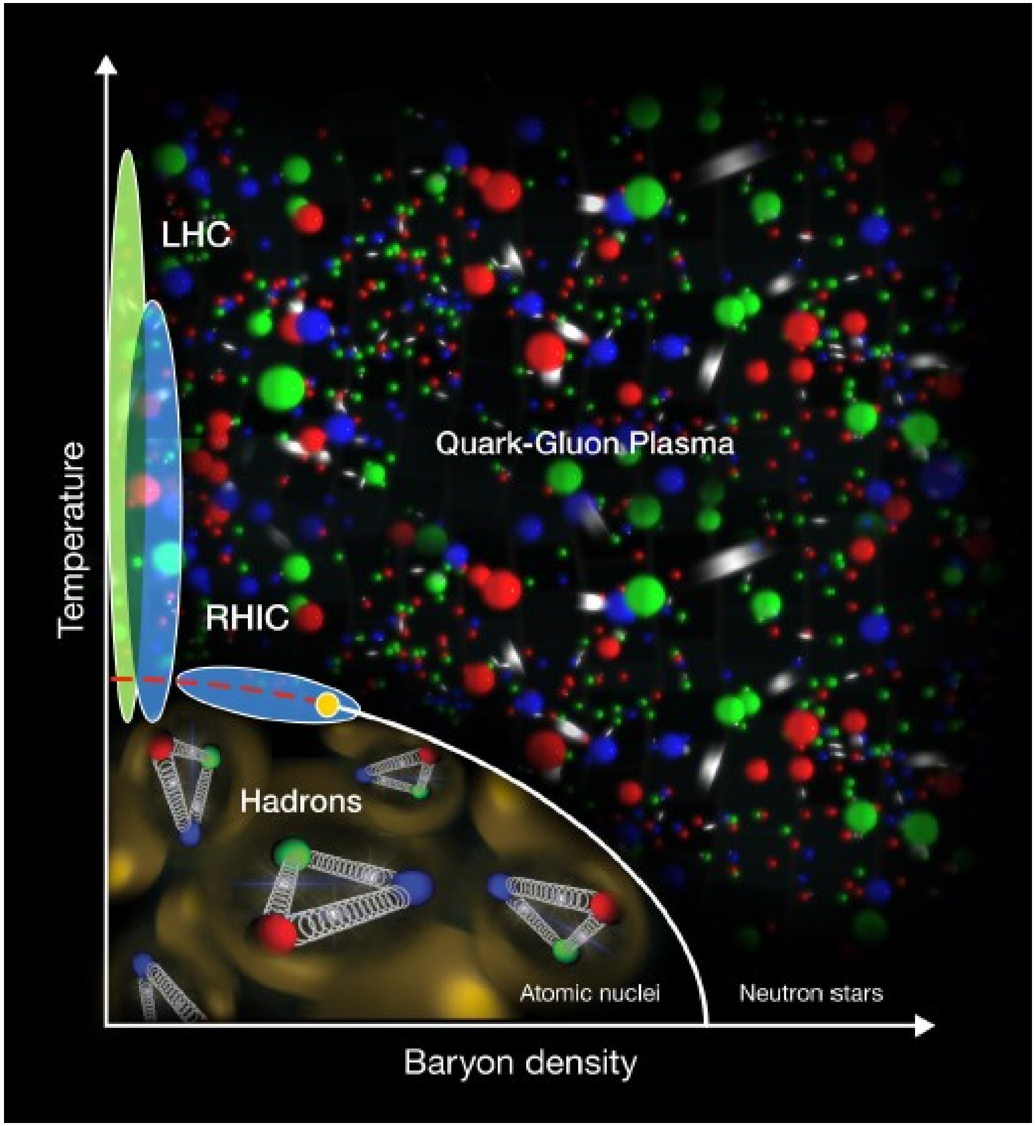
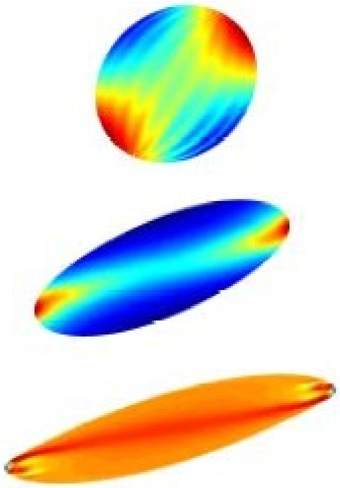
What you will see in this talk

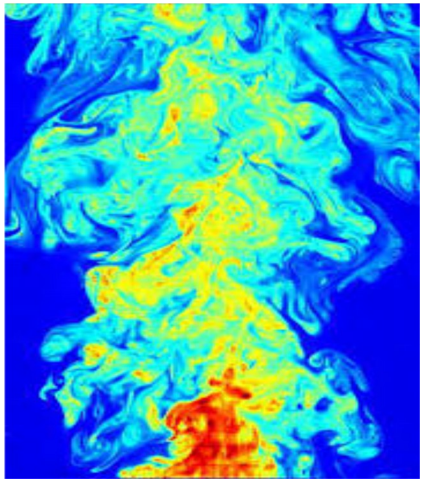
- ✔ Motivation
- ✔ Fluid-dynamical modeling of heavy-ion collisions at finite net-charge
- ✔ Diffusion in relativistic fluid dynamics
- ✔ Conclusions and perspectives

nuclear matter under extreme conditions

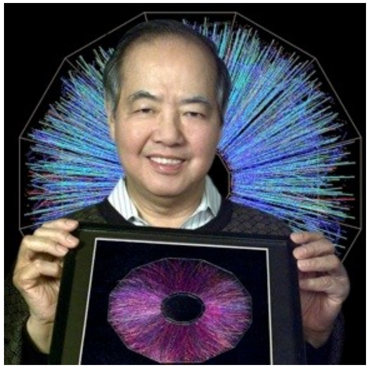
- *QCD phase diagram (thermodynamic properties)*
- *Non equilibrium phenomena (transport properties)*



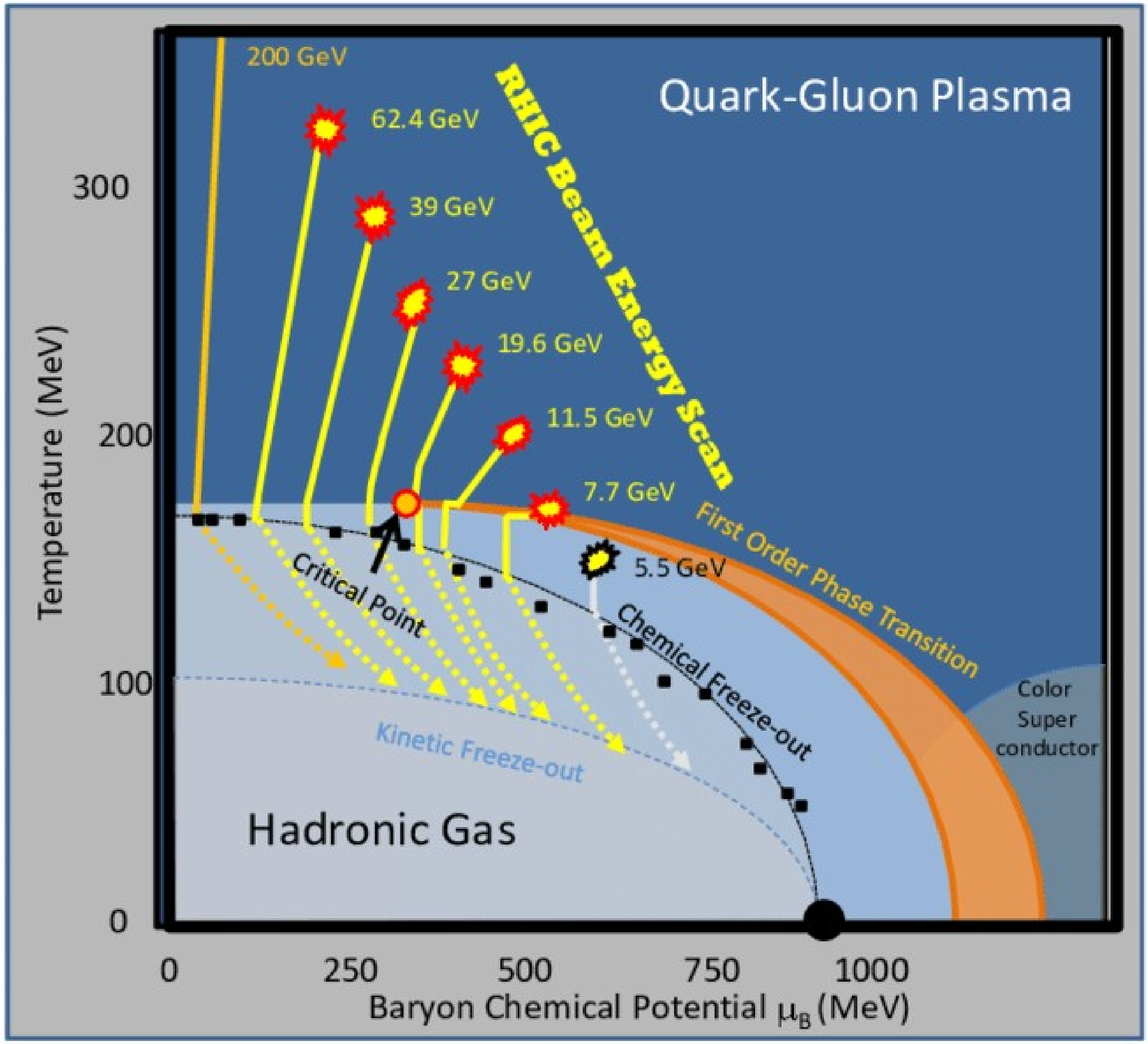




Goal of Heavy-Ion Collisions: *Produce* and *study* QCD matter near (local) equilibrium



T.D. Lee



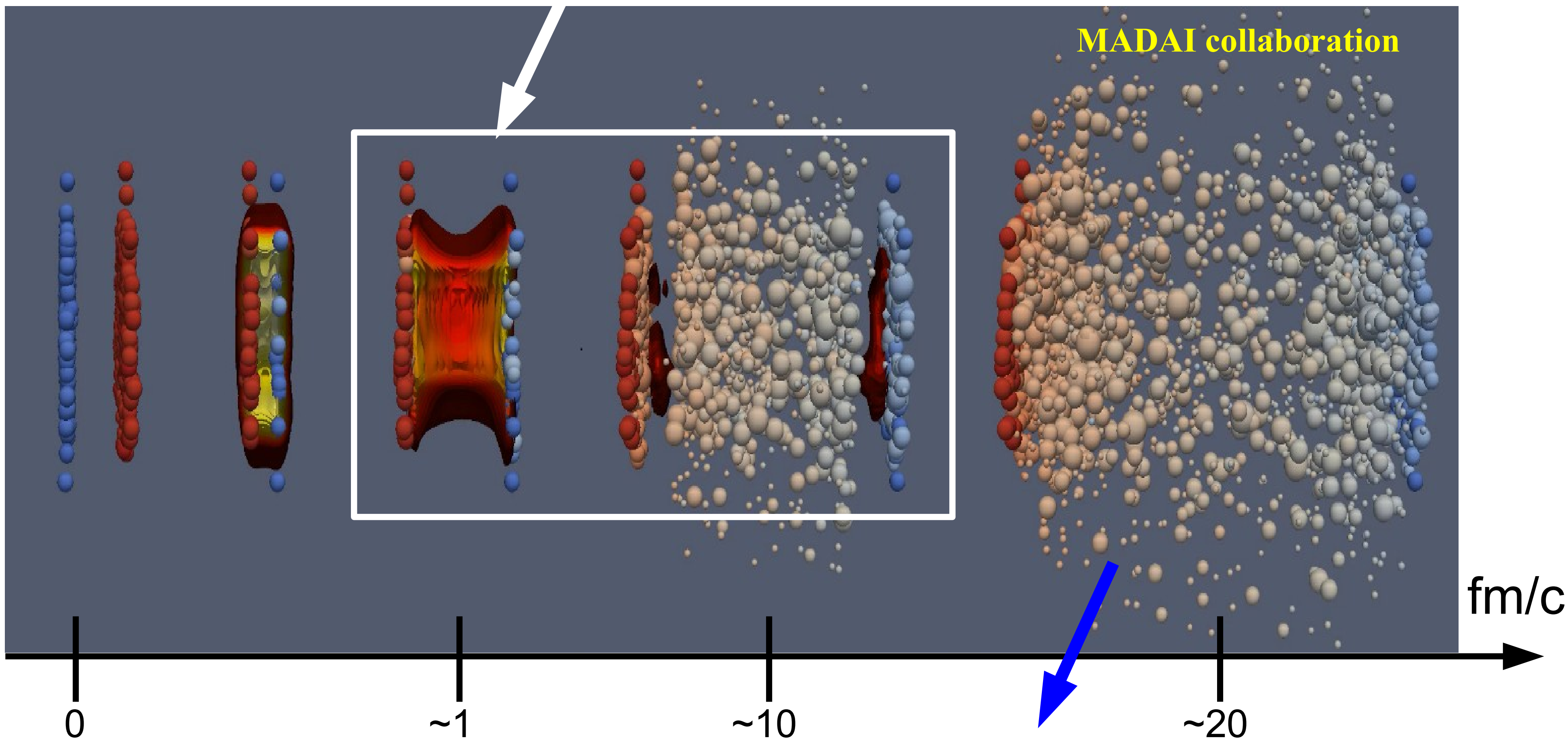
<u>Challenge</u>

- Extract thermodynamic <u>*and*</u> transport properties of QCD matter
- Experiment is not to test QCD, but to <u>understand</u> it
- Going towards finite net charge density -- BES

Heavy Ion Collisions

QCD matter is only created *transiently* ~10 fm/c

Assumption: *fluid-dynamical expansion*

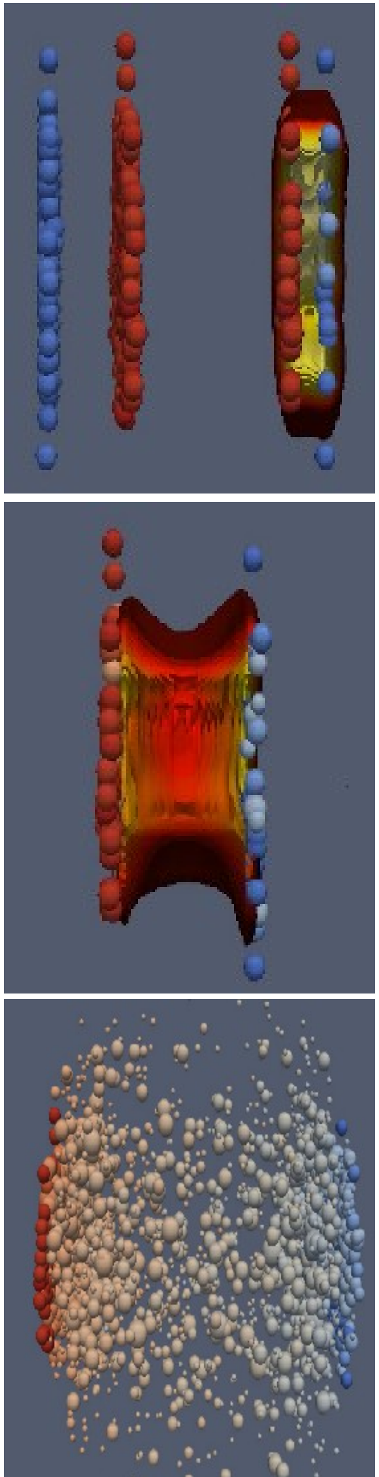


measured: hadrons, leptons, and photons

We must model the whole collision!

Properties of matter must be reverse-engineered

Current theoretical description



1) Initial state and "pre-equilibrium" dynamics:

- description of early-time dynamics and "thermalization"
- initial condition for hydrodynamic evolution

(approach) thermalization (?)

2) Fluid-dynamical expansion of QGP and Hadron Gas

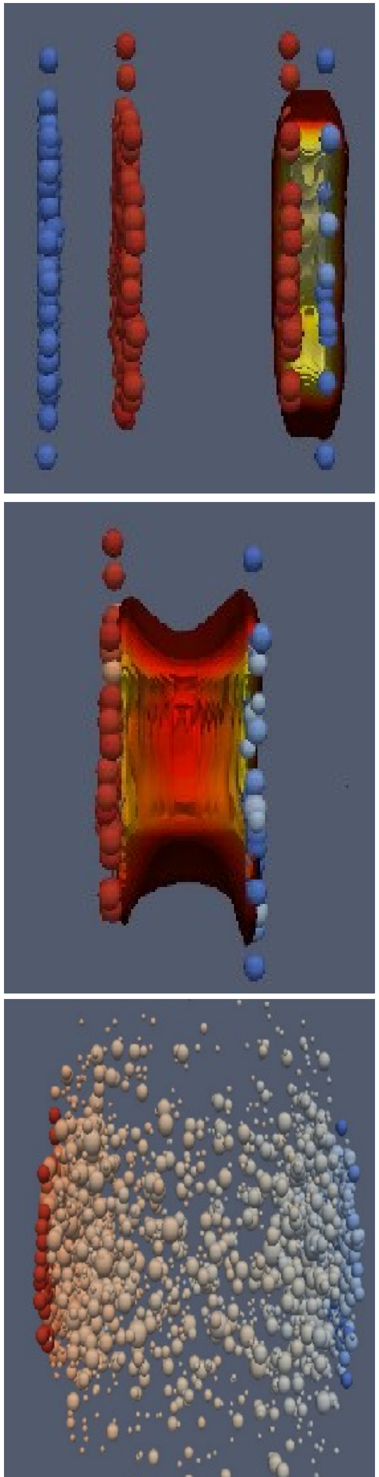
- Phase transition
- Matter described by EoS and transport coefficients

shear and bulk viscosity, charge diffusion, relaxation times ...

fluid elements converted to particles

3) Transport description of Hadron Gas

- Late stage description using the *hadron resonance gas* model – using cross sections and decay probabilities



During last 15 years, developed at n_B=0

- Focus has <u>not</u> been in extracting EoS

  ➝ Extraction of transport properties (<u>shear</u> and bulk viscosities)

  ➝ Understanding the initial state

- Fluid-dynamical models have evolved dramatically in the last 20 years:

  inclusion of dissipation (2006),
  event-by-event fluctuations (2010),
  sub-nucleonic fluctuations (2012), ...

Relativistic fluid dynamics

Effective theory describing the dynamics of a system over long-times and long-distances

Conservation laws

+

Equation of state

+

simple constitutive relations

Basics of fluid dynamics (Landau frame)

Conservation laws

energy-momentum conservation

$$\boxed{\partial_\mu T^{\mu\nu} = 0}$$

Net charge conservation

$$\begin{aligned} \partial_\mu N_s^\mu &= 0 \\ \partial_\mu N_e^\mu &= 0 \\ \partial_\mu N_b^\mu &= 0 \end{aligned}$$

strangeness

electric charge

Baryon number

Tensor decomposition

$$\begin{aligned} N_q^\mu &= n_q u^\mu + n_q^\mu \\ T^{\mu\nu} &= \varepsilon u^\mu u^\nu - (P_0 + \Pi)\,\Delta^{\mu\nu} + \pi^{\mu\nu} \end{aligned}$$

net-charge diffusion 4-current

Bulk viscous pressure

Shear stress tensor

Projection operator: $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

Equation of state

Thermodynamic pressure: $P_0 = \underbrace{P_0\left(T, \mu_b, \mu_e, \mu_s\right)}_{\textit{not known!}}$

Taylor expansion up to 4th order: $$\frac{P}{T^4} = \frac{P_0}{T^4} + \sum_{l,m,n} \underbrace{\frac{\chi^{B,Q,S}_{l,m,n}}{l!m!n!}}_{\text{lQCD}} \left(\frac{\mu_B}{T}\right)^l \left(\frac{\mu_Q}{T}\right)^m \left(\frac{\mu_S}{T}\right)^n$$

- matched to *hadron resonance gas* model at small T
- matched to Stefan-Boltzmann limit at large T

- Prescription employed by:

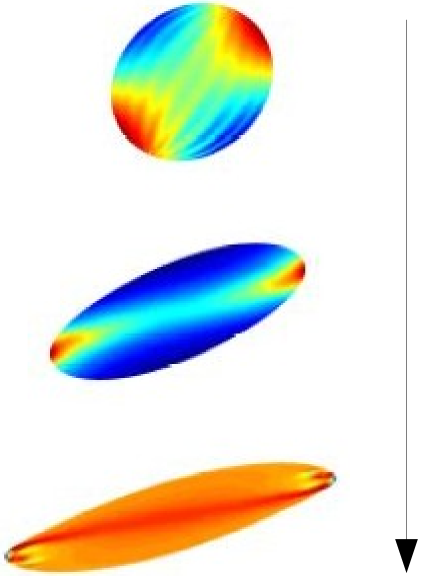
Monnai, Schenke, Shen, PRC 100, 024907 (2019)
Noronha-Hostler, Parotto, Ratti, Stafford, PRC 100, 064910 (2019)

Relativistic Navier-Stokes theory

Shear Viscosity

(Resistance to deformation)

$$\pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle}$$



$$\eta\left(T,\mu_q\right)$$

Bulk Viscosity

(Resistance to expansion)

$$\Pi = -\zeta\nabla_\mu u^\mu$$

$$\zeta\left(T,\mu_q\right)$$

Net-Charge Diffusion

$$n_q^\mu = \kappa_q\nabla^\mu\frac{\mu_q}{T}$$

$$\kappa_q\left(T,\mu_q\right)$$

Relativistic Navier-Stokes theory

Shear Viscosity

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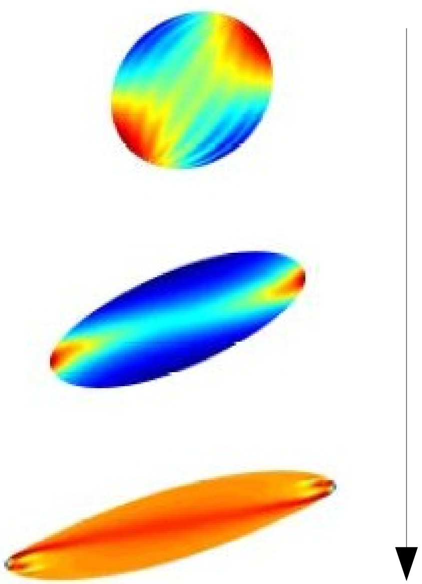

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Bulk Viscosity

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$$\Pi = -\zeta\nabla_\mu u^\mu$$

$$\zeta\left(T,\mu_q\right)$$

Net-Charge Diffusion

$$n_q^\mu = \kappa_q\nabla^\mu\frac{\mu_q}{T}$$

$$\kappa_q\left(T,\mu_q\right)$$

How does the critical point affect these coefficients?

Must include **Net-Charge Diffusion**

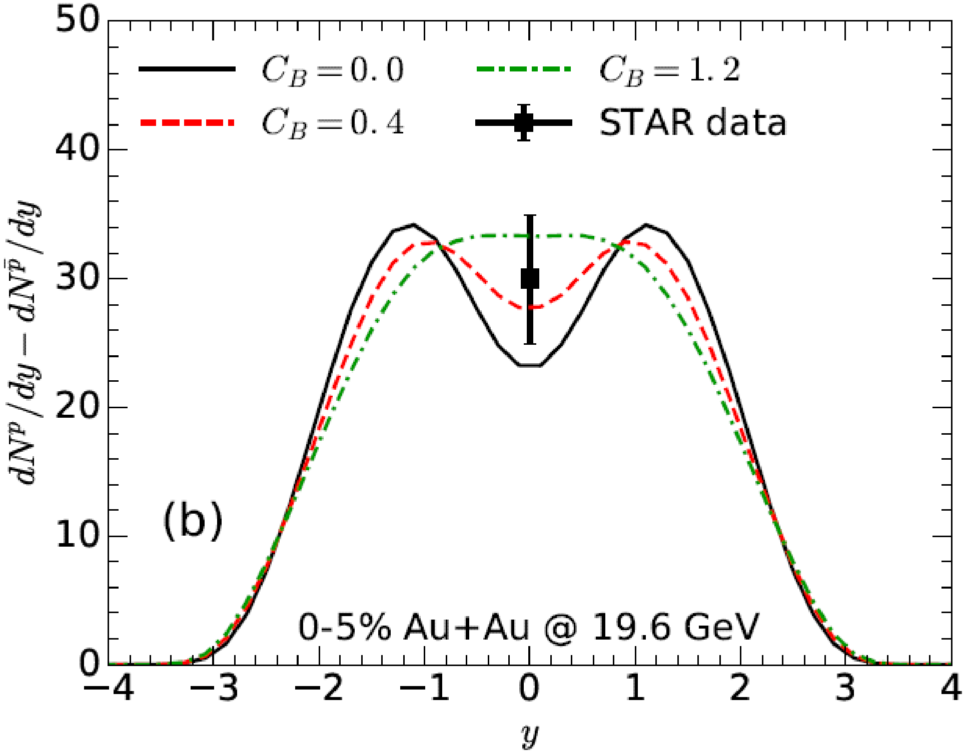
$$\boxed{j_q^\mu = \kappa_q \nabla^\mu \alpha_q}$$

Denicol *et al*, PRC98 (2018) no.3, 034916

$\alpha = \mu/T$

thermal potential

Net proton rapidity distribution



- C_B regulates the magnitude of the net-baryon diffusion coefficient

$$\kappa_B = \frac{C_B}{T} n_B \left(\frac{1}{3} \coth\left(\frac{\mu_B}{T}\right) - \frac{n_B T}{e + \mathcal{P}} \right)$$

- Clear effect of net charge diffusion

Must include **Net-Charge Diffusion**

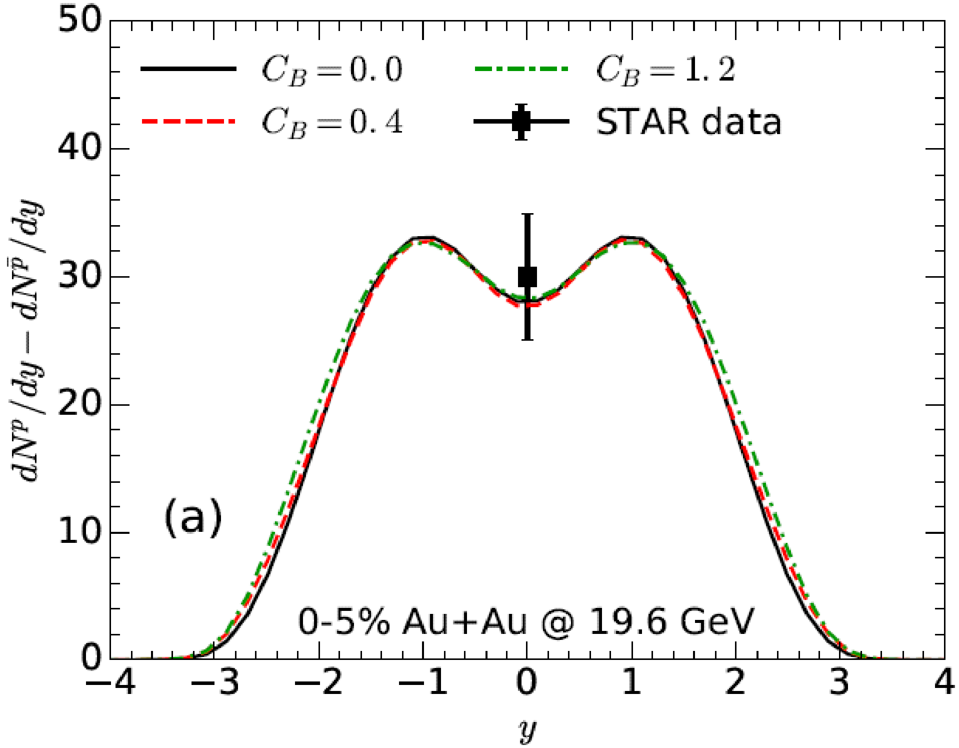
$$\boxed{j_q^\mu = \kappa_q \nabla^\mu \alpha_q}$$

Denicol *et al*, PRC98 (2018) no.3, 034916

α = μ/T

thermal potential

Net proton rapidity distribution



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- Effects can be compensated by modifying the initial condition

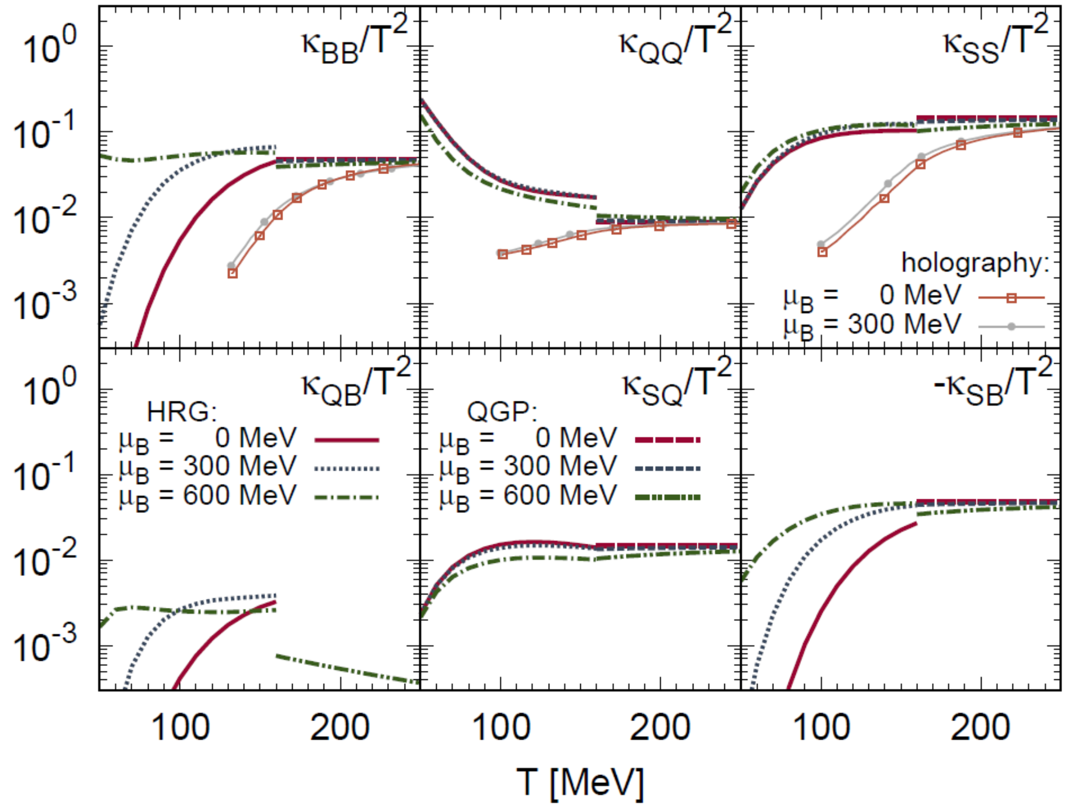
Our system has at least *3 conserved charges:* baryon number, strangeness, electric charge

The diffusion currents are coupled!

$$\begin{pmatrix} j^\mu_B \\ j^\mu_Q \\ j^\mu_S \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \cdot \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

M. Greif *et al*, PRL 120 (2018) no.24, 242301

- *first estimates from kinetic theory*
- provide information on *effective* degrees of freedom of QCD



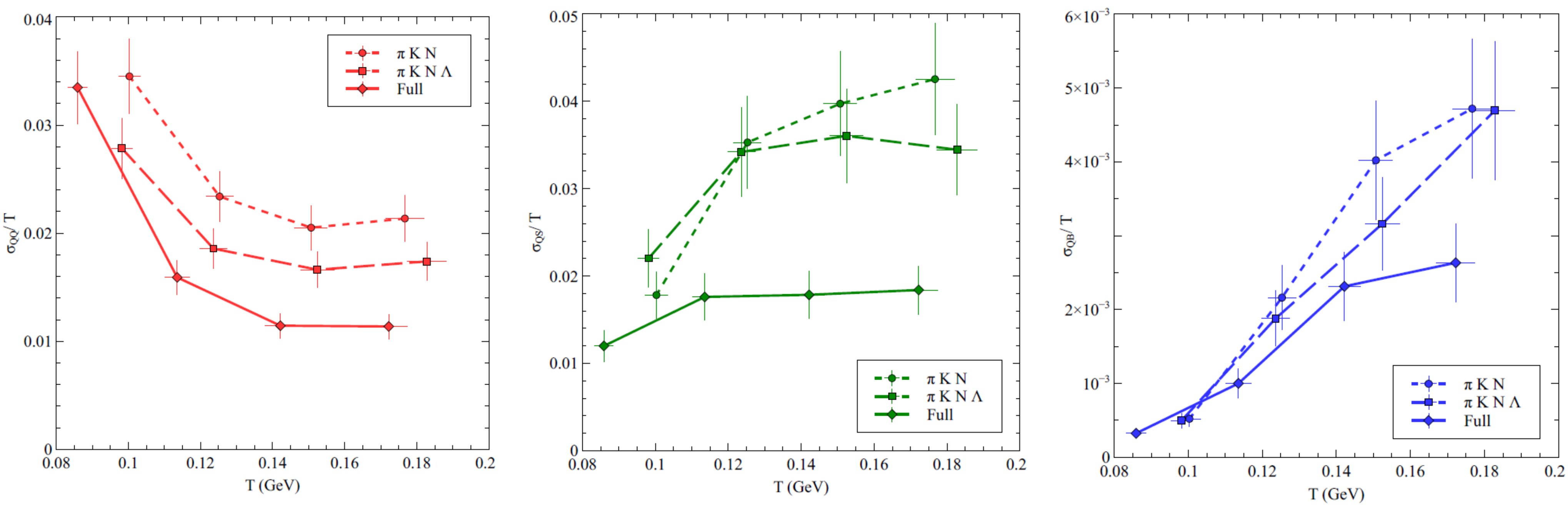
Cross-Conductivity

Rose *et al*, Phys.Rev.D 101 (2020) 11, 114028

- Constituents of bulk nuclear matter carry a multitude of conserved charges
- an electric field will also generate currents in baryon number and strangeness

$$\mathbf{j}_Q = \sigma_{QQ}\mathbf{E},$$
$$\mathbf{j}_B = \sigma_{QB}\mathbf{E},$$
$$\mathbf{j}_S = \sigma_{QS}\mathbf{E}.$$

Calculations using SMASH



Relativistic Navier-Stokes theory

Shear Viscosity

(Resistance to deformation)

$$\boxed{\pi^{\mu\nu} \quad = \quad 2\eta\nabla^{\langle\mu}u^{\nu\rangle}}$$

Bulk Viscosity

(Resistance to expansion)

$$\boxed{\Pi \quad = \quad -\zeta\nabla_{\mu}u^{\mu}}$$

Net-Charge Diffusion

$$\boxed{q^{\mu} \quad = \quad \kappa\nabla^{\mu}\frac{\mu_B}{T}}$$

- Equations violate causality and display unphysical instabilities

  Global equilibrium state is linearly unstable

  Hiscock and Lindblom, Phys.Rev.D 31 (1985) 725-733

- Relativistic Navier-Stokes theory cannot be used to model relativistic fluids

- This is the reason why it took the field so long to add dissipation into the models

What we solve is not "traditional" fluid dynamics

Causality: constitutive relations for the dissipative currents cannot be imposed

Dynamical equation, e.g. Israel-Stewart theory

Denicol *et al*, Phys.Rev.D 85 (2012) 114047

$$
\boxed{
\begin{aligned}
\tau_n \dot{n}^{\langle\mu\rangle} + n^\mu &= \kappa_n \nabla^\mu \alpha_B - n_\nu \omega^{\nu\mu} - \delta_{nn} n^\mu \theta + \ell_{n\pi} \Delta^{\mu\nu} \nabla_\lambda \pi^\lambda_\nu \\
&\quad - \tau_{n\pi} \pi^{\mu\nu} \nabla_\nu P - \lambda_{nn} n_\nu \sigma^{\mu\nu} - \lambda_{n\pi} \pi^{\mu\nu} \nabla_\nu \alpha_B, \\
\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} &= 2\eta\sigma^{\mu\nu} + 2\tau_\pi \pi_\lambda^{\langle\mu} \omega^{\nu\rangle\lambda} - \delta_{\pi\pi} \pi^{\mu\nu}\theta - \tau_{\pi\pi} \pi^{\lambda\langle\mu} \sigma_\lambda^{\nu\rangle} \\
&\quad - \tau_{\pi n} n^{\langle\mu} \nabla^{\nu\rangle} P + \ell_{\pi n} \nabla^{\langle\mu} n^{\nu\rangle} + \lambda_{\pi n} n^{\langle\mu} \nabla^{\nu\rangle} \alpha_B,
\end{aligned}
}
$$

relaxation times

Higher-order terms

These theories can be constructed to be causal and stable

Linear Stability and Causality conditions
(one conserved charge)

$$\begin{aligned}\tau_n \dot{n}^{\langle\mu\rangle} + n^\mu &= \kappa_n \nabla^\mu \alpha_B - n_\nu \omega^{\nu\mu} - \delta_{nn} n^\mu \theta + \ell_{n\pi} \Delta^{\mu\nu} \nabla_\lambda \pi^\lambda_\nu \\ &\quad - \tau_{n\pi} \pi^{\mu\nu} \nabla_\nu P - \lambda_{nn} n_\nu \sigma^{\mu\nu} - \lambda_{n\pi} \pi^{\mu\nu} \nabla_\nu \alpha_B, \\ \tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} &= 2\eta\sigma^{\mu\nu} + 2\tau_\pi \pi_\lambda^{\langle\mu} \omega^{\nu\rangle\lambda} - \delta_{\pi\pi} \pi^{\mu\nu} \theta - \tau_{\pi\pi} \pi^{\lambda\langle\mu} \sigma_\lambda^{\nu\rangle} \\ &\quad - \tau_{\pi n} n^{\langle\mu} \nabla^{\nu\rangle} P + \ell_{\pi n} \nabla^{\langle\mu} n^{\nu\rangle} + \lambda_{\pi n} n^{\langle\mu} \nabla^{\nu\rangle} \alpha_B,\end{aligned}$$

Brito and Denicol, Phys.Rev.D 102 (2020) 11, 116009

same as before

$$\boxed{\tau_\pi \geq \frac{2\eta}{\varepsilon_0 + P_0},} \qquad \boxed{\tau_n \geq \frac{\kappa_n}{\bar{n}_B},}$$

$$\boxed{|\ell_{\pi n}\ell_{n\pi}| \leq \frac{3}{2}\left(\tau_\pi - \frac{2\eta}{\varepsilon_0 + P_0}\right)\left(\tau_n - \frac{\kappa_n}{\bar{n}_B}\right)}$$

Our systems have at least *3 conserved charges:* baryon number, strangeness, electric charge

- Causal and stable equations of motion are still not fully understood

- A natural ansatz is

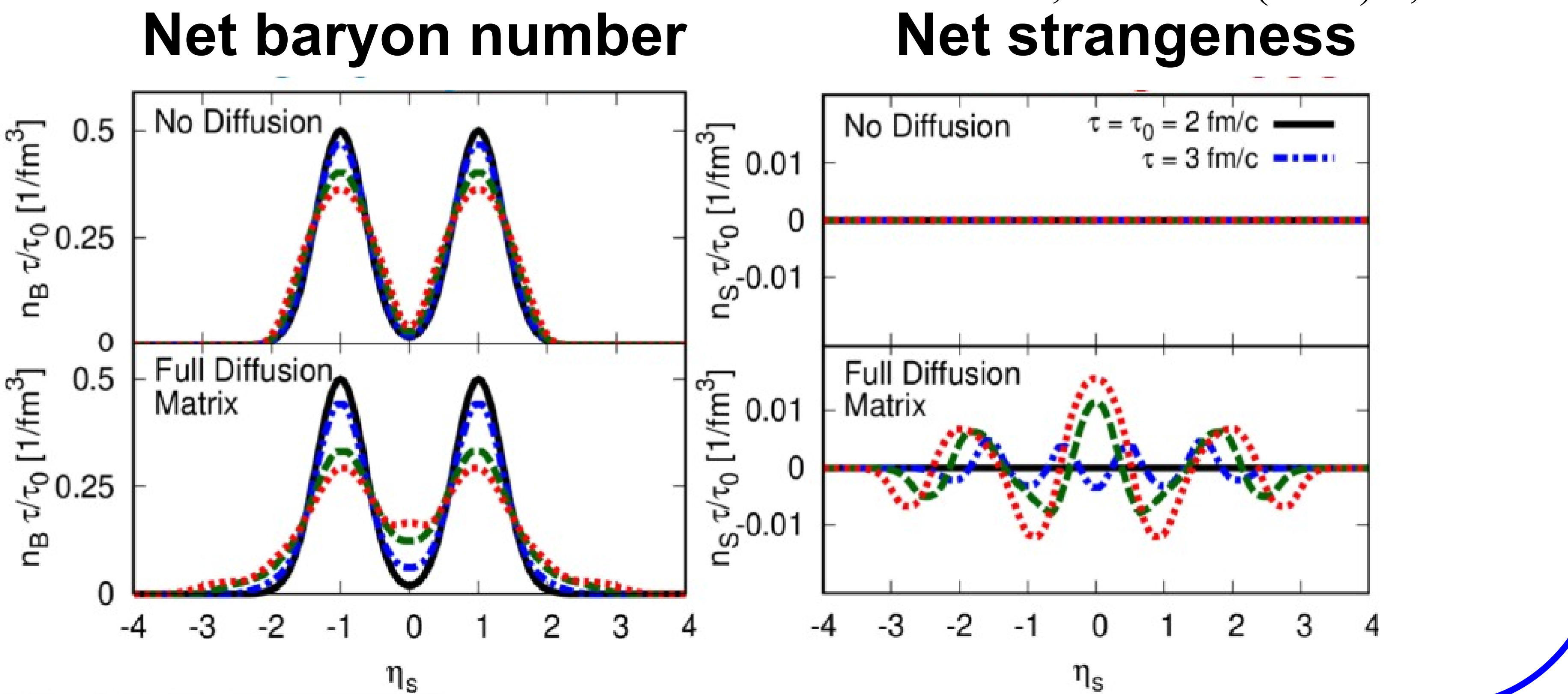
$$\boxed{\begin{aligned} \tau_q\, \Delta^{\mu}{}_{\nu} \mathcal{D} j^{\nu}_q + j^{\mu}_q &= \sum_{q'} \kappa_{qq'} \nabla^{\mu} \alpha_{q'} \\ \tau_\pi\, \Delta^{\mu\nu}_{\alpha\beta} \mathcal{D} \pi^{\alpha\beta} + \pi^{\mu\nu} &= 2\eta\sigma^{\mu\nu} \end{aligned}}$$

- But this could be more complicated ...

Our systems have at least *<u>3 conserved charges:</u>* baryon number, strangeness, electric charge

Effects explored in 1+1D simulations

J. Fotakis *et al*, PRD 101 (2020) 7, 076007



diffusion does affect the dynamics, in particular of net-strangeness

Conclusions and outlook

Fluid-dynamical models that describe low energy heavy ion collisions are under construction – but appear to be able to fit the data

- Many transport coefficients appear at finite μ_B. Very difficult to include and extract them...

but they may be crucial in identifying a phase transition

- Causal theories for multiple conserved charges must still be derived from kinetic theory