

Low energy constituent quark couplings from a dynamical approach (wide view)

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OUTLINE

- Motivations

- Quark-antiquark interaction mediated by (“non-perturbative”) 1-gluon exchange

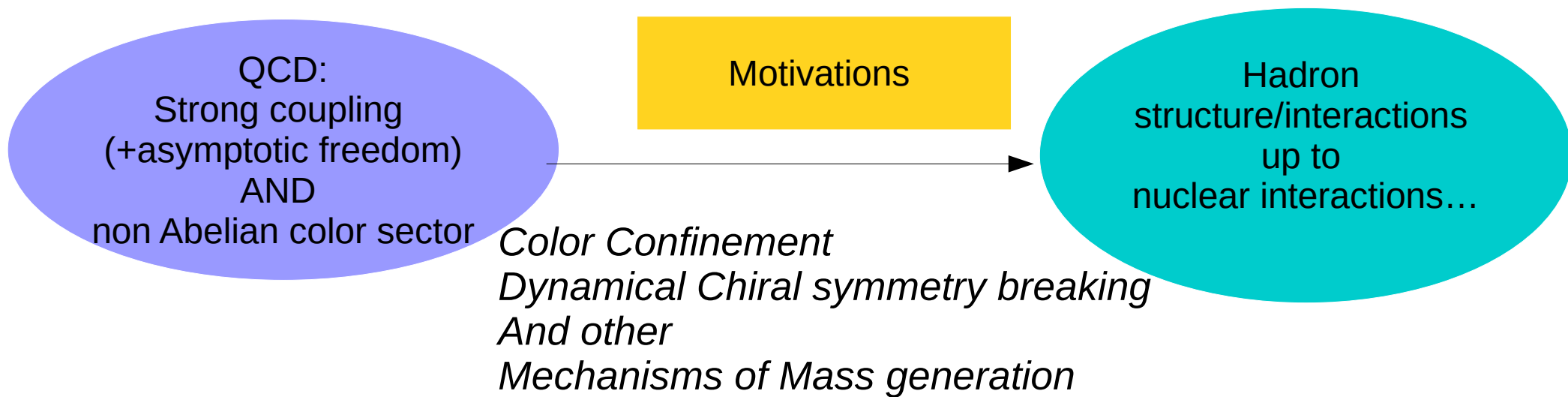
And analytical methods → mesons states, gluon cloud, pion cloud for constituent quarks

- Results for coupling constants, form factors, averaged quadratic radii, large N_c EFT

(- modifications in hadrons under weak magnetic fields)

- Wess-Zumino-Witten type couplings

- Final remarks



How (in a broad way) do fundamental degrees of freedom (flavor)

survive and show up in hadron and nuclear observables?

There are many works
So that it impossible to quote
Most of them

To address such question *without a precise and complete understanding* of all the (fundamental) mechanisms

mixed information of QCD-theorems + phenomenology → EFT+models
EFT usually with larger predictive power than effective models

Constituent Quark Model (CQM)

GellMann-Zweig
And many others

In these models: “dressed” (constituent) quark masses
are responsible for the hadrons masses

They are assumed to be “quasiparticles” (not propagating(?))
And it works...

**Quark masses (from the Higgs)
are much smaller than needed
to describe hadron masses**

Mechanisms of mass generation :
Dynamical Chiral Symmetry Breaking
Associated to other effects

$$\langle \bar{q}_R q_L + \bar{q}_L q_R \rangle \simeq -(250 \text{ MeV})^3$$

Other works/talks in this conference

From constituent quarks

$$M^*_u \sim 360 \text{ MeV}$$

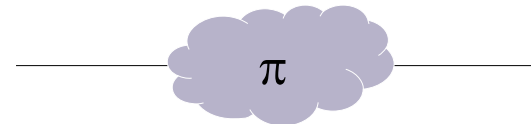
$$M^*_d \sim 370 \text{ MeV}$$

$$M^*_s \sim 510 \text{ MeV}$$

**Most of Hadrons Spectra
support or suggest the idea
(original quark models/
eightfold way)**

It can receive corrections:
Diquarks
Virtual n-quarks states
from Fock space
etc

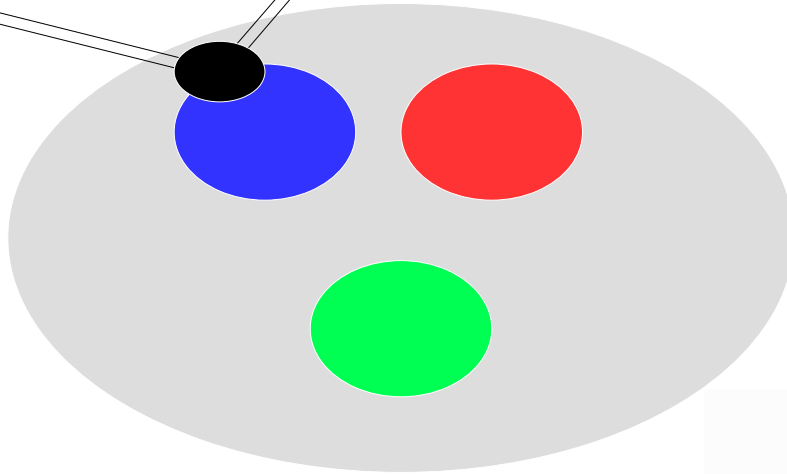
Constituent quarks:
two main possible contributions
Usually considered separately
Maybe “competing” contributions



$$m_{qqb} = m_{qb} + m_q + \Delta_{spin}^1$$

$$m_{qqq} = m_q + m_{q'} + m_{q''} + \Delta_{spin}^2$$

Meson- baryon interactions



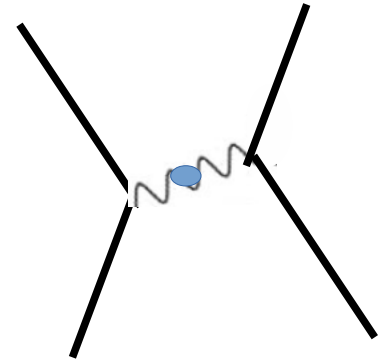
Details of interactions and dynamics
Of constituent quarks

Can help to understand further their contribution to
Dynamics inside hadrons
(even if their role as quasiparticles is not clear)

Investigation of Parton at high energies
test this picture

The complete understanding of the emergence of constituent quarks requires
Considering a huge amount of diagrams from QCD processes
(quarks and gluons at strong coupling limit) → running quark and gluon masses
(not considered in this talk, see next talk for that)

The idea is to investigate
leading term of the QCD quark-effective action
Obtained from QCD Lagrangian

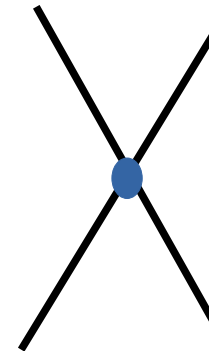


The quark-quark interaction mediated by ONE gluon exchange

WITH however an improvement due to “non perturbative” (non Abelian) effects

This present approach provides, to some extent, similar results to the QCD-SDE at the rainbow ladder approach

And also similar to the NJL model



$$G_{\text{NJL}} \sim 1/M_g^2$$

QCD Effective action → quark antiquark interaction

One leading term of the QCD effective action (quarks)

$$Z = N \int \mathcal{D}[\bar{\psi}, \psi] \exp i \int_x \left[\bar{\psi}(i\not{\partial} - m)\psi - \frac{g^2}{2} \int_y j_\mu^b(x) \tilde{R}_{bc}^{\mu\nu}(x-y) j_\nu^c(y) + \bar{\psi}J + J^*\psi \right]$$

Quark-gluon (running)
Coupling Constant
(external input)

Gluon propagator as external input

* non perturbative one → DChSB

* to some extent non Abelian
dynamics accounted

Equal current
quark mass
Flavor SU(2) or SU(3)

In the 1980's and 1990's by considering
bosonization-type approaches **for mesons: Chiral Perturbation theory**
Has been derived from quark-antiquark interaction by many
With works in the following decades
Roberts, Praschifka, Cahill, Ebert, Reinhardt, Volkov
Holdom and others

General approach For the flavor structure

Quark field $\rightarrow$ Background quark + (sea+mesons) quark field

Quark-antiquark mesons and sea quarks

Dressed by gluon cloud
Becomes
a constituent quark

Emergence
Of
Pion cloud

Auxiliary fields for all the
Dirac-flavor-color quark currents
Quark-antiquark mesons:
[color singlet ones
leading by $1/N_c$]

This improves usual
One loop background field method
By taking into account
Dynamical Chiral Symmetry Breaking and M^*
FLB,
EPJA (2016, 2018)+PRD (2018)

The NJL-limit
is also directly
obtained

Mechanism for masses considered here
Basically: Dynamical chiral symmetry breaking

Although it may involve other mechanism

Since

- coupling constants are assumed to be large
- Gluon with effective masses

Solution for scalar condensate,
This can lead to Schwinger Dyson eqs.

$$\frac{\partial S_{eff}}{\partial \phi_q} = 0.$$

Chiral scalar condensate = it corrects quark mass = DChSB

But it easily takes into account

Other effects such as trace anomaly

$$\tilde{M}^* = m + \langle S \rangle (P_R \tilde{U} + P_L \tilde{U}^\dagger)$$

Vacuum becomes infinitely degenerated

**Scalar field
is eliminated by chiral rotation,
With resulting
non linear pion dynamics**

Scalar mesons:
Unsolved problem
not considered
- exotic states?

Effective action and Large quark effective mass expansion

(large mass + derivative expansion at leading order)

Vector and axial mesons

$$S_{eff} = -i \text{Tr} \ln \left\{ i \left[S_0^{-1}(x-y) + \Xi_v(x-y) + \Xi_s(x-y) + \sum_q a_q \Gamma_q j_q(x,y) \right] \right\},$$

$$\tilde{S}_0^{-1}(x-y) = (i\not{D} - M^*) \delta(x-y).$$

This is a NJL-like quark propagator
The effect of running quark mass+Z(q)
Well known for results SD-Eqs (Bennich's talk)

All the constituent quark
Currents with Dirac/flavor
Structures (from Fierz)
And corresponding
Component of gluon kernel

Sum of (background)
Constituent quarks currents

Non linear realization Goldstone boson:
many possible pion field definitions

$$\Xi_s(x-y) = F[P_R(U - \tilde{U}) + P_L(U^\dagger - \tilde{U}^\dagger)] \delta(x-y),$$

Scheme for complete renormalization
(Yukawa model)
A.A.Nogueira+F.L.B.
2103.05133

In some versions:
Of constituent quark
model

Large Nc EFT

With a pion cloud

S.Weinberg, (2010)
to cope * CQM - Manohar-Georgi form
And * Large Nc- 't Hooft]

$$\begin{aligned}\mathcal{L}_{\text{eff}} = & -\frac{1}{4g^2}\text{Tr}\{F_{\mu\nu}F^{\mu\nu}\} - \frac{1}{g^2}[\bar{\psi}(D^\mu\gamma_\mu + m)\psi] \\ & - \frac{F_\pi^2}{2}\mathcal{D}_\mu\vec{\pi}\cdot\mathcal{D}^\mu\vec{\pi} - \frac{2ig_A}{g^2}(\bar{\psi}\gamma_5\gamma^\mu\vec{t}\psi)\cdot\mathcal{D}_\mu\vec{\pi} \\ & - c_1(\mathcal{D}_\mu\vec{\pi}\cdot\mathcal{D}^\mu\vec{\pi})^2 - c_2(\mathcal{D}_\mu\vec{\pi}\cdot\mathcal{D}_\nu\vec{\pi}) \\ & \times (\mathcal{D}^\mu\vec{\pi}\cdot\mathcal{D}^\nu\vec{\pi}).\end{aligned}$$

In the Weinberg's version of the
Large Nc EFT

There is only one mass term
But ChPT requires others

Also without Photon couplings

Constituent gluons coupling
+
Pion vector -coupling

Leading terms
chiral perturbation theory

This EFT (local pion field)
without and with electromagnetic couplings
+ chiral symmetry breaking terms:
F.L.B. Eur. Phys. Journ. (2018)
F.L.B. Eur. Phys. Journ. (2016)

Values for lec's obtained
Can be numerically nearly equal to
1-loop ChPT
(truncated quark propagator)

Leading pion (or pseudoscalars) couplings to Constituent quarks

F.L.B.
Phys. Rev. D 99, 014001 (2019).

Pion-quark

$$\text{where } Z_{\pm} = \frac{1}{2}(U \pm U^{\dagger})$$

Large quark masses
And
Very Long wavelength limit..

$$L_{q-\pi} = M_3 j_s + g_{2j_s} F \text{tr}_F Z_+ j_s + g_{ps} F \text{tr}_F (\sigma_i Z_-) j_{ps}^i$$

$$+ i2g_V \text{tr}_F (\sigma_i \partial_{\mu} Z_+) j_i^{V,\mu}$$

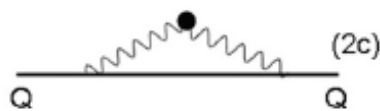
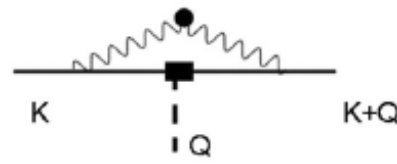
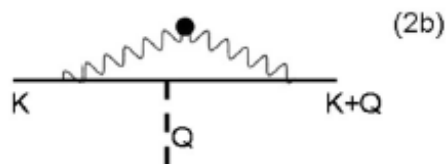
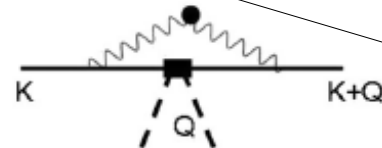
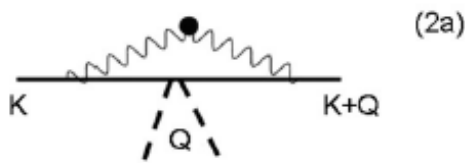
$$+ i2g_A \text{tr}_F (\sigma_i \partial_{\mu} Z_-) j_{A,\mu}^i,$$

$$- G_{ps}^p \frac{\partial_{\mu} \partial^{\mu} \pi^i(Q)}{M^*} \bar{\psi} i \gamma_5 \sigma^i \psi - G_s^p \frac{\partial_{\mu} \partial^{\mu} \pi^2(Q)}{M^*} \bar{\psi} \psi,$$

All types of pion couplings
emerge

→ This contributes for the
Vector current (induced V)

→ This Contributes
for the axial current
(induced PS)



Well known relations between the coupling constants

And other relations usually in good agreement with phenomenology

All ultraviolet finite !

(For UV and IR
regular gluon propagators)

Goldberger Treiman type
Relations
-quark level

$$\begin{array}{ll} i) \quad \frac{g_{ps}}{g_{2js}} = -1, & ii) \quad \frac{g'_A}{g'_V} = 1, \\ iii) \quad \frac{g_{ps}}{g'_A} \sim \frac{M^*}{F}, & iv) \quad \frac{g_{2js}}{g'_V} \sim -\frac{M^*}{F}. \end{array}$$

There is a very large amount of work done to calculate
Most of the nucleon form factors
Corresponding to these CQM form factors/couplings
= the shape of the resulting form factors is even similar to the
Experimental ones

However I'll not show here,
I would be unfair to cite few of them not the others, maybe I don't know all

For Numerical estimates

These effective propagators include a
Quark-gluon (running) coupling constant value (GAP eq. for DChSB)

So far: two
effective gluon propagators

$$D_I(k) = \frac{8\pi^2}{\omega^4} D e^{-k^2/\omega^2} + \frac{8\pi^2 \gamma_m E(k^2)}{\ln [\tau + (1 + k^2/\Lambda_{\text{QCD}}^2)^2]},$$

Propagator derived from Tandy- Maris
(transversal)

$$D_{II}(k) = \frac{K_F}{(k^2 + M_k^2)^2},$$

Cornwall = confining effective propagator
(longitudinal)

Ambiguity in defining the quark-gluon coupling strength
Renormalization scale
To use g_A to settle overall (re)normalization

How "good" is a constituent quark form factor In comparison with nucleon form factor?

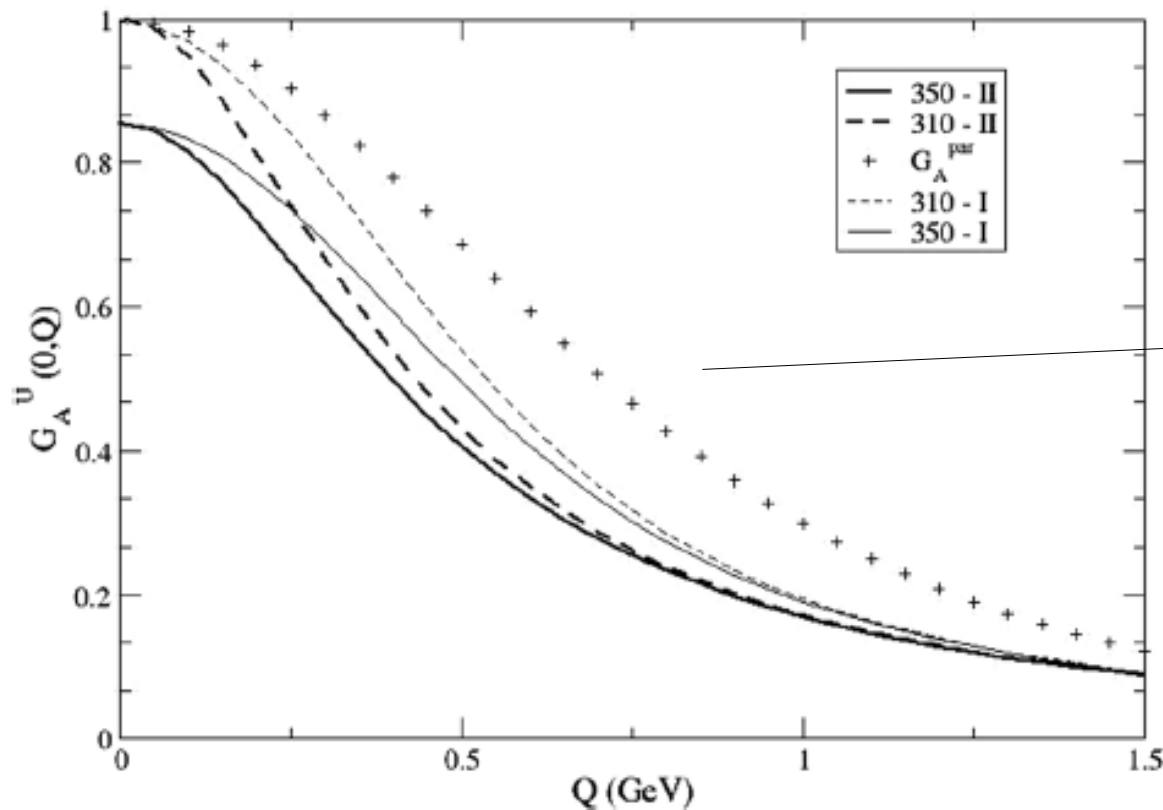
eg. Axial form factor

In such a comparison several aspects can be addressed

Nucleon observable as an average of constituent quark ones

Effect of SDE: running quark mass and $Z(q) \rightarrow$ reproduce experimental nucleon form factor

Form factors are quite similar: example the axial form factor (pion coupling)



Usual fitting of
NUCLEON
Axial form factor:

$$G_A^{\text{par}}(Q^2) = \frac{G_0}{\left(1 + \frac{Q^2}{M_A^2}\right)^2},$$

$$M_A = 1.1 \text{ GeV}$$

Const. Quark Axial radius

Assuming constituent quark masses are large enough as to justify a sort of “non-relativistic” 3-dim definition of the

constituent quark radii
and mesons radii

$$\sqrt{\langle r^2 \rangle_{CQ}} \simeq 0.2\text{--}0.3 \text{ fm} \mid$$

Petronzio et al (2003)
Vogl-Weise, (1991)

$$\langle r_{\text{a.m.}}^2 \rangle_A \sim 0.4\text{--}0.2 \text{ fm}^2,$$

$$\langle r^2 \rangle_A^U = -6 \frac{dG_A^U(0, Q)}{dQ^2} \Big|_{Q=0} = \langle r^2 \rangle_V^U,$$

$$\langle r^2 \rangle_{\text{ps}} = -6 \frac{dG_{\text{ps}}^U(0, Q)}{dQ^2} \Big|_{Q=0} = \langle r^2 \rangle_{2js},$$

Pion radii

$$\langle r^2 \rangle_{\pi}^s \sim 0.481\text{--}0.637 \text{ fm}^2,$$

$$\langle r^2 \rangle_{\pi}^V \sim 0.310\text{--}0.494 \text{ fm}^2.$$

S. Aoki et al, (2017) *Review of Lattice results concerning low-energy particle physics*, Eur. Phys. J. C **77**, 112.

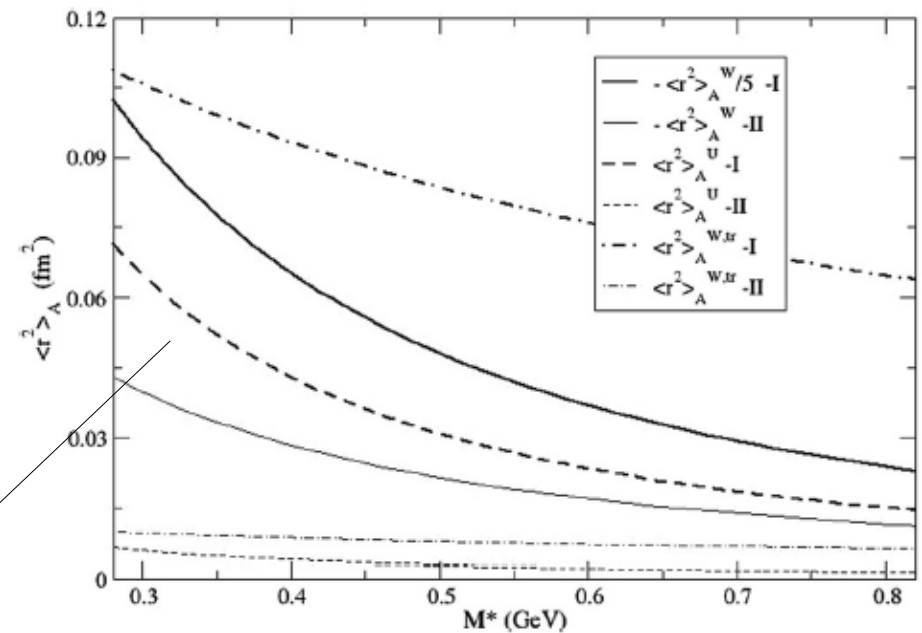
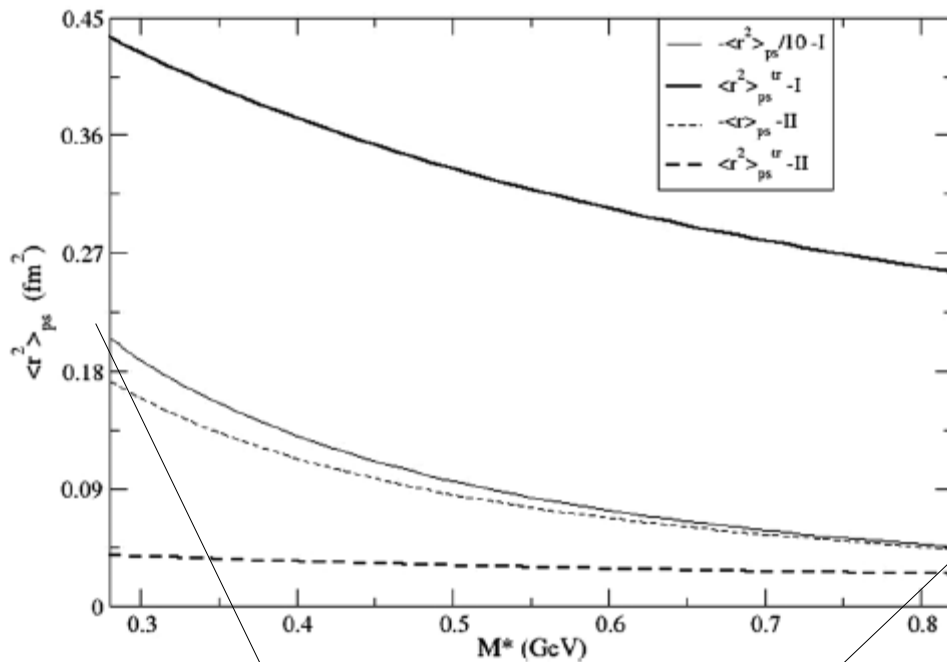
Averaged quadratic radii

$$\langle r^2 \rangle_A^U = -6 \frac{dG_A^U(0, Q)}{dQ^2} \Big|_{Q=0} = \langle r^2 \rangle_V^U,$$

$$\langle r^2 \rangle_{ps} = -6 \frac{dG_{ps}^U(0, Q)}{dQ^2} \Big|_{Q=0} = \langle r^2 \rangle_{2js},$$

At this level:

Chirally symmetric related
A-V
S-PS



$$\langle r^2 \rangle_{ps} \gg \langle r^2 \rangle_A^U$$

Emergence of the
Pion cloud for long range interactions
As compared to the axial/vector channels

Leading vector and axial mesons couplings to Constituent quarks

PHYS. REV. D 97, 054025 (2018)

Very Long wavelength limit..

$$\mathcal{L}_{v-q} = g_{r1} \left(V_i^\mu(x) j_\mu^{V,i}(x) + (1 + \delta_A) \bar{A}_i^\mu(x) j_\mu^{A,i}(x) \right) -$$

$$+ g_{v1} \left(V^\mu(x) j_\mu(x) + (1 + \delta_A) \bar{A}_\mu(x) j_A^\mu(x) \right)$$

Minimal couplings
Proposed by Sakurai (1960)

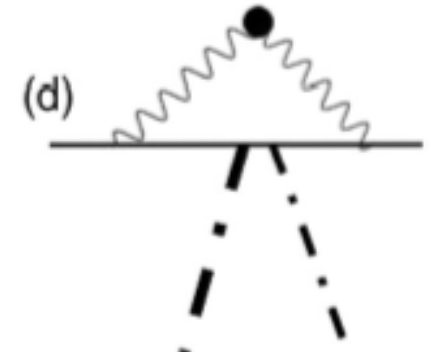
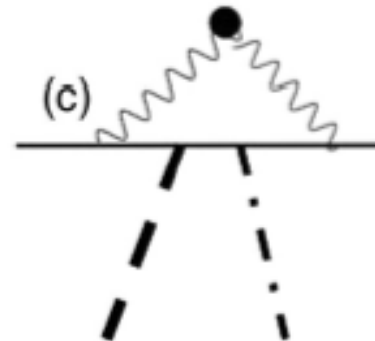
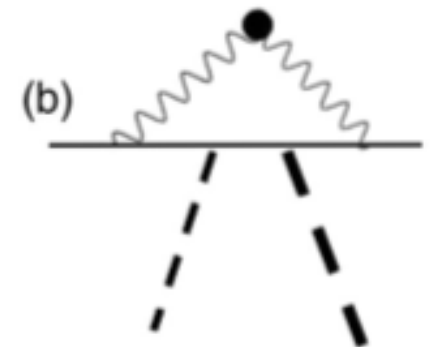
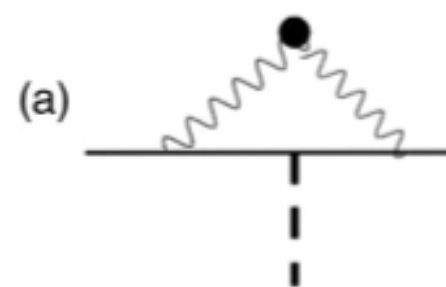
Leading
And
Next leading
couplings

$\rho(770) [h]$

$a_1(1260) [$

$\omega(782)$

$f_1(1285)$



Next leading couplings:

Suppressed by
 $1/M^*$ or $1/M^{*2}$

(that might not be good in higher energies)

VERY LARGE QUARK MASS LIMIT
 And longwavelength limit come together

At higher energies however
 (close to T_c)

M^* should decrease (restoration)
 Expansion $1/M^*$ may not be good

$$\begin{aligned}\mathcal{L}_{2v-j} = & g_{va-j} \left([V_\mu \bar{A}_i^\mu + V_\mu^i \bar{A}^\mu] j_{ps}^i + [V_\mu^i \bar{A}_i^\mu + V_\mu \bar{A}^\mu] j_{ps} \right) \\ & + g_{va-j} \left([(V_\mu^2 + (1 + \delta_2) \bar{A}_\mu^2) + (V_\mu^{i2} + (1 + \delta_2) (\bar{A}_i^\mu)^2)] j_s + [V^\mu V_\mu^i + (1 + \delta_A) \bar{A}_\mu \bar{A}_i^\mu] j_s^i \right) \\ & + g_{Fjs} \left[((\mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} + \mathcal{F}_{\mu\nu}^A \mathcal{F}_A^{\mu\nu}) + (\mathcal{F}_{\mu\nu}^i \mathcal{F}_i^{\mu\nu} + \mathcal{F}_{\mu\nu}^{A,i} \mathcal{F}_{A,i}^{\mu\nu})) j_s \right. \\ & + (\mathcal{F}_{\mu\nu}^i \mathcal{F}_A^{\mu\nu} + \mathcal{F}_{\mu\nu} \mathcal{F}_{A,i}^{\mu\nu}) j_p^i + (\mathcal{F}_{\mu\nu}^i \mathcal{F}^{\mu\nu} + \mathcal{F}_{\mu\nu}^{A,i} \mathcal{F}_A^{\mu\nu}) j_s^i + (\mathcal{F}_{\mu\nu} \mathcal{F}_A^{\mu\nu} + \mathcal{F}_{\mu\nu} \mathcal{F}_A^{\mu\nu}) j_p \left. \right] \\ & + g_{\epsilon v} i \epsilon_{ijk} (\mathcal{F}_{\mu\nu}^i V_j^\nu + \mathcal{F}_{\mu\nu}^{i,A} \bar{A}_j^\nu) j_{k,V}^\mu + g_{\epsilon v} i \epsilon_{ijk} (\mathcal{F}_{\mu\nu}^i \bar{A}_j^\nu + \mathcal{F}_{\mu\nu}^{i,A} \bar{V}_j^\nu) j_{k,A}^\mu\end{aligned}$$

$$\frac{g_{va-j}}{g_{r1}} \sim \frac{3}{2M^*}, \quad \frac{g_{Fjs}}{g_{r1}} \sim \frac{3}{2M^{*3}}.$$

$$\frac{g_{\epsilon v}}{g_{va-j}} = -\frac{1}{M^*}.$$

Several of these
 Interactions
 might be seen
 as mesons-mixings
 Induced by
 quark-currents

Relatively few coupling constants

Nearly along the UNIVERSALITY
 Hypothesis

$$\mathcal{F}_{\mu\nu}^i = \partial_\mu V_\nu^i - \partial_\nu V_\mu^i,$$

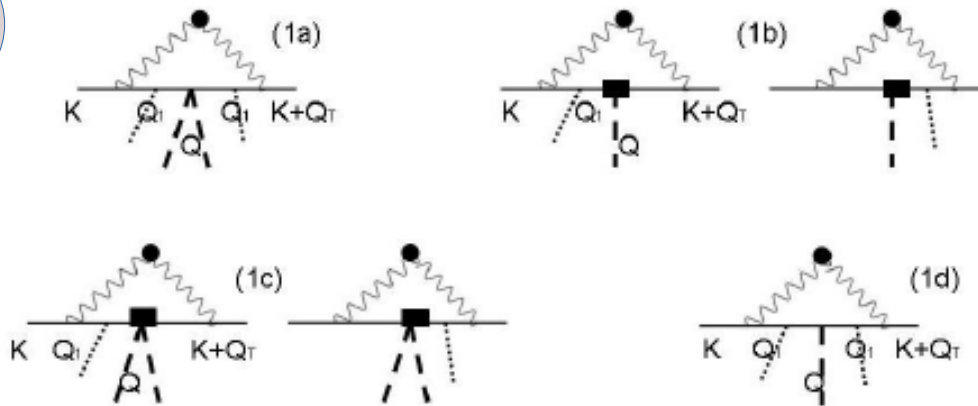
$$\mathcal{F}_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu,$$

$$\mathcal{G}_{\mu\nu}^i = \partial_\mu \bar{A}_\nu^i - \partial_\nu \bar{A}_\mu^i,$$

$$\mathcal{G}_{\mu\nu} = \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu.$$

Effect of "weak" magnetic field: $eB \ll M^{*2}$

1



The photon field interacting with hadrons (B is very strong-photon field)

And

2

Leading Landau orbits

$$G(k) = S_0(k) + S_1(k)(eB_0)$$

FLB + W.F.deSousa, Journ.of Phys. G (2020)

Poster by T.H.Moreira

About Strong B
For the NJL model

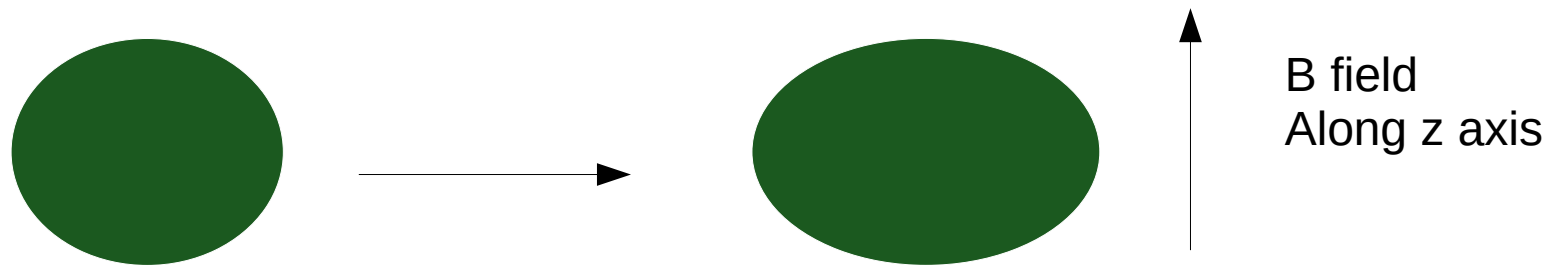
Corrections to radii due to weak magnetic fields
In the plane perpendicular to the
Magnetic field:

Isospin /Chiral symmetry breaking by electromagnetic coupling– leading terms

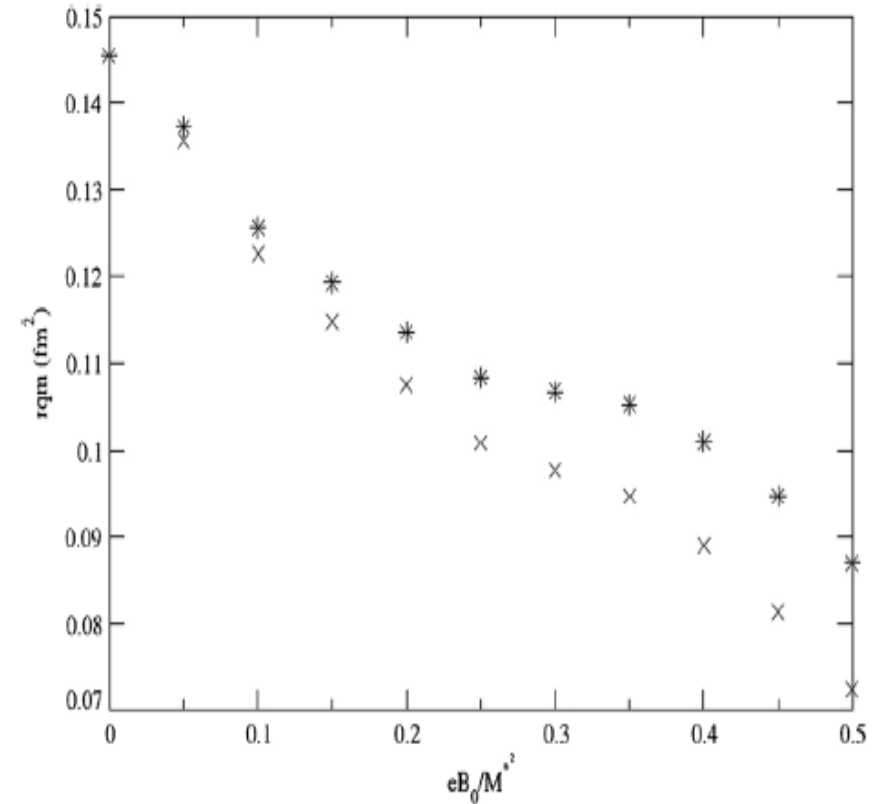
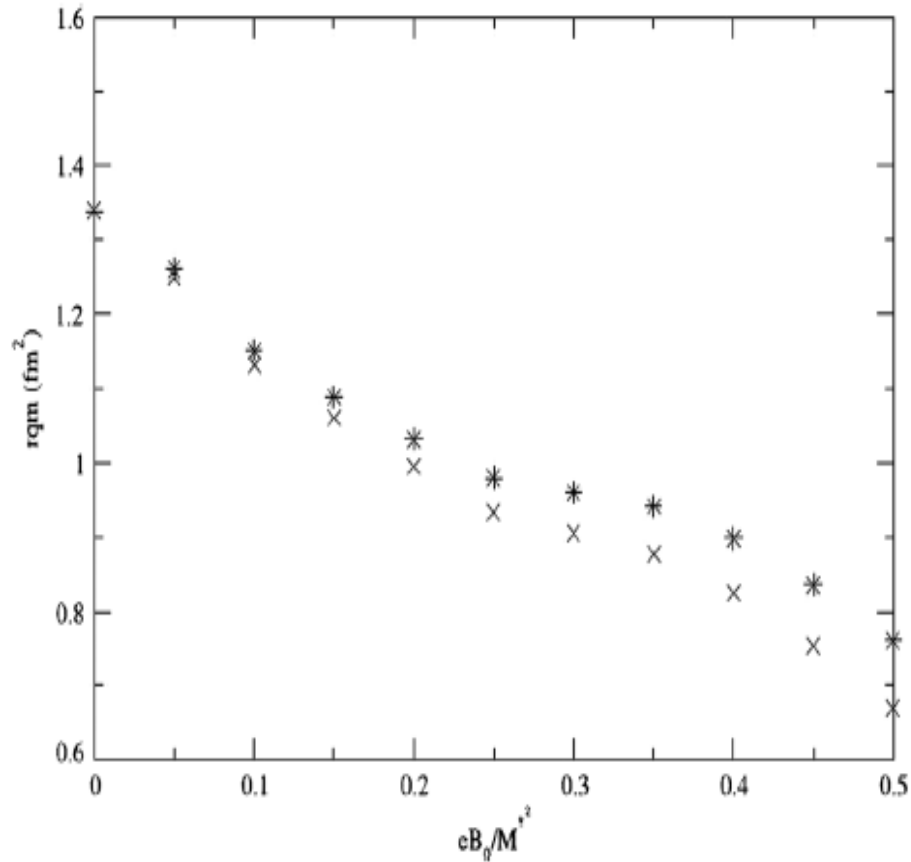
$$\Delta \langle r^2 \rangle_V^{\pi} \Big|_{x-y} = 3 \Delta \langle r^2 \rangle_A \Big|_{x-y},$$

$$\Delta \langle r^2 \rangle_s^{\pi} \Big|_{x-y} = \frac{5}{12} \Delta \langle r^2 \rangle_{ps} \Big|_{x-y}.$$

Deviations
Of the order Of $\left(\frac{eB_0}{M^{*2}}\right)^2$



Average quadratic radii (vector and axial mesons) dependence on (weak) magnetic field



PRD97 (2018)

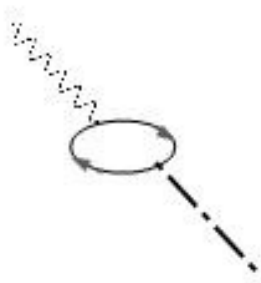
Mixing vector mesons (Or axial mesons) And Vector Meson Dominance (VMD) From effective (Lagrangian) couplings

Mixing

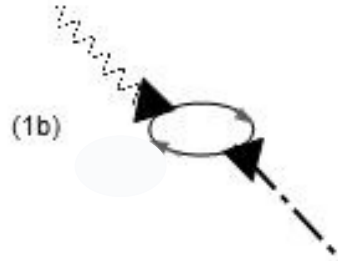
Coleman-Glashow (1964)
Goldhaber, Fox, Quigg, (1969)

$$\langle \rho^0 | H | \omega \rangle = -4520 \pm 600 \text{ MeV}^2.$$

(1a)



(1b)



Vector meson dominance:
Different MIXING couplings
between rho/omega and photon

$$\mathcal{L}_{\text{VMD}} = -g_{\rho A} V_3^\mu A_\mu - g_{\omega A} V_\mu A^\mu - g_{F\rho} \mathcal{F}_{\mu\nu}^3 F^{\mu\nu} - g_{F\omega} \mathcal{F}_{\mu\nu} F^{\mu\nu},$$

$$g_{\rho A} = 3g_{\omega A} :$$

$$g_{F\rho} = 3g_{F\omega} :$$

This procedure eliminates
photon mass

from expansion of determinant

And the non gauge invariant VMD couplings

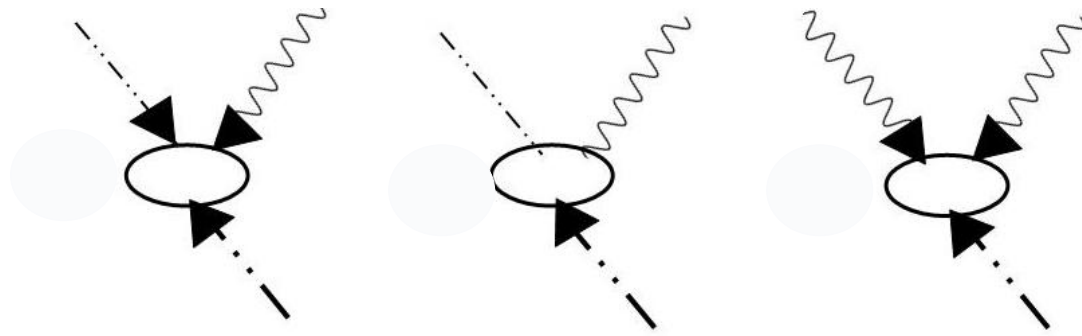
FLB, JPG (2020)

Kroll, B. Lee, Zumino

$$A_\mu = c\tilde{A}_\mu,$$

$$V_\mu^3 = \tilde{V}_\mu^3 + \frac{ce}{2f_v} \tilde{A}_\mu,$$

$$V_\mu = \tilde{V}_\mu + \frac{ce}{6f_v} \tilde{A}_\mu,$$



$$\mathcal{L}_F = g_{F\rho\omega}(F^{\mu\nu}\mathcal{F}_{\nu\rho}^3\mathcal{F}_\mu^\rho + F^{\mu\nu}\mathcal{G}_{\nu\rho}^3\mathcal{G}_\mu^\rho) - g_{FF\omega}F_{\mu\nu}F^{\nu\rho}\mathcal{F}_\rho^\mu - g_{FF\rho}F_{\mu\nu}F^{\nu\rho}\mathcal{F}_\rho^{3,\mu} \\ + \epsilon_{ij3} [g_{m1}F_{\mu\nu}V_i^\mu V_j^\nu + g_{m1A}F_{\mu\nu}\bar{A}_i^\mu \bar{A}_j^\nu] - \epsilon_{ij3} [g_{m1}\mathcal{F}_{\mu\nu}^i A^\mu V_j^\nu + g_{m1A}\mathcal{G}_{\mu\nu}^i A^\mu \bar{A}_j^\nu]$$

Mechanism for
Vector or axial
mesons mixings

Leading vector mesons and photon interactions

Yield both:
Vector mesons mixings and VMD

Induced by background photons

FLB, JPG (2020)

BCKF Photon interaction contributions to pion form factor

- No magnetic field in the e^+e^- reactions
- Background photons might induce rho-omega mixing

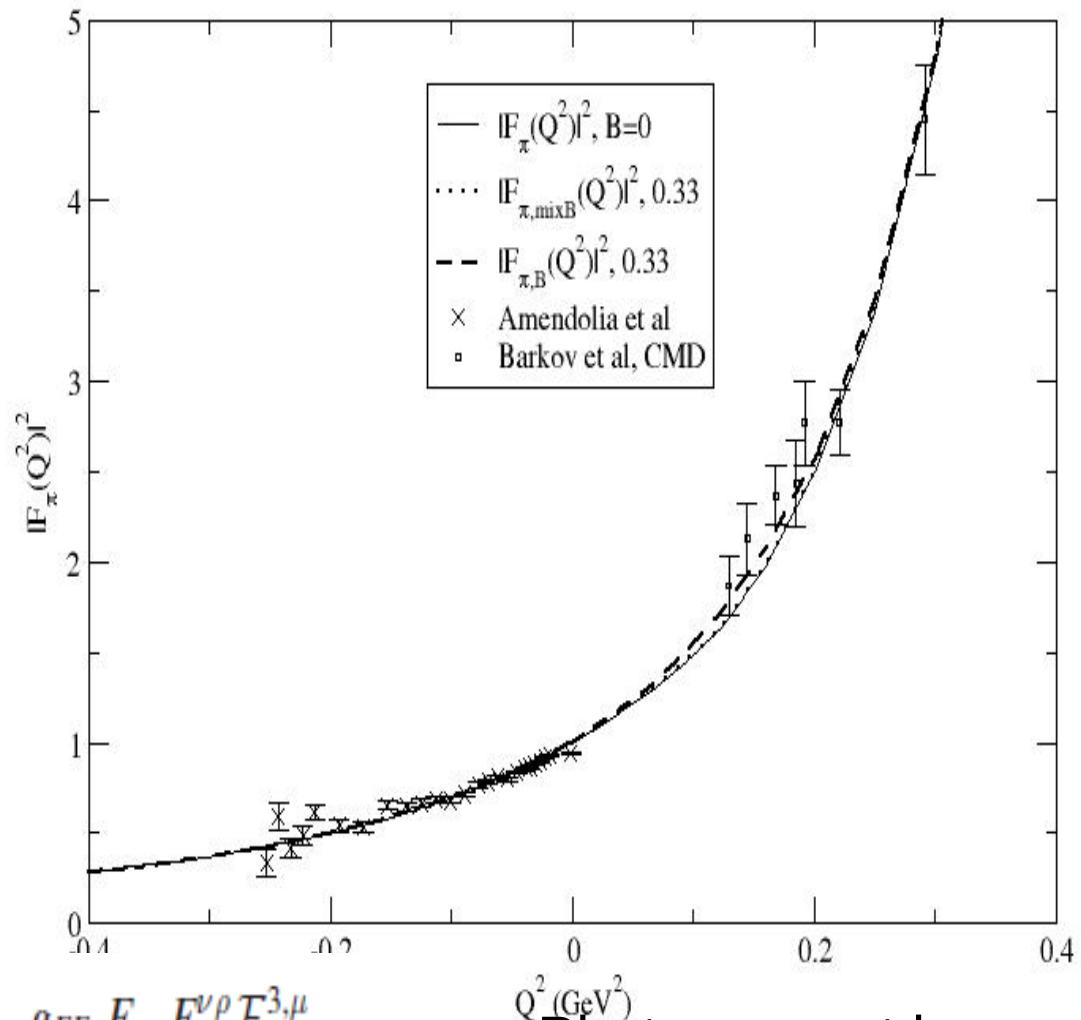
$$e^+e^- \rightarrow \pi^+\pi^-$$

mixing

VMD

$$\mathcal{L}_F = g_{F\rho\omega}(F^{\mu\nu}\mathcal{F}_{\nu\rho}^3\mathcal{F}_\mu^\rho + F^{\mu\nu}\mathcal{G}_{\nu\rho}^3\mathcal{G}_\mu^\rho) - g_{FF\omega}F_{\mu\nu}F^{\nu\rho}\mathcal{F}_\rho^\mu - g_{FF\rho}F_{\mu\nu}F^{\nu\rho}\mathcal{F}_\rho^{3,\mu}$$

$$\mathcal{L}_F^B = \left(\frac{eB_0}{M^{*2}}\right) g_{F\rho\omega}^B (\mathcal{F}_{\mu=2,\rho}^{i=3}\mathcal{F}_{\nu=1}^\rho + \mathcal{G}_{\mu=2,\rho}^{i=3}\mathcal{G}_{\nu=1}^\rho) - \left(\frac{eB_0}{M^{*2}}\right) g_{FF\rho}^B F^{\mu=2,\rho}\mathcal{F}_\rho^{i=3,\nu=1} - \left(\frac{eB_0}{M^{*2}}\right) g_{FF\omega}^B F^{\mu=2,\rho}\mathcal{F}_\rho^{\nu=1}$$



Photons must be extremely abundant..
To be tested:
Do onshell photons induce Vector mesons mixings?

Unusual ("anomalous"?)

Vector mesons couplings to the axial quark current

arXiv:2109.02203

K=quark incoming momentum

Q=meson incoming momentum

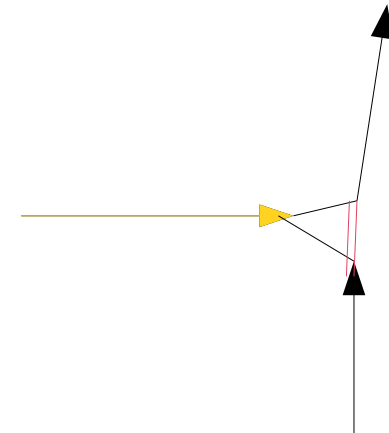
$$\mathcal{L}_{vja} = i\delta_{ij}\epsilon^{\sigma\rho\mu\nu}F^{vja}(K,Q)K_\sigma\mathcal{F}_{\rho\mu}^i(Q)j_\nu^{A,j}(K,K+Q) + i\epsilon^{\sigma\rho\mu\nu}F^{vja}(K,Q)K_\sigma\mathcal{F}_{\rho\mu}(Q)j_\nu^A(K,K+Q)$$

$$\mathcal{F}_{\rho\mu}^i(Q) = Q_\rho V_\mu^i(Q) - Q_\mu V_\rho^i(Q), \quad \mathcal{F}_{\rho\mu}(Q) = Q_\rho V_\mu(Q) - Q_\mu V_\rho(Q).$$

Small with respect to
The pion axial coupling

$$\frac{F_{vja}(K,Q)}{G_A(K,Q)} = \frac{1}{4M^*F}.$$

But with similar (equal)
Momentum dependence to the
constituent quark axial form factor



This is similar to Wess-
Zumino-Witten term
That might lead to
Quantization of currents

$$n\Gamma = -\frac{i}{240\pi^2} \int d^4K d^4Q \epsilon_{\sigma\rho\mu\nu}F^{vja}(K,Q)K_\sigma\mathcal{F}_{\rho\mu}^i(Q)j_\nu^{A,i}(K,K+Q),$$

Final remarks

The method has a starting point the QCD Lagrangian to obtain hadrons and their couplings
It yields many aspects of low energy (maybe intermediary energies) phenomenology
With good resulting numerical values for observables

To reproduce observables: corresponding SDE+BetheSalpeter-eqs must be supplemented
Testing Constituent quark model for global properties

It allows to consider further effects dynamically and systematically

It allows directly and naturally to compute in medium effects

On going:

-Several on going/planned directions, for example:

* Role of strong magnetic field AND chromo-electromagnetic fields in mesons and CQM
And for flavor $SU(N) \times SU(N)$ up to $N_f=5$ (beauty), with some improved methods

* To include other states from Fock space,

* to calculate mesons mixings (mechanism from 1loop flavor symmetry breaking)

- Anomalous (WZW) couplings and vector/axial currents

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Thank you for your attention!

#StaySafe