Connecting b→s anomalies with neutrino masses, dark matter and charged LFV

Jean Orloff Laboratoire de Physique de Clermont



Based on: JHEP 1811 (2018) 011 arXiv:1806.10146

In collaboration with G. Kumar, C. Hati, and A. M. Teixeira



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Outline

- Motivations and approach Beyond the Standard Model
- Neutrino mass models crash review: tree & loop
- Lepton Flavour Universality Violation (LFUV) in B decays?
- A minimal model connecting these with Dark Matter (DM)
- Parameters and pheno constraints:
 - * Neutrino masses
 - * DM relic density
 - * Charged Lepton Flavour Violation (cLFV)

Introduction

The Standard Model (SM): Highly successful but incomplete ...

Hundreds of theoretical models with various th./aesthetical motivations :

- Flavour puzzle
- Unification of interactions
- Hierarchy Problem
- Matter-antimatter asymmetry
- ▶ ???

Strategy:

- Start from solid BSM evidence: Neutrino Oscillations!!!
 - => neutrino masses
 - => New physics beyond the SM (SM neutrinos are strictly massless)
- ▶ If possible, help the Dark Matter (DM) problem
- Seek further guidance from (prelim.) experimental anomalies: B-decays, $(g-2)_{\mu}$, ...



Neutrino mass models

Neutrino Mass Models: Dirac mass term

Simplest implementation of observed neutrino kinematical mass:

$$\mathcal{L}_D = Y_{D,ij} ar{
u}_{R,i} \phi^\dagger L_j + h.c.$$

requires 3 new fields vR,i with no SM charge (?check?)

• However nothing, except (anomalous) *L*-number, then forbids $\mathcal{L}_{M} = M_{ij}\bar{\nu}_{R,i}^{c}\nu_{R,j} + h.c.$

lifting Dirac degeneracy: in terms of Majorana spinors $N = \nu_R + C \bar{\nu}_R^T$

$$\mathcal{L}_{M} + \mathcal{L}_{D} = \frac{1}{2} \begin{pmatrix} \overline{\nu} & \overline{N} \end{pmatrix} \begin{pmatrix} 0 & m_{D} \\ m_{D} & M \end{pmatrix} \begin{pmatrix} \nu \\ N \end{pmatrix}$$

▶ In the limit of large $M \approx MN$, see-saw formula :



Step back: without new fields, need effective dim. 5 operator

• Weinberg operator
$$\mathcal{L}_W = \frac{m_{\nu}}{v^2} (L^T i \sigma_2 \phi) C(\phi^T i \sigma_2 L)$$

3 possible **renormalisable** «blow-up» by **tree-level** $< \phi >$ single field exchange, giving $m_{\nu} \sim m_{new}^{-1(2)}$:



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Weinberg operator

 Type III: SU(2)-triplet fermion Σ (replace N by Σ, also Majorana)
 neutral component ~ Type I (+ extra cLFV)



 $<\phi>$

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- ▶ **Type II:** SU(2)-triplet scalar Δ





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In all cases, small m_v for $Y \sim O(1)$ require large, out of reach, m_{new}

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- N and S₂ are odd under a Z₂
 - * N is stable (=> DM candidate)
 - * forbids Dirac mass term (and type I seesaw)





Neutrino Mass Models : radiative = loop(s) inside Weinberg

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- Allows an interesting link between neutrino mass, and DM



 Φ/H

Η

 h^+

Η

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- Allows an interesting link between neutrino mass, and DM
- Somewhat artificial: why S1, S2? Why no seesaw?



 Φ/H

Η

 h^+

Η

Beyond the Standard Model: hints at LFUV?

 $\sim 2.5\sigma$ deviations from the SM Lepton Flavour Universality in B meson decays New physics ?



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« Too few muons » ?

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Phenomenological bottom-up approach to LFUV

Simplest working solutions for LFUV anomalies :

Scalar **leptoquark** triplet: (3, 3, 1/3); Vector leptoquark singlet: (3, 1, 2/3); ...

Our model: SM + 2 Scalar LQ + Triplet Majorana fermions

	Field	$SU(3)_C \times SU(2)_L \times U(1)_Y$	Z_2	
Fermions	$Q_L \equiv (u, d)_L^T$	(3 , 2 , 1/6)	1	
(3 gen.)	u_R	$(3,1,\ 2/3)$	1	
	d_R	(3 , 1 , -1/3)	1	
	$\ell_L \equiv (\nu, e)_L^T$	(1, 2, -1/2)	1	
	e_R	(1, 1, -1)	1	A rôle for each BSM field:
	Σ_R	(1, 3, 0)	-1	ightarrow contains DM candidate
Scalars	Н	(1, 2, 1/2)	1	
	h_1	(3 , 3 , -1/3)	1	ightarrow controls B-anomalies
	h_2	(3 , 3 , -1/3)	-1	\rightarrow allows KNT ν masses

 $Z_2\,$ symmetry makes lightest \varSigma_R stable, and forbids type III seesaw

 $\rightarrow \nu$ masses « feel » B anomalies

Couplings:

$$\mathcal{L} \supset \qquad + y_{ij} \bar{Q}_L^{C\,i} \epsilon(\vec{\tau}.\vec{h}_1) L_L^j + z_{ij} \bar{Q}_L^{C\,i} \epsilon(\vec{\tau}.\vec{h}_1)^{\dagger} Q_L^j + \underbrace{\tilde{y}_{ij}(\vec{\tau}.\vec{\Sigma})}_R^{C\,i,ab}[\epsilon(\vec{\tau}.\vec{h}_2)\epsilon^T]^{ba} d_R^j \\ - \frac{1}{2} \overline{\Sigma^C}^i M_{ij}^{\Sigma} \Sigma^j + \text{h.c.} - V(H, h_1, h_2) \supset \lambda_h (h_1.h_2)^2$$

incorrect (net needed)

$$m_{\nu}^{\text{diag}} = U^T y^T m_d \tilde{y}^T F(\lambda_h, m_{\Sigma}, m_{h_{1,2}}) \tilde{y} m_d y U$$

A parameterisation à la Casas-Ibarra

$$\left(\sqrt{m_{\nu}^{diag}}^{-1}U^T y^T m_d \tilde{y}^T \sqrt{F}\right)\left(\sqrt{F} \tilde{y} m_d y U \sqrt{m_{\nu}^{diag}}^{-1}\right) = \mathbb{1} = \mathcal{R}^T \mathcal{R}$$

$$\tilde{y} = F^{-1/2} \mathcal{R} \sqrt{m_{\nu}^{diag}} U^{\dagger} y^{-1} m_d^{-1}$$

 ${\cal R}$ is a complex orthogonal matrix determining ${ ilde y}$

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A viable dark matter candidate

Electroweak radiative corrections: $m(\Sigma \pm) - m(\Sigma 0) \sim 166$ MeV Cirelli, Fornengo, Strumia'06

$$Z_2$$
 symmetry $=> \Sigma_R$ is odd $=>$

 Σ^0 (the lightest "exotic" stable state) = dark matter candidate

Σ_R co-annihilate via gauge interactions

Neutral current anomalies: RK and RK*

$$\mathcal{H}_{\text{eff}} \supset -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=7,8,9,10,P,S,\dots} \left(C_i(\mu) \mathcal{O}_i + C_i'(\mu) \mathcal{O}_i' \right)$$

0.006 R_K and $R_{K^*}@1\sigma$, $M_{LQ}=1.5$ TeV Relevant operators for: $b \to s \ell^- \ell'^+$ 0.20 $\mathcal{O}_9^{\ell\ell'} = \frac{e^2}{(4\pi)^2} \left(\bar{s}\gamma^{\mu} P_L b\right) (\bar{\ell}\gamma_{\mu}\ell') \,,$ $y_{\rm be}y_{\rm se}=2\times10^{-5}$ 0.005 0.15 $\eta d\eta h$ $\mathcal{O}_{10}^{\ell\ell'} = \frac{e^2}{(4\pi)^2} \left(\bar{s}\gamma^{\mu} P_L b\right) (\bar{\ell}\gamma_{\mu}\gamma_5\ell')$ 0.004 $y_{\mathrm{b}\mu}y_{\mathrm{s}\mu}$ 0.05 0.00 0.10 0.05 0.15 0.20 $C_{9}^{\ell\ell'} = -C_{10}^{\ell\ell'}$ 0.003 $y_{s\mu}$ Taking the 1σ best fit to R_K and R_{K^*} data 0.002 $-1.4 \lesssim 2 \operatorname{Re}[C_{9.\mathrm{NP}}^{\mu\mu} - C_{9.\mathrm{NP}}^{ee}] \lesssim -0.8$ 0.001∟ 0 see e.g. Hiller, Nisandzic 2017, Capdevila et al. 2018 0.00002 0.00004 0.00006 0.00008 Hurth et al. 2016, Bečirević et al. 2015, ...

 $y_{\rm be}y_{\rm se}$

$$C_9^{\ell\ell'} = -C_{10}^{\ell\ell'} = \frac{\pi v^2}{\alpha_e V_{tb} V_{ts}^*} \frac{y_{b\ell'} y_{s\ell}^*}{m_{h_1}^2}$$

Je

Charged current anomalies: R_D and R_{D*}

$$\mathcal{H}_{\text{eff}}(d_k \to u_j \ell \bar{\nu}_i) = \frac{4G_F}{\sqrt{2}} V_{cb} \left[U_{\ell i} - \frac{v^2}{4V_{cb} m_{h_1}^2} (yU)_{ki} (Vy^*)_{j\ell} \right] \left(\bar{u}_j \gamma^\mu P_L d_k \right) \left(\bar{\ell} \gamma_\mu P_L \nu_i \right)$$

Using
$$x_{j\ell} = (v^2/4V_{cb}m_{h_1}^2) (Vy^*)_{j\ell}$$

If this signal is confirmed, this minimal model needs to be extended!

Eg: include additional leptoquark $R_2 = (3, 2, 7/6) / S_1 = (3, 1, 1/3)$? See for example: Becirevic et al. 18

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Crivellin et al. 17 14

Details: flavour structure of scalar triplet LQ

How to implement a flavour structure for y ?

$$y \sim \begin{pmatrix} \epsilon^{n_{11}} & \epsilon^{n_{12}} & \epsilon^{n_{13}} \\ \epsilon^{n_{21}} & \epsilon^{n_{22}} & \epsilon^{n_{23}} \\ \epsilon^{n_{31}} & \epsilon^{n_{32}} & \epsilon^{n_{33}} \end{pmatrix}$$

Hierarchy parameter ϵ inspired by Froggatt-Nielsen/flavour symmetry/...

To select a benchmark ϵ , we parametrise $R_{K(*)}$ data best fit value as $(m_{h1} \sim 1.5 \text{ TeV})$

 $\epsilon^4 \sim y_{22}y_{32} \sim 2.1555 \times 10^{-3} \quad n_{22} + n_{32} = 4 \implies \epsilon \sim 0.215 \quad \ \ \sim \sin \theta_C \simeq 0.22!$

Textures consistent with all the constraints from flavour violation:

CR ($\mu - e$, N), $K \rightarrow \pi \nu \bar{\nu}$ amongst the most stringent constraints	$ \begin{array}{c} \text{Texture type I} \\ \begin{pmatrix} \times & \times & \times \\ \times & \epsilon^3 & \times \\ \times & \epsilon & \times \end{pmatrix} \end{array} $	Texture type II $\begin{pmatrix} \times & \times & \times \\ \times & \epsilon^2 & \times \\ \times & \epsilon^2 & \times \end{pmatrix}$	Texture type III $ \begin{pmatrix} \times & \times & \times \\ \times & \epsilon & \times \\ \times & \epsilon^3 & \times \end{pmatrix} $
Generic allowed Textures	$ \begin{pmatrix} \epsilon^4 & \epsilon^{\geq 5} & \epsilon^{\geq 2} \\ \epsilon^{\geq 3} & \epsilon^3 & \epsilon^{\geq 4} \\ \epsilon^{\geq 4} & \epsilon & \epsilon^{\geq 1} \end{pmatrix} $	$ \begin{pmatrix} \epsilon^{6} & \epsilon^{\geq 4} & \epsilon^{\geq 3} \\ \epsilon^{\geq 5} & \epsilon^{2} & \epsilon^{\geq 3} \\ \epsilon^{\geq 3} & \epsilon^{2} & \epsilon^{\geq 1} \end{pmatrix} $	$ \begin{pmatrix} \epsilon^5 & \epsilon^{\geq 5} & \epsilon^{\geq 4} \\ \epsilon^4 & \epsilon & \epsilon^{\geq 2} \\ \epsilon^{\geq 4} & \epsilon^3 & \epsilon^{\geq 1} \end{pmatrix} $

Prospects for flavour violating (LFV) processes

Prospects for flavour violating (LFV) processes

Exciting possibilities to probe leptoquark coupling textures at experiments! Jean Orloff GdRv'19@CENBG

Connection between LFUV and charged LFV

The textures give direct correlations between LFUV data with charged LFV

Current upper bounds (solid colors) on Charged LFV processes translates into an upper bound on leptoquark masses within collider reach

Impose neutrino oscillation data

Texture type I

 $\begin{array}{c} \times & \times & \times \\ \times & \epsilon^3 & \times \\ & & \epsilon & \times \end{array}$

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$$\tilde{y} = F^{-1/2} \mathcal{R} \sqrt{m_{\nu}^{diag}} U^{\dagger} y^{-1} m_d^{-1}$$

 10^{-1} Three generations m_{Σ} : 2.5, 3.5, 4.5 TeV $m_{h_2} \sim 3 \text{ TeV}$ $m_{h_1} \sim 1.5 \text{ TeV}$ 10-11 Lightest neutrino mass 0.001 eV **CR(µ-e,Au)** 10⁻¹² Global best fit values for other oscillation parameters Scan for y, R consistent with 10^{-14} perturbativity: $\begin{array}{c} y, \tilde{y} \lesssim 4\pi \\ y, \tilde{y} \gtrsim 4\pi \end{array}$ 10⁻¹⁵ $1. \times 10^{-10}$ 2. $\times 10^{-10}$ $5. \times 10^{-10}$ $BR(K^+ \rightarrow \pi^+ v \overline{v})$ $1. \times 10^{-1}$ (IT (**he**, **Au**) 1.×10⁻¹. 1.×10-Texture type III $\begin{pmatrix} \times & \times & \times \\ \times & \epsilon & \times \\ \times & \epsilon^3 & \times \end{pmatrix}$ Texture type II 10^{-9} $1. \times 10^{-14}$ **Ruled out** 10-13 1. × 10⁻¹⁵ 1.5×10^{-10} 3 x 10 4×10^{-1} 10^{-1} $BR(K^+ \rightarrow \pi^+ v \overline{v})$ 10-11 10-10 10-6 10-9 10^{-5} $BR(K^+ \rightarrow \pi^+ v \overline{v})$ Jean Orloff GdRv'19@CENBG

Concluding Remarks

We considered a simple scalar leptoquark extension

- [SM + 2 Scalar LQ + Triplet Majorana fermion (3 gen)] allowing to:
- 1. Accommodate the latest data on neutrino oscillation parameters
- 2. Explain the $R_{K(*)}$ anomalies
- 3. Account for a correct relic abundance for dark matter
- 4. Consistent with the bounds on the leptoquark couplings from the relevant leptonic and semi-leptonic meson decays, neutral meson anti-meson oscillations, and CLFV processes
- 5. Exciting prospects for probing the model in future CLFV experiments:

 $^{*}\,\mu-e$ conversion in nuclei and radiative decays $\mu \to e\gamma,\,\tau \to \mu\gamma$, $e\gamma,\,\mu \to 3e$ and $\tau \to 3\mu$

• Open issues:

- * Consistent UV completion ?
- * Implementing a mechanism for baryogenesis
- * Computation of EDMs (two-loop)

Backup I: Full Lagrangian

 $\mathcal{L} = \mathcal{L}_{\text{int}}^{\text{SM}} + \mathcal{L}_{\text{int}}^{h,\Sigma} + \mathcal{L}_{\text{mass}}^{\Sigma} - V_{\text{scalar}}^{H,h}$ $\mathcal{L}_{\rm int}^{h,\Sigma} = y_{ij} \bar{Q}_L^{C\,i} \,\epsilon \,(\vec{\tau}.\vec{h}_1) \,L_L^j + z_{ij} \bar{Q}_L^{C\,i} \,\epsilon \,(\vec{\tau}.\vec{h}_1)^{\dagger} \,Q_L^j + \tilde{y}_{ij} \,\overline{(\vec{\tau}.\vec{\Sigma})}_R^{C\,i,ab} [\epsilon \,(\vec{\tau}.\vec{h}_2) \,\epsilon^T]^{ba} \,d_R^j + \text{H.c.}\,,$ $\mathcal{L}_{\text{mass}}^{\Sigma} = -\frac{1}{2} \overline{\Sigma^C}^i M_{ij}^{\Sigma} \Sigma^j.$ $V(H,h_1,h_2) = \mu_H^2 H^{\dagger} H + \frac{1}{2} \lambda_H |H^{\dagger} H|^2 + \mu_{h_1}^2 \operatorname{Tr}[h_1^{\dagger} h_1] + \mu_{h_2}^2 \operatorname{Tr}[h_2^{\dagger} h_2] + \mu_{h_2}^2 \operatorname{Tr}[h_2^2 h$ $+ \frac{1}{8} \lambda_{h_1} [\operatorname{Tr}(h_1^{\dagger} h_1)]^2 + \frac{1}{8} \lambda_{h_2} [\operatorname{Tr}(h_2^{\dagger} h_2)]^2 + \frac{1}{4} \lambda_{h_1}' \operatorname{Tr}[(h_1^{\dagger} h_1)]^2 + \frac{1}{4} \lambda_{h_2}' \operatorname{Tr}[(h_2^{\dagger} h_2)]^2$ + $\frac{1}{2}\lambda_{Hh_1}(H^{\dagger}H)\operatorname{Tr}[h_1^{\dagger}h_1] + \frac{1}{2}\lambda'_{Hh_1}\sum_{i=1}^{3}(H^{\dagger}\tau_i H)\operatorname{Tr}[h_1^{\dagger}\tau_i h_1] +$ + $\frac{1}{2}\lambda_{Hh_2}(H^{\dagger}H)\operatorname{Tr}[h_2^{\dagger}h_2] + \frac{1}{2}\lambda'_{Hh_2}\sum_{i=1}^{5}(H^{\dagger}\tau_i H)\operatorname{Tr}[h_2^{\dagger}\tau_i h_2] +$ + $\frac{1}{4}\lambda_h \operatorname{Tr}[h_1^{\dagger}h_2]^2 + \frac{1}{8}\lambda'_h [\operatorname{Tr}(h_1^{\dagger}h_2)]^2 + \frac{1}{4}\lambda''_h \operatorname{Tr}[h_1^{\dagger}h_1] \operatorname{Tr}[h_2^{\dagger}h_2] + \mathrm{H.c.}$ $1 ({}_{\mu}(1) ; {}_{\mu}(2)) = {}_{\mu}-2/3 = 1 ({}_{\mu}(1) ; {}_{\mu}(2)) = {}_{\mu}1/3 = {}_{\mu}(3) (i = 1, 2)$, 4/3

$$h_{j}^{-} = \frac{1}{\sqrt{2}} \left(h_{j}^{(-)} - i h_{j}^{(-)} \right), \quad h_{j}^{-} = \frac{1}{\sqrt{2}} \left(h_{j}^{(-)} + i h_{j}^{(-)} \right), \quad h_{j}^{-} = h_{j}^{(-)} \quad (j = 1, 2);$$

$$\Sigma^{+} = \frac{1}{\sqrt{2}} \left(\Sigma^{(1)} - i \Sigma^{(2)} \right), \quad \Sigma^{-} = \frac{1}{\sqrt{2}} \left(\Sigma^{(1)} + i \Sigma^{(2)} \right), \quad \Sigma^{0} = \Sigma^{(3)} \quad \text{(for the 3 generations)}.$$

Backup II: Relevant Lagrangian for neutrino masses

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Backup III: Dark matter co-annihilation channels

$$x_f = \ln \frac{0.038 \ g_{\text{eff}} \ m_{\text{Pl}} \ m_{\Sigma} \ \left\langle \sigma_{\text{eff}} | v | \right\rangle}{g_*^{1/2} x_f^{1/2}} \quad g_{\text{eff}} = g_0 + 2g_{\pm} \left(1 + \frac{\Delta_{m_{\Sigma}}}{m_{\Sigma}} \right)^{3/2} \exp\left(-\frac{\Delta_{m_{\Sigma}}}{m_{\Sigma}} x_f\right)$$

$$\Omega h^2 = \frac{1.07 \times 10^9 x_f}{g_*^{1/2} m_{\rm Pl}({\rm GeV}) I_a} \quad I_a = x_f \int_{x_f}^\infty x^{-2} a_{\rm eff} dx$$

The thermally averaged effective cross section $\langle \sigma_{\rm eff} | v | \rangle$ is given by

$$\begin{split} \langle \sigma_{\text{eff}} | v | \rangle &= \frac{g_0^2}{g_{\text{eff}}^2} \sigma(\Sigma^0 \Sigma^0) | v | + 4 \frac{g_0 g_{\pm}}{g_{\text{eff}}^2} \sigma(\Sigma^0 \Sigma^{\pm}) | v | \left(1 + \frac{\Delta_{m_{\Sigma}}}{m_{\Sigma}} \right)^{3/2} \exp\left(-\frac{\Delta_{m_{\Sigma}}}{m_{\Sigma}} x_f \right) \\ &+ \frac{g_{\pm}^2}{g_{\text{eff}}^2} [2\sigma(\Sigma^+ \Sigma^-) | v | + 2\sigma(\Sigma^{\pm} \Sigma^{\pm}) | v |] \left(1 + \frac{\Delta_{m_{\Sigma}}}{m_{\Sigma}} \right)^3 \exp\left(-2\frac{\Delta_{m_{\Sigma}}}{m_{\Sigma}} x_f \right). \\ \hline s\text{-channel} & t\text{-channel} \\ \Sigma^0 \Sigma^{\pm} \to W^{\pm} \to W^{\pm} W^0, W^{\pm} H, \bar{f} f \prime & \Sigma^0 \Sigma^0 \to W^+ W^- \quad \Sigma^0 \Sigma^{\pm} \to W^{\pm} W^0 \\ \Sigma^+ \Sigma^- \to W^0 \to W^+ W^-, W^0 H, \bar{f} f & \Sigma^{\pm} \Sigma^{\pm} \to W^{\pm} W^{\pm} \\ \Sigma^+ \Sigma^- \to W^0 W^0 (W^+ W^-) \end{split}$$

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Backup IV:Some important constraints from the mesonic observables

Observables	SM prediction	Experimental data	
$DD(V^{+} \rightarrow \pi^{+} u\bar{u})$	$(9.4 \pm 1.0) \times 10^{-11}$ (Purea et al.)	$17.3^{+11.5}_{-10.5} \times 10^{-11}$ (E949)	
$DR(K^+ \to \pi^+ \nu \nu)$	$(8.4 \pm 1.0) \times 10^{-11}$ (Buras et al.)	$< 11 \times 10^{-10}$ (Na62)	
$\mathrm{BR}(K_L \to \pi^0 \nu \bar{\nu})$	$(3.4 \pm 0.6) \times 10^{-11}$ (Buras et al.)	$\leq 2.6 \times 10^{-8}$ (E391a)	
$R_K^{\nu\nu}, R_{K^*}^{\nu\nu}$	$B^{\nu\nu} = 1$	$R_K^{\nu\nu} < 3.9$ (Belle)	
$(B \to K^{(*)} \nu \bar{\nu})$	$n_{K^{(*)}} - 1$	$R_{K^*}^{\nu\nu} < 2.7$ (Belle)	
$B_s^0 - \bar{B}_s^0$	$\Delta_s = \Delta_s e^{i\phi_s} = 1$	$ \Delta_s = 1.01^{+0.17}_{-0.10}$ (CKMfitter),	
(mixing parameters)	$\phi_s = 0$	$\phi_s[^\circ] = 1.3^{+2.3}_{-2.3}$ (CKMfitter)	
$K^0 - \bar{K}^0$			
$\Delta m_K/(10^{-15}{ m GeV})$	3.1(1.2) (Brod et al.)	3.484(6) (PDG)	
$BR(K_L \to \mu e)$		$< 4.7 \times 10^{-12}$ (PDG)	
$BR(B_s \to \mu e)$		$< 1.1 \times 10^{-8}$ (PDG)	

Backup V: LFV: current limits and future sensitivities

cLFV process	Current experimental bound	Future sensitivity	$\left. \right] \left[\begin{array}{c} d(u) \\ \bullet \end{array} \right] \left[\begin{array}{c} \ell \\ \bullet \end{array} \right] \left[\begin{array}{c} d(u) \\ \bullet \end{array} \right] \left[\begin{array}{c} \ell \\ \bullet \end{array} \right] \left[\begin{array}{c} d(u) \\ \bullet \end{array} \right] \left[\begin{array}{c} \ell \\ \bullet \end{array} \right] \left[\begin{array}{c} d(u) \\ \\ \end{array} \right] \left[\begin{array}{c} d(u) \\ \end{array} \\ \\ \\ \left[\begin{array}{c} d(u) \end{array} \right] \left[\begin{array}{c} d(u) \\ \end{array} \\ \\ \\ \\ \left[\begin{array}{c} d(u) \end{array} \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \left[\begin{array}{c} d(u) \\ \end{array} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \\ \\ \\ \\ \\ \\ $
$BR(\mu \to e\gamma)$	$4.2 \times 10^{-13} \text{ (MEG)}$	$6 \times 10^{-14} \text{ (MEG II)}$	
$BR(\tau \to e\gamma)$	$3.3 \times 10^{-8} (BaBar)$	10^{-9} (Super B)	3 (1/3
$BR(\tau \to \mu \gamma)$	$4.4 \times 10^{-8} (BaBar)$	10^{-9} (Super B)	
$\Box BR(\mu \to 3e)$	1.0×10^{-12} (SINDRUM)	$10^{-15(-16)}$ (Mu3e)	ℓ' $d'(u')$
$BR(\tau \to 3e)$	2.7×10^{-8} (Belle)	10^{-9} (Super B)	$\mu \rightarrow e$ Conversion
$BR(\tau \to 3\mu)$	3.3×10^{-8} (Belle)	10^{-9} (Super B)	
$\boxed{\mathrm{CR}(\mu - e, \mathrm{N})}$	7×10^{-13} (Au, SINDRUM)	10^{-14} (SiC, DeeMe)	
		$10^{-15(-17)}$ (Al, COMET)	
		3×10^{-17} (Al, Mu2e)	
		10^{-18} (Ti, PRISM/PRIME)	