# *Theory of Lepton Flavour and CP*

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$CP^3$-Origins, University of Southern Denmark

GDR Neutrino Meeting,

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# ***Outline***

- ways to correlate leptonic mixing angles and CP phases
- possibilities to fix neutrino masses in addition
- options to relate leptonic CP violation to the quark CP phase

# Experimental data

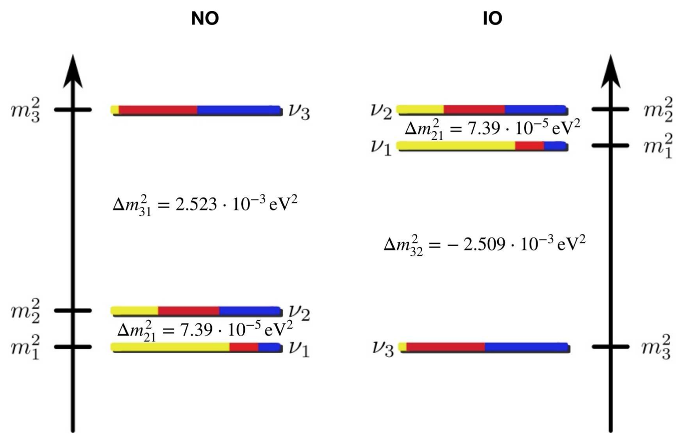
Summary of current knowledge about lepton mixing *(NuFIT ('19))*

$$||U_{\text{PMNS}}|| \approx \begin{pmatrix} 0.82 & 0.55 & 0.15 \\ 0.31 & 0.60 & 0.74 \\ 0.48 & 0.58 & 0.66 \end{pmatrix} \quad \text{[NO]}$$

and hint for CP violation: $\delta \approx 222^\circ$ , $\alpha =$ ? , $\beta =$ ?

# Experimental data

Summary of current knowledge about neutrino masses

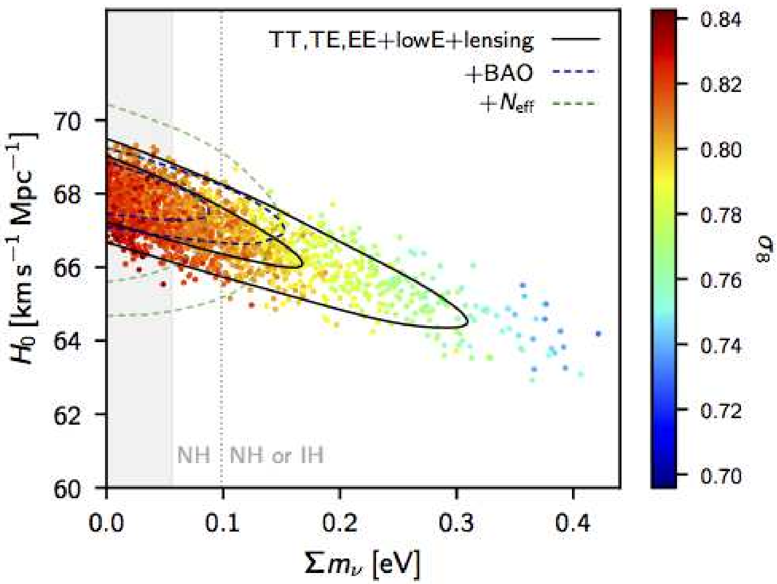



Their ordering is unknown, although NO seems preferred.

*(NuFIT ('19))*

Their absolute scale is also unknown.

*(Planck ('18))*



# *Lepton mixing*

Examples:

- flavour symmetry $G_f$ and its breaking as explanation
- flavour symmetry $G_f$ and CP and its breaking as explanation

# Lepton mixing

Flavour symmetry $G_f$ and its breaking as explanation

*(Lam ('07,'08), Blum/H/Lindner ('07))*

$$G_f$$

$$\swarrow \qquad\qquad \searrow$$

charged leptons $\qquad\qquad$ neutrinos

$G_e$ $\qquad\qquad$ $G_\nu$

$U_e$ $\qquad\qquad$ $U_\nu$

$$\searrow \qquad\qquad \swarrow$$

$$U_{\text{PMNS}} = U_e^\dagger U_\nu$$

Note: Masses do not play a role in this approach.

# *Lepton mixing*

You expect to fix

- the 3 lepton mixing angles
- the Dirac phase $\delta$

up to permutations of rows and columns of the PMNS mixing matrix, since masses are not fixed.

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Example for $G_f$ for $\theta_{13} \neq 0$, $\theta_{23} \neq \frac{\pi}{4}$

***(de Adelhart Toorop/Feruglio/H ('11))***

- $G_f = \Delta(384)$
- $G_e = Z_3$
- $G_\nu = Z_2 \times Z_2$

# *Lepton mixing*

***(de Adelhart Toorop/Feruglio/H ('11))***

$$||U_{\text{PMNS}}|| \approx \begin{pmatrix} 0.81 & 0.58 & 0.11 \\ 0.31 & 0.58 & 0.75 \\ 0.50 & 0.58 & 0.65 \end{pmatrix}$$

and $\sin\delta = 0$

This result is generic.

Non-trivial values of $\delta$

from combination of flavour and CP symmetries.

With less stringent assumptions on flavour (and residual) symmetries non-trivial CP violation can also be achieved.
(see e.g. ***Hernandez/Smirnov ('12)***)

# Lepton mixing

Flavour symmetry $G_f$ and CP and its breaking as explanation

*(Feruglio/H/Ziegler ('12))*

$G_f$ and CP

$\swarrow \qquad\qquad \searrow$

charged leptons | neutrinos

$G_e$ | $G_\nu = Z_2 \times$ CP

$U_e$ | $U_\nu = \Omega_\nu R(\theta) K_\nu$

$\searrow \qquad\qquad \swarrow$

$$U_{\text{PMNS}} = U_e^\dagger \Omega_\nu R(\theta) K_\nu$$

Note: Masses do not play a role in this approach.

# *Lepton mixing*

You expect to express

- the 3 lepton mixing angles
- all CP phases $\delta$, $\alpha$ and $\beta$

in terms of one single real parameter $\theta$ and up to permutations of rows and columns of the PMNS mixing matrix, since one has one $Z_2$ only and masses are not fixed.

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Example: study of series of $G_f$ with CP

***(H/Meroni/Molinaro ('14))***

- $G_f = \Delta(3\,n^2), \Delta(6\,n^2)$ and CP
- $G_e = Z_3$
- $G_\nu = Z_2 \times$ CP

4 different types of mixing patterns with different characteristics

# *Lepton mixing*

*(H/Meroni/Molinaro ('14))*

- fix $\theta$ and choice of $Z_2$ to accommodate lepton mixing angles
- large $\delta$ follows

$$|\sin\delta| \gtrsim 0.71$$

- Majorana phases $\alpha$, $\beta$ depend on CP symmetry $X(s)$ only

$$|\sin\alpha| = |\sin\beta| = |\sin 6\,\phi_s| \quad \text{with} \quad \phi_s = \frac{\pi s}{n} \quad \text{and} \quad s = 0, ..., n-1$$

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Correlations among lepton mixing angles are

$$\sin^2\theta_{12} = \frac{1}{3}\left(\frac{1 - 3\,\sin^2\theta_{13}}{1 - \sin^2\theta_{13}}\right)$$

and

$$\sin^2\theta_{23} \approx \frac{1}{2} - \sqrt{2}\,\cos\left(\frac{3\,\pi\,s}{8}\right)\,\sin\theta_{13}$$

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There are also sum rules for the CP phases

$$\left(1 - 2\,\sin^2\theta_{23}\right)^2 \approx 8\,\sin^2\theta_{13}\,\cos^2\delta$$

and

$$|\sin\alpha| = |\sin\beta| \approx \left|\sin\delta\,\left(\frac{1 - 2\,\sin^2\theta_{23}}{\sqrt{2}\,\sin\theta_{13}}\right)\right|$$

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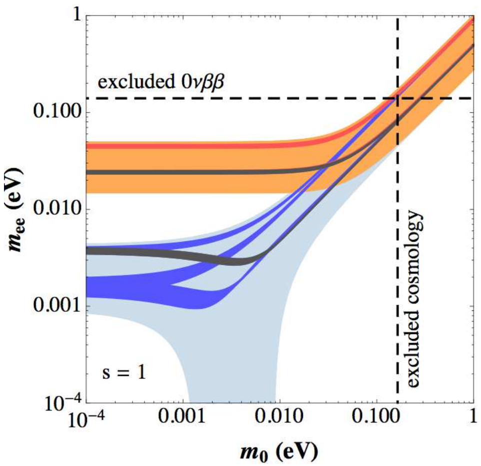
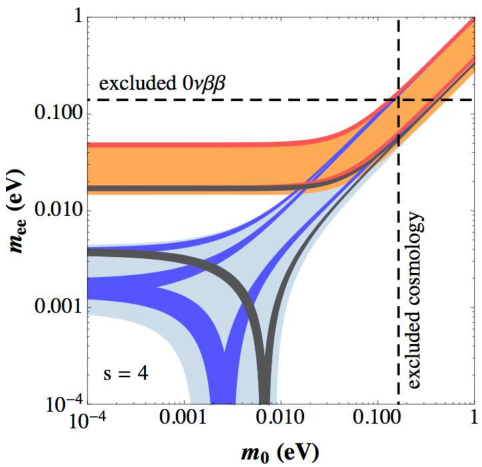
for $n$=8:

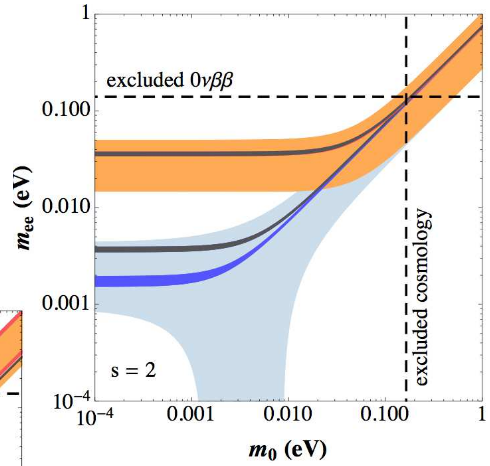
| $s$ | $\sin^2\theta_{13}$ | $\sin^2\theta_{12}$ | $\sin^2\theta_{23}$ | $\sin\delta$ | $\sin\alpha = \sin\beta$ |
|---|---|---|---|---|---|
| $s=1$ | $0.0220$ | $0.318$ | $0.579$ | $0.936$ | $-1/\sqrt{2}$ |
| | $0.0220$ | $0.318$ | $0.421$ | $-0.936$ | $-1/\sqrt{2}$ |
| $s=2$ | $0.0216$ | $0.319$ | $0.645$ | $-0.739$ | $1$ |
| $s=4$ | $0.0220$ | $0.318$ | $0.5$ | $\mp 1$ | $0$ |

# *Lepton mixing*

Results for neutrinoless double beta decay ***(H/Molinaro ('16))***



# *Beyond lepton mixing*

## Unflavoured leptogenesis

***(H/Molinaro ('16))***

- consider scenario with flavour and CP symmetry and 3 RH neutrinos $N_i$ forming $\mathbf{3}$
- baryon asymmetry of the Universe

$$Y_B = (8.65 \pm 0.09) \times 10^{-11}$$ ***(Planck ('15))***

is generated through unflavoured leptogenesis

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- $Y_B$ is generated through unflavoured leptogenesis
- we assume

$$-Y_D\, \bar{l}\, H^c N - \frac{1}{2}\overline{N^c} M_R N$$

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$$-Y_D \, \bar{l} \, H^c N - \frac{1}{2} \overline{N^c} M_R N$$

  - Dirac Yukawa coupling $Y_D$ invariant under flavour and CP
  - RH neutrino mass matrix $M_R$ invariant under residual symmetry $G_\nu$

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Why?

- otherwise no CP asymmetry is achieved
- always arise in explicit models

# Beyond lepton mixing

## Unflavoured leptogenesis

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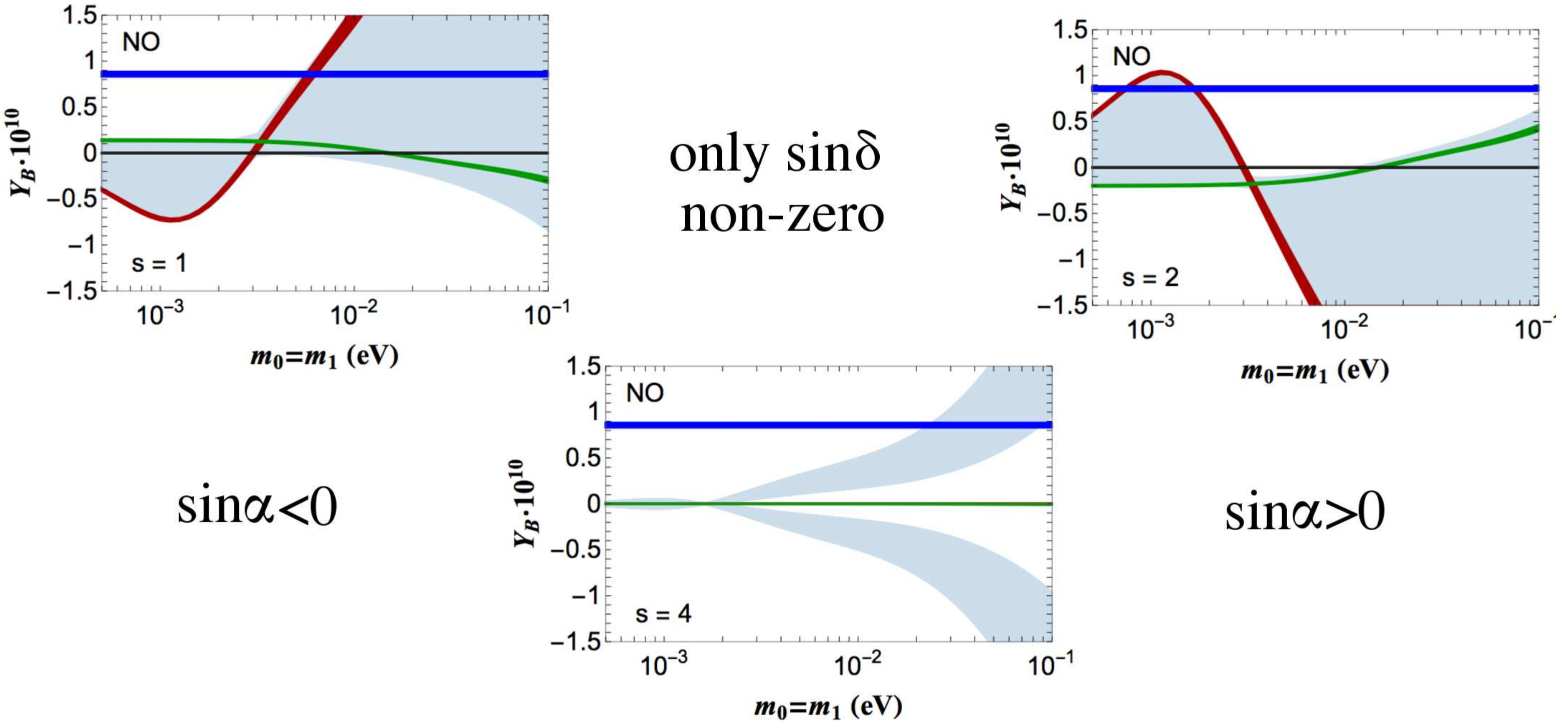


For further works on flavour and CP symmetries and leptogenesis see e.g. ***Mohapatra/Nishi ('15), Chen/Ding/King ('16)***.

# *Neutrino masses and lepton mixing*

Examples:

- flavour symmetry $G_f$ (and CP) and its breaking
- modular invariance

# *Neutrino masses and lepton mixing*

Use flavour symmetry $G_f$ (and CP) and its breaking to also constrain neutrino mass spectrum,
either one massless neutrino ***(Joshipura/Patel ('13,'14))***
or a pair of degenerate neutrinos ***(Joshipura/Patel ('14,'15,'18))***

Assume in the following neutrinos are Majorana particles.

# Neutrino masses and lepton mixing

*(Joshipura/Patel ('13,'14))*

One massless neutrino

- requires that residual symmetry for neutrinos is larger than $Z_2$
- leads thus to $G_f$ being a subgroup of $U(3)$ rather than $SU(3)$
- is, however, difficult to reconcile with acceptable lepton mixing

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I.e. only for

- $G_f = [[432, 239]]$
- $G_e = Z_4$
- $G_\nu = Z_4$

a mixing pattern close to experimental data follows.

# *Neutrino masses and lepton mixing*

***(Joshipura/Patel ('13,'14))***

The mixing pattern is

$$||U_{\text{PMNS}}|| \approx \begin{pmatrix} 0.77 & 0.61 & 0.18 \\ 0.5 & 0.40 & 0.77 \\ 0.40 & 0.68 & 0.61 \end{pmatrix}$$

and $\delta$ is trivial.

Neutrinos are normally ordered with $m_1 = 0$.

Degenerate neutrinos always require some correction.

# *Neutrino masses and lepton mixing*

Use modular invariance to fix neutrino masses and lepton mixing

***(Feruglio ('17), Criado/Feruglio ('18))***

- consider globally supersymmetric theory
- impose modular invariance on this theory, meaning
  - the modulus $\tau$ transforms as

$$\tau \quad \rightarrow \quad \gamma\,\tau = \frac{a\,\tau + b}{c\,\tau + d} \quad \text{with} \quad ad - bc = 1$$

  - chiral superfields $\phi^{(I)}$ transform as

$$\phi^{(I)} \quad \rightarrow \quad (c\,\tau + d)^{k_I}\,\rho^{(I)}(\gamma)\,\phi^{(I)}$$

with $k_I$ being the weight, $\rho^{(I)}$ the representation of $\Gamma_N = \Gamma/\Gamma(N)$

# *Neutrino masses and lepton mixing*

***(Feruglio ('17), Criado/Feruglio ('18))***

- the superpotential has weight zero and is invariant under $\Gamma_N$
- Yukawa couplings must be modular forms
  (in a representation $\rho_f$, with certain (even) weight $k_f$
  and of level $N$)

$$f_i(\gamma\,\tau) = (c\,\tau + d)^{k_f}\;\rho_f(\gamma)_{ij} f_j(\tau)$$

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Choose

- $N$, e.g. $N = 3$, meaning $\Gamma_3 \simeq A_4$
- representations $\rho^{(I)}$ of different chiral superfields
- additional fields (flavons), if needed
- weights $k_I$ of different chiral superfields

# *Neutrino masses and lepton mixing*

*(Feruglio ('17), Criado/Feruglio ('18))*

Example:

- charged leptons: $L \sim (\mathbf{3}, -1)$, $e_i^c \sim ([\mathbf{1}, \mathbf{1}'', \mathbf{1}'], -2)$, $h_d \sim (\mathbf{1}, 0)$
- flavon $\varphi \sim (\mathbf{3}, 3)$ needed for charged lepton masses
- postulate: $\langle \varphi \rangle \propto (1, 0, \epsilon)$

$$M_e \propto \begin{pmatrix} a & 0 & c\,\epsilon \\ a\,\epsilon & b & 0 \\ 0 & b\,\epsilon & c \end{pmatrix}$$

# Neutrino masses and lepton mixing

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- flavon $\varphi \sim (\mathbf{3}, 3)$ needed for charged lepton masses
- postulate: $\langle\varphi\rangle \propto (1, 0, \epsilon)$
- results: charged lepton masses adjusted by 3 couplings,
  small contribution to $U_{\text{PMNS}}$

# Neutrino masses and lepton mixing

(Feruglio ('17), Criado/Feruglio ('18))

Example:

- neutrinos get mass from Weinberg operator
- neutrinos: $L \sim (\mathbf{3}, -1)$, $h_u \sim (\mathbf{1}, 0)$
- couplings of Weinberg operator are modular forms with weight 2 and of level 3,
  i.e. 3 holomorphic functions $Y_i$ of $\tau$ that fulfil constraint $Y_2(\tau)^2 + 2\,Y_1(\tau)\,Y_3(\tau) = 0$

$$M_\nu \propto \begin{pmatrix} 2\,Y_1(\tau) & -Y_3(\tau) & -Y_2(\tau) \\ -Y_3(\tau) & 2\,Y_2(\tau) & -Y_1(\tau) \\ -Y_2(\tau) & -Y_1(\tau) & 2\,Y_3(\tau) \end{pmatrix}$$

with e.g. $Y_1(\tau) = 1 + 12\,q + 36\,q^2 ...$ and $q = e^{i\,2\,\pi\,\tau}$.

# *Neutrino masses and lepton mixing*

*(Feruglio ('17), Criado/Feruglio ('18))*

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  i.e. 3 holomorphic functions $Y_i$ of $\tau$ that fulfil constraint
  $Y_2(\tau)^2 + 2\,Y_1(\tau)\,Y_3(\tau) = 0$
- postulate: $\langle\tau\rangle \approx 0.0117 + i\,0.9948$
- results: neutrino masses are inversely ordered,
  all mixing angles are in agreement with data,
  $\delta$ as well and Majorana phases are also fixed

# *Neutrino masses and lepton mixing*

***(Feruglio ('17), Criado/Feruglio ('18))***

Example:

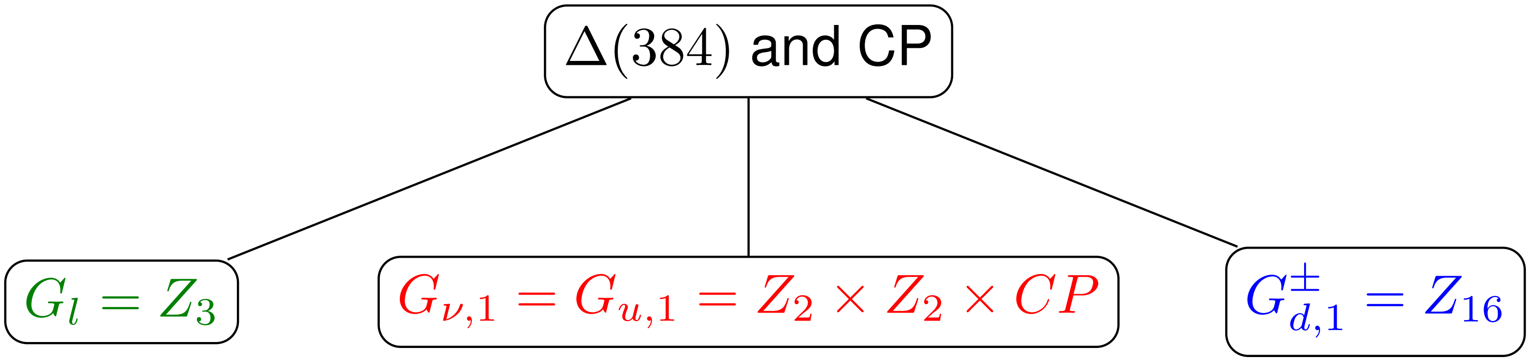
| | best value |
|---|---|
| $r \equiv \lvert\Delta m^2_{sol}/\Delta m^2_{atm}\rvert$ | $0.0302(11)$ |
| $m_3/m_2$ | $0.0150(5)$ |
| $\sin^2\theta_{12}$ | $0.304(17)$ |
| $\sin^2\theta_{13}$ | $0.0217(8)$ |
| $\sin^2\theta_{23}$ | $0.577(4)$ |
| $\delta/\pi$ | $1.529(3)$ |
| $\alpha_{21}/\pi$ | $0.135(6)$ |
| $\alpha_{31}/\pi$ | $1.728(18)$ |

Modular symmetries have been combined with CP ***(Novichkov et al. ('19))***.

# Connections to quark sector

Same flavour symmetry $G_f$ and CP for leptons and quarks

***(H/König ('18))***



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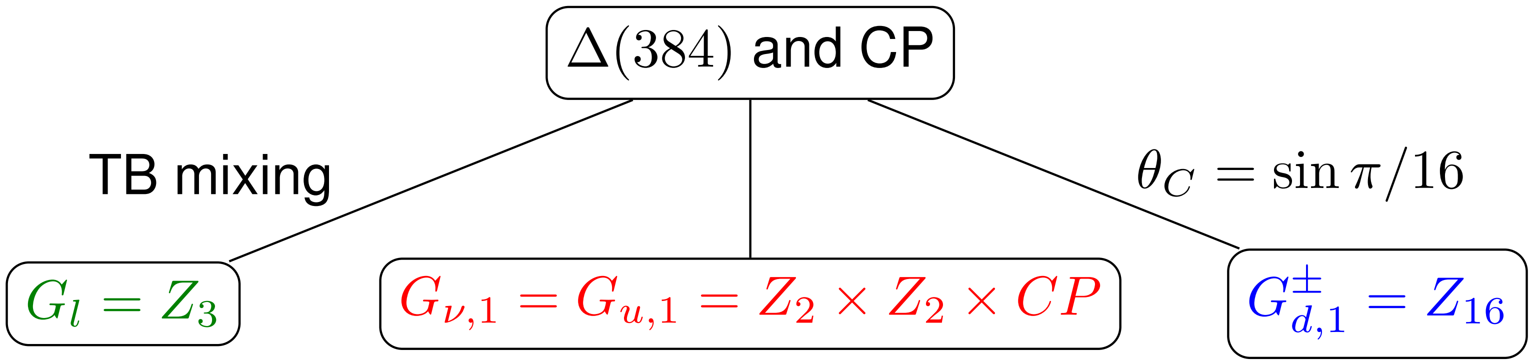

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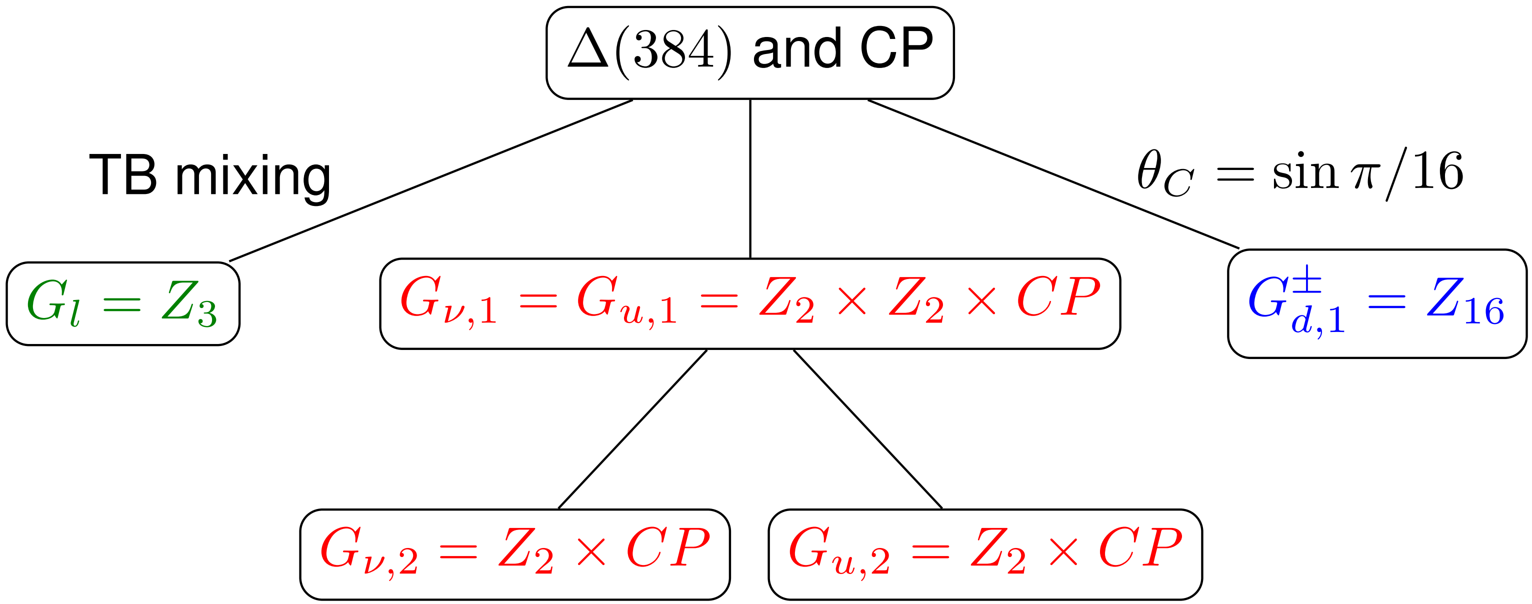

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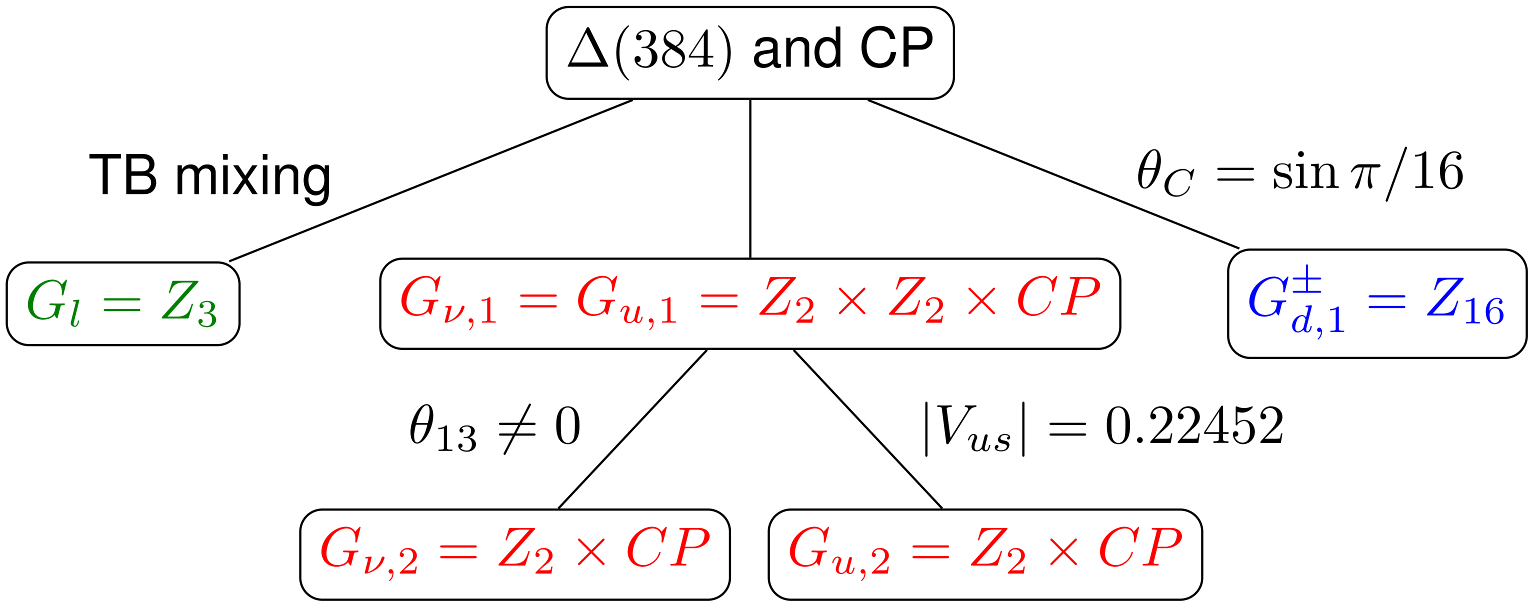

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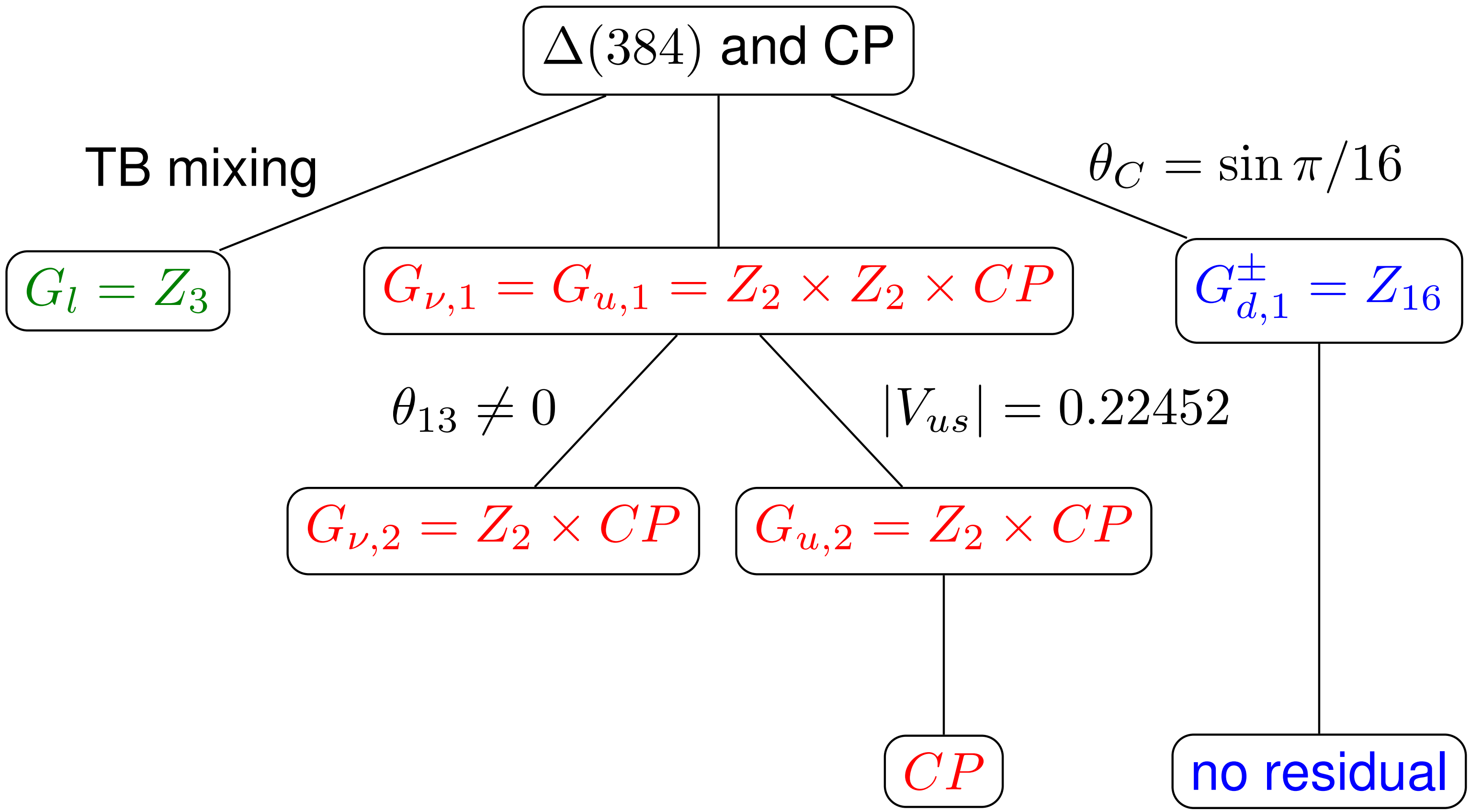

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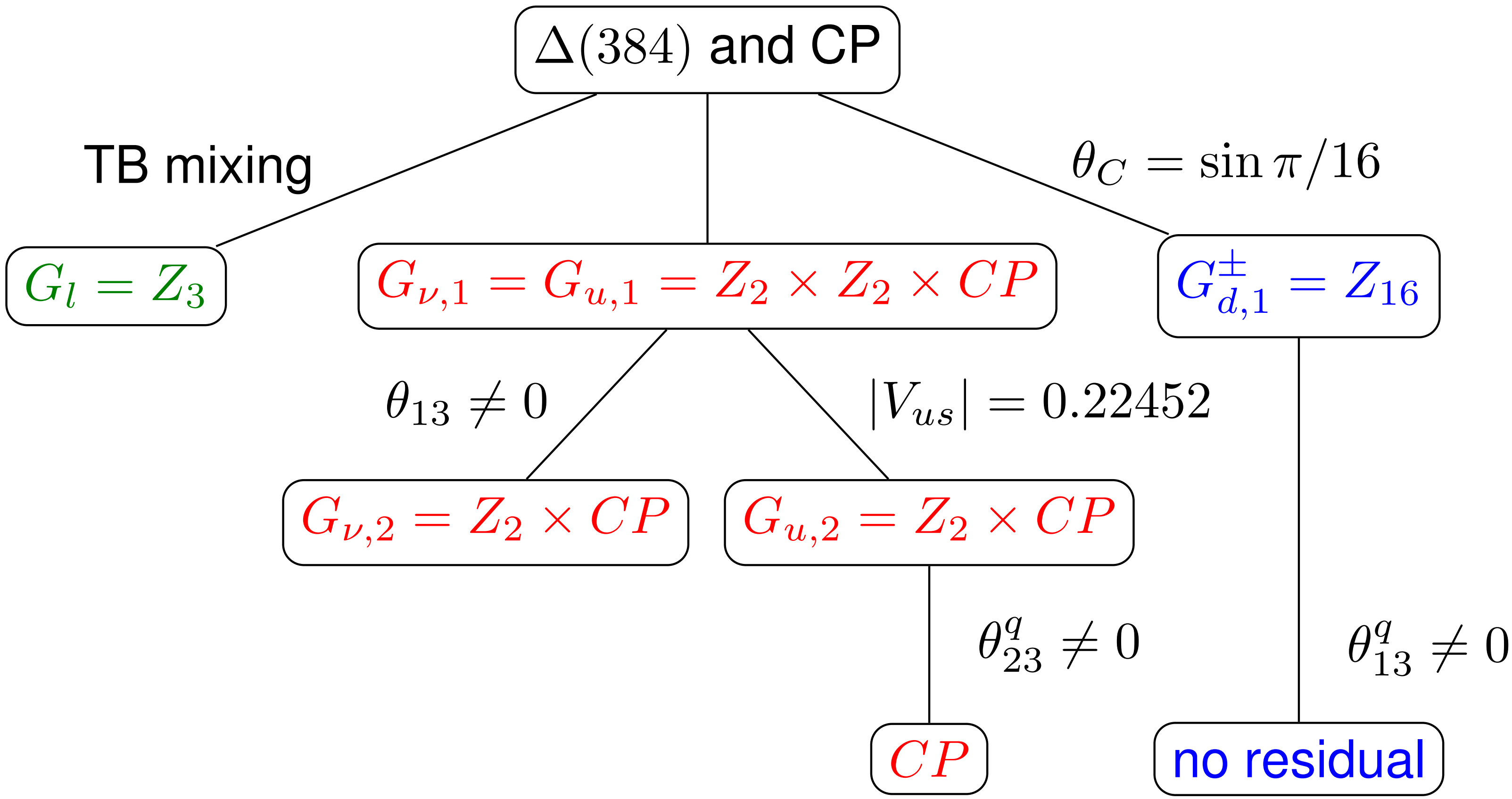

# Connections to quark sector

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Interesting result:

CP violation among quarks is correlated with lepton sector

$$|\sin\delta^q| \approx \left|\sin\left(\frac{\pi\, k}{8} + \frac{3\,\pi\, s}{8} \mp \frac{\pi}{16} + \psi_{d,13}\right)\right|$$

where

- input from lepton sector

  i) $k = 0$ is needed for getting TB mixing after 1st step

  ii) $s = 7$ is selected by data on $\theta_{23}$

- $\mp\frac{\pi}{16}$ refers to 2 choices $G^{\pm}_{d,1} = Z_{16}$
- $\psi_{d,13}$ arises as free parameter from breaking of $G^{\pm}_{d,1}$

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Use $k=0$ in order to formulate this expression as

sum rule involving $\delta^q$, $\theta_C$ and $\delta$, $\theta_{13}$, $\theta_{23}$

$$|\sin\delta^q| \approx \left|\sin\delta\, \cos\left(\theta_C \mp \psi_{d,13}\right) \pm \left(\frac{1-2\,\sin^2\theta_{23}}{2\,\sqrt{2}\,\sin\theta_{13}}\right)\, \sin\left(\theta_C \mp \psi_{d,13}\right)\right|$$

and for vanishing $\psi_{d,13}$

$$|\sin\delta^q| \approx \left|\sin\delta\, \cos\left(\theta_C\right) \pm \left(\frac{1-2\,\sin^2\theta_{23}}{2\,\sqrt{2}\,\sin\theta_{13}}\right)\, \sin\left(\theta_C\right)\right|$$

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- for $k = 0$, $s = 7$ and $\psi_{d,13} = 0$ we obtain

$$J^q_{\text{CP}} \approx 3.29 \times 10^{-5} \quad \text{for} \quad G^{+}_{d,1}$$
$$J^q_{\text{CP}} \approx 2.79 \times 10^{-5} \quad \text{for} \quad G^{-}_{d,1}$$

  which should be compared to *(PDG ('18))*

$$J^q_{\text{CP}} = (3.18 \pm 0.15) \times 10^{-5}$$

- for small $\psi_{d,13}$ best fit value of $J^q_{\text{CP}}$ can be achieved

# Conclusions

- lepton mixing angles can be understood with the help of flavour symmetries
- extensions with CP allow to constrain all leptonic CP phases
- further symmetries can fix neutrino masses as well
- eventually, CP violation among leptons and quarks can be related

Thank you for your attention.