Loop-corrected Higgs Masses in the NMSSM[1] with Inverse Seesaw Mechanism

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[1]Next-to-Minimal Supersymmetric Standard Model

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Motivations

The **Standard Model** (SM) is currently our best description of particle physics. But it cannot answer some questions.

Motivations

The **Standard Model** (SM) is currently our best description of particle physics. But it cannot answer some questions.

About dark matter

Observations:

- Dark matter should exist.
- Dark matter **cannot** be baryonic matter, neutrinos, or anything **in the SM** [a].

Questions:

1. **What is dark matter?**

   → See Trung's seminar.

[a] Gelmini, TASI 2014 Lectures, arXiv:1502.01320

Motivations

The **Standard Model** (SM) is currently our best description of particle physics. But it cannot answer some questions.

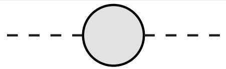
About Higgs boson

Observation:

- In 2012, a SM-like Higgs boson was discovered with a mass of 125 GeV [a,b].

Questions:

2. **Why is the mass of Higgs boson so small?**
   - New Physics should exist and appear at an energy scale $\gg$ 125 GeV, e.g. a new particle with mass $m_N \gg$ 125 GeV.
   - The Higgs' mass would depend on m_N through quantum correction

$\delta m_h^2 \sim$  $\sim m_N^2 \gg (125\ \text{GeV})^2.$

[a] ATLAS collaboration (2012), arXiv:1207.7214.
[b] CMS collaboration (2012), arXiv:1207.7235.

Motivations

The **Standard Model** (SM) is currently our best description of particle physics. But it cannot answer some questions.

About neutrinos

Observation:

- In 1998, neutrino oscillation was observed ⇒ **neutrinos have masses** [a].
- In 2019, the upper bound for neutrinos' masses was 1.1 eV [b].

$$0 < m_\nu < 1.1 \text{ eV}. \quad (1)$$

Questions:

3. **How do neutrinos get their masses?**
4. **Why are neutrinos' masses so small?**

[a] Super-Kamiokande Collaboration (1998), arXiv:hep-ex/9807003.
[b] KATRIN Collaboration (2019), arXiv:1909.06048.

Motivations

About dark matter

1. **What is dark matter?** (See Trung's seminar)

About Higgs boson

2. **Why is the mass of Higgs boson so small?**

$\Rightarrow$ **Supersymmetric models**.

About neutrinos

3. **How does neutrinos get its mass?**
4. **Why are neutrinos' masses so small?**

$\Rightarrow$ **Seesaw mechanism**.

Contents

Higgs sector in supersymmetric theories

The largest possible symmetry of nature

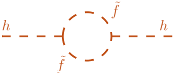
Supersymmetry = SuperPoincaré $\times$ Internal symmetry.[a]

[a] S.Coleman *et al.* (1967); R.Haag *et al.* (1975)

SuperPoincaré $= \underbrace{\text{Lorentz transformation} \times \text{translation}}_{\text{Poincaré symmetry}} \times$ (fermion $\leftrightarrow$ boson).

In supersymmetry conserving theories,

- Each **fermion** has a partnering **boson** with the same $\overbrace{\text{quantum numbers}}^{\text{mass, charge, etc.}}$, called a superpartner, and vice versa. They are combined in a **superfield**.
- No quadratic quantum correction.



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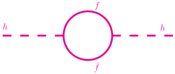

$\sim \underbrace{m_{\tilde{f}}^2 - m_f^2}_{0} + \cdots$

- For convenience, interactions are encoded in the **superpotential** W.
- Supersymmetry should be **broken** at electroweak scale.

Higgs sector in supersymmetric theories

Next-to-Minimal Supersymmetric Standard Model (NMSSM)

Contain 2 Higgs double H_u and H_d.

A Higgs singlet superfield $\hat{S}$ to solve μ-problem.

Scale-independent = Z_3-symmetry.

The superpotential is

$$\begin{aligned} W_{NMSSM} = &\hat{u}^c \boldsymbol{Y}_u \hat{u}_L \hat{H}_u^0 - \hat{u}^c \boldsymbol{Y}_u \hat{d}_L \hat{H}_u^+ - \hat{d}^c \boldsymbol{Y}_d \hat{u}_L \hat{H}_d^- + \hat{d}^c \boldsymbol{Y}_d \hat{d}_L \hat{H}_d^0 \\ &- \hat{e}^c \boldsymbol{Y}_e \hat{\nu}_L \hat{H}_d^- + \hat{e}^c \boldsymbol{Y}_e \hat{e}_L \hat{H}_d^0 + \lambda \hat{S}(\hat{H}_u^+ \hat{H}_d^- - \hat{H}_u^0 \hat{H}_d^0) + \frac{\kappa}{3}\hat{S}^3. \end{aligned} \tag{2}$$

Dynamically generate μ through electroweak symmetry breaking (EWSB):

$$\lambda \hat{S}(\hat{H}_u^+ \hat{H}_d^- - \hat{H}_u^0 \hat{H}_d^0) \xrightarrow{EWSB} \underbrace{\lambda \frac{v_s e^{i\varphi_s}}{\sqrt{2}}}_{\mu}(\hat{H}_u^+ \hat{H}_d^- - \hat{H}_u^0 \hat{H}_d^0). \tag{3}$$

Some features of NMSSM:

- Higgs masses can be calculated from other parameters, resulting in 5 neutral Higgs bosons $h_i, i = 1, ..., 5$ with $m_{h_1} \leq ... \leq m_{h_5}$, and 2 charged Higgs bosons $H^{\pm}$.
- There is one SM-like Higgs boson. Its mass is upper-bounded at tree level.

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Inverse Seesaw Mechanism

Seesaw mechanisms attempt to answer

3. How does neutrinos get its mass?
4. Why are neutrinos' masses so small?

[3] electroweak symmetry breaking

Inverse Seesaw Mechanism

Seesaw mechanisms attempt to answer

3. How does neutrinos get its mass?
4. Why are neutrinos' masses so small?

Neutrinos get its mass from Yukawa interaction ⇒ need right-handed neutrinos.

$$\mathcal{L}^{\text{Yukawa}} = -Y^{\nu}\bar{\nu}_L \phi \nu_R + h.c. \xrightarrow{\text{EWSB}^3} \mathcal{L}^D = -\underbrace{m_D \bar{\nu}_L \nu_R + h.c.}_{\text{Dirac mass term}}. \tag{4}$$

Right-handed neutrinos are neutral ⇒ can have Majorana mass terms.

$$\mathcal{L}^M = -\frac{m_M}{2}(\bar{\nu}_L \nu_L^c + \bar{\nu}_R \nu_R^c) + h.c. \tag{5}$$

Mass term of neutrinos

$$\mathcal{L}^{\nu \text{ mass}} = \mathcal{L}^D + \mathcal{L}^M = -\frac{1}{2}\begin{pmatrix}\bar{\nu}_L & \bar{\nu}_R^c\end{pmatrix} \underbrace{\begin{pmatrix} 0 & m_D \\ m_D & m_M \end{pmatrix}}_{\text{Mass matrix}} \begin{pmatrix}\nu_L^c \\ \nu_R\end{pmatrix} + h.c. \tag{6}$$

[3] electroweak symmetry breaking

Inverse Seesaw Mechanism

Seesaw mechanism

Diagonalizing the mass matrix gives [4]

$$m_{\nu \text{ light}} \approx \frac{{m_D}^2}{m_M}\ , \quad m_{\nu \text{ heavy}} \approx m_M. \tag{7}$$

m_D comes from electroweak symmetry breaking $\Rightarrow m_D \sim v \sim 100$ GeV.
m_M is a free parameter, independent of the rest of SM $\Rightarrow m_M \sim$ scale of New Physics $\sim 10^{14}$ GeV.

$$\Longrightarrow m_{\nu \text{ light}} \approx \frac{{m_D}^2}{m_M} \sim 0.1 \text{ eV} \tag{8}$$

$\Rightarrow$ Seesaw mechanism can explain why the masses of neutrinos are so small.

But, heavy neutrinos **cannot be detected** by experiment.

[4] A "$-$" sign has been absorbed into the fields.

Inverse Seesaw Mechanism

Motivations

- Answer: how neutrinos are massive; why their masses are so small.
- Heavy neutrinos are on TeV scale, can be tested by future experiments

Inverse Seesaw mechanism: right-handed neutrino + neutrino singlet X.

[5] I.Gogoladze *et al*, Phys.Lett. B672 (2009) 235-239

Inverse Seesaw Mechanism

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Inverse Seesaw mechanism: right-handed neutrino + neutrino singlet X.
NMSSM with Inverse Seesaw: Introduce 3 generations of superfields $\hat{\nu}_R^c$ and $\hat{X}$

Superfields	Z_3-charge [5]	Lepton number
$\hat{\nu}_R^c$	ω^2	-1
$\hat{X}$	ω	1

Table 1: Z_3-charge and lepton number of newly introduced fields with $\omega = e^{2\pi i/3}$

[5] I.Gogoladze *et al*, Phys.Lett. B672 (2009) 235-239

Inverse seesaw mechanism

The superpotential is

$$W = W_{NMSSM} + \hat{\nu}_R^c \mathbf{Y}_\nu \hat{\nu}_L \hat{H}_u^0 - \hat{\nu}_R^c \mathbf{Y}_\nu \hat{e}_L \hat{H}_u^+ + \hat{S}\hat{X}\boldsymbol{\lambda}_X \hat{X} + \hat{\nu}_R^c \boldsymbol{\mu}_X \hat{X}. \tag{9}$$

In the basis (ν_L, ν_R^c, X), the mass matrix is

$$\boldsymbol{M}_\nu = \begin{pmatrix} \mathbf{0} & v_u \mathbf{Y}_\nu & \mathbf{0} \\ v_u \mathbf{Y}_\nu^T & \mathbf{0} & {\boldsymbol{\mu}_X}^T \\ \mathbf{0} & \boldsymbol{\mu}_X & v_s \boldsymbol{\lambda}_X \end{pmatrix}.$$

$\boldsymbol{\lambda}_X$ violates lepton number and $\boldsymbol{\mu}_X$ is the only new dimensionful parameter
$\Rightarrow$ $\boldsymbol{\lambda}_X$ is suppressed and $\boldsymbol{\mu}_X$ can be on TeV scale.
$\Rightarrow$ can be tested by experiment.

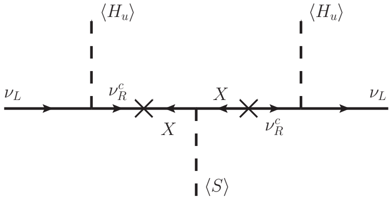

Figure 1: Inverse seesaw mechanism in NMSSM

The mass matrix for light neutrinos

$$\boldsymbol{m}_\nu \approx v_u^2 v_s \mathbf{Y}_\nu {\boldsymbol{\mu}_X}^{-1} \boldsymbol{\lambda}_X ({\boldsymbol{\mu}_X}^{-1})^T \mathbf{Y}_\nu^T \sim 0.1 \text{ eV}.$$

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Higgs sector in supersymmetric theories

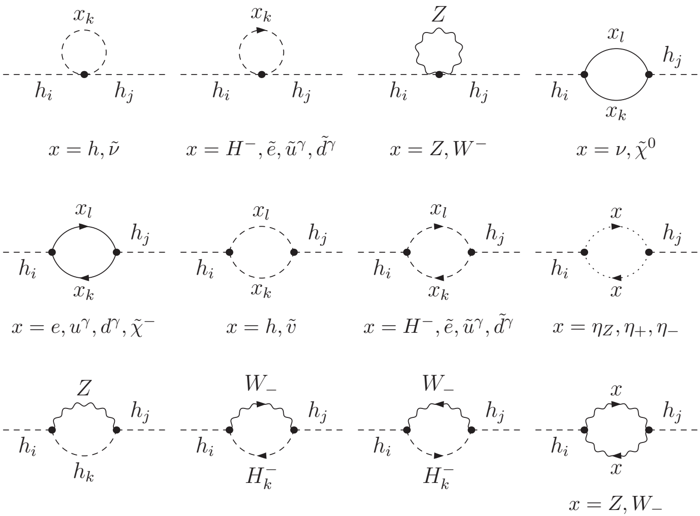


Figure 2: Neutral Higgs self-energy diagrams contributing to the Higgs masses. These diagrams are ultraviolet (UV) divergent.

Higgs sector in supersymmetric theories

Two methods were used to eliminate UV-divergence.

- Dimensional reduction (DRED)

Dimension	$D = 4 - 2\epsilon$	4
Quantity	$p^\mu, x^\mu, g^{\mu\nu}$	A^μ, γ^μ

- Renormalization using counterterm formalism:

$$m_0 = m + \delta m \quad , \quad \Phi_0 = (1 + \frac{1}{2}\delta Z_\Phi)\Phi$$

The counterterms are determined using a mixture of these condition schemes

  - On-shell (OS) scheme: $t_{h_d}, t_{h_u}, t_{h_s}, t_{a_d}, t_{a_s}, M_W^2, M_Z^2, M_{H^\pm}^2, v$
  - $\overline{DR}$ scheme[6]: $\tan\beta, v_s, |\lambda|, |\kappa|, \mathrm{Re}\, A_\kappa, \varphi_\lambda, \varphi_\kappa, \varphi_u, \varphi_s$

[6] counterterms are proportional to $\Delta = 1/\epsilon - \gamma_E + \ln 4\pi$

Contents

Constraints

HIGGS DATA

The SM-like Higgs boson mass $m_h = 125 \pm 3$ GeV [7].
Higgs' decay width and effective coupling in agreement with LEP, Tevatron and LHC data encoded in `HiggsBounds-5`[8] and HiggsSignal[9] within 2σ significance.

[7] Particle Data Group collaboration (2020)
[8] Bechtle, Dercks, Klingl, Stefaniak, Weiglein, and Wittbrodt (2020), arXiv:2006.06007.
[9] Bechtle, Heinemeyer, Stal, Stefaniak, Weiglein (2014), arXiv:1305.1933.

Constraints

LIGHT NEUTRINOS DATA

Neutrino oscillations [10]

Parameter	3σ
Δm^2_{21} $\left[10^{-5}\ \text{eV}^2\right]$	$6.82 \to 8.04$
$\Delta m^2_{31(23)}$ $\left[10^{-3}\ \text{eV}^2\right]$	$2.435 \to 2.598$
	$(2.581 \to 2.414)$
$\sin^2\theta_{12}$	$0.269 \to 0.343$
$\sin^2\theta_{23}$ (normal hierarchy)	$0.415 \to 0.616$
$\sin^2\theta_{23}$ (inverted hierarchy)	$0.419 \to 0.617$
$\sin^2\theta_{13}$ (normal hierarchy)	$0.02032 \to 0.02410$
$\sin^2\theta_{13}$ (inverted hierarchy)	$0.02052 \to 0.02428$

Cosmological constrain [11]

$$\sum_{i=1}^{3} m_{\nu_i} < 0.12\ \text{eV}.$$

Deviation from unitarity

$\tilde{U}_{PMNS} = (I - \eta)U_{PMNS}$[12]

$$\sqrt{2|\eta_{ee}|} < 0.050,\ \sqrt{2|\eta_{\mu\mu}|} < 0.021,$$
$$\sqrt{2|\eta_{\tau\tau}|} < 0.075,\ \sqrt{2|\eta_{e\mu}|} < 0.026,$$
$$\sqrt{2|\eta_{e\tau}|} < 0.052,\ \sqrt{2|\eta_{\mu\tau}|} < 0.035.$$

[10] NuFIT5.0; Esteban, Gonzalez-Garcia, Maltoni, Schwetz,Zhou (2020), arXiv:2007.14792
[11] Planck Collaboration (2018), arXiv:1807.06209
[12] E. Fernandez-Martinez *et al.*, (2016) arXiv:1605.08774;
J.Baglio *et al.*, (2017) arXiv:1612.06403

Constraints

LEPTON FLAVOUR VIOLATIONS

- **Flavour** = generation
- Experiment: **neutrino oscillation** $\Rightarrow$ Flavour violation in leptonic sector

Particle	Flavour
$e^-(e^+)$	$+1_e(-1_e)$
$\nu_e(\bar{\nu}_e)$	$+1_e(-1_e)$
$\mu^-(\mu^+)$	$+1_\mu(-1_\mu)$
$\nu_\mu(\bar{\nu}_\mu)$	$+1_\mu(-1_\mu)$
$\tau^-(\tau^+)$	$+1_\tau(-1_\tau)$
$\nu_\tau(\bar{\nu}_\tau)$	$+1_\tau(-1_\tau)$

Table 2: Flavour quantum number of leptons

Constraints

LEPTON FLAVOUR VIOLATIONS

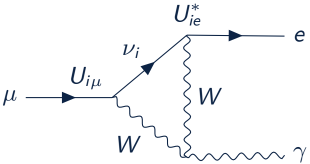
- **Flavour** = generation
- Experiment: **neutrino oscillation** $\Rightarrow$ Flavour violation in leptonic sector
- Highly suppressed in the SM due to **GIM**[13] **mechanism**.
- GIM mechanism: suppression due to the **unitarity** of mixing matrix.

[13] Glashow-Iliopoulos-Maiani

Constraints

LEPTON FLAVOUR VIOLATIONS

- **Flavour** = generation
- Experiment: **neutrino oscillation** ⇒ Flavour violation in leptonic sector
- Highly suppressed in the SM due to **GIM[13] mechanism**.
- GIM mechanism: suppression due to the **unitarity** of mixing matrix. E.g.



In the SM, the amplitude of this diagram is

$$\mathcal{M} = U^*_{ie} U_{i\mu} \tilde{\mathcal{M}}\left(\frac{m^2_{\nu_i}}{m^2_W}, \ldots\right)$$

$$\approx \underbrace{U^*_{ie} U_{i\mu}}_{=(U^\dagger U)_{e\mu}=0} \tilde{\mathcal{M}}(0,\ldots) + U^*_{ie} U_{i\mu} \underbrace{\frac{m^2_{\nu_i}}{m^2_W}}_{\sim 10^{-24}} \tilde{\mathcal{M}}'(0,\ldots)$$

⇒ Chance of happening: $BR(\mu \to e\gamma) \sim 10^{-55}$!

[13] Glashow-Iliopoulos-Maiani

Lepton Flavour violation (LFV)

Observable	Upper limit
$BR(\mu^- \to e^-\gamma)$	$< 4.2 \times 10^{-13}$
$BR(\tau^- \to e^-\gamma)$	$< 3.3 \times 10^{-8}$
$BR(\tau^- \to \mu^-\gamma)$	$< 4.4 \times 10^{-8}$
$BR(\mu^- \to e^-e^+e^-)$	$< 1.0 \times 10^{-12}$
$BR(\tau^- \to e^-e^+e^-)$	$< 2.7 \times 10^{-8}$
$BR(\tau^- \to e^-\mu^+\mu^-)$	$< 2.7 \times 10^{-8}$
$BR(\tau^- \to e^+\mu^-\mu^-)$	$< 1.7 \times 10^{-8}$
$BR(\tau^- \to \mu^-e^+e^-)$	$< 1.8 \times 10^{-8}$
$BR(\tau^- \to \mu^+e^-e^+)$	$< 1.5 \times 10^{-8}$
$BR(\tau^- \to \mu^-\mu^+\mu^-)$	$< 2.1 \times 10^{-8}$

Table 3: Experimental bound for LFV at 90% confident level[14]

$\Rightarrow$ **LFV can be used to test and constrain models**

[14] Particle Data Group (2018)

LFV in NMSSM with inverse seesaw

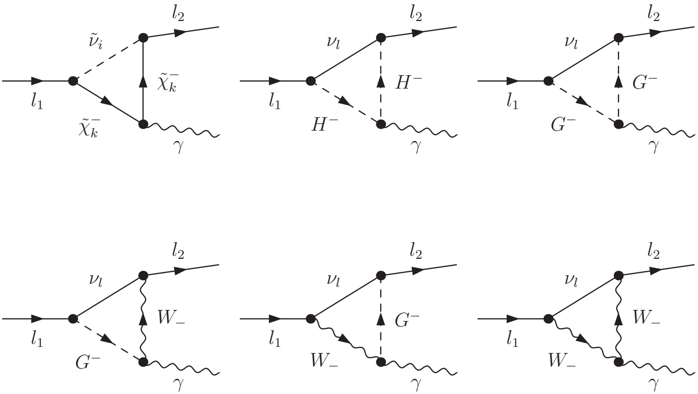


Figure 3: Diagrams involving neutrino sector that contributes to $\mu \to e\gamma$

Contents

Numerical analysis

We implemented new contributions to the public code `NMSSMCALC` [15].
Input parameters are taken in accordance with experimental data if available or motivated by previous papers [16].
In this presentation, we probe the free parameters μ_X, λ_X (from inverse seesaw) and A_t (from NMSSM). Other parameters are kept fixed.

- μ_X dictates the masses of heavy neutrinos, $\mu_X \sim m_{\nu\,\mathrm{heavy}}$.
- λ_X controls the effect of lepton flavour violation.
- A_t is the soft-breaking parameter in the stop sector. It has significant effect on the correction to the Higgs bosons masses.

[15] arXiv:1903.11358; Phys. Rev. D 85, 075024; JHEP 1210 (2012) 122; Comput.Phys.Commun. 185 (2014) no.12, 3372-3391; Nucl.Phys. B901 (2015) 526-555; Comput.Phys.Commun. 108 (1998) 56-74; Comput.Phys.Commun. 238 (2019) 214-231

[16] arXiv:1903.11358; JHEP 1704 (2017) 038; Nucl.Phys. B618 (2001) 171-204

Numerical analysis

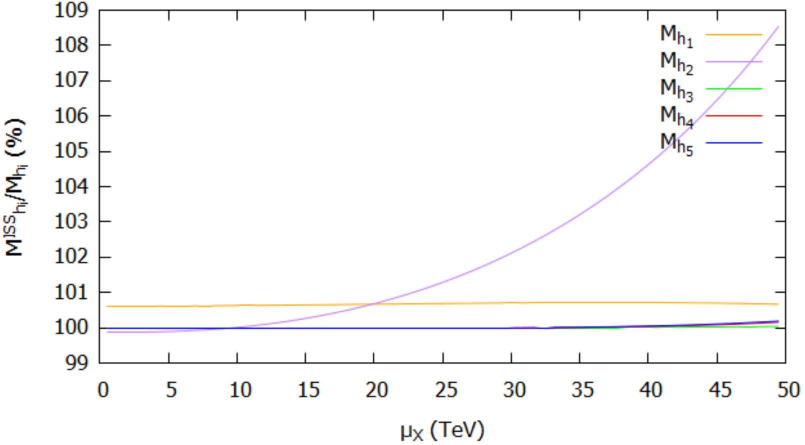


Figure 4: The dependence of corrected Higgs masses on μ_X (heavy neutrino masses) in percentage of correction. $M_{h_i}^{ISS}$ is the corrected Higgs mass with Inverse seesaw; M_{h_i} is without inverse seesaw. A correction up to 9% can be seen for SM-like Higgs h_2.

Numerical analysis

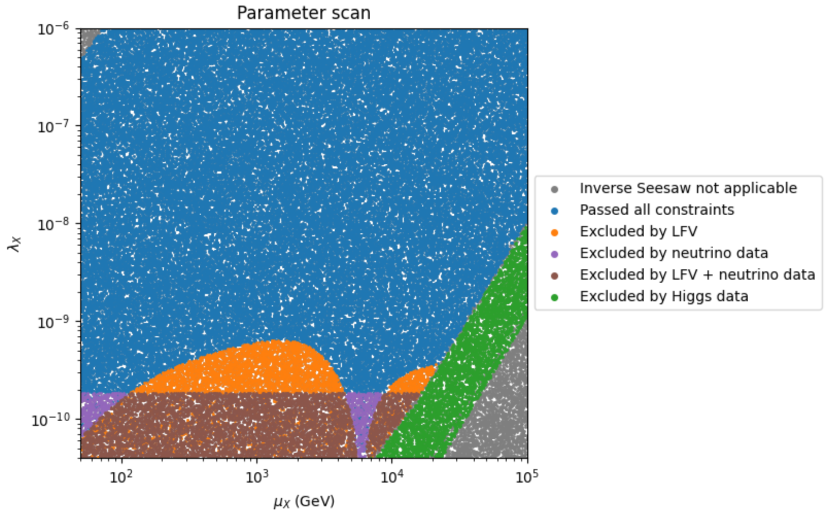


Figure 5: Parameter scan using 100,000 points. Only λ_X and μ_X changes. The bounds are taken from Baglio, Weiland (2017), arXiv:1612.06403.

Numerical analysis

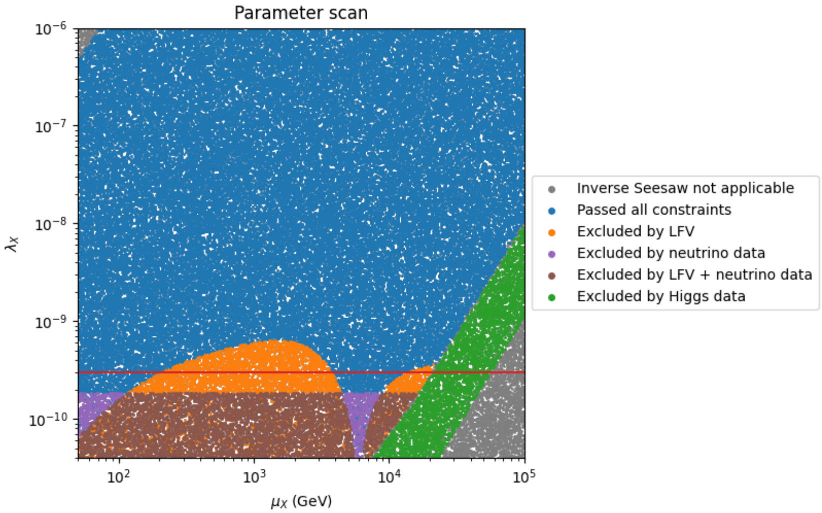


Figure 6: Parameter scan using 100,000 points. Only λ_X and μ_X changes. The bounds are taken from Baglio, Weiland (2017), arXiv:1612.06403.

Numerical analysis

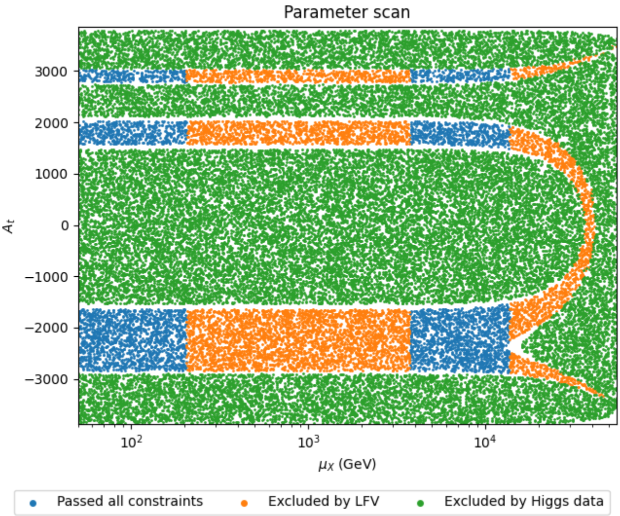


Figure 7: Parameter scan using 50,000 points. λ_X is kept fixed at $3 \cdot 10^{-9}$. Neutrino sector has sizeable effect on the Higgs sector for large μ_X.

Conclusion and outlook

- NMSSM was introduced to answer some problems in SM.
- Inverse seesaw mechanism was introduced to generate small mass for neutrino while keeping heavy neutrinos at TeV scale.
- Constraints such as Higgs and neutrino data, lepton flavour violation and oblique parameters were taken into account.
- Phenomenological analysis of inverse seesaw mechanism in NMSSM and its effect on the Higgs masses was analysed.
- Inverse seesaw mechanism has a sizeable contribution to the Higgs masses for large μ_X.

Outlook: Further analysis can be done on the relation between the Higgs sector and neutrino sector. We are finalizing our numerical analysis for publication.

Acknowledgments:

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Thank you for your listening

Back up slides

Supersymmetry

Symmetries of the Standard Model:

- Poincaré symmetry = Lorentz symmetry × translation symmetry.
- $SU_C(3) \times SU_L(2) \times U_Y(1)$.

How to go beyond the Standard Model? Expand its symmetry.
What is the largest possible symmetry group allowed in QFT?

Supersymmetry

Symmetries of the Standard Model:

- Poincaré symmetry = Lorentz symmetry $\times$ translation symmetry.
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How to go beyond the Standard Model? Expand its symmetry.
What is the largest possible symmetry group allowed in QFT?

The largest possible symmetry of nature

Supersymmetry = SuperPoincaré $\times$ Internal symmetry.[a]

[a] S.Coleman *et al.* (1967); R.Haag *et al.* (1975)

SuperPoincaré $= \underbrace{\text{Lorentz transformation} \times \text{translation}}_{\text{Poincaré symmetry}} \times$ (fermion $\leftrightarrow$ boson).

Internal symmetry: symmetries not involving spacetime (e.g. $SU(5)$, $U_Q(1)$, $SU_C(3) \times SU_L(2) \times U_Y(1)$, etc.).

Supersymmetry

Generators of SuperPoincaré group: $\overbrace{P^\mu, M^{\mu\nu}}^{\text{Poincaré group}}$, $\overbrace{Q, \bar{Q}}^{\text{fermion} \leftrightarrow \text{boson}}$, with

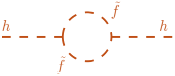
$$Q \left|\text{Boson}\right\rangle = \left|\text{Fermion}\right\rangle , Q \left|\text{Fermion}\right\rangle = \left|\text{Boson}\right\rangle . \tag{10}$$

In supersymmetry conserving theories,

- Each **fermion** has a partnering **boson** with the same $\overbrace{\text{quantum numbers}}^{\text{mass, charge, etc.}}$, called a superpartner, and vice versa. They are combined in a **superfield**.

Superfield		spin 0	spin 1/2
$\hat{e}_L$	$\Leftrightarrow$	$\tilde{e}_L$	e_L
lepton superfields		**s**leptons	leptons

- No quadratic quantum correction.



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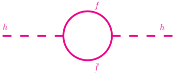

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Minimal supersymmetric Standard Model (MSSM)

There are many supersymmetric theories.
Minimal supersymmetric Standard Model (MSSM): the most similar to the SM. The number of newly introduced fields is kept minimal.

[17] "^" is the corresponding superfield

Minimal supersymmetric Standard Model (MSSM)

There are many supersymmetric theories.
Minimal supersymmetric Standard Model (MSSM): the most similar to the SM. The number of newly introduced fields is kept minimal.

Contains 2 Higgs doublets H_u, H_d.
The superpotential for this model is [17]

$$\begin{aligned} W_{MSSM} = {} & \hat{u}^c \boldsymbol{Y}_u \hat{u}_L \hat{H}_u^0 - \hat{u}^c \boldsymbol{Y}_u \hat{d}_L \hat{H}_u^+ - \hat{d}^c \boldsymbol{Y}_d \hat{u}_L \hat{H}_d^- + \hat{d}^c \boldsymbol{Y}_d \hat{d}_L \hat{H}_d^0 \\ & - \hat{e}^c \boldsymbol{Y}_e \hat{\nu}_L \hat{H}_d^- + \hat{e}^c \boldsymbol{Y}_e \hat{e}_L \hat{H}_d^0 + \mu(\hat{H}_u^+ \hat{H}_d^- - \hat{H}_u^0 \hat{H}_d^0). \end{aligned} \tag{11}$$

Remarks:

- The SM-like Higgs boson mass is upper-bounded at tree-level,

$$m_h^2 \leq M_Z^2 \cos^2 2\beta. \tag{12}$$

- Fine-tuning μ-problem: μ should be on supersymmetric scale. But Higgs' masses (small) depends on μ (much larger).

[17] "^" is the corresponding superfield