Phenomenology with Unitarily Inequivalent Vacua and Long-Range Forces: Mass-shift

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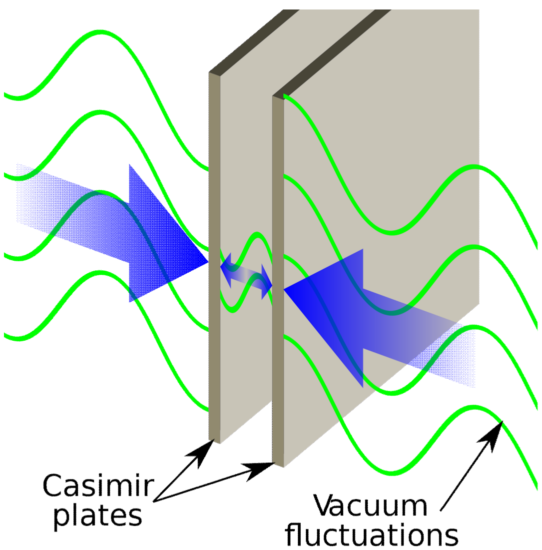

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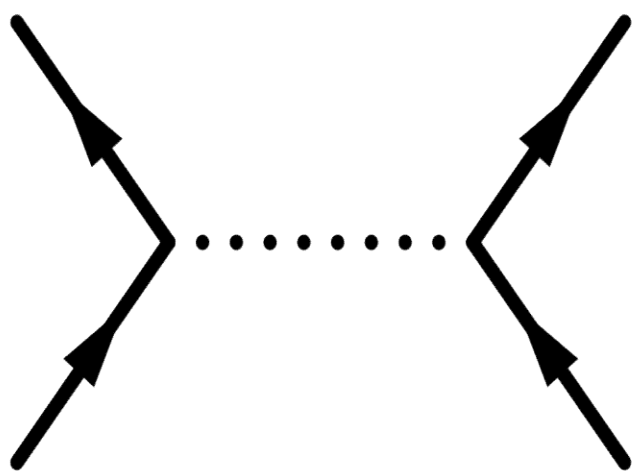

https://en.wikipedia.org/wiki/Virtual_particle#/media/File:Momentum_exchange.svg

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Motivation: Non-Trivial Vacuum

$$|\boldsymbol{P_{new}}\rangle = \widehat{\boldsymbol{\phi}}\,|\mathbf{0}\rangle$$

- Spontaneous Broken Symmetry [1]
- Non-Perturbative Methods in Field Theory [2]
- Haag's theorem and unitarily inequivalent vacua [2]
- A new origin for breaking Lorentz symmetry

Motivation: Mass-Shift

- Mass-generating mechanism
- Higgs phase transition [3]
- Did the particle mass change more than one time?
- Can we know about these instances from today experiments?

How does this type of new physics manifest in Long-Range Forces?

Outline

I. **Mass-Shift: Scalar Boson**
   A. Toy model
   B. Representation of Fields and Vacua
II. **Low-energy Signatures and Long-Range Forces**
   A. Condensate Density
   B. One-Boson Exchange Potential
      1. Feynman Propagator
      2. Potential with Mass-shift
   C. Casimir Force
III. **Discussion and Future Prospects**

Mass-shift: Toy model with Scalar Boson

- Before the transition

$$\mathcal{L}_1^{\text{free}} = \frac{1}{2}\partial_\mu \phi(x)\partial^\mu \phi(x) - \frac{1}{2}m_1^2\phi^2(x)$$

- After the transition

$$\mathcal{L}_2^{\text{free}} = \frac{1}{2}\partial_\mu \phi(x)\partial^\mu \phi(x) - \frac{1}{2}m_2^2\phi^2(x)$$

- Time Evolution Generators

$$H_i = \int d^3r \left[\frac{1}{2}\Pi^2(x) + \frac{1}{2}\nabla^2\phi(x) + \frac{1}{2}m_i^2\phi^2(x)\right]$$

Mass-shift: Toy model with Scalar Boson

- Canonically quantized, we obtain

$$\phi(\vec{r}) = \begin{cases} \phi_1(\vec{r}) \equiv \frac{1}{\sqrt{V}} \sum_{\vec{k}} \frac{1}{\sqrt{2\omega_1}} [a_1 + a^\dagger_{-,1}] e^{i\vec{k}\cdot\vec{r}}, & \text{before the transition,} \\ \phi_2(\vec{r}) \equiv \frac{1}{\sqrt{V}} \sum_{\vec{k}} \frac{1}{\sqrt{2\omega_2}} [a_2 + a^\dagger_{-,2}] e^{i\vec{k}\cdot\vec{r}}, & \text{after the transition,} \end{cases}$$

- Two different vacua to minimize 2 Hamiltonians

$$a_i |0\rangle_i \equiv 0$$

Mass-shift: Toy model with Scalar Boson

- Which vacuum is the current physical vacuum? $|0\rangle_1$ or $|0\rangle_2$?
- Little is known about quantum phase transition [3]
- Should vacuum state be the lowest energy state? (only necessarily true in non-SSB context [4])

Abrupt Mass-shift:

Chronological order: $|0\rangle_1$ is the physical vacuum.

Mass-shift: Representation of Fields and Vacua

- Field matching

$$\phi(\vec{r}) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \frac{1}{\sqrt{2\omega_1}} [a^\dagger_{-,1} + a_1] e^{i\vec{k}\cdot\vec{r}} = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \frac{1}{\sqrt{2\omega_2}} [a^\dagger_{-,2} + a_2] e^{i\vec{k}\cdot\vec{r}},$$

$$\Pi(\vec{r}) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \sqrt{\frac{\omega_1}{2}} [a^\dagger_{-,1} - a_1] e^{i\vec{k}\cdot\vec{r}} = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \sqrt{\frac{\omega_2}{2}} [a^\dagger_{-,2} - a_2] e^{i\vec{k}\cdot\vec{r}}.$$

- Bogolyubov transformation

$$\begin{pmatrix} a^\dagger_{-,2} \\ a_2 \end{pmatrix} = \begin{pmatrix} u & v \\ v & u \end{pmatrix} \begin{pmatrix} a^\dagger_{-,1} \\ a_1 \end{pmatrix}$$

$$u = \frac{1}{2}\left(\sqrt{\frac{\omega_1}{\omega_2}} + \sqrt{\frac{\omega_2}{\omega_1}}\right) \quad v = \frac{1}{2}\left(\sqrt{\frac{\omega_2}{\omega_1}} - \sqrt{\frac{\omega_1}{\omega_2}}\right)$$

Mass-shift: Representation of Fields and Vacua

$$|0\rangle_1 = \prod_{\vec{k}} [c_0(\vec{k})|0\rangle_2 + c_2(\vec{k})|1_{\vec{k}} 1_{-\vec{k}}\rangle_2 + c_4(\vec{k})|2_{\vec{k}} 2_{-\vec{k}}\rangle_2 + \cdots],$$

- Containing pairs of zero-momentum particles
- Evolving non-trivially in time governed by $\widehat{H}_2$
- No longer Lorentz-invariant due to time dependence
- Analog of metric-varying vacuum in expanding universes [5]

Condensate Density

- The number density

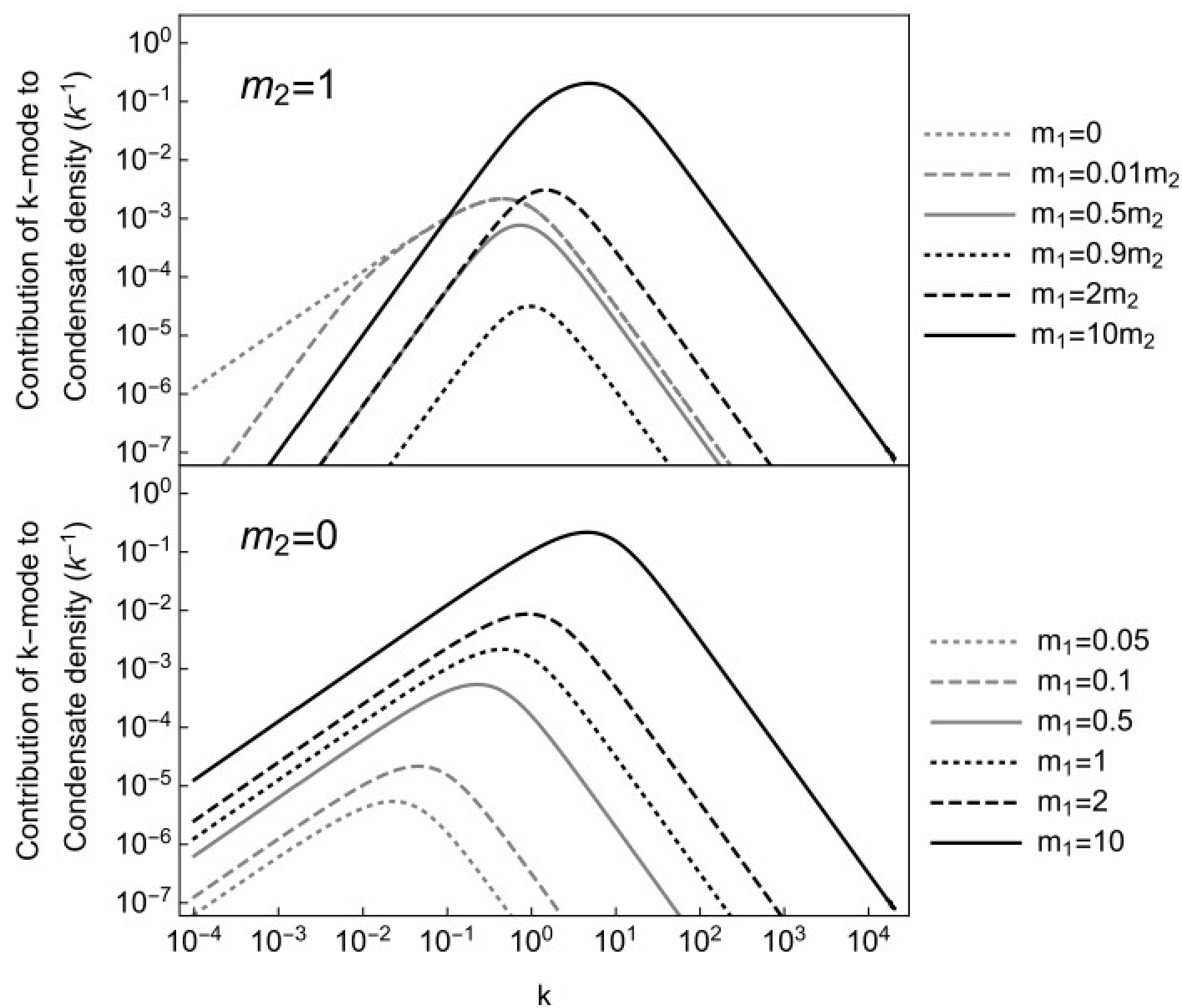
$$\rho = \frac{\langle \hat{N} \rangle}{V}$$

- Contribution of each k-mode to the number density

$$\rho(k) = \frac{k^2}{8\pi^2} \frac{(\omega_2 - \omega_1)^2}{\omega_1 \omega_2}$$

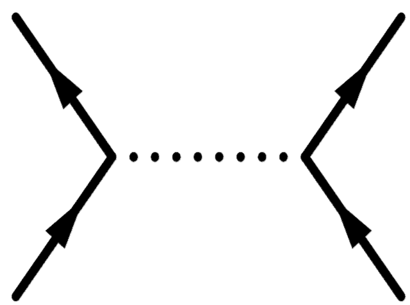
- Candidates for new physics of dark matter and dark energy

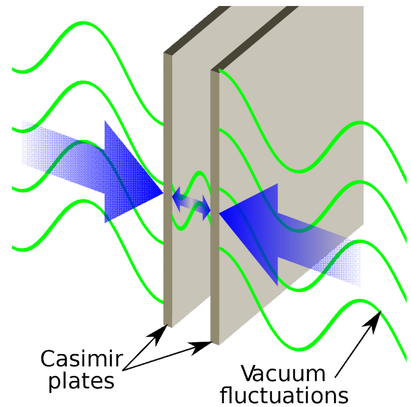
Condensate Density



Long-Range Forces

- Consequence of quantum vacuum process [6]
- Sensitive to non-trivial vacuum structures





One-Boson Exchange Potential (OBEP)

- Virtual particle exchange
- Coupling constant dependence
- Perturbative effect

Casimir Force

- Vacuum energy shift
- Boundary conditions on fields
- Non-perturbative effect

Long-Range Forces: OBEP

- Feynman Propagator

$$\Delta_F(x-y) = {}_1\langle 0|\mathcal{T}[\phi(x)\phi(y)]|0\rangle_1$$

- Mixture of positive and negative energy states

$$\Delta_F(x) = \frac{1}{(2\pi)^3}\int d^3k \frac{e^{i\vec{k}\cdot\vec{x}}(v+u)}{2\omega_2}[\Theta(x^0)(ve^{i\omega_2 t} + ue^{-i\omega_2 t}) + \Theta(-x^0)(ue^{i\omega_2 t} + ve^{-i\omega_2 t})].$$

- Momentum space Feynman Propagator

$$\Delta_F(p) = \frac{\omega_2}{\omega_1}\frac{i}{p^2 - m_2^2 + i\epsilon}.$$

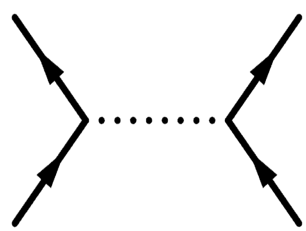
$$d^4p \to \frac{\omega_2}{\omega_1} d^4p$$

Long-Range Forces: OBEP

- Potential from Feynman amplitude

$$V(r) = \int \frac{d^3k}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{r}} i\mathcal{M}(p)|_{(0,\vec{k})}$$

- Using Feynman rule

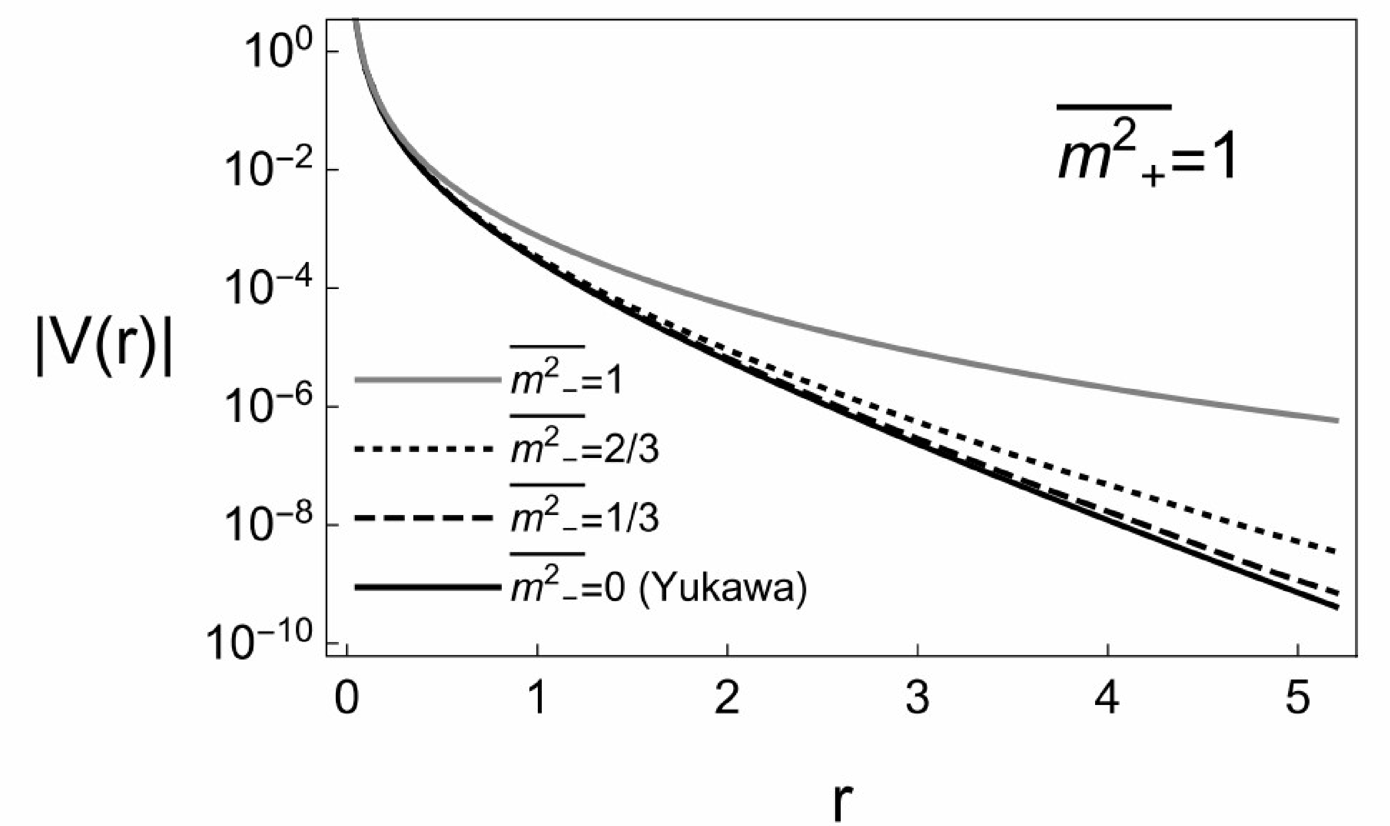


$$\mathcal{M} = \frac{-ig_1 g_2}{p^2 - m_2^2} \frac{\omega_2}{\omega_1}$$

- One-Boson Exchange Potential

$$V(r) = -\frac{2g_1 g_2}{(2\pi)^2 r} \int_0^\infty dk \frac{k \sin kr}{\sqrt{k^2 + m_1^2}\sqrt{k^2 + m_2^2}}$$

Long-Range Forces: OBEP



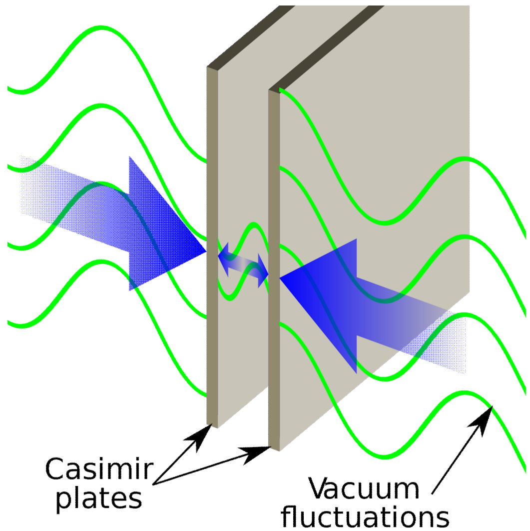
- $m_1 = 0$ or $m_2 = 0$

$$-g_1 g_2\left[\frac{I_0(m_1 r) - L_0(m_1 r)}{4\pi r}\right]$$

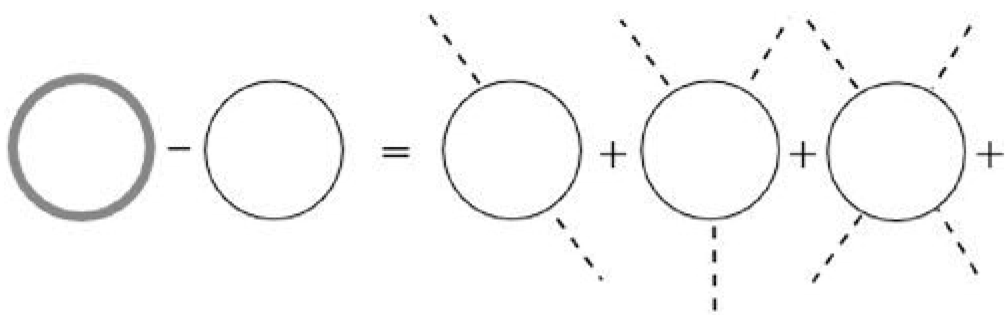
- $m_1, m_2 \neq 0$

$$-\frac{g_1 g_2 e^{-\sqrt{\overline{m_+^2}}r}}{4\pi r} + g_1 g_2\sqrt{\overline{m_+^2}}\left(1+\sqrt{\overline{m_+^2}}r\right)\frac{e^{-\sqrt{\overline{m_+^2}}r}}{64\pi}\left(\frac{\overline{m_-^2}}{\overline{m_+^2}}\right)^2 + \mathcal{O}\left[\left(\frac{\overline{m_-^2}}{\overline{m_+^2}}\right)^4\right]$$

Long-Range Forces: Casimir Force



- Arise from the boundary conditions on the field due to interaction



R. L. Jaffe, Phys. Rev. D **72**, 021301(R) (2005)

- Strong coupling limit due to the many-body nature [6,7]

Long-Range Forces: Casimir Force

- Consider the stress-energy tensor $T_{\alpha\beta}$ of a scalar field

$$T_{\alpha\beta} = -\frac{\partial\phi}{\partial x^\alpha}\frac{\partial\phi}{\partial x^\beta} + \frac{1}{2}\delta_{\alpha\beta}\left[\vec{\nabla}\phi(x)\cdot\vec{\nabla}\phi(x) - \left(\frac{\partial\phi(x)}{\partial t}\right)^2 + m_2^2\phi^2(x)\right]$$

- Pressure from the vacuum at the boundary

$$P_{\rm net}(x = L/2) = {}_1\langle 0|\left[\lim_{x\to(L/2)^+} T_{11}(x) - \lim_{x\to(L/2)^-} T_{11}(x)\right]|0\rangle_1$$

- Exactly the Casimir force from the m_1 scalar field

$$P_{\rm net}(x = L/2) = -\frac{m_1^4}{2\pi^2}\sum_{s=1}^{\infty}\frac{1}{2m_1Ls}\left[K_3(2m_1Ls) - \frac{1}{2m_1Ls}K_2(2m_1Ls)\right]$$

- No m_2-dependence!

Discussion

- OBEP is a dynamical process, thus it depends on the mass in the Hamiltonian

- Difference between OBEP and Casimir Force

- Casimir force considered here is only the strong-coupling leading order [7]

- Both should have complicated dependences on m_1 and m_2

Discussion and Future Prospects

- **Applicability in the Standard Model**
  - Little is known about Higgs phase transition [3]
- **Other manifestation of mass-shift**
  - Vacuum changes while field representation remains (our OBEP is applicable while the Casimir force will be different)
  - Vacuum changes slowly after the mass-shift into $|0\rangle_2$
    → Strength OBEP varies through time, peaking at the present
- **Mass-shift in particle mixing [8]**
  - There is an ambiguity for the vacuum mass in particle oscillating phenomenon
  - Possible new physics hidden in the vacuum for neutrino sector
- **Unitarily inequivalent vacua as a new arena for phenomenological work in BSM**

Thank you!

Reference: Q. Le Thien, D. E. Krause, Mod. Phys. Lett. A 35 (17), 2050139

References

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