ONE–LOOP THREE–POINT FEYNMAN INTEGRALS & ONE–LOOP CONTRIBUTIONS TO $H \to Z\gamma$ IN TERMS OF HYPERGEOMETRIC FUNCTIONS

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Collaborating with **Dr. Phan Hong Khiem**

Based on:

1. Khiem Hong Phan and **Dzung Tri Tran**, Progress of Theoretical and Experimental Physics (PTEP), Volume 2019, Issue 6, June 2019, 063B01.
2. Khiem Hong Phan and **Dzung Tri Tran**, Progress of Theoretical and Experimental Physics (PTEP), Volume 2020, Issue 5, May 2020, 053B08.

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Contents

Introductions

Future Experimental Programs

- High-Luminosity Large Hadron Collider (HL-LHC).[a]
- International Linear Collider (ILC).[b]

[a] ATLAS Collaboration, arXiv:1307.7292 [hep-ex]; CMS Collaboration, arXiv:1307.7135 [hep-ex].
[b] H. Baer et al., arXiv:1306.6352 [hep-ph].

Main missions

- Measuring precisely the properties of the Higgs boson, top quark, and vector bosons.
- Discovering the nature of the Higgs sector.
- Searching for signals and effects of Physics Beyond the Standard Model (BSM).

Theoretical Predictions

- Matching the high precision of experimental data in the near future.
- Including high-order corrections such as one-loop multi-leg integrals and higher-loop integrals at a general scale and mass assignments (external momentum & internal masses).

Introductions

One-loop Feynman integrals in the general space-time dimension d

- Playing a crucial role for building blocks of computations:
  - Two-loop or higher-loop corrections.
  - Higher terms in the ε-expansions at space-time dimension $d = 4 - 2\varepsilon$.

Higgs decay processes as an applicable consideration

- The channels $H \to \gamma\gamma, Z\gamma$ are the most important at LHC for the following reasons:
  - The channels arise at first from one-loop Feynman diagrams.
  - The decay widths are sensitive to New Physics in which new heavy particles may exchange in the loop diagrams.
- Theoretical calculations for one-loop and higher-loop decay amplitudes of Higgs channels play important roles in:
  - Controlling the standard model (SM) background.
  - Constraining the physical parameters in many beyond the standard models (BSM).

Outlines

One-loop three-point Feynman integrals

- New analytic formulas[a] in the general space-time dimension d.
  - The evaluations are performed in general configuration and several special cases: internal masses and external momenta.
  - The analytic results are expressed in terms of hypergeometric Gauss ${}_2F_1$, ${}_3F_2$, and Appell F_1 functions which are useful for general ε-expansions in $d = 4 - 2\varepsilon$.
- These analytic results are cross-checked with other papers that are available for several special cases expanded the results up to ε^0 (or setting $d = 4$ at beginning).

[a] K. H. Phan and D. T. Tran, Prog. Theor. Exp. Phys. 063B01 (2019)

$H \to \gamma\gamma, Z\gamma$ decay processes as hypergeometric representation applications

- Total amplitude in processes is performed by using Feynman diagrams in Unitary gauge.
- New analytic formulas[b] for Form factors at general d by applying Davydychev's tensor one-loop reduction method and Integration-by-parts method (IBP).

[b] K. H. Phan and D. T. Tran, Prog. Theor. Exp. Phys. 053B08 (2020)

Scalar one-loop three-point Feynman integrals

Based on:

[K. H. Phan and D. T. Tran, Prog. Theor. Exp. Phys. 063B01 (2019)]

Research approach

Scalar one-loop three-point Feynman integrals

- C_0 is a function of p_1^2, p_2^2, p_3^2 and m_1^2, m_2^2, m_3^2 with d is space-time dimension.

$$C_0(\{p_i^2\};\{m_i^2\}) = \int \frac{\mathrm{d}^d k}{i\pi^{d/2}} \frac{1}{\left[(k+q_1)^2 - m_2^2 + i\rho\right]\left[(k+q_2)^2 - m_3^2 + i\rho\right]\left[(k+q_3)^2 - m_1^2 + i\rho\right]}$$

- The internal (loop) momentum is k and the external momenta are p_i $(i = 1,2,3)$.
- The momenta of internal lines are $q_i = \sum_{j=1}^{i} p_j$, so on $q_3 = \sum_{i=1}^{3} p_i = 0$ (thanks to momentum conservation) and m_i $(i = 1,2,3)$ are the internal masses.

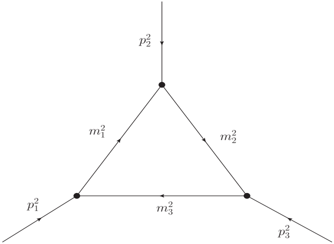

Figure 1 : One-loop triangle diagrams

Research approach

Scalar one-loop three-point Feynman integrals

- C_0 has known that an algebraically compact expression and a numerically stable representation for Feynman diagrams.
- The analytic results for C_0 will be expressed in terms of the ratio of these kinematic variables such as the determinants of Caylay and Gram matrices:

$$
\begin{aligned}
M_{123}^2 &= -S_{123}/G_{123}\,(G_{123} \neq 0) \\
M_{ij}^2 &= -S_{ij}/G_{ij}\,(G_{ij} \neq 0) \text{ for } i,j = 1,2,3
\end{aligned}
$$

- The determinant of Cayley matrix: One-loop three- and two-point functions

$$
S_{123} = \begin{vmatrix} 2m_1^2 & -p_1^2 + m_1^2 + m_2^2 & -p_3^2 + m_1^2 + m_3^2 \\ -p_1^2 + m_1^2 + m_2^2 & 2m_2^2 & -p_2^2 + m_2^2 + m_3^2 \\ -p_3^2 + m_1^2 + m_3^2 & -p_2^2 + m_2^2 + m_3^2 & 2m_3^2 \end{vmatrix} \quad \text{and} \quad
\begin{aligned}
S_{12} &= -\lambda(p_1^2, m_1^2, m_2^2), \\
S_{13} &= -\lambda(p_3^2, m_1^2, m_3^2), \\
S_{23} &= -\lambda(p_2^2, m_2^2, m_3^2).
\end{aligned}
$$

- The determinant of Gram matrix: One-loop three- and two-point functions

$$
G_{123} = 2\lambda(p_1^2, p_3^2, p_2^2) \quad \text{and} \quad G_{12} = -4p_1^2,\, G_{13} = -4p_3^2,\, G_{23} = -4p_2^2
$$

- Källen function is defined by

$$
\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz
$$

Research approach

Scalar one-loop three-point Feynman integrals

- Performing Feynman parametrization, then integrating over loop-momentum and one of Feynman parameters.
- Arriving at two-fold integral

$$C_0(\{p_i^2\};\{m_i^2\}) = -\Gamma\left(3-\frac{d}{2}\right)\int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \left[Ax^2 + By^2 + 2Cxy + Dx + Ey + F - \mathrm{i}\rho\right]^{\frac{d}{2}-3}$$

- These coefficients are shown

$$A = p_1^2 \qquad B = p_3^2 \qquad C = -p_1 p_3 = (p_1^2 + p_3^2 - p_2^2)/2$$
$$D = -(p_1^2 + m_1^2 - m_2^2) \qquad E = -(p_3^2 + m_1^2 - m_3^2) \qquad F = m_1^2$$

- Two light-like momentum e.g. $p_1^2 = 0, p_3^2 = 0$ and One light-like momentum e.g. $p_3^2 = 0$ with several special cases in the internal masses e.g. $m_1^2 = m_2^2 = m_3^2 = 0$ or m^2 are considered by Master integral.
- Generalizing this method for the general case in which $p_i^2 \neq 0$ and $m_i^2 \neq 0$ $(i = 1, 2, 3)$.

Master integral

- Considering the follow Master integral with $p_i^2 \neq 0$ and $i, j, k = 1, 2, 3$ as

$$\mathcal{K} = \int_0^1 \mathrm{d}x \, \frac{\left[p_i^2 x^2 - \left(p_i^2 + m_i^2 - m_j^2\right)x + m_i^2 - \mathrm{i}\rho\right]^{d/2-2}}{x - x_k}$$

- Using the Mellin-Barnes relation which is provided that $|\mathrm{Arg}(z)| < \pi$

$$(1+z)^{-\lambda} = \frac{1}{2\pi\mathrm{i}} \int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty} \mathrm{d}s \, \frac{\Gamma(-s)\Gamma(\lambda+s)}{\Gamma(\lambda)} z^s$$

- Decomposing the integrand $\mathcal{K}$ with the Feynman parameter integral $\mathcal{H}$ in terms of Gauss hypergeometric functions ${}_2F_1$ as follows

$$\mathcal{K} = \mathcal{H} \times \int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty} \frac{\mathrm{d}s}{2\pi\mathrm{i}} \frac{\Gamma(-s)\Gamma(2-d/2+s)}{\Gamma(2-d/2)} \left(M_{ij}^2 - \mathrm{i}\rho\right)^{d/2-2}$$

- Evaluating following Mellin-Barnes integral by closing integration contour on the right side of imaginary axis in s-complex plane and calculating residua of sequence pole $\Gamma(-s)$.

$$\int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty} \mathrm{d}s \, \frac{\Gamma(-s)\Gamma(a+s)\Gamma(b+s)}{\Gamma(c+s)} \frac{(-x)^s}{2\pi\mathrm{i}} {}_2F_1\left[a+s, b'; c+s; y\right]$$

- The master integral $\mathcal{K}$ is presented as a series of Appell F_1 functions with $i, j, k = 1, 2, 3$

$$\mathcal{K} = -\frac{x_{ij}(x_k - x_{ij})}{(x_k - x_{ij})^2 - x_{ij}^2} (m_i^2)^{d/2-2} F_1\left[1; 1, 2-d/2; 3/2; -\frac{x_{ij}^2}{(x_k - x_{ij})^2 - x_{ij}^2}, \frac{p_i^2 x_{ij}^2}{p_i^2 x_{ij}^2 + M_{ij}^2 - \mathrm{i}\rho}\right]$$

$$+ \frac{x_{ij}^2}{2[(x_k - x_{ij})^2 - x_{ij}^2]} (m_i^2)^{d/2-2} F_1\left[1; 1, 2-d/2; 2; -\frac{x_{ij}^2}{(x_k - x_{ij})^2 - x_{ij}^2}, \frac{p_i^2 x_{ij}^2}{p_i^2 x_{ij}^2 + M_{ij}^2 - \mathrm{i}\rho}\right] - \{x_{ij} \leftrightarrow x_{ji}; x_k \leftrightarrow 1 - x_k\}$$

General case

Scalar one-loop three-point Feynman integrals

- Generalizing Master integral for general case in which $p_i^2 \neq 0$, $m_i^2 \neq 0$ $(i = 1, 2, 3)$ follows the idea in paper[a]. The C_0's integral can be written in the compact form;

$$C_0(\{p_i^2\}; \{m_i^2\}) = \frac{\Gamma(2 - d/2)}{\lambda^{1/2}(p_1^2, p_2^2, p_3^2)} [b_3 - J_{312} - J_{123} - J_{231}]$$

- The term b_3 is given as

$$b_3 = -\sum_{l=1}^{3} \frac{\left(M_{123}^2 - \mathrm{i}\rho\right)^{d/2-2}}{y_l} {}_2F_1 [1, 1; 2, 1/y_l]$$

- For instant, one takes J_{312} as an example, then the same way for J_{123} and J_{231}. The analytic result of integral J_{312} will be written as follows

$$\begin{aligned} J_{312} = & \left(\frac{\partial_1 S_{13}}{G_{13}}\right) \frac{\partial_2 S_{123}}{\sqrt{8 G_{123}}} \frac{\left(M_{13}^2 - \mathrm{i}\rho\right)^{d/2-2}}{M_{123}^2 - M_{13}^2} F_1 \left[\frac{1}{2}; 1, 2 - \frac{d}{2}; \frac{3}{2}; \frac{M_{13}^2 - m_3^2}{M_{13}^2 - M_{123}^2}, 1 - \frac{m_3^2}{M_{13}^2}\right] \\ & + \left(\frac{\partial_3 S_{13}}{G_{13}}\right) \frac{\partial_2 S_{123}}{\sqrt{8 G_{123}}} \frac{\left(M_{13}^2 - \mathrm{i}\rho\right)^{d/2-2}}{M_{123}^2 - M_{13}^2} F_1 \left[\frac{1}{2}; 1, 2 - \frac{d}{2}; \frac{3}{2}; \frac{M_{13}^2 - m_1^2}{M_{13}^2 - M_{123}^2}, 1 - \frac{m_1^2}{M_{13}^2}\right] \\ & + \frac{M_{13}^2 - m_3^2}{2\left(M_{123}^2 - M_{13}^2\right)} \left(M_{13}^2 - \mathrm{i}\rho\right)^{d/2-2} F_1 \left[1; 1, 2 - \frac{d}{2}; 2; \frac{M_{13}^2 - m_3^2}{M_{13}^2 - M_{123}^2}, 1 - \frac{m_3^2}{M_{13}^2}\right] \\ & - \frac{M_{13}^2 - m_1^2}{2\left(M_{123}^2 - M_{13}^2\right)} \left(M_{13}^2 - \mathrm{i}\rho\right)^{d/2-2} F_1 \left[1; 1, 2 - \frac{d}{2}; 2; \frac{M_{13}^2 - m_1^2}{M_{13}^2 - M_{123}^2}, 1 - \frac{m_1^2}{M_{13}^2}\right] \end{aligned}$$

[a] G. 't Hooft and M. J. G. Veltman, Nuclear Physics **B 153** (1979)

One-loop contributions to $H \to Z\gamma$ process

<u>Based on:</u>

[K. H. Phan and D. T. Tran, Prog. Theor. Exp. Phys. 053B08 (2020)]

Research approach

Tensor one-loop N-point Feynman integrals

- In general, tensor one-loop N-point Feynman integrals with tensor rank M and external momenta (internal masses) p_i (m_i) for $i = 1, 2, \ldots,$ N-external momenta are defined as

$$J_{N,\mu_1\mu_2\ldots\mu_M}(d;\{\nu_1,\nu_2,\ldots,\nu_N\}) \equiv J_{N,\mu_1\mu_2\ldots\mu_M}(d;\{\nu_1,\nu_2,\ldots,\nu_N\};\{p_ip_j;m_i^2\}) =$$
$$\int \frac{d^dk}{i\pi^{d/2}} \frac{k_{\mu_1}k_{\mu_2}\ldots k_{\mu_M}}{[(k+q_1)^2-m_1^2+i\rho]^{\nu_1}[(k+q_2)^2-m_2^2+i\rho]^{\nu_2}\ldots[(k+q_N)^2-m_N^2+i\rho]^{\nu_N}}$$

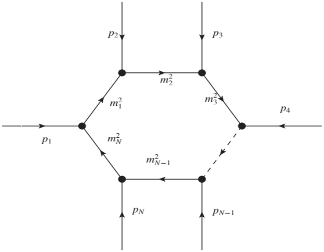

Figure 2 : Generic Feynman diagrams at one-loop

Research approach

Tensor one-loop reduction

- [A. Denner, Fortsch. Phys. 41, 307 (1993)]
  - Performing in space-time $d = 4 - 2\varepsilon$ at ε^0-expansions or set $d = 4$ at beginning.
  - The results cannot be extended to higher-loop contributions to the decay channels.
- [A. I. Davydychev, Phys. Lett. B 263 (1991) 107]
  - Carrying out in general space-time d.
  - Scalar one-loop functions at arbitrary d are able to gain stable numerical results.
  - The higher-order ε-expansion results in $d = 4 - 2\varepsilon$ are needed in computing for the decay channels at two- and higher-loop Feynman diagram contributions.

$\Longrightarrow$ In this presentation, we follow Davydychev's reduction method !!!

Research approach

Tensor one-loop N-point Feynman integrals

- Applying Tensor reduction method[a], tensor one-loop integrals can be reduced to scalar functions with the shifted space-time dimension $d + 2(M - \lambda)$ and raising the indices of propagators $\{\nu_i + \kappa_i\}$ for $i = 1, 2, \ldots, N$.

$$J_{N,\mu_1\mu_2\cdots\mu_M}(d; \{\nu_1, \nu_2, \cdots, \nu_N\}) = \sum_{\lambda,\kappa_1,\ldots,\kappa_N} \left(-\frac{1}{2}\right)^{\lambda} \Big\{[g]^{\lambda}[q_1]^{\kappa_1}[q_2]^{\kappa_2}\cdots[q_N]^{\kappa_N}\Big\}_{\mu_1\mu_2\cdots\mu_M}$$
$$\times(\nu_1)_{\kappa_1}(\nu_2)_{\kappa_2}\cdots(\nu_N)_{\kappa_N}\, J_N(d + 2(M - \lambda); \{\nu_1 + \kappa_1, \nu_2 + \kappa_2, \cdots, \nu_N + \kappa_N\})$$

- The arbitrary scalar integrals $J_N(d; \{\nu_1, \nu_2, \cdots, \nu_N\})$ will be casted into subset of Master integrals by using Integration-by-part method (IBP)[b].

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[a] A. I. Davydychev, Phys. Lett. B **263** (1991) 107

[b] F.V. Tkachov, Phys. Lett. **B100** (1981) 65; K.G. Chetyrkin and F.V. Tkachov, Nucl. Phys. **B192** (1981) 159

Research approach

Tensor one-loop three-point Feynman integrals appearing in $H \to Z\gamma$ process

$$J_{3,\mu_1\mu_2\ldots\mu_M}(d;\{\nu_1,\nu_2,\nu_3\}) = \int \frac{\mathrm{d}^d k}{i\pi^{d/2}} \frac{k_{\mu_1}k_{\mu_2}\ldots k_{\mu_M}}{[(k+q_2)^2 - M^2]^{\nu_1}[(k+p)^2 - M^2]^{\nu_2}(k^2 - M^2)^{\nu_3}}$$

- Some kinematic invariances related $H \to Z\gamma$ process are defined by $q_1^2 = M_Z^2, q_2^2 = 0, p^2 = (q_1+q_2)^2 = M_H^2, p_2^2 = M_Z^2, 0$ and internal masses $M^2 = m_f^2, M_W^2$.

Integration-by-parts method (IBP)

- Appling IBP relations on the momentum of three internal lines in the integrand of the scalar one-loop three-point Feynman integral $J_3(d;\{\nu_1,\nu_2,\nu_3\};p_2^2,M_H^2,M^2)$.
- The system of equations is written in terms of the standard notation for increasing and lowering operators has been used, e.g. $\mathbf{j}^{\pm} J_3(d;\{\nu_1,\nu_2,\nu_3\}) = J_3(d;\{\nu_j \pm 1\})$ arrived as

$$\begin{cases} (d-2\nu_1-\nu_2-\nu_3)\mathbf{1} - \nu_2\mathbf{1}^-\mathbf{2}^+ - \nu_3\mathbf{1}^-\mathbf{3}^+ = \nu_1(2M^2)\mathbf{1}^+ + \nu_2(2M^2-q_1^2)\mathbf{2}^+ + \nu_3(2M^2-q_2^2)\mathbf{3}^+ \\ (d-\nu_1-2\nu_2-\nu_3)\mathbf{1} - \nu_1\mathbf{1}^+\mathbf{2}^- - \nu_3\mathbf{2}^-\mathbf{3}^+ = \nu_1(2M^2-q_1^2)\mathbf{1}^+ + \nu_2(2M^2)\mathbf{2}^+ + \nu_3(2M^2-p^2)\mathbf{3}^+ \\ (d-\nu_1-\nu_2-2\nu_3)\mathbf{1} - \nu_1\mathbf{1}^+\mathbf{3}^- - \nu_2\mathbf{2}^+\mathbf{3}^- = \nu_1(2M^2-q_2^2)\mathbf{1}^+ + \nu_2(2M^2-p^2)\mathbf{2}^+ + \nu_3(2M^2)\mathbf{3}^+ \end{cases}$$

- Master integrals $J_3(d;\{\nu_1,\nu_2,\nu_3\};p_2^2,M_H^2,M^2)$ are considered by solving the above system of equations in several special cases and then can be presented in terms of hypergeometric functions ${}_3F_2$.

Research approach

Master integrals involved $H \to Z\gamma$ decay process in case $\nu_1 = \nu_2 = \nu_3 = 1$

$$J_3(d;\{1,2,1\};p_2^2,M_H^2,M^2) = \frac{2}{(p_2^2-M_H^2)} J_2(d;\{2,1\},M_H^2,M^2) + \frac{2}{(M_H^2-p_2^2)} J_2(d;\{2,1\},p_2^2,M^2)$$

$$J_3(d;\{2,1,1\};p_2^2,M_H^2,M^2) = \frac{(d-4)M_H^2}{2M^2(M_H^2-p_2^2)} J_3(d;\{1,1,1\};p_2^2,M_H^2,M^2) + \frac{2}{(p_2^2-M_H^2)} J_2(d;\{2,1\},0,M^2) + \frac{M_H^2(4M^2-M_H^2)}{M^2(M_H^2-p_2^2)^2} J_2(d;\{2,1\},M_H^2,M^2) + \frac{p_2^2M_H^2-2M^2(p_2^2+M_H^2)}{M^2(M_H^2-p_2^2)^2} J_2(d;\{2,1\},p_2^2,M^2)$$

$$J_3(d;\{1,1,2\};p_2^2,M_H^2,M^2) = \frac{(d-4)p_2^2}{2M^2(p_2^2-M_H^2)} J_3(d;\{1,1,1\};p_2^2,M_H^2,M^2) + \frac{2}{(M_H^2-p_2^2)} J_2(d;\{2,1\},0,M^2) + \frac{p_2^2M_H^2-2M^2(M_H^2+p_2^2)}{M^2(M_H^2-p_2^2)^2} J_2(d;\{2,1\},M_H^2,M^2) + \frac{p_2^2(4M^2-p_2^2)}{M^2(M_H^2-p_2^2)^2} J_2(d;\{2,1\},p_2^2,M^2)$$

Research approach

Master integrals involved $H \to Z\gamma$ decay process in case $\nu_1 = 1, \nu_2 = 2, \nu_3 = 1$

$$J_3(d;\{1,3,1\};p_2^2,M_H^2,M^2) = \frac{1}{2(p_2^2-M_H^2)} J_2(d;\{2,2\},M_H^2,M^2) + \frac{1}{2(M_H^2-p_2^2)} J_2(d;\{2,2\},p_2^2,M^2) + \frac{1}{(p_2^2-M_H^2)} J_2(d;\{3,1\},M_H^2,M^2) + \frac{1}{(M_H^2-p_2^2)} J_2(d;\{3,1\},p_2^2,M^2)$$

$$J_3(d;\{2,2,1\};p_2^2,M_H^2,M^2) = \frac{(4-d)}{2M^2(M_H^2-p_2^2)} J_3(d;\{1,1,1\};p_2^2,M_H^2,M^2)$$
$$+\frac{M_H^2\left(4M^2-M_H^2\right)}{2M^2(M_H^2-p_2^2)^2}\left[J_2(d;\{2,2\},M_H^2,M^2)+2J_2(d;\{3,1\},M_H^2,M^2)\right] + \frac{(6-d)M_H^2}{M^2(M_H^2-p_2^2)^2} J_2(d;\{2,1\},M_H^2,M^2)$$
$$+\frac{p_2^2M_H^2-2M^2(M_H^2+p_2^2)}{2M^2(M_H^2-p_2^2)^2}\left[J_2(d;\{2,2\},p_2^2,M^2)+2J_2(d;\{3,1\},p_2^2,M^2)\right] + \frac{(d-5)M_H^2-p_2^2}{M^2(M_H^2-p_2^2)^2} J_2(d;\{2,1\},p_2^2,M^2)$$

$$J_3(d;\{1,2,2\};p_2^2,M_H^2,M^2) = \frac{(d-4)}{2M^2(M_H^2-p_2^2)} J_3(d;\{1,1,1\};p_2^2,M_H^2,M^2)$$
$$+\frac{p_2^2M_H^2-2M^2(M_H^2+p_2^2)}{2M^2(M_H^2-p_2^2)^2}\left[J_2(d;\{2,2\},M_H^2,M^2)+2J_2(d;\{3,1\},M_H^2,M^2)\right] + \frac{(6-d)p_2^2}{M^2(M_H^2-p_2^2)^2} J_2(d;\{2,1\},p_2^2,M^2)$$
$$+\frac{p_2^2\left(4M^2-p_2^2\right)}{2M^2(M_H^2-p_2^2)^2}\left[J_2(d;\{2,2\},p_2^2,M^2)+2J_2(d;\{3,1\},p_2^2,M^2)\right] + \frac{(d-5)p_2^2-M_H^2}{M^2(M_H^2-p_2^2)^2} J_2(d;\{2,1\},M_H^2,M^2)$$

Research approach

Master integrals involved $H \to Z\gamma$ decay process

- The analytic results for the scalar one-loop Feynman integrals[a] are performed in terms of hypergeometric ${}_3F_2$ functions.

- Scalar one-loop one-point functions with the arbitrary propagator index ν

$$J_1(d; \{\nu\}; M^2) = (-1)^{\nu} \frac{\Gamma(\nu - d/2)}{\Gamma(\nu)} (M^2)^{d/2-\nu}$$

- Scalar one-loop two-point functions with the general propagator indexes ν_1, ν_2

$$J_2(d; \{\nu_1, \nu_2\}; p^2, M^2) = (-1)^{N_2} \frac{\Gamma(N_2 - d/2)}{\Gamma(N_2)} (M^2)^{d/2-N_2} \; {}_3F_2\left[\begin{matrix} \nu_1, \nu_2, N_2 - d/2\,; \\ \frac{N_2}{2}, \frac{N_2+1}{2}\,; \end{matrix} \; \frac{p^2}{4M^2} \right]$$

- In the case involved $H \to Z\gamma$ decay process, we confirmed some related internal masses and external momenta $m_1^2 = m_2^2 = M^2 = m_f^2, M_W^2$; $p^2 = 0, M_H^2, M_Z^2$ and $N_2 = \nu_1 + \nu_2$.

[a] K. H. Phan and T. Riemann, Phys. Lett. B 791, 257 (2019); K. H. Phan and D. T. Tran, Prog. Theor. Exp. Phys. 063B01 (2019); K. H. Phan and D. T. Tran, Prog. Theor. Exp. Phys. 053B08 (2020); K. H. Phan, Eur. Phys. J. C 80 (2020) 5, 414.

Research approach

Master integrals involved $H \to Z\gamma$ decay process

- Scalar one-loop three-point functions with some specific indices of propagators

$$\frac{J_3(d;\{1,1,1\};p_2^2,M_H^2,M^2)}{\Gamma(2-d/2)} = \frac{(d-4)M_H^2}{4(M_H^2-p_2^2)}(M^2)^{d/2-3}\left\{ {}_3F_2\left[\begin{matrix}1,1,3-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_H^2}{4M^2}\right] - {}_3F_2\left[\begin{matrix}1,1,3-d/2;\\ 3/2,2;\end{matrix}\ \frac{p_2^2}{4M^2}\right]\right\}$$

$$\frac{J_3(d;\{1,2,1\};p_2^2,M_H^2,M^2)}{\Gamma(2-d/2)} = \frac{(4-d)}{2(M_H^2-p_2^2)}(M^2)^{d/2-3}\left\{ {}_3F_2\left[\begin{matrix}1,2,3-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_H^2}{4M^2}\right] - {}_3F_2\left[\begin{matrix}1,2,3-d/2;\\ 3/2,2;\end{matrix}\ \frac{p_2^2}{4M^2}\right]\right\}$$

$$\frac{J_3(d;\{1,3,1\};p_2^2,M_H^2,M^2)}{(6-d)\,\Gamma(2-d/2)} = \frac{(d-4)}{16(M_H^2-p_2^2)}(M^2)^{d/2-4}\left\{ {}_3F_2\left[\begin{matrix}1,2,4-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_H^2}{4M^2}\right] - {}_3F_2\left[\begin{matrix}1,2,4-d/2;\\ 3/2,2;\end{matrix}\ \frac{p_2^2}{4M^2}\right]\right\}$$

$$\frac{J_3(d;\{2,2,1\};p_2^2,M_H^2,M^2)}{\Gamma(2-d/2)} = (4-d)\,(M^2)^{d/2-4}\times$$

$$\times\left\{ \frac{(6-d)\,M_H^2\left(4M^2-M_H^2\right)}{16M^2(M_H^2-p_2^2)^2}\,{}_3F_2\left[\begin{matrix}1,2,4-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_H^2}{4M^2}\right] - \frac{\left[2M_H^2(d-5)-2p_2^2\right]}{8(M_H^2-p_2^2)^2}\,{}_3F_2\left[\begin{matrix}1,2,3-d/2;\\ 3/2,2;\end{matrix}\ \frac{p_2^2}{4M^2}\right]\right.$$

$$+\frac{(6-d)\left[M_H^2p_2^2-2M^2(M_H^2+p_2^2)\right]}{16M^2(M_H^2-p_2^2)^2}\,{}_3F_2\left[\begin{matrix}1,2,4-d/2;\\ 3/2,2;\end{matrix}\ \frac{p_2^2}{4M^2}\right] + \frac{(4-d)\,p_2^2}{8(M_H^2-p_2^2)^2}\,{}_3F_2\left[\begin{matrix}1,1,3-d/2;\\ 3/2,2;\end{matrix}\ \frac{p_2^2}{4M^2}\right]$$

$$\left. -\frac{(6-d)\,M_H^2}{4(M_H^2-p_2^2)^2}\,{}_3F_2\left[\begin{matrix}1,2,3-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_H^2}{4M^2}\right] + \frac{(d-4)\,M_H^2}{8(M_H^2-p_2^2)^2}\,{}_3F_2\left[\begin{matrix}1,1,3-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_H^2}{4M^2}\right]\right\}$$

Feynman diagrams involved $H \to Z\gamma$ in Unitary gauge

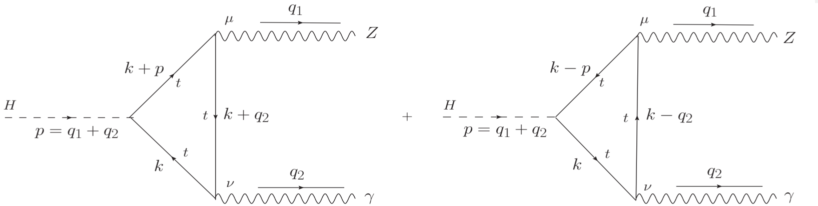

Figure 3 : Feynman diagrams contributing to the $H \to Z\gamma$ decay through Top quark loop in Unitary gauge.

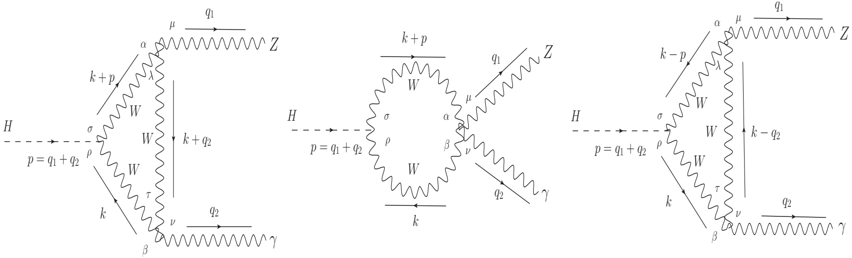

Figure 4 : Feynman diagrams contributing to the $H \to Z\gamma$ decay through W boson loop in Unitary gauge.

Form factors contributing to $H \to Z\gamma$ process

Total amplitude

- Using the symbolic-manipulation Package-X[a] to handle all Dirac and Tensor algebra in d dimension, the total amplitude $\mathcal{M}_{H\to Z\gamma}$ will be expressed in terms of Form factors with reflecting the Lorentz invariant structure and the content of gauge symmetry as follows

$$
\begin{aligned}
i\mathcal{M}_{H\to Z\gamma} &= i\mathcal{M}_{\mu\nu}\, \varepsilon_1^{\mu *}(q_1)\varepsilon_2^{\nu *}(q_2) \\
&= \left[F_{00}\, g_{\mu\nu} + \sum_{i,j=1}^{2} F_{ij}\, q_{i,\mu} q_{j,\nu} + F_5 \times i\epsilon_{\mu\nu\alpha\beta} q_1^{\alpha} q_2^{\beta} \right] \varepsilon_1^{\mu *}(q_1)\varepsilon_2^{\nu *}(q_2)
\end{aligned}
$$

- Related kinematic invariant: $q_1^2 = M_Z^2,\ q_2^2 = 0,\ p^2 = (q_1+q_2)^2 = M_H^2$.
- The polarization vectors of the Z boson and the photon γ: $\varepsilon_1^{\mu *}$ and $\varepsilon_2^{\nu *}$.
- On-shell condition for external photon $\varepsilon_2^{\nu *}(q_2) q_{2,\nu} = 0 \Rightarrow F_{12,22} = 0$.
- Following Ward identity $F_{11} = 0,\ F_{00} = -(q_1 \cdot q_2)\, F_{21} = (M_Z^2 - M_H^2)/2 \times F_{21}$.
- Summing all top quark loop diagrams $F_5 = 0$.

$$\Longrightarrow F_{11}, F_{12}, F_{22} \text{ and } F_5 \text{ do not contribute to } H \to Z\gamma \text{ process !!!}$$

---

[a] H. H. Patel, Comput. Phys. Commun. **197** (2015) 276

Form factors contributing to $H \to Z\gamma$ process

Total amplitude

- The total amplitude for this decay process is then casted into the form of

$$i\mathcal{M}_{H\to Z\gamma} = \frac{e^3}{\sin\theta_W M_W} \mathcal{F}_{H\to Z\gamma}(d; M_H^2, M_Z^2, M_W^2, m_f^2)\Big[q_{2,\mu} q_{1,\nu} - (q_1 \cdot q_2)\, g_{\mu\nu}\Big]\varepsilon_1^{\mu *}(q_1)\varepsilon_2^{\nu *}(q_2)$$

- $\mathcal{F}_{H\to Z\gamma}(d; M_H^2, M_Z^2, M_W^2, m_f^2)$ which are form factors and can be derived from F_{00} and F_{21} are decomposed in terms of W-loop and top-loop contributions as follows

$$\mathcal{F}_{H\to Z\gamma}(d; M_H^2, M_Z^2, M_W^2, m_f^2) = \cot\theta_W\, \mathcal{F}^{(W)}_{H\to Z\gamma}(d; M_H^2, M_Z^2, M_W^2) + \sum_f \frac{Q_f N_C}{e}\left(\lambda_1^f + \lambda_2^f\right)\mathcal{F}^{(f)}_{H\to Z\gamma}(d; M_H^2, M_Z^2, m_f^2)$$

  - I_f^3, Q_f and m_f are iso-spin, electric charge, mass of fermions f in the loops respectively.
  - θ_W is Weinberg angle and N_C is a color factor for fermions which will be 1 for leptons and 3 for quarks.

Form factors $\mathcal{F}^{(W)}_{H\to Z\gamma}$ [First representation]

- For the first time, one-loop form factors for the decay process are shown which are valid at arbitrary space–time dimension d.

First representation: Form factors are derived from F_{00} in terms of ${}_3F_2$ hypergeometric functions for W-loop contribution.

$$\frac{\mathcal{F}^{(W)}_{H\to Z\gamma}(d; M_H^2, M_Z^2, M_W^2)}{\Gamma(2-d/2)} = \frac{(M_W^2)^{d/2-2}}{(4\pi)^{d/2}\, M_W^2 (M_Z^2 - M_H^2)^2} \times$$

$$\times \Bigg\{ (4-d)(M_Z^2 - 4M_W^2)(M_H^2 - M_Z^2) \left(M_H^2 \, {}_3F_2\left[\begin{matrix} 1,1,3-d/2; \\ 3/2,2; \end{matrix} \; \frac{M_H^2}{4M_W^2} \right] - M_Z^2 \, {}_3F_2\left[\begin{matrix} 1,1,3-d/2; \\ 3/2,2; \end{matrix} \; \frac{M_Z^2}{4M_W^2} \right] \right)$$

$$+ \left[2M_W^2 (M_H^2 - M_Z^2) - M_H^2 M_Z^2 + 4M_W^4 (d-1) \right] \times$$

$$\times \left((M_H^2 - M_Z^2) \, {}_3F_2\left[\begin{matrix} 2,1,2-d/2; \\ 3/2,2; \end{matrix} \; \frac{M_H^2}{4M_W^2} \right] \right.$$

$$\left. + M_Z^2 \, {}_3F_2\left[\begin{matrix} 1,1,2-d/2; \\ 3/2,2; \end{matrix} \; \frac{M_Z^2}{4M_W^2} \right] - M_H^2 \, {}_3F_2\left[\begin{matrix} 1,1,2-d/2; \\ 3/2,2; \end{matrix} \; \frac{M_H^2}{4M_W^2} \right] \right) \Bigg\}$$

Form factors $\mathcal{F}^{(t)}_{H\to Z\gamma}$ [First representation]

First representation: Form factors are derived from F_{00} in terms of ${}_3F_2$ hypergeometric functions for Top-loop contribution.

$$\frac{\mathcal{F}^{(t)}_{H\to Z\gamma}(d; M_H^2, M_Z^2, m_t^2)}{\Gamma(2-d/2)} = \frac{(m_t^2)^{d/2-2}}{(4\pi)^{d/2}\,(M_Z^2-M_H^2)^2} \times$$

$$\times\Big\{ (4-d)\,M_H^2(M_H^2-M_Z^2)\,{}_3F_2\left[\begin{matrix}1,1,3-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_H^2}{4m_t^2}\right] + 8M_H^2 m_t^2\,{}_3F_2\left[\begin{matrix}1,1,2-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_H^2}{4m_t^2}\right]$$

$$+(4-d)\,M_Z^2(M_Z^2-M_H^2)\,{}_3F_2\left[\begin{matrix}1,1,3-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_Z^2}{4m_t^2}\right] - 8M_Z^2 m_t^2\,{}_3F_2\left[\begin{matrix}1,1,2-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_Z^2}{4m_t^2}\right]$$

$$-8(M_H^2-M_Z^2)m_t^2\,{}_3F_2\left[\begin{matrix}1,1,2-d/2;\\ 3/2,1;\end{matrix}\ \frac{M_H^2}{4m_t^2}\right]\Big\}$$

- The form factors $\mathcal{F}^{(f)}_{H\to Z\gamma}(d; M_H^2, M_Z^2, m_f^2)$ for fermions are obtained by replacing $m_t \to m_f$.
- For fermion masses are smaller than $M_H/2$, the argument of hypergeometric functions ${}_3F_2$ will be $|M_H^2/4m_f^2| > 1$. Applying analytic continuation for ${}_3F_2$ functions appearing in the form factors $\mathcal{F}^{(f)}_{H\to Z\gamma}(d; M_H^2, M_Z^2, m_f^2)$.
- In the limit $d \to 4$, we confirm that the terms in curly bracket of right hand side result of $\mathcal{F}^{(W)}_{H\to Z\gamma}(d; M_H^2, M_Z^2, M_W^2)$ and $\mathcal{F}^{(t)}_{H\to Z\gamma}(d; M_H^2, M_Z^2, m_t^2)$ will tend to zero.
- It means that the form factors always stay finite in the limit.

Form factors $\mathcal{F}^{(W)}_{H\to Z\gamma}$ [Second representation]

Second representation: Form factors are derived from F_{21} for W-loop contribution.

$$\frac{\mathcal{F}^{(W)}_{H\to Z\gamma}(d; M_H^2, M_Z^2, M_W^2)}{\Gamma(2-d/2)} = \frac{(M_W^2)^{d/2-2}}{(4\pi)^{d/2}\, M_W^4 (M_Z^2 - M_H^2)^2} \times$$

$$\times \Bigg\{ (4-d)\, M_W^2 (M_Z^2 - 4M_W^2)(M_H^2 - M_Z^2) \left(M_H^2\, {}_3F_2\left[\begin{matrix} 1,1,3-d/2; \\ 3/2,2; \end{matrix} \frac{M_H^2}{4M_W^2}\right] - M_Z^2\, {}_3F_2\left[\begin{matrix} 1,1,3-d/2; \\ 3/2,2; \end{matrix} \frac{M_Z^2}{4M_W^2}\right]\right)$$

$$+ \left[2M_W^4(M_H^2 - M_Z^2) - M_H^2 M_W^2 M_Z^2 + 4M_W^6(d-1)\right] \times$$

$$\times \Bigg[\frac{M_H^2\left(6M_W^2 - M_H^2\right) - 2M_W^2 M_Z^2}{2M_W^2}\, {}_3F_2\left[\begin{matrix} 2,1,2-d/2; \\ 3/2,2; \end{matrix} \frac{M_H^2}{4M_W^2}\right] + \frac{M_H^2(M_Z^2 - 4M_W^2)}{2M_W^2}\, {}_3F_2\left[\begin{matrix} 2,1,2-d/2; \\ 3/2,2; \end{matrix} \frac{M_Z^2}{4M_W^2}\right]$$

$$-2M_H^2 \left({}_3F_2\left[\begin{matrix} 2,1,2-d/2; \\ 3/2,2; \end{matrix} \frac{M_H^2}{4M_W^2}\right] - \frac{M_H^2}{6M_W^2}\, {}_3F_2\left[\begin{matrix} 3,2,2-d/2; \\ 5/2,3; \end{matrix} \frac{M_H^2}{4M_W^2}\right]\right)$$

$$+ \frac{2M_H^2(d-1) - 2M_Z^2}{(d-2)} \left({}_3F_2\left[\begin{matrix} 2,1,2-d/2; \\ 3/2,2; \end{matrix} \frac{M_Z^2}{4M_W^2}\right] - \frac{M_Z^2}{6M_W^2}\, {}_3F_2\left[\begin{matrix} 3,2,2-d/2; \\ 5/2,3; \end{matrix} \frac{M_Z^2}{4M_W^2}\right]\right)$$

$$+ \frac{d}{(2-d)} \left(M_H^2\, {}_3F_2\left[\begin{matrix} 1,1,2-d/2; \\ 3/2,2; \end{matrix} \frac{M_H^2}{4M_W^2}\right] - \frac{M_H^4}{12M_W^2}\, {}_3F_2\left[\begin{matrix} 2,2,2-d/2; \\ 5/2,3; \end{matrix} \frac{M_H^2}{4M_W^2}\right]\right)$$

$$+ \frac{d}{(d-2)} \left(M_Z^2\, {}_3F_2\left[\begin{matrix} 1,1,2-d/2; \\ 3/2,2; \end{matrix} \frac{M_Z^2}{4M_W^2}\right] - \frac{M_Z^4}{12M_W^2}\, {}_3F_2\left[\begin{matrix} 2,2,2-d/2; \\ 5/2,3; \end{matrix} \frac{M_Z^2}{4M_W^2}\right]\right) \Bigg] \Bigg\}$$

- In the limit $d \to 4$, we also confirm that the form factors always stay in finite.

Form factors $\mathcal{F}^{(t)}_{H\to Z\gamma}$ [Second representation]

Second representation: Form factors are derived from F_{21} for Top-loop contribution.

$$
\begin{aligned}
\frac{\mathcal{F}^{(t)}_{H\to Z\gamma}(d;M_H^2,M_Z^2,m_t^2)}{\Gamma(2-d/2)} = \Bigg\{ & \frac{4M_H^2\left(4m_t^2-M_H^2\right)}{3}\left({}_3F_2\left[\begin{matrix}2,2,2-d/2;\\ 5/2,2;\end{matrix}\ \frac{M_H^2}{4m_t^2}\right] + 2\ {}_3F_2\left[\begin{matrix}3,1,2-d/2;\\ 5/2,2;\end{matrix}\ \frac{M_H^2}{4m_t^2}\right]\right) \\
& + \frac{4M_H^2M_Z^2-8m_t^2(M_H^2+M_Z^2)}{3}\left({}_3F_2\left[\begin{matrix}2,2,2-d/2;\\ 5/2,2;\end{matrix}\ \frac{M_Z^2}{4m_t^2}\right] + 2\ {}_3F_2\left[\begin{matrix}3,1,2-d/2;\\ 5/2,2;\end{matrix}\ \frac{M_Z^2}{4m_t^2}\right]\right) \\
& -16M_H^2m_t^2\left({}_3F_2\left[\begin{matrix}2,1,2-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_H^2}{4m_t^2}\right] - \frac{M_H^2}{6m_t^2}\ {}_3F_2\left[\begin{matrix}3,2,2-d/2;\\ 5/2,3;\end{matrix}\ \frac{M_H^2}{4m_t^2}\right]\right) \\
& + \frac{16M_H^2m_t^2(d-1)-16M_Z^2m_t^2}{(d-2)}\left({}_3F_2\left[\begin{matrix}2,1,2-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_Z^2}{4m_t^2}\right] - \frac{M_Z^2}{6m_t^2}\ {}_3F_2\left[\begin{matrix}3,2,2-d/2;\\ 5/2,3;\end{matrix}\ \frac{M_Z^2}{4m_t^2}\right]\right) \\
& + \frac{8M_H^2m_t^2\,d}{(2-d)}\left({}_3F_2\left[\begin{matrix}1,1,2-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_H^2}{4m_t^2}\right] - \frac{M_H^2}{12m_t^2}\ {}_3F_2\left[\begin{matrix}2,2,2-d/2;\\ 5/2,3;\end{matrix}\ \frac{M_H^2}{4m_t^2}\right]\right) \\
& + \frac{8M_Z^2m_t^2\,d}{(d-2)}\left({}_3F_2\left[\begin{matrix}1,1,2-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_Z^2}{4m_t^2}\right] - \frac{M_Z^2}{12m_t^2}\ {}_3F_2\left[\begin{matrix}2,2,2-d/2;\\ 5/2,3;\end{matrix}\ \frac{M_Z^2}{4m_t^2}\right]\right) \\
& +8m_t^2(M_H^2-M_Z^2)\left({}_3F_2\left[\begin{matrix}2,1,2-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_H^2}{4m_t^2}\right] - {}_3F_2\left[\begin{matrix}2,1,2-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_Z^2}{4m_t^2}\right]\right) \\
+(d-4)(M_H^2-M_Z^2)\Bigg(& M_H^2\ {}_3F_2\left[\begin{matrix}1,1,3-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_H^2}{4m_t^2}\right] - M_Z^2\ {}_3F_2\left[\begin{matrix}1,1,3-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_Z^2}{4m_t^2}\right]\Bigg)\Bigg\} \frac{(m_t^2)^{d/2-2}}{(4\pi)^{d/2}(M_Z^2-M_H^2)^2}
\end{aligned}
$$

Form factors contributing to $H \to \gamma\gamma$ process

One-loop contributions to $H \to \gamma\gamma$ process reduction

- Taking some limits $M_Z^2 \to 0$, and $\lambda_1^f = eQ_f$, $\lambda_2^f, \lambda_3^f \to 0$, the total amplitude of the decay $H \to Z\gamma$ will be reduced to that of the decay $H \to \gamma\gamma$ as follows

$$i\mathcal{M}_{H\to\gamma\gamma} = \frac{e^2 g}{M_W}\mathcal{F}_{H\to\gamma\gamma}(d; M_H^2, M_W^2, m_f^2)\Big[(q_1\cdot q_2)\,g_{\mu\nu} - q_{2,\mu}q_{1,\nu}\Big]\varepsilon_1^{\mu *}(q_1)\varepsilon_2^{\nu *}(q_2)$$

$$\mathcal{F}_{H\to\gamma\gamma}(d; M_H^2, M_W^2, m_f^2) = \mathcal{F}^{(W)}_{H\to\gamma\gamma}(d; M_H^2, M_W^2) + \sum_f N_C Q_f^2 \mathcal{F}^{(f)}_{H\to\gamma\gamma}(d; M_H^2, m_f^2)$$

- Form factor for W-loop contribution:

$$\frac{\mathcal{F}^{(W)}_{H\to\gamma\gamma}(d; M_H^2, M_W^2)}{\Gamma(2-d/2)} = \frac{(M_W^2)^{d/2-2}}{(4\pi)^{d/2}}\Bigg\{\left(2+\frac{M_H^2}{M_W^2}\right){}_3F_2\left[\begin{matrix}2,1,2-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_H^2}{4M_W^2}\right]$$
$$-\left[4+\frac{M_H^2}{M_W^2}+4(d-1)\frac{M_W^2}{M_H^2}\right]{}_3F_2\left[\begin{matrix}1,1,2-d/2;\\ 3/2,1;\end{matrix}\ \frac{M_H^2}{4M_W^2}\right]$$
$$+\left[2+4(d-1)\frac{M_W^2}{M_H^2}\right]{}_3F_2\left[\begin{matrix}1,1,2-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_H^2}{4M_W^2}\right] - 4(d-4)\ {}_3F_2\left[\begin{matrix}1,1,3-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_H^2}{4M_W^2}\right]\Bigg\}$$

- Form factor for top-loop contribution:

$$\frac{\mathcal{F}^{(t)}_{H\to\gamma\gamma}(d; M_H^2, m_t^2)}{\Gamma(2-d/2)} = \frac{(m_t^2)^{d/2-2}}{(4\pi)^{d/2}}\Bigg\{-\frac{8m_t^2}{M_H^2}\,{}_3F_2\left[\begin{matrix}1,1,2-d/2;\\ 3/2,1;\end{matrix}\ \frac{M_H^2}{4m_t^2}\right]$$
$$+(4-d)\ {}_3F_2\left[\begin{matrix}1,1,3-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_H^2}{4m_t^2}\right] + \frac{8m_t^2}{M_H^2}\,{}_3F_2\left[\begin{matrix}1,1,2-d/2;\\ 3/2,2;\end{matrix}\ \frac{M_H^2}{4m_t^2}\right]\Bigg\}$$

Numerical results

Numerical confirmation for Ward identity at general d

- Setting $M_H = 125$ GeV, $M_Z = 91.2$ GeV, $m_t = 173.5$ GeV and $M_W = 80.4$ GeV. Our numerical results are generated by using package `NumEXP`[a] for numerical ϵ-expansions hypergeometric functions.
- Confirming two representations for form factors at general d. It means that we confirm numerically Ward indentity at general d. Two representations for form factors are perfect agreement to last digit for $3.5 \leq d \leq 5.5$:

d	$\mathcal{F}^{(t)}_{H\to Z\gamma}(d; M_H^2, M_Z^2, m_t^2)$ in 1st Rep. $\mathcal{F}^{(t)}_{H\to Z\gamma}(d; M_H^2, M_Z^2, m_t^2)$ in 2nd Rep.	$\mathcal{F}^{(W)}_{H\to Z\gamma}(d; M_H^2, M_Z^2, M_W^2)$ in 1st Rep. $\mathcal{F}^{(W)}_{H\to Z\gamma}(d; M_H^2, M_Z^2, M_W^2)$ in 2nd Rep.
3.5	−0.00117666222408164570889597705142 −0.00117666222408164570889597705142	−0.00924203129694608232780754562475 −0.00924203129694608232780754562475
4.5	−0.0756076123635421878866551078159 −0.0756076123635421878866551078159	−0.211488266331639234594811276488 −0.211488266331639234594811276488
5.0	−0.754001360017782779626359989943 −0.754001360017782779626359989943	−1.26786296363083047430009124220 −1.26786296363083047430009124220
5.5	−10.6345811567309032438825219401 −10.6345811567309032438825219401	−10.8040444333273283701507434992 −10.8040444333273283701507434992

Numerical confirmations for two representations of form factors involving to Top-loop diagrams at arbitrary d.

Numerical confirmations for two representations for form factors involving to W-loop diagrams at arbitrary d.

[a] Z. W. Huang and J. Liu, Comput. Phys. Commun. **184** (2013) 1973

Numerical results

Numerical confirmation for ϵ-expansions in space-time dimension $d = 4 - 2\varepsilon$

- Performing higher-order ϵ-expansion for the form factors up to ϵ^5 and comparing our results with paper[a] ($F^{SM}_{21,W}$ and $F^{SM}_{21,t}$) at ϵ^0-expansion terms. Giving a perfect agreement between two results at ϵ^0-expansion.
- It is important to note that higher-power ϵ-expansions for the form factors are our new results.

$$F^{SM}_{21,W} = -0.0418477713507083034768633206537\ \epsilon^0 + \mathcal{O}(\epsilon);$$

$$\begin{aligned}\mathcal{F}^{(W)}_{H\to Z\gamma}(d = 4 - 2\epsilon; M_H^2, M_Z^2, M_W^2) &= \\ = -0.0418477713507083034768633206537\ \epsilon^0 \\ +0.260913488721110921277821252790\ \epsilon^1 \\ -0.849415964842831522240990065525\ \epsilon^2 \\ +1.93196240724203383916822579654\ \epsilon^3 \\ -3.46717780533875010127157401115\ \epsilon^4 \\ +5.25914558345954670519178485415\ \epsilon^5 \\ +\mathcal{O}(\epsilon^6).\end{aligned}$$

$$F^{SM}_{21,t} = -0.00894937919735623466782637004746\ \epsilon^0 + \mathcal{O}(\epsilon);$$

$$\begin{aligned}\mathcal{F}^{(t)}_{H\to Z\gamma}(d = 4 - 2\epsilon; M_H^2, M_Z^2, m_t^2) &= \\ = -0.00894937919735623466782637004746\ \epsilon^0 \\ +0.0742785979879735824790115497100\ \epsilon^1 \\ -0.315615957203796781182876228270\ \epsilon^2 \\ +0.917527446546694361353843959657\ \epsilon^3 \\ -2.05845003852606360149227809637\ \epsilon^4 \\ +3.81281647820690166355588887060\ \epsilon^5 \\ + \mathcal{O}(\epsilon^6).\end{aligned}$$

[a] L. T. Hue, A. B. Arbuzov, T. T. Hong, T. P. Nguyen, D. T. Si and H. N. Long, Eur. Phys. J. C **78** (2018)

Conclusions and Outlooks

Conclusions

- Having presented the analytic solutions for scalar one-loop three-point Feynman integrals in arbitrary space-time dimensions (d) at general configuration and several special cases of internal masses and external momenta.
- The results have been presented in terms of generalized hypergeometric functions such as Gauss ${}_2F_1$ and Appell F_1 functions. The hypergeometric presentation can be performed higher-order ε-expansion.
- Having applied this method for computing one-loop contributions to $H \to Z\gamma$ process. For the first time, we have presented the form factors that are valid in general space-time dimension (d) and have confirmed again previous work which have been valid at ε^0-expansions in space-time dimension $d = 4 - 2\varepsilon$.
- It is important to note that higher-power ϵ-expansions for the form factors are our new results when they are needed in computing for higher-loop Feynman integral contributions to the decay channel.

Conclusions and Outlooks

Outlooks

- In near future works, it is expected that improving hypergeometric representation approach for scalar one-loop four-point and two-loop massive integrals.
- Additionally, the method for $H \to Z\gamma$ process can be extended to evaluate several one-loop contributions to Higgs decay to $Zf\bar{f}$, $f\bar{f}\gamma$ (working in progress), etc., within the SM and many BSMs.

THANK YOU VERY MUCH FOR YOUR ATTENTION !!!