Contents

Limit on the mass of particle dark matter

Lower bound on fermionic dark matter

- Pauli exclusion principle

→ The phase space density is bounded by the internal d.o.f

$f(x,p) < g$ → $m_{\rm DM} \gtrsim 70\ {\rm eV}$ (aka. Tremaine-Gunn limit)

Lower bound on bosonic dark matter

- The de Broglie wavelength of dark matter should be smaller than the core scale

$$\lambda_{\rm dB}^{\rm DM} = \frac{2\pi}{m_{\rm DM} v} \lesssim 1\ {\rm kpc} \quad \rightarrow \quad m_{\rm DM} \gtrsim 10^{-22}\ {\rm eV}$$

Upper bound on particle dark matter

- The Compton wavelength of dark matter should be larger than the Schwarzschild radius

$$\lambda_{\rm C}^{\rm DM} = \frac{2\pi}{m_{\rm DM}} \gtrsim r_S = 2G m_{\rm DM} \quad \rightarrow \quad m_{\rm DM} \lesssim M_{\rm Pl} \simeq 10^{19}\ {\rm GeV}$$

Popular dark matter candidates

- Weakly Interacting Massive Particles (WIMPs)

  (10 GeV – 100 TeV)

  - When phenomenologists discuss dark matter physics, WIMPs are implicitly assumed in many cases

- Light dark matter (keV – 10 GeV)

- Axion, Axion-Like Particle (ALP) (10^{-22} eV – 10^{-17} eV)

- [Primodial Black Hole (PBH)]

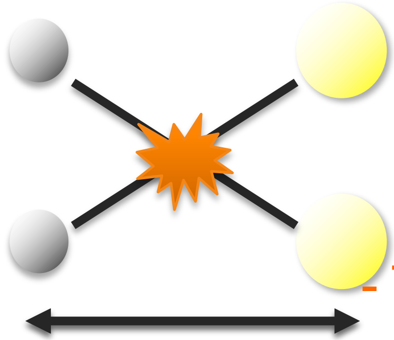
  - Not an elementary particle

  - Must have formed before BBN

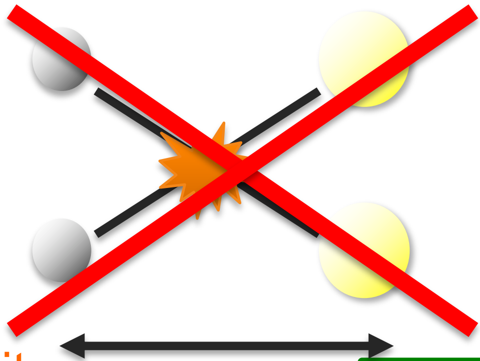
- etc.

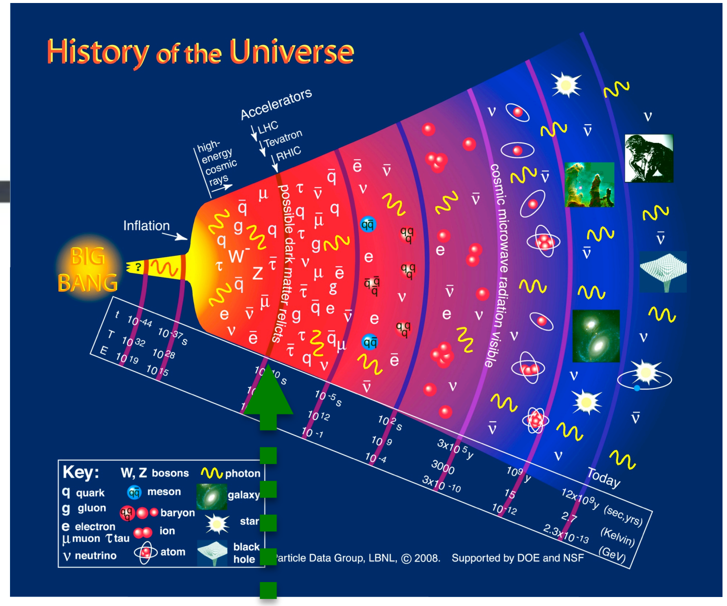
WIMP miracle

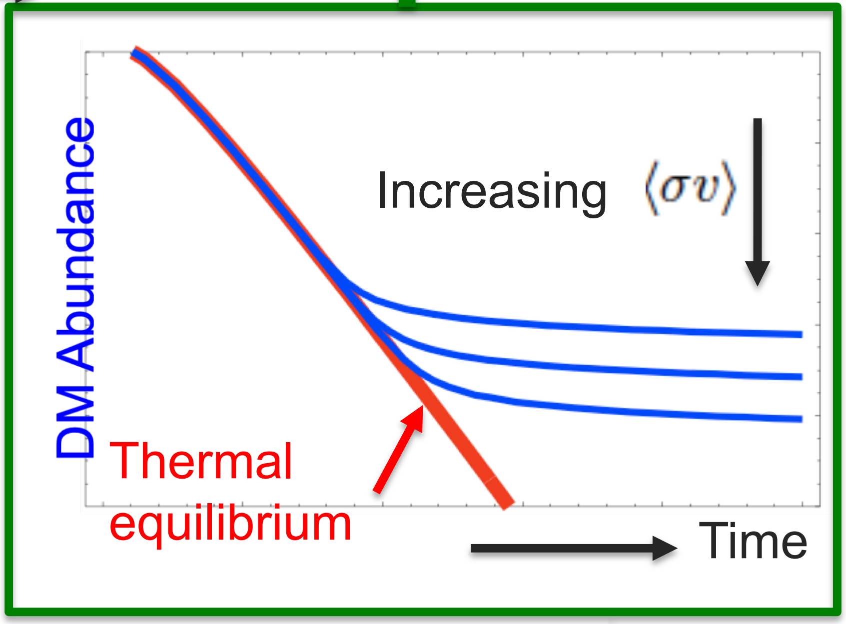
Thermal production of dark matter



- Temp. drops
- Number density decreases







Prediction: $\Omega_\chi h^2 \sim 0.1 \left(\frac{1\,\text{pb}}{\langle \sigma v \rangle} \right)$

σ : ann. cross section $\quad v$: velocity

- CDM abundance: $\Omega_{\rm CDM} \simeq 0.25$
- For WIMP DM: $\langle \sigma v \rangle \sim \frac{\pi \alpha}{m^2}$

$m \sim \mathcal{O}(1)$ TeV **New Physics at the TeV scale** [MSSM, UED, ...]

Boltzmann equation

- The evolution of a phase space distribution function $f(\boldsymbol{x}, \boldsymbol{p})$ is described by the Boltzmann equation:

$$L[f] = C[f]$$

- Liouville operator: Change of the phase space

- Collision operator: Action on the phase space under collision

WIMPs χ

- After simplifications and computations, the Boltzmann eq. reduces to the equation for the WIMP number density n_χ

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma v \rangle (n_\chi^2 - n_{\chi,\mathrm{eq}}^2)$$

$\langle \sigma v \rangle$: Thermal average of the annihilation cross section times relative velocity

$n_{\chi,\mathrm{eq}}$: Equilibrium number density

Boltzmann equation

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma v\rangle(n_\chi^2 - n_{\chi,\mathrm{eq}}^2)$$

Dilution by Hubble expansion

Depletion by annihilation $\chi\chi \to ff$

Creation from inverse process $ff \to \chi\chi$

- $\Gamma = n_\chi\langle\sigma v\rangle$ describes the interaction rate of the WIMP

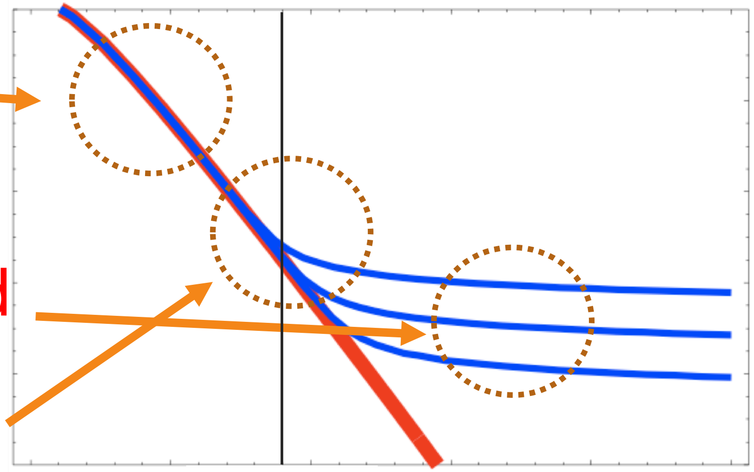
- If $\Gamma > H$

  $n_\chi = n_{\chi,\mathrm{eq}}$ Coupled

- If $\Gamma < H$

  n_χ is diluted by expansion Decoupled

- Annihilations freeze out when $\Gamma \sim H$



$T = T_F$

Present relic density

$$\Omega_\chi h^2 = m_\chi \frac{n_{\chi,0}}{s_0} s_0 \frac{h^2}{\rho_c}$$

Present entropy density

$$s_0 = 2.9 \times 10^3 \text{ cm}^3$$

After freeze-out, $\frac{n_\chi}{s}$ is fixed:

$$\frac{n_{\chi,0}}{s_0} = \left.\frac{n_\chi}{s}\right|_{T=T_F}$$

Hot relics

- Particles such that freeze out occurs when they are relativistic and $\frac{n_\chi}{s}$ is not changing

Neutrinos as hot relics

$$\Omega_\nu h^2 = \frac{m_\nu}{90 \text{ eV}}$$

Cold relics

- Particles such that freeze out occurs when they are non-relativistic and $\frac{n_\chi}{s}$ is rapidly changing

- Introducing $Y_\chi = n_\chi / s$ $x = m_\chi / T$, the Boltzmann eq. is

$$\frac{\mathrm{d}Y_\chi}{\mathrm{d}x} = -\frac{\langle \sigma v \rangle s}{Hx}\left(1 - \frac{x}{3g_{*s}}\frac{\mathrm{d}g_{*s}}{\mathrm{d}x}\right)\left(Y_\chi^2 - Y_{\mathrm{eq}}^2\right) \qquad Y_{\chi,\mathrm{eq}} = \frac{45}{4\sqrt{2}\pi^{7/2}}\frac{g_\chi}{g_{*s}}x^{3/2}\mathrm{e}^{-x}$$

$$\Rightarrow \quad Y_\chi(x \to \infty) = \frac{1}{(4\pi/\sqrt{90})m_\chi M_{\mathrm{Pl}} I(x_F)} \qquad I(x_F) = \int_{x_F}^{\infty} dx \frac{\sqrt{g_{*\mathrm{eff}}}\langle \sigma v \rangle}{x^2}$$

$$\Rightarrow \quad \Omega_\chi h^2 \sim 0.1 \left(\frac{1\ \mathrm{pb}}{\langle \sigma v \rangle}\right)$$

Coannihilation

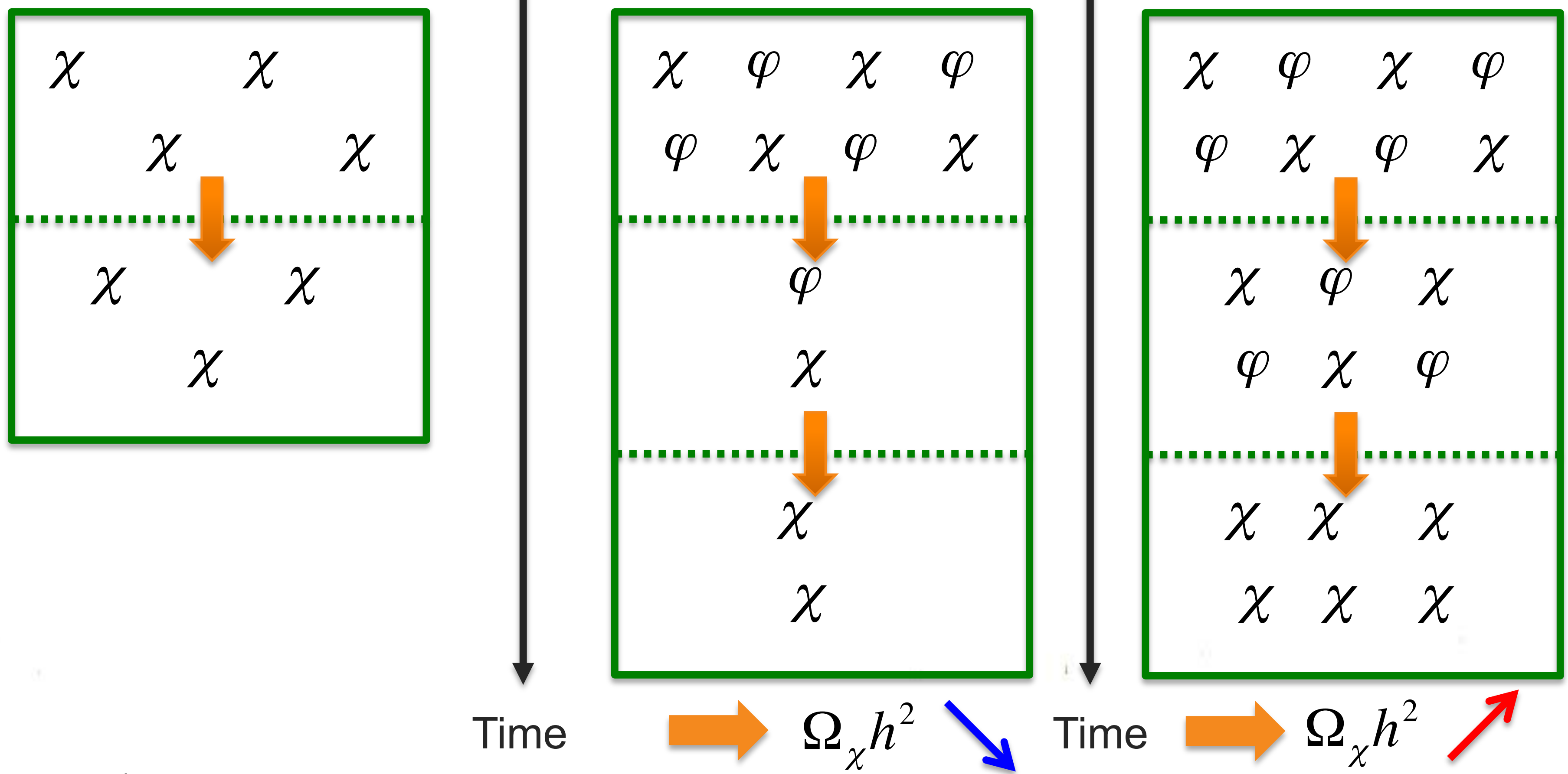
Only self-annihilation

$$\chi\chi \to SM, SM$$

Coannihilation with the next lightest particle φ

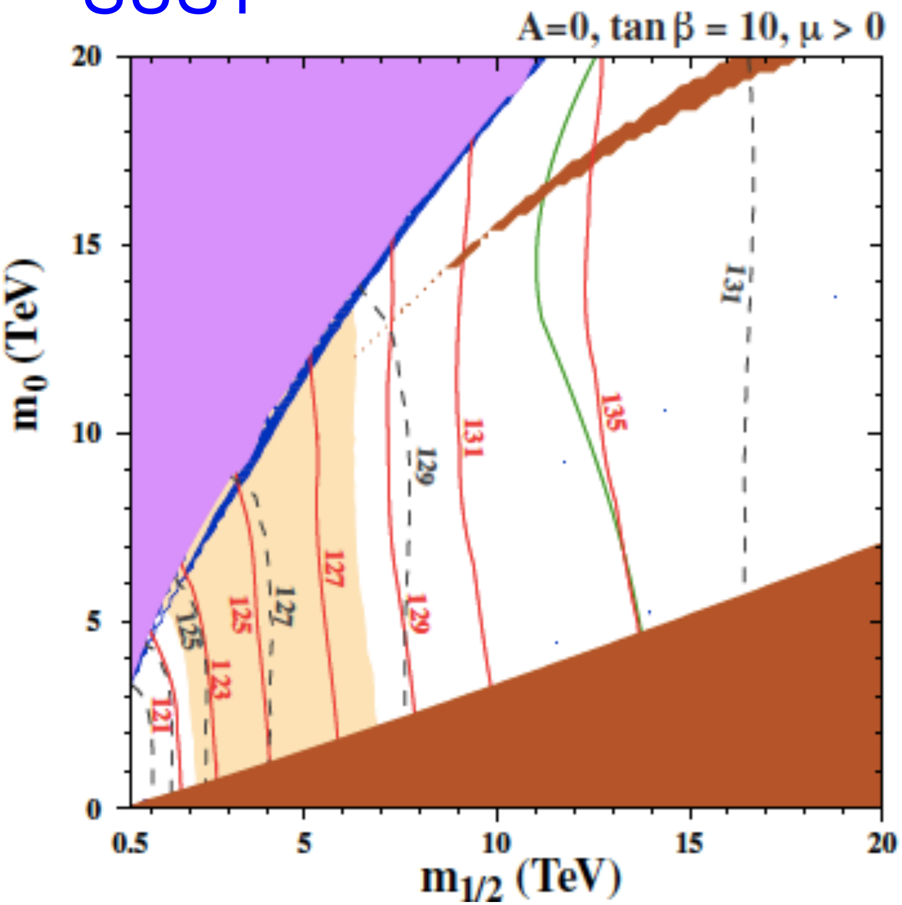
$$\chi\varphi, \varphi\varphi \to SM, SM$$

- For $\sigma_{co} >> \sigma_{self}$
- For $\sigma_{co} \approx \sigma_{self}$



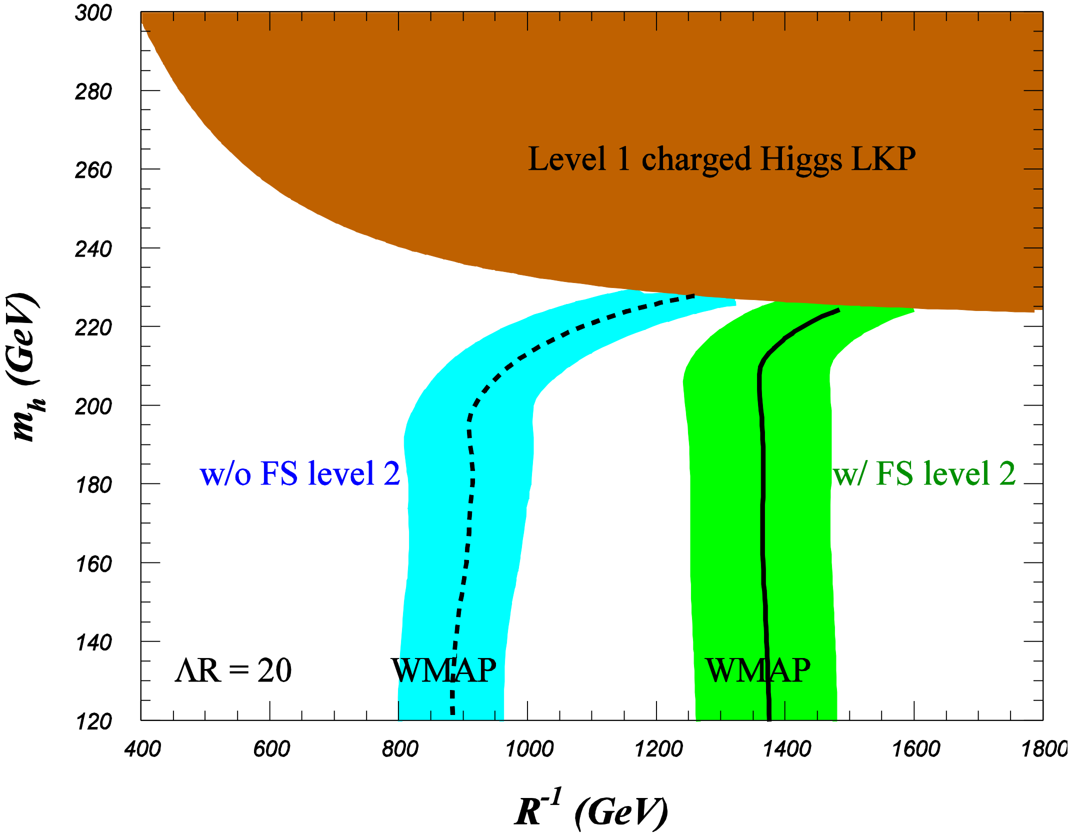
WIMP abundance in new physics models

SUSY



[Bagnaschi et al. EPJC79, no.2, 149 (2019)]

UED



[Belanger, MK, Pukhov, JCAP02, 009 (2012)]

The WIMP abundance constrains the mass scale of new physics