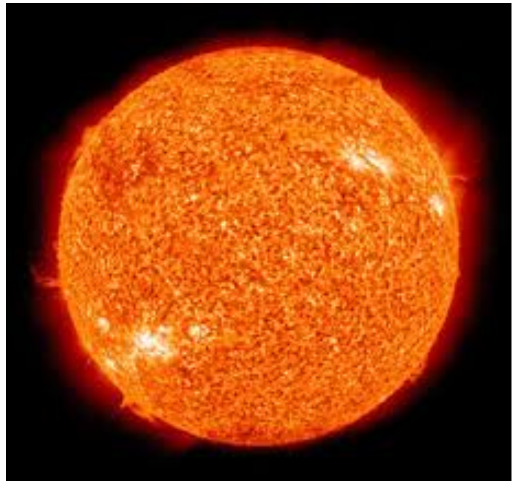
Contents

How large is the Universe?

- Mountain (10^3 m)

- Earth (10^7 m)

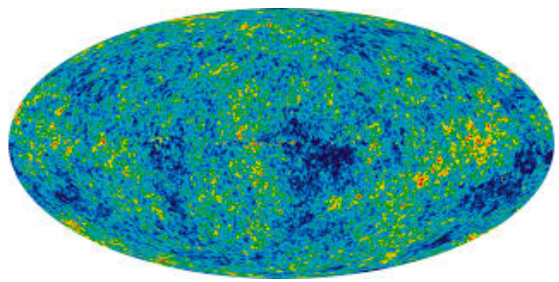
- Sun (10^9 m)



- Galaxy (10^{21} m) (Andromeda Galaxy)

- Galaxy cluster (10^{23} m) (Virgo Cluster)

- CMB (10^{27} m)



Units

Astronomical unit (AU)

- Originally distance from the Earth to the Sun

$$1\ \mathrm{AU} = 1.50 \times 10^{11}\ \mathrm{m}$$

Light-year (ly)

- Distance that light travels in a year

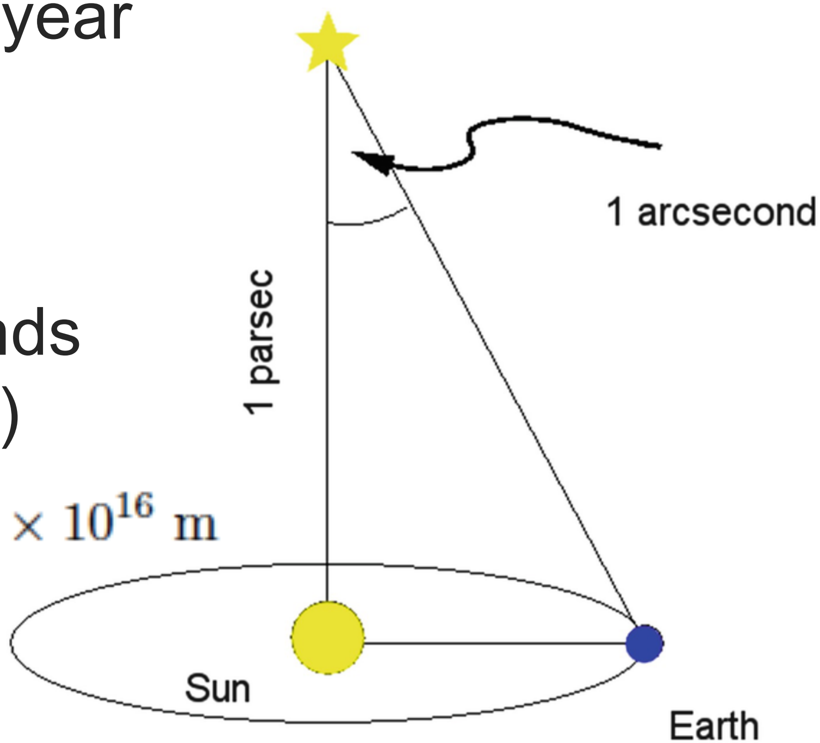
$$1\ \mathrm{ly} = 6.32 \times 10^{4}\ \mathrm{AU} = 9.46 \times 10^{15}\ \mathrm{m}$$

Parsec (pc)

- Distance at which 1 AU subtends 1 arcsecond (= 1/3600 degree)

$$1\ \mathrm{pc} = 3.26\ \mathrm{ly} = 2.07 \times 10^{5}\ \mathrm{AU} = 3.09 \times 10^{16}\ \mathrm{m}$$

$$1\ \mathrm{Mpc} = 10^{3}\ \mathrm{kpc} = 10^{6}\ \mathrm{pc}$$



[springer]

Physical constants

Newton gravitational constant

$$G = 6.67 \times 10^{-11}\ \mathrm{m^3 kg^{-1} s^{-2}}$$

Planck mass

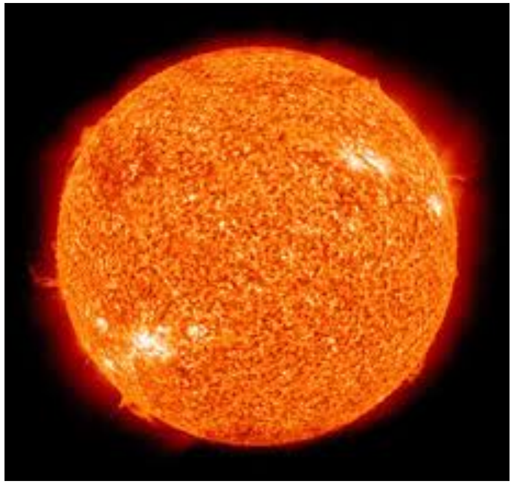
$$m_{\mathrm{Pl}} = \sqrt{\frac{\hbar c}{G}}$$

Solar mass

$$M_\odot = 1.99 \times 10^{30}\ \mathrm{kg}$$

Solar luminosity

$$L_\odot = 3.83 \times 10^{26}\ \mathrm{W}$$



Mass-to-light ratio

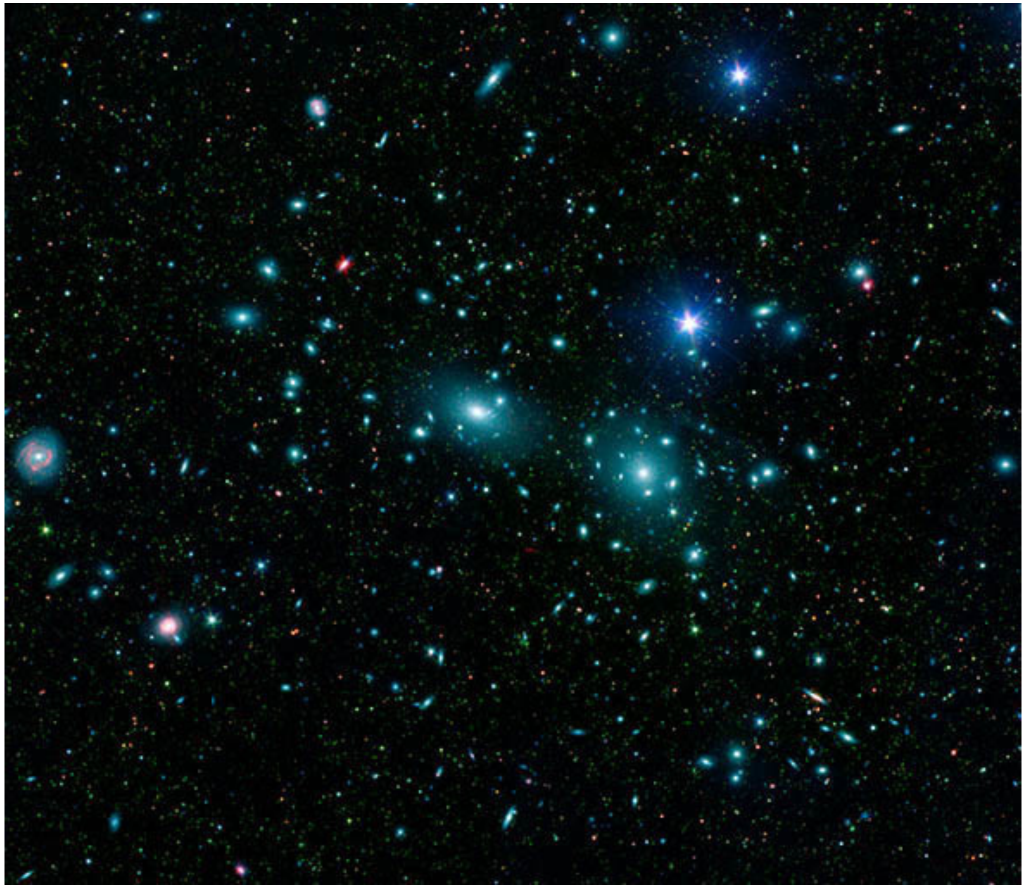
Zwicky (1933)

- The galaxies of the Coma Cluster move too fast compared to the amount of the luminous mass

  * Galaxy clusters: largest gravitationally bound structures containing 100-1000 galaxies

Existence of unseen matter (dark matter)

Coma Cluster



[spitzer.caltech.edu]

Mass-to-light ratio

Virial theorem

- Relation between the time average of the kinetic energy and that of the potential energy of a bound system
- For the gravitational potential $V \propto 1/r$,

$$\langle T \rangle = -\frac{1}{2}\langle V \rangle$$

$$\langle T \rangle \sim M v^2 \qquad\qquad \langle V \rangle \sim -\frac{GM^2}{R}$$

Coma cluster

- Velocity dispersion: $v^2 = 5 \times 10^{11}\ \mathrm{m^2 s^{-2}}$
- Radius: $R = 10^{22}\ \mathrm{m}$
- Luminosity: $L = 8.5 \times 10^{10} L_\odot$

→ Mass-to-light ratio: $M/L \sim 400 M_\odot / L_\odot$

Rotation curve

* Rotation curve: plot of orbital speeds vs. radial distance

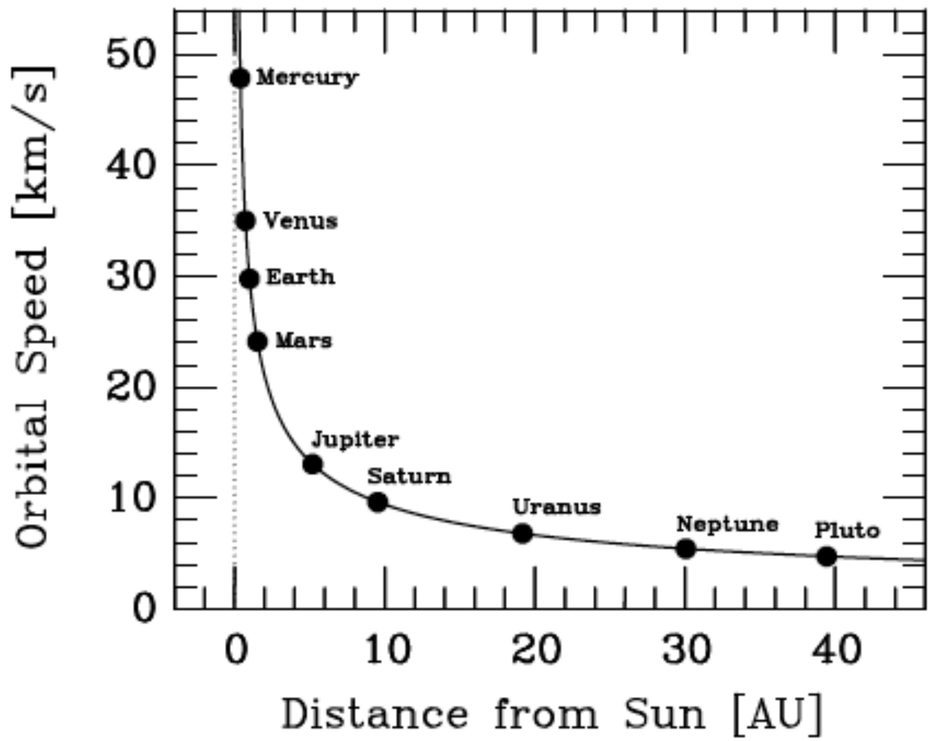
Solar system

- Consider a planet w/ mass m at distance r from the Sun, moving w/ orbital velocity v
- Balance between centrifugal and gravitational forces:

$$m\frac{v^2}{r} = G\frac{M_\odot m}{r^2}$$

$$\Rightarrow \quad v = \sqrt{\frac{GM_\odot}{r}}$$

Rotation curve of planets



[Raffelt, hep-ph/9712538]

Rotation curve

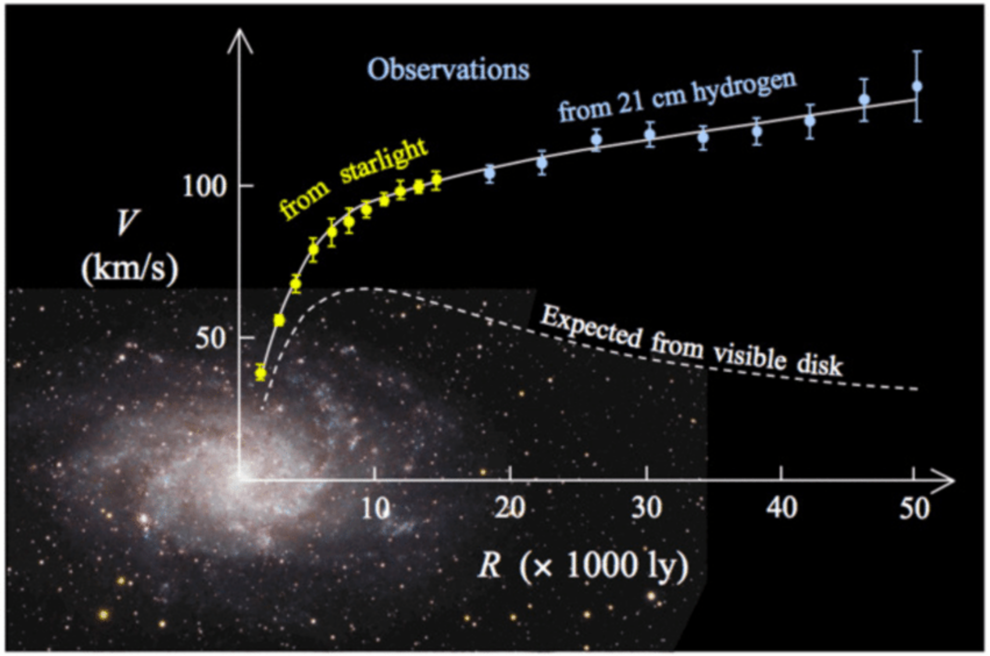
Spiral galaxies

- Orbital velocity of a star w/ mass m at distance r from the galactic center

$$v = \sqrt{\frac{GM(<r)}{r}}$$

- Observation:

Rotation curve of spiral galaxy M33



[researchgate.net]

For large distances from the galactic center

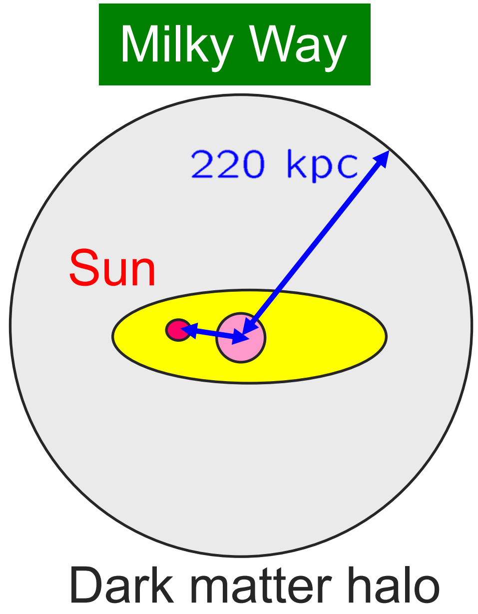
$$v \sim \text{const.} \Rightarrow M(<r) \propto r$$

⇒ **Most of the mass of galaxy is from the invisible galactic halo**

Profile of dark halos

- Assuming sphericity for the halo mass density $\rho = \rho(r)$,

$$M(<r) = 4\pi \int_0^r dr' r'^2 \rho(r')$$



Isothermal profile

- For a constant velocity, $\rho(r) \propto r^{-2}$
- For avoiding the central singularity, we often assume

$$\rho(r) = \rho_0 \frac{R_0^2 + a^2}{r^2 + a^2}$$

- Case of the Milky way
  - Galactocentric radius of the Sun: $R_0 = 8.5 \text{ kpc}$
  - Core radius of the halo: $a = 5 \text{ kpc}$
  - Local density in the solar system: $\rho_0 = 0.2 - 0.8 \text{ GeV cm}^{-3}$

    (Canonical value in the literature: $\rho_0 = 0.3 \text{ GeV cm}^{-3}$)

Profile of dark halos

- Simulations have been performed to obtain the actual profile, but have not converged

Proposed analytic profiles

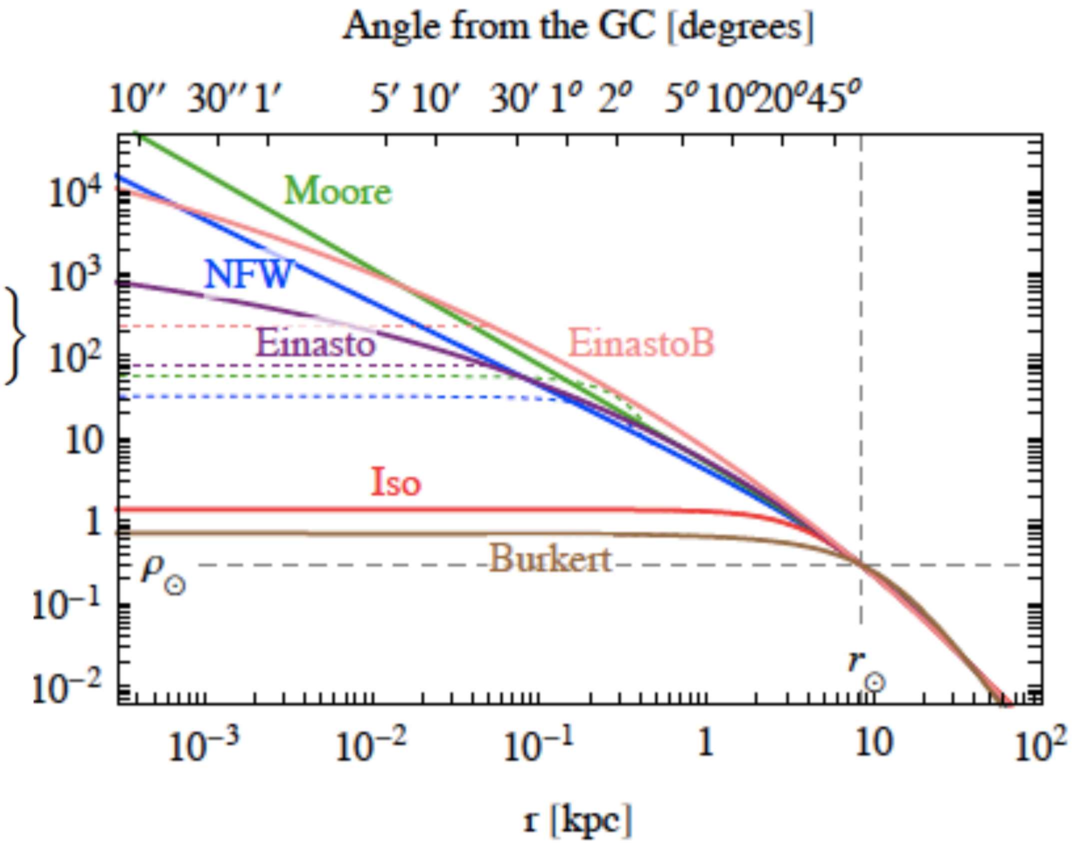
$$\text{NFW}: \quad \rho_{\rm NFW}(r) = \rho_s \frac{r_s}{r}\left(1+\frac{r}{r_s}\right)^{-2}$$

$$\text{Einasto}: \quad \rho_{\rm Ein}(r) = \rho_s \exp\left\{-\frac{2}{\alpha}\left[\left(\frac{r}{r_s}\right)^{\alpha}-1\right]\right\}$$

$$\text{Isothermal}: \quad \rho_{\rm Iso}(r) = \frac{\rho_s}{1+(r/r_s)^2}$$

$$\text{Burkert}: \quad \rho_{\rm Bur}(r) = \frac{\rho_s}{(1+r/r_s)(1+(r/r_s)^2)}$$

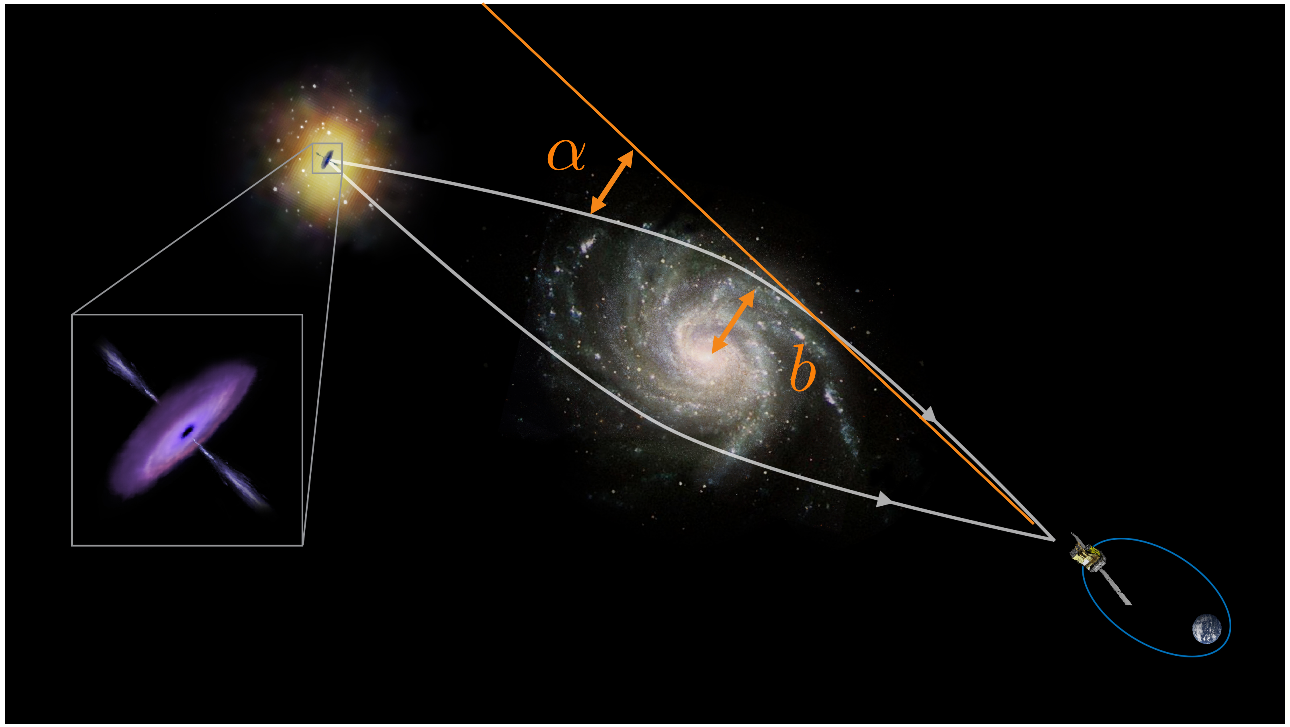
$$\text{Moore}: \quad \rho_{\rm Moo}(r) = \rho_s \left(\frac{r_s}{r}\right)^{1.16}\left(1+\frac{r}{r_s}\right)^{-1.84}$$



[Cirelli et al., JCAP 03, 051 (2011)]

Gravitational lensing

- Deflection of light caused by the gravitational field of a (very large) mass
- First test of Einstein's general relativity



b : Distance of closest approach

M : Mass of a galaxy or galaxy cluster

- Bending angle:

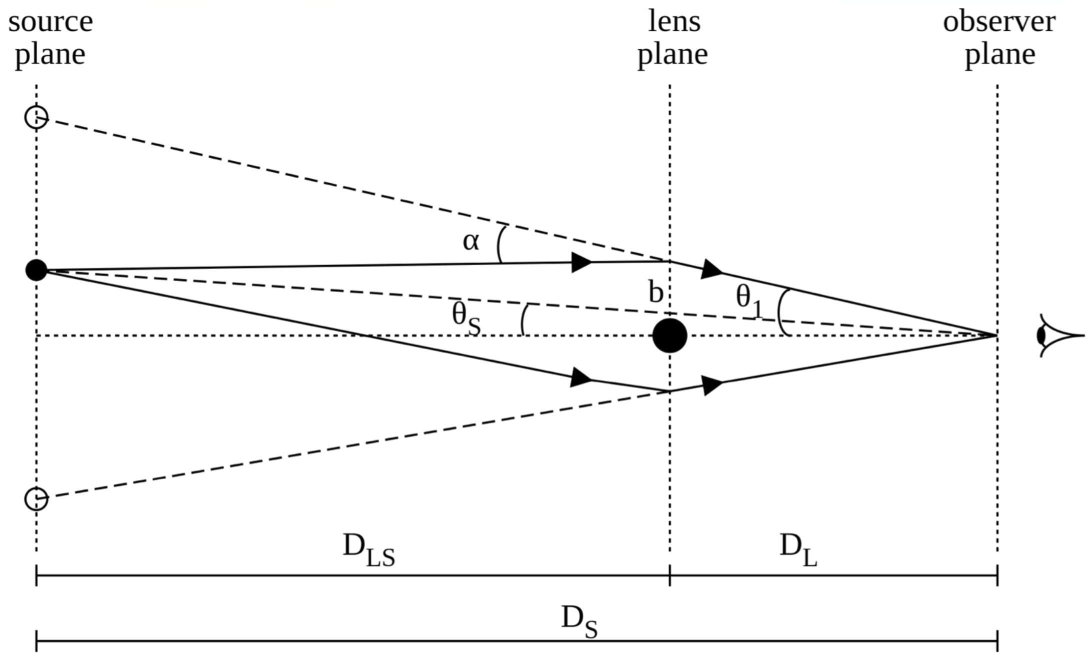
$$\alpha = \frac{4GM}{b}$$

Gravitational lensing

Two images

$$\theta_1 = \frac{1}{2}\left(\theta_S \pm \sqrt{\theta_S^2 + 4\theta_E^2}\right)$$

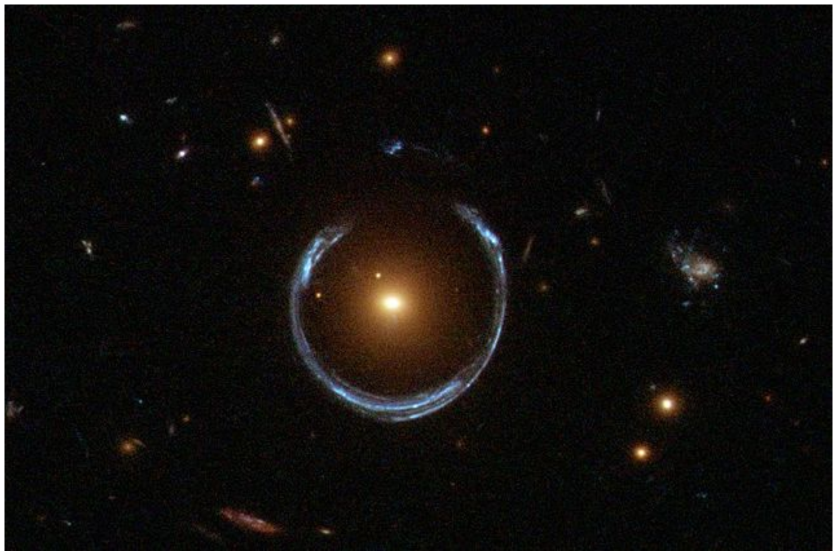
$$\theta_E = \sqrt{\frac{4GMD_{LS}}{D_S D_L}}.$$



Einstein ring

- In the collider case ($\theta_S = 0$), a ring image appears

$$\theta_1 = \theta_E$$



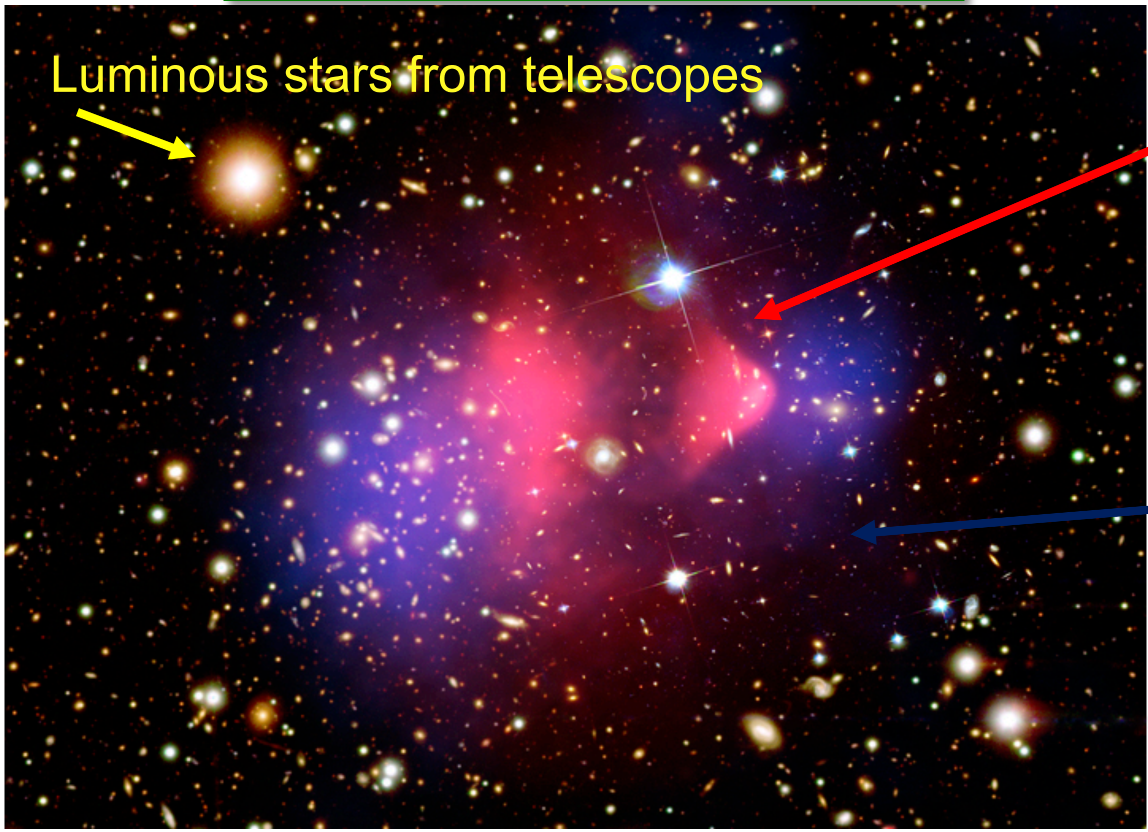
[wikipedia]

Bullet cluster

- Lensing galaxies or clusters are extended objects

➡ Analysis of the multiple images of distant sources gives the mass distribution of the foreground galaxy or cluster

Galaxy cluster 1E 0657-558



[chandra.harvard.edu]

- Two clusters having passed through each other

Hot gas (ordinary matter)

(from satellite X-ray measurement)

Slowed due to EM int.

Dark matter

(from gravitational lensing)

- The separation cannot be explained by modified gravity

Cosmological scales

Cosmological parameters

- Friedmann equation

$$\sum_i \Omega_i + \Omega_\Lambda - 1 = \frac{k}{R^2 H^2}$$

Ω_i : Density parameters for species i

Ω_Λ : Density parameter for the cosmological constant

k : Curvature constant

R : Scale factor

H : Hubble expansion rate

Cosmological scales

- Data of cosmological observations depends on the cosmological parameters
- Present data most notably from CMB anisotropy prefers the Lambda cold dark matter (Lambda-CDM) model:

  - Curvature: $k = 0$

  - Cold (non-relativistic), non-baryonic dark matter:

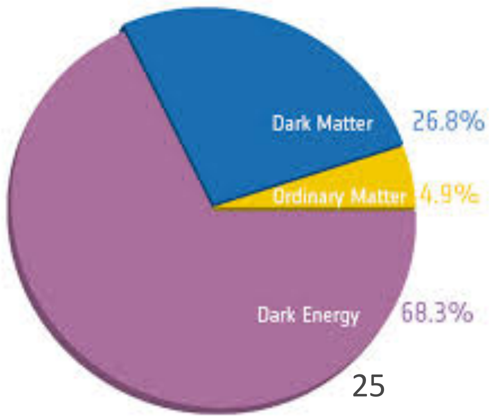
    $\Omega_{\rm CDM} \simeq 0.25$

  - Baryonic matter including baryonic dark matter:

    $\Omega_{\rm baryon} \simeq 0.05$

  - Cosmological constant: $\Omega_\Lambda \simeq 0.70$

We need a new theory with cold dark matter



Properties of dark matter

- Electrically neutral (or extremely weakly charged)
- Very weakly (at least gravitationally) interacting with ordinary matter
- Stable (or very long-lived as long as the Universe)
- Cold (i.e. non-relativistic)
- Non-baryonic
- Around 25% of the energy density of the Universe

Dark matter candidates must fit these conditions