Dark Matter (Kakizaki) - Exercises

1. Consider a particle moving in a central potential

$$V = kr^n, \tag{1}$$

where k is a constant parameter. Show that the virial theorem reduces to

$$\langle T \rangle = \frac{n}{2} \langle V \rangle. \tag{2}$$

Hint: Consider the time derivative of the scalar product of the momentum and position vectors,

$$G \equiv \boldsymbol{p} \cdot \boldsymbol{r}. \tag{3}$$

2. The bending angle of photons passing by a point mass M at a distance of closest approach b is given by

$$\alpha = \frac{4GM}{b}. \tag{4}$$

Show that in general the observer observes two images of the source of light with angles,

$$\theta_1 = \frac{1}{2}\left(\theta_S \pm \sqrt{\theta_S^2 + 4\theta_E^2}\right), \tag{5}$$

where

$$\theta_E = \sqrt{\frac{4GMD_{\mathrm{LS}}}{D_S D_L}}. \tag{6}$$