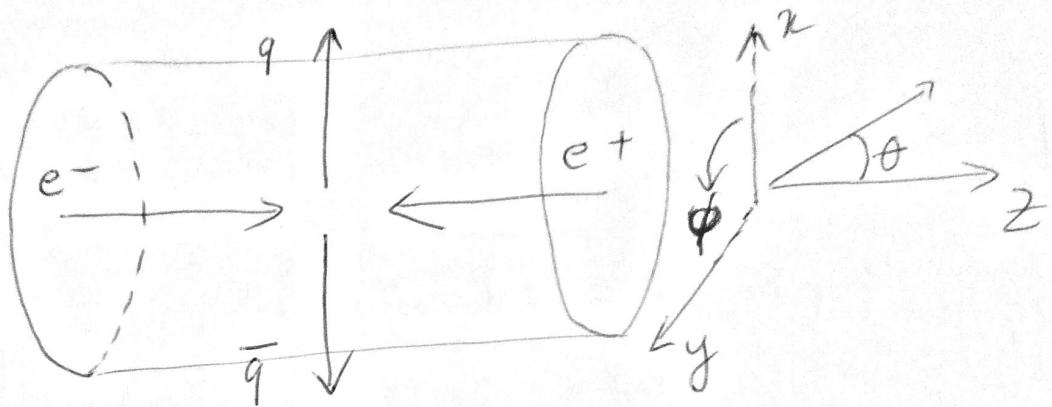
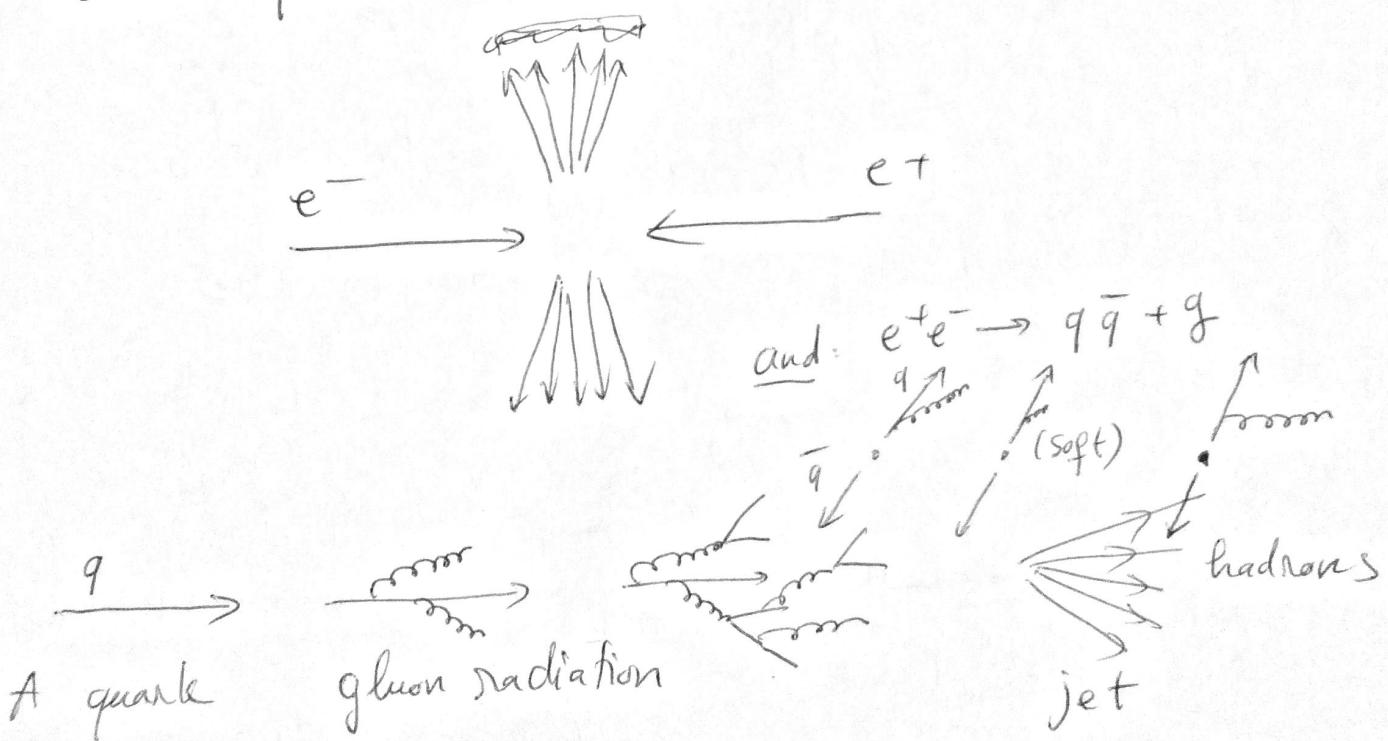


Jet

$\sum_{\text{hadrons}} \rightarrow$ no angular dependence.
What happens here: \sqrt{s} is large



each quark \rightarrow hadrons :



A jet = A Set of partons = A Set of hadrons

partons/hadrons should be close enough together.

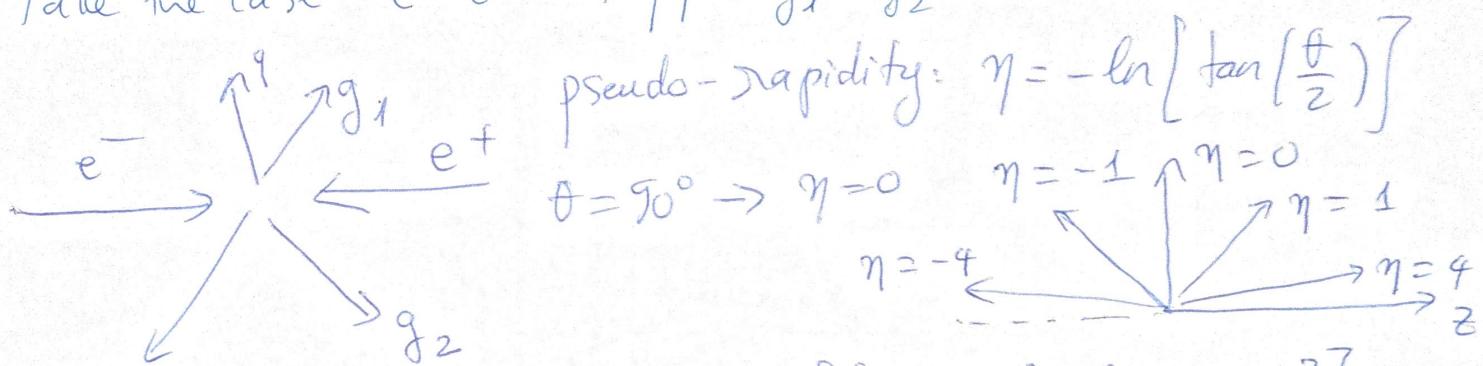
$$\Delta R(i,j) = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2} < R_c \rightarrow \text{combine}$$

R_c : jet-radius parameter

L3-1

Jet definition

Take the case $e^+ e^- \rightarrow \bar{q}q + g_1 + g_2$ as an example.



$$d_{ij} = \min \left(E_{T,ii}^2, E_{T,jj}^2 \right) \frac{[(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2]}{R^2}$$

$$E_{T,i} = |\vec{P}_{T,i}| = \sqrt{P_{x,i}^2 + P_{y,i}^2}$$

$$d_i = E_{T,i}^2 \quad (i = 1, 2, 3, 4)$$

$$d_{\min} = \min \{ d_i, d_{ij} \}$$

If $d_{\min} \in \{ d_{ij} \}$ (say $d_{\min} = d_{qg_1}$) \Rightarrow merge i and j

$$\Rightarrow p_{ik}^\mu = p_i^\mu + p_j^\mu \Rightarrow \text{calculate } (\eta_k, \phi_k)$$

$\Rightarrow k$ is now a new jet candidate.

$\Rightarrow k$ is now a new jet candidate.

If $d_{\min} \in \{ d_i \}$, say $d_{\min} = d_l \Rightarrow l$ is not mergable.

\Rightarrow remove l from the list and add l to the list of jets.

Repeat the above steps with 3 candidates.

More on jet algorithm: Ellis and Soper, hep-ph/9305266;

Seymour hep-ph/9707338; Cacciari, Salam, Soyez,

EPJC (2012) 72: 1896 (FastJet user manual), ...

$$\text{Theory: } P_{\text{jet}}^{\mu} = \sum_{i \in \text{Partons}} P_i^{\mu}$$

$$\text{Exp: } P_{\text{jet}}^{\mu} = \sum_{i \in \text{hadrons}} P_i^{\mu}$$

fat jet: R_c is large. (≈ 0.8)

Normal jet: $R_c \approx 0.4$

$\sigma_{e^+e^- \rightarrow \text{hadrons}}$: finite

$$\sigma_{\text{hadrons}} = \underbrace{\sigma_{2\text{-jet}}}_{\text{finite}} + \underbrace{\sigma_{3\text{-jet}}}_{\text{finite}} + \dots$$

Note: $\sigma_{q\bar{q}}$ is NOT finite.

How to calculate $\sigma_{2\text{-jet}}$ from theory?

$$\sigma_{2\text{-jet}} = \sigma_{q\bar{q}} + \sigma_{q\bar{q}g} + \sigma_{q\bar{q}+2g} + \dots : \text{finite}$$

$$\sigma_{3\text{-jet}} = \sigma_{q\bar{q}g} + \sigma_{q\bar{q}+2g} + \dots : \text{finite}$$

→ prove the existence
of gluon.

[Soft and Collinear divergences.]

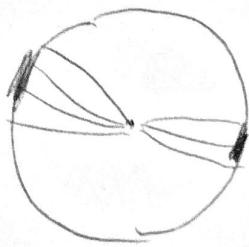
2-jet exclusive: only 2-jet events

2-jet inclusive: include all events with at least 2 jets

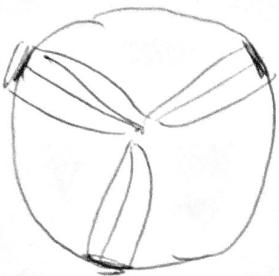
$$\sigma_{\text{2-jet}}^{\text{inc.}} = \sigma_{\text{2-jet}}^{\text{exc.}} + \sigma_{\text{3-jet}}^{\text{inc.}}$$

$$\sigma_{\text{N-jet}}^{\text{inc.}} = \sigma_{\text{N-jet}}^{\text{exc.}} + \sigma_{(N+1)\text{-jet}}^{\text{inc.}}$$

Experiment:



2-jet



3-jet



4-jet

$\underbrace{\qquad}_{q\bar{q} \text{ dominant}}$

$\underbrace{\qquad}_{\text{gluon discovery}}$

jet cross section : $e^+e^- \rightarrow \text{hadrons}$

Tree-level (leading order)

$$q_a + q_b \rightarrow p_1 + p_2 + \dots + p_N$$

N partons in the final state.

$$\Rightarrow \sigma_{LO} = \int d\phi_N \overline{|M_{ab \rightarrow N}^0(q_a, p_i)|^2} \cdot F_J^{(N)}(\{p_i\})$$

F_J : jet function

$F_J^{(N)}$ includes the requirement of the number of jets, experimental cuts, R_c (jet-radius parameter), jet algorithm (k_T or anti- k_T , ...).

And, F_J has to be ~~not~~ soft and collinear safe. This means that the value of F_J has to be independent of the number of soft and collinear particles in the final state. We must have

$$F_J^{(N+1)} \rightarrow F_J^{(N)}$$

in any case where $(N+1)$ -parton and (N) -parton configurations are kinematically degenerate.

Next - to leading order :

$$\sigma_{\text{Virtual}} = \int d\phi_N 2 \operatorname{Re} \left[\overline{M_V \cdot M_0^*} \right]^{(N)} (q_a, p_i) \times F_J^{(N)} (\{p_i\})$$

$$\sigma_{\text{Real}} = \int d\phi_{N+1} \left| \overline{M_{0,N+1}} (q_a, p_i) \right|^2 \times F_J^{(N+1)} (\{p_i\})$$

$$\Rightarrow \sigma_{\text{NLO}} = \sigma_{\text{LO}} + \sigma_{\text{Virtual}} + \sigma_{\text{Real}}$$

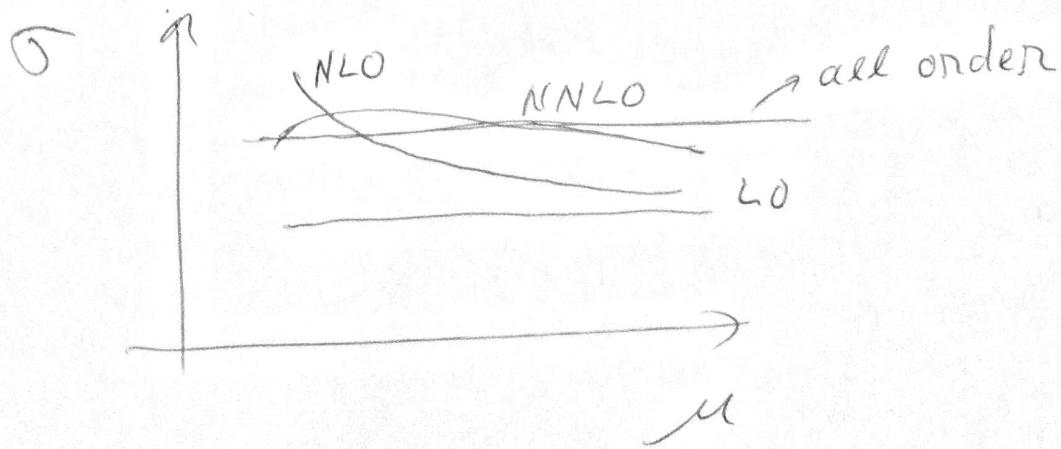
is IR safe.

UV divergences are removed by renormalization

Soft and Collinear divergences are cancelled

between σ_{Virt} and σ_{Real} .

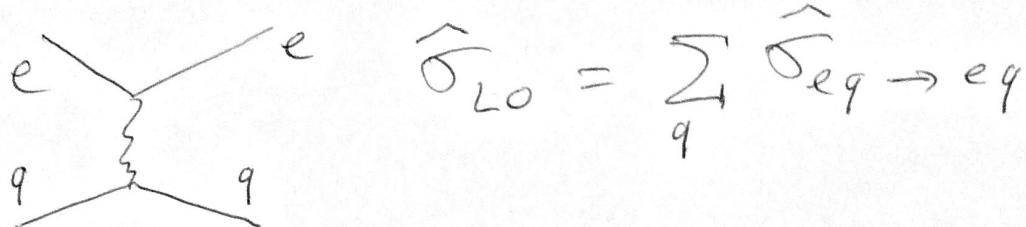
But $\sigma_{\text{NLO}} = \sigma_{\text{NLO}} [\alpha_s(\mu)]$: depends on $\mu = \mu_R$.



Jet cross section: $e + p \rightarrow e + \text{hadrons}$

$p = \text{A set of partons } (u, d, c, s, b, g, + \bar{q}_c)$:

partonic level:



Proton level:

$$\sigma_{ep} = \sum_q \int_0^1 dx f_q(x, \mu_F^2) \hat{\sigma}_{eq}$$

$f_q(x, \mu_F^2)$ is the density of parton of type q in the proton.

$$e(p_a) + p(p_b) \rightarrow e(p_1) + \text{hadrons}$$

$$\rightarrow e(p_a) + q(p'_b) \rightarrow e(p_1) + \text{hadrons}$$

$p'_b = x p_b$: x is called "momentum fraction".

and $0 \leq x \leq 1$.

$$\sqrt{s} = (p_a + p_b)^2 \approx^2 p_a p_b$$

$$\sqrt{s} = (p_a + p'_b)^2 = 2 p_a p'_b = 2 x p_a p_b = x \sqrt{s}$$

μ_F : factorization scale. (arbitrary mass scale)

Like μ_R , μ_F occurs when quantum corrections are taken into account.

L3.6

Like $\lambda_S(\mu)$, we cannot calculate the value of λ_S from the Lagrangian.

We cannot calculate the density function $f_q(x, \mu_F^2)$ from the Lagrangian. They have to be fitted from the data at some energy scale.

There is a differential equation about the dependence of $f_q(x, \mu_F^2)$ on μ_F , like the running eq. of λ_S . This equation can be calculated order by order from the Lagrangian perturbatively.

Why f_q depends on μ_F ?

+ $\lambda_S \sim \mu_R$ because of UV divergences
(we have to subtract this divergence at some energy scale).

1) $f \sim \mu_F$ because of collinear divergences related to initial state radiation.
(we have to subtract this divergence at the scale μ_F)

To cancel this μ_F dependence, we have to add a collinear counter term order by order in the calculation. Fixed-order calculation still depends on μ_F .

L3.7

$$\sigma_{pp} = \sum_{a,b} \int_0^1 dx_1 f_a(x_1, \mu_F^2) \int_0^1 dx_2 f_b(x_2, \mu_F^2) \times \hat{\sigma}(a+b \rightarrow \text{hadrons})$$

$$a, b = \{u, d, c, s, b, g, +\bar{q}_i\}$$

$$p(p_a) + p(p_b) \rightarrow \text{hadrons}$$

$$S = (p_a + p_b)^2 = 2 p_a p_b$$

$$a(q_a) + a(q_b) \rightarrow \text{hadrons} ; q_a = x_1 p_a ; q_b = x_2 p_b$$

$$\hat{S} = (q_a + q_b)^2 = 2 q_a q_b = 2 x_1 x_2 S$$

\Rightarrow Calculate σ_{pp} numerically.

Monte-Carlo integration.