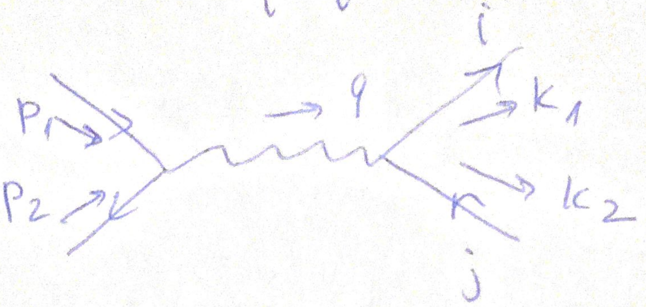
$e^- + e^+ \longrightarrow q_i + \bar{q}_i$ at tree level: $m_e \simeq m_q \simeq 0$ $(\ll q^2)$



$q = k_1 + k_2$

$s = q^2$

$$\mathcal{M} = \bar{v}_{\lambda_2}(p_2)\, ie\gamma^\mu\, u_{\lambda_1}(p_1)\, \frac{-i}{q^2}\left[g_{\mu\nu} - (1-\eta)\frac{q_\mu q_\nu}{q^2}\right]$$

$$\times\, \bar{u}_{\rho_1}(k_1)\, ieQ_i\gamma^\nu\, v_{\rho_2}(k_2)\, \delta_{ij}$$

$$= \frac{ie^2 Q_i}{q^2}\, \bar{v}_{\lambda_2}(p_2)\gamma^\mu u_{\lambda_1}(p_1)\left[g_{\mu\nu} - (1-\eta)\frac{q_\mu q_\nu}{q^2}\right]$$

$$\times\, \bar{u}_{\rho_1}(k_1)\gamma^\nu v_{\rho_2}(k_2)\, \delta_{ij}$$

Note: $q_\nu\, \bar{u}_{\rho_1}(k_1)\gamma^\nu v_{\rho_2}(k_2) = 0$

$$\Rightarrow \mathcal{M} = \delta_{ij}\frac{ie^2 Q_i}{q^2}\left[\bar{v}_{\lambda_2}(p_2)\gamma^\mu u_{\lambda_1}(p_1)\right]\left[\bar{u}_{\rho_1}(k_1)\gamma_\mu v_{\rho_2}(k_2)\right]$$

$$\mathcal{M}^* = -\delta_{ij}\frac{ie^2 Q_i}{q^2}\left[\bar{u}_{\lambda_1}(p_1)\gamma^\nu v_{\lambda_2}(p_2)\right]\left[\bar{v}_{\rho_2}(k_2)\gamma_\nu u_{\rho_1}(k_1)\right]$$

$$|\mathcal{M}|^2 = \frac{e^4 Q_i^2}{N_c q^4}\left[\bar{v}_{\lambda_2}(p_2)\gamma^\mu u_{\lambda_1}(p_1)\bar{u}_{\lambda_1}(p_1)\gamma^\nu v_{\lambda_2}(p_2)\right]$$

$$\times\left[\bar{u}_{\rho_1}(k_1)\gamma_\mu v_{\rho_2}(k_2)\bar{v}_{\rho_2}(k_2)\gamma_\nu u_{\rho_1}(k_1)\right]$$

(Summed over colors)

$$|\overline{M}|^2 = \frac{1}{4}\sum_{\lambda_1,\lambda_2,\rho_1,\rho_2} |M|^2$$

$$= \frac{N_c}{4}\frac{e^4 Q_i^2}{q^4}\sum_{\lambda_1,\lambda_2,\rho_1,\rho_2}\left[\bar{v}_{\lambda_2}\gamma^\mu \not{p}_1 \gamma^\nu v_{\lambda_2}\right]$$

$$\times\left[\bar{u}_{\rho_1}\gamma_\mu \not{k}_2 \gamma_\nu u_{\rho_1}\right]$$

Where we have used: $\sum_{\lambda_1} u_{\lambda_1}(p_1)\,\bar{u}_{\lambda_1}(p_1) = \not{p}_1$

$$\sum_{\rho_2} v_{\rho_2}(k_2)\,\bar{v}_{\rho_2}(k_2) = \not{k}_2$$

Note: $\left[\bar{v}_{\lambda_2}\gamma^\mu \not{p}_1 \gamma^\nu v_{\lambda_2}\right] = \text{number} = Tr\left[\bar{v}_{\lambda_2}\gamma^\mu \not{p}_1 \gamma^\nu v_{\lambda_2}\right]$

$$= Tr\left[v_{\lambda_2}(p_2)\,\bar{v}_{\lambda_2}(p_2)\gamma^\mu \not{p}_1 \gamma^\nu\right]$$

$$= Tr\left[\not{p}_2 \gamma^\mu \not{p}_1 \gamma^\nu\right]$$

Similarly $\left[\bar{u}_{\rho_1}(k_1)\gamma_\mu \not{k}_2 \gamma_\nu u_{\rho_1}(k_1)\right] = Tr\left[\not{k}_1 \gamma_\mu \not{k}_2 \gamma_\nu\right]$

Use: $Tr(\gamma_\mu \gamma_\nu) = 4 g_{\mu\nu}$

$$Tr(\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\rho) = 4(g_{\mu\nu}g_{\lambda\rho} + g_{\mu\rho}g_{\nu\lambda} - g_{\mu\lambda}g_{\nu\rho})$$

$$\Rightarrow |\overline{M}|^2 = \frac{N_c 4 e^4 Q_i^2}{q^4}\left(p_2^\mu p_1^\nu + p_2^\nu p_1^\mu - p_1 p_2 g^{\mu\nu}\right)$$

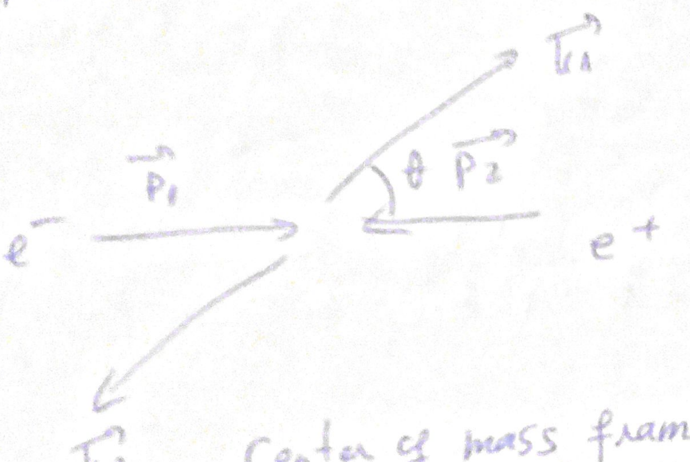
$$\times\left(k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu} - k_1 k_2 g_{\mu\nu}\right)$$

- Cross section:

$$\sigma = \frac{1}{2s} \int \frac{d^3\vec{k_1}}{(2\pi)^3 2k_{10}} \frac{d^3\vec{k_2}}{(2\pi)^3 2k_{20}} \times (2\pi)^4 \delta^4(k_1 + k_2 - p_1 - p_2) \times |\overline{M}|^2$$

Use: $\displaystyle\int \frac{d^3\vec{p}}{2p_0} = \int d^4p\, \theta(p_0)\, \delta(p^2 - m^2)$

$$\Rightarrow \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \cdot \frac{|\vec{k_1}|}{|\vec{p_1}|} \cdot |\overline{M}|^2$$



Center of mass frame:

$$\vec{p_1} + \vec{p_2} = \vec{k_1} + \vec{k_2} = 0$$

$$\Rightarrow |\vec{p_1}| = |\vec{p_2}| = |\vec{k_1}| = |\vec{k_2}| = \frac{\sqrt{s}}{2}$$

$$\Rightarrow \frac{d\sigma}{d\Omega} = N_c Q_i^2 \frac{\alpha^2}{4s}(1 + \cos^2\theta) \;;\quad \alpha = e^2/4\pi$$

$$\sigma_{total} = Q_i^2 \frac{4\pi\alpha^2}{3s} \cdot N_c$$

Sum over all quark flavors:

$$\sigma_{total} = \frac{4\pi\alpha^2}{3s} \left(\sum_{i=1}^{n_f} Q_i^2 \right) \cdot N_c$$

Sensitive to N_c, Q_i^2 and n_f

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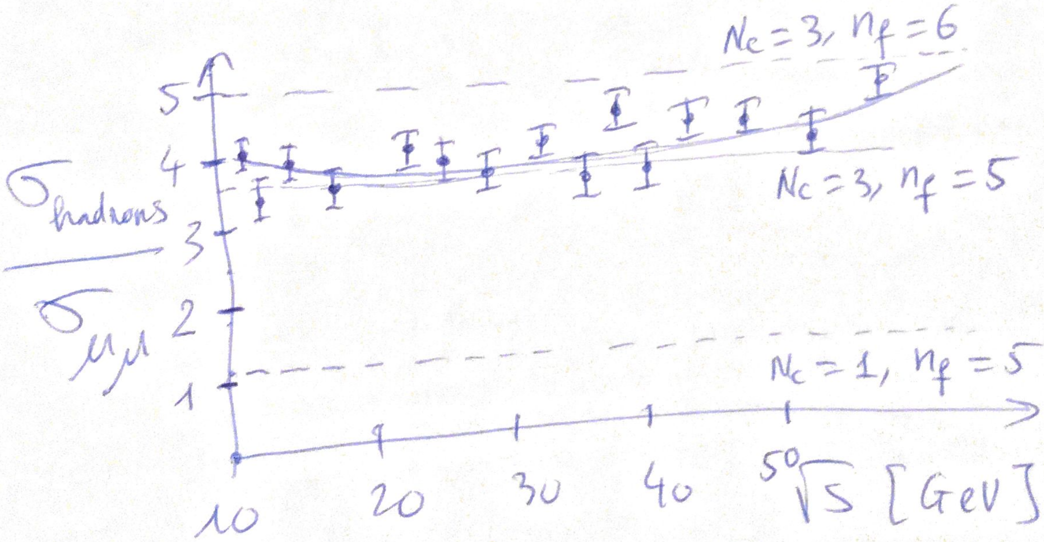
For $e^+e^- \to \mu^+\mu^-$: $N_c = 1$, $Q_i^2 = 1$, $n_f = 1$

$$\to \sigma_{\mu\mu} = \frac{4\pi\alpha^2}{3s}$$

$$\Rightarrow R = \frac{\sigma_{hadrons}}{\sigma_{\mu\mu}} = N_c \cdot \sum_{i=1}^{n_f} Q_i^2$$

If $\sqrt{s} < 2m_t \Rightarrow n_f = 5$ $\quad (m_t \simeq 172\ \text{GeV})$

$$\Rightarrow R = 3\left(Q_u^2 + Q_d^2 + Q_c^2 + Q_s^2 + Q_b^2\right)$$
$$= 3\left(\frac{4}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9} + \frac{1}{9}\right) = 3 \cdot \frac{11}{9} = \frac{11}{3} \simeq 3.667$$



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