Introduction to QCD

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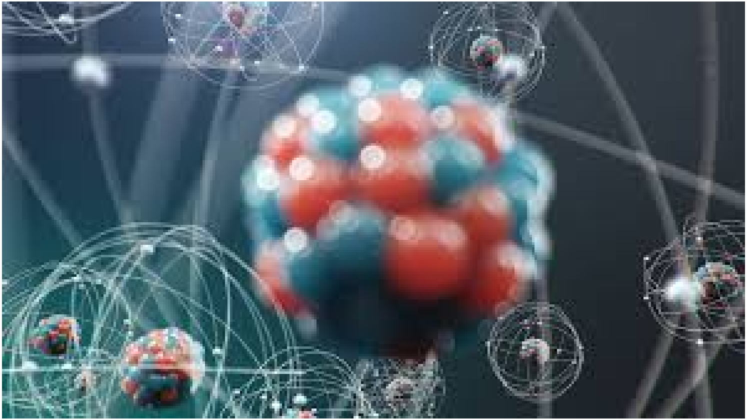
Outline

- History of QCD
- QCD Lagrangian (Black board)
- Feynman rules (Black board)
- Cross section (Black board)
- Quantum corrections: UV and IR divergences (Black board)
- ...

Why are nuclei stable?

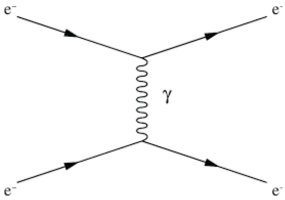


Source: foronuclear.org

- electromagnetic interaction
- strong nuclear interaction

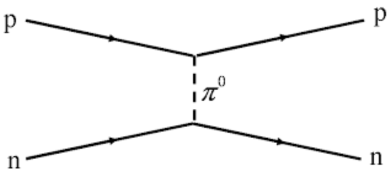
Strong nuclear interaction

- PHOTON is the messenger of electromagnetic interaction



Source: researchgate.net

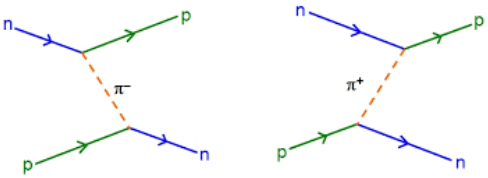
- Messenger of the strong interaction between nucleons ?



Source: Wikipedia

How many pions ?

- π^0, $\pi^\pm$:

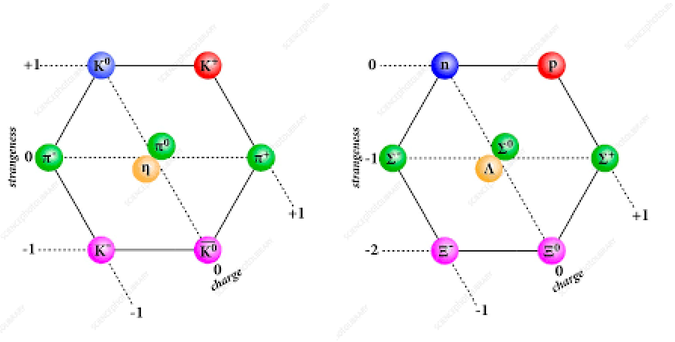


Source: cronodon.com

- Pions, proton, neutron are fundamental ?

Periodic table of particles

- 1940-1960: many mesons and new baryons were discovered. They are very similar to nucleons and pions.



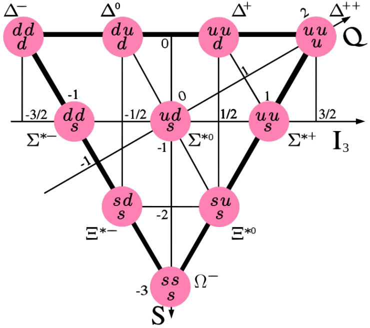
Source: sciencephoto.com

- Periodic table of particles !

A simple idea

Gell-Mann and Zweig (1964):

All those hundreds of hadrons are made up of 3 quarks: u, d, s.



Source: Wikipedia

$\rightsquigarrow Q_u = +2/3, Q_d = Q_s = -1/3$ with Spin $= 1/2$.

Colour

Consider e.g. the Δ^{++} resonance:

$$|\Delta^{++}, J_3 = 3/2 >= |u \uparrow, u \uparrow, u \uparrow>, \quad (1)$$

which contradicts Fermi-Dirac statistics.
Solution [Han and Nambu 1965]:

$$|\Delta^{++}, J_3 = 3/2 >= \epsilon_{ijk}|u^i \uparrow, u^j \uparrow, u^k \uparrow>, \quad (2)$$

where $i, j, k = 1, 2, 3$ are color indices.

Quarks exist in different color states, but hadrons are colorless.

Invariance

Lorentz transformation: $(t, x, y, z) \longrightarrow (t', x', y', z')$

$$s^2 = s^\mu s_\mu = t^2 - x^2 - y^2 - z^2 \tag{3}$$

Color transformation: $(a, b, c) \longrightarrow (a', b', c')$

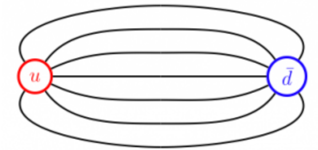
$$\begin{aligned} \text{meson} &= \frac{1}{\sqrt{3}}\delta_{ij} q^i \bar{q}^j = \frac{1}{\sqrt{3}}(q^1\bar{q}^1 + q^2\bar{q}^2 + q^3\bar{q}^3), \end{aligned} \tag{4}$$

$$\begin{aligned} \text{baryon} &= \frac{1}{\sqrt{6}}\epsilon_{ijk} q^i q^j q^k \\ &= \frac{1}{\sqrt{6}}[q^1q^2q^3 - q^1q^3q^2 - q^2q^1q^3 \\ &\quad + q^2q^3q^1 + q^3q^1q^2 - q^3q^2q^1] \end{aligned} \tag{5}$$

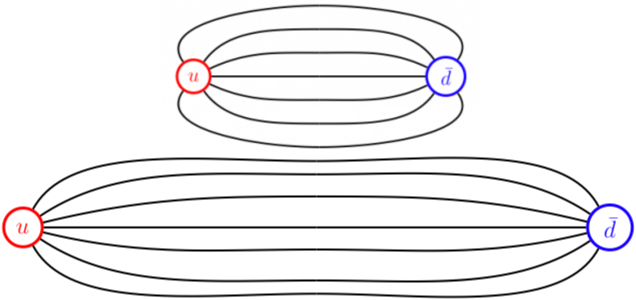
$\rightsquigarrow$ many ways to create colorless states (color singlets) !

All hadron states and physical observables are color singlets.

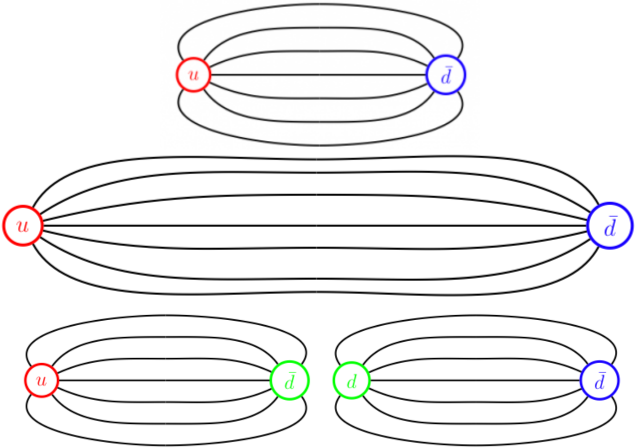
Color confinement



Color confinement



Color confinement



[Source: http://www.quantumdiaries.org]

Group and representation

See C.H. Nam's lecture.

SU(3) group

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \xrightarrow{U_\alpha} \psi' = \begin{pmatrix} \psi_1' \\ \psi_2' \\ \psi_3' \end{pmatrix}, \quad U_\alpha = \exp\left(i \sum_{a=1}^{8} T_a \alpha_a \right), \quad T_a = \lambda_a/2, \tag{6}$$

Gell-Mann matrices:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

$G = \{U_\alpha\}$ is called the $SU(3)$ group of unitary matrices.
If U_α are $d \times d$ matrices $\rightsquigarrow$ d representation ($d = 3, 8, 10, \ldots$).
All representations: $[T_a^{(d)}, T_b^{(d)}] = i f_{abc} T_c^{(d)}$, $a, b, c = 1, \ldots, 8$.
f_{abc}: real and totally anti-symmetric.

$SU(3)_F$ vs. $SU(3)_C$

<u>$SU(3)_F$</u>: flavor space

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \tag{7}$$

$\rightsquigarrow$ explanation of periodic tables of hadrons (octet and decuplet representations).

<u>$SU(3)_C$</u>: color space

$$q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \tag{8}$$

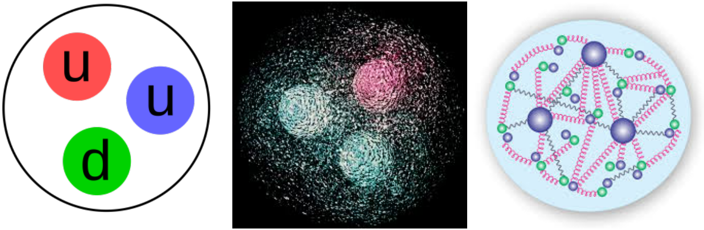
$\rightsquigarrow$ hadrons are colorless ($SU(3)_C$ singlets).

Interaction between quarks is the origin of the strong nuclear force !

MODELS of the proton

"... the imagination of nature is far, far greater than the imagination of man."

– Richard Feynman –



[Source: quantumdiaries.org, fineartamerica.com, physicstoday.scitation.org]

- Feynman (1969): hadrons are made of point-like constituents, termed partons.
- Bjorken scaling: ep scattering at high energies $\rightsquigarrow$ partons behave as independent particles.

Partons = (quarks, gluons).

Gluons = messengers of the strong interaction between quarks.

Next steps

- Quantum chromodynamics (QCD): quantum field theory of quarks and gluons (Lagrangian, perturbative approach)
- From quarks and gluons to hadrons

$\longrightarrow$ Black board !

References

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