Der Wissenschaftsfonds.

# *Scalar-field dark matter versus standard CDM: looking for deviance*

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**Li, Shapiro, Rindler-Daller 2017 PRD, 96, 063505 (arXiv: 1611.07961)**
**Li, Rindler-Daller, Shapiro 2014 PRD, 89, 083536 (arXiv: 1310.6061)**

# Scalar-field dark matter (SFDM)

**"a zoo of (similar) animals"**

Ultralight axions (**ALPs**)

(axion-like potential)

Bose-Einstein-condensed DM (**BECDM**)

**fuzzy / quantum wave** DM, **free** SFDM

(no self-interaction)

**repulsive / fluid** DM

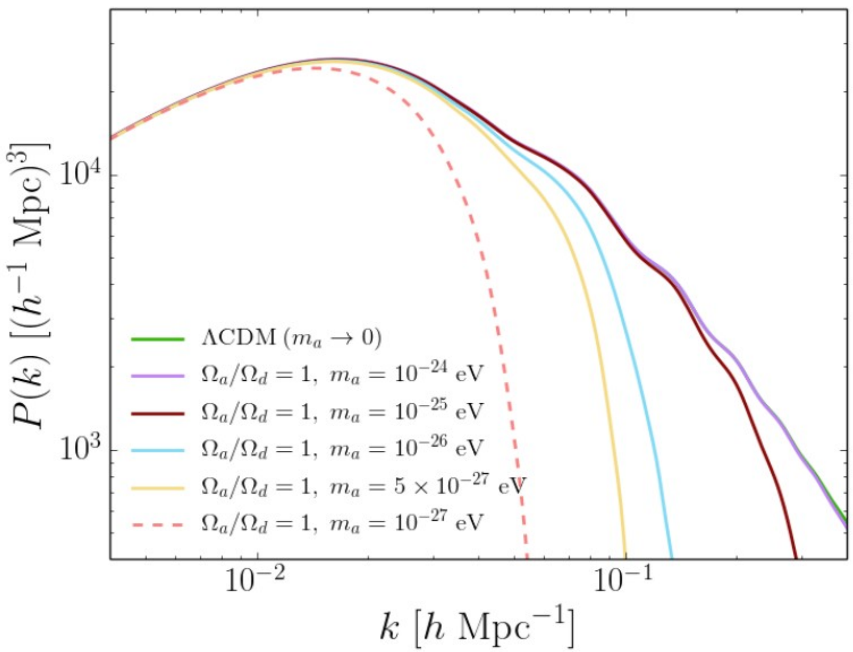
(strong, positive self-interaction)

They all obey a similar EoM, if an effective classical field description is adopted; its physics gives rise to a minimum clustering scale. To "resolve" galactic small-scale problems, need ultralight particles

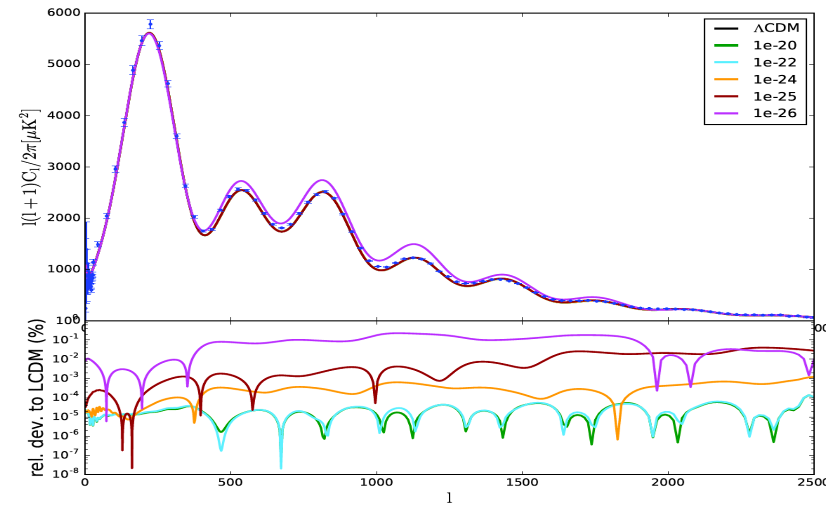
$$10^{-23}\ \mathrm{eV} \lesssim m \lesssim 10^{-20}\ \mathrm{eV}$$

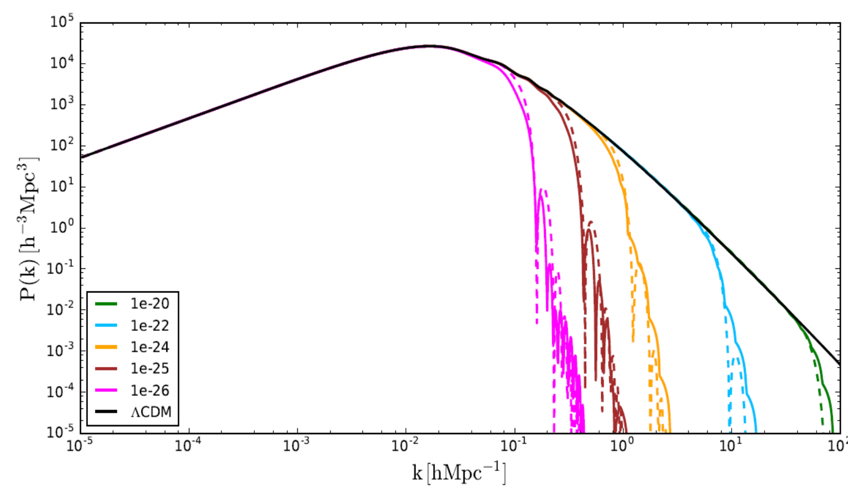
# SFDM and power spectra

Hlozek et al (2015)




Ureña-Lopez & Gonzalez-Morales (2016)



***Note:*** *my talk focuses on deviations from CDM on a grand scale, in fact wrt the evolution of the background universe !!*

*How much "deviance" is allowed ?*

→ *Neff and GWs help to find out*

→ *constrain SFDM particle parameters*

# Scalar Field Dark Matter (SFDM)

*real or complex scalar field* $\psi$ ( *model-dependent)*

$$\mathscr{L} = \frac{\hbar^2}{2m} g^{\mu\nu} \partial_\mu \psi^* \partial_\nu \psi - V(\psi)$$

units: $[\mathscr{L}]$ = [eV/cm³], [ψ] = cm $^{-3/2}$ , (+,-,-,-)

*V(ψ) is model-dependent*

QCD axion, ALPs: $V_a = f_a^2\, m_a^2(t)\ [1 - \cos{(a/f_a)}]$

→ upon expansion: quadratic (+ quartic)

**phi^4 - potential:** $V(\psi) = \frac{1}{2} mc^2 |\psi|^2 + \frac{\lambda}{2} |\psi|^4$ $\lambda = \hat{\lambda} \frac{\hbar^3}{m^2 c}$

*Quartic term:* λ is an energy-independent coupling constant, λ > 0: repulsive, λ < 0: attractive

→ ***fundamental SFDM parameters:*** **m and λ**

# Scalar Field Dark Matter (SFDM)

*if ψ is complex* → U(1)-symmetry, particle number conserved
→ no self-annihilation !

$$\rho_{SFDM,0} = n_{SFDM,0} mc^2 = \Omega_{DM} \rho_{crit,0}$$

if ψ is *real* → no U(1) symmetry, self-annihilation, but particle number *approximately* conserved in the non-relativistic limit

Equation of states (EOS) encountered: „oscillation" $0 < w < 1/3$
„slow-roll" $w = -1$
„fast-roll" $w = 1$

Eventually, in order to behave **„CDM-like" ($w_{average}$ = 0) :** need quadratic term
(all models require $w_{average} = 0$ after $z_{eq}$ → imposes important constraint !)

# Equations of motion (EoM)

Klein-Gordon equation for the SFDM field ψ

$$g^{\mu\nu}\partial_\mu\partial_\nu\psi - g^{\mu\nu}\Gamma^\sigma{}_{\mu\nu}\partial_\sigma\psi + \frac{m^2c^2}{\hbar^2}\psi + \frac{2\lambda m}{\hbar^2}|\psi|^2\psi = 0$$

...which is minimally coupled to GR $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$

A standard flat FLRW background Universe is usually assumed.

( side remark: In the „CDM-like“ SFDM-dominated epoch, well within the horizon, the non-relativistic limits yield a nonlinear Gross-Pitaevskii („Schrödinger“) equation:)

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi + \lambda|\psi|^2\psi + m\Phi\psi$$

$$\Delta\Phi = 4\pi G m|\psi|^2$$

# Evolution of background SFDM in an FLRW Universe

Compare size of SF oscillation freq **ω** to Hubble expansion rate **H**

- Fast oscillation regime („oscillation“):

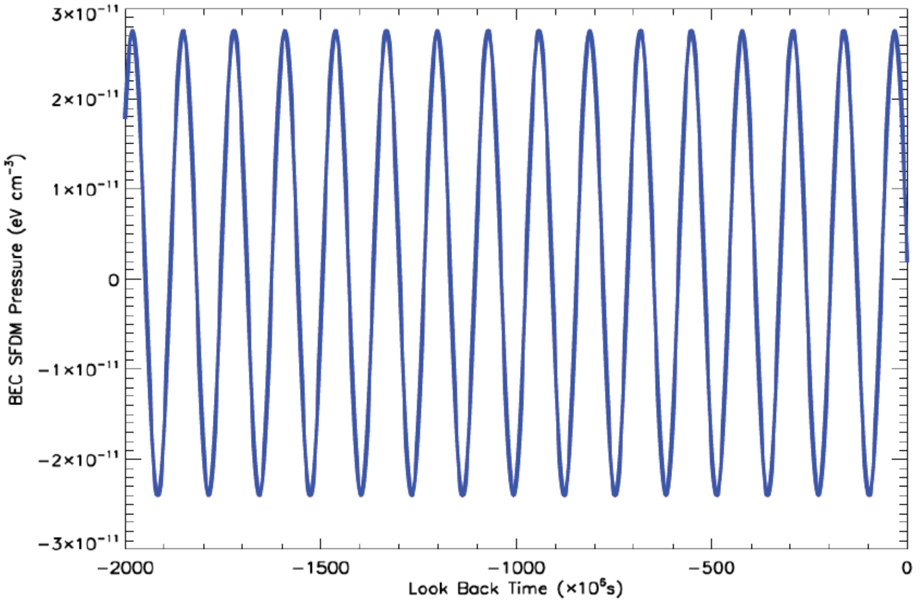
  **ω / H >> 1**

  disp.relation: ω = ω (V),

  e.g.

$$\omega = \frac{mc^2}{\hbar}\sqrt{1+\frac{2\lambda}{mc^2}|\psi|^2}$$

  "easy"




- Slow oscillation regime:

  **ω / H << 1**

  "hard"

  kinetic energy $\equiv$ 0: w = - 1 CC EOS ("constant energy density")

  kinetic energy ≠ 0: w = 1 stiff EOS ("kination", „stiff phase“)

***Non-standard expansion histories and consequences for structure formation !***

# real vs. complex SFDM

- **Real** e.g. QCD axion, axion-like particles:
  first $w = -1$ (*CC phase*), later $w = 0$ (*CDM-like phase*)

- **Complex** e.g. our model, Arbey et al.(2002), Boyle et al.(2003):
  first $w = 1$ (*stiff phase*), later $w = 0$ (*CDM-like phase*)
  if $\lambda > 0 \rightarrow w = 1/3$ (intermediate *rad.-like phase*)

**Real:** whether EOS stiffer than w = 0 depends on choice of potential and initial condition. That choice is usually set the same than for axions → CC

**Complex:** the requirement of setting the (conserved) charge density to the present-day DM abundance leads naturally to the stiff phase !

- SFDM with $w \gtrsim 1/3$ affects $\Delta N_{eff} = N_{eff} - 3.046$ !

  While $w = 0$ required not later than $z_{eq}$, deviations are allowed before BBN, but lead to constraints on the SFDM model !

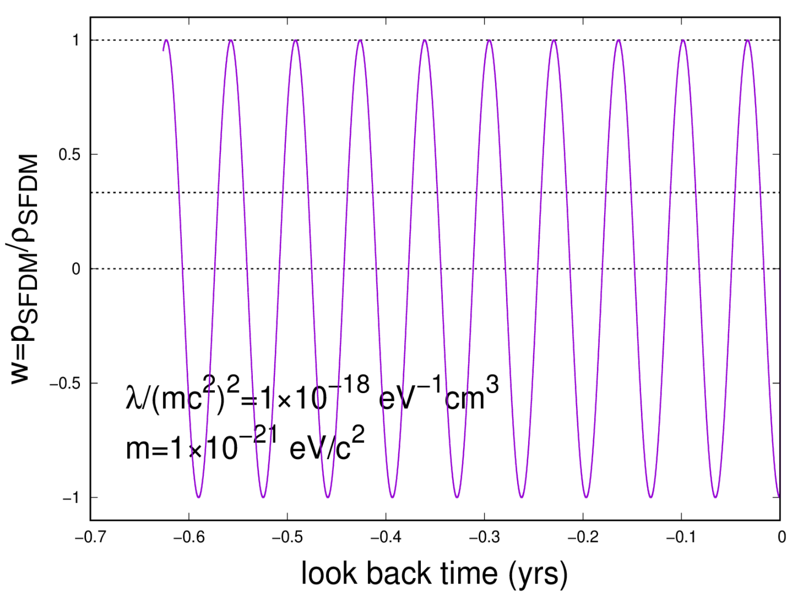
# Field oscillations of SFDM: real vs. complex

e.g. in the "CDM-like" SFDM-dominated epoch ("matter domination"):

*the average w oscillates around zero, however:*

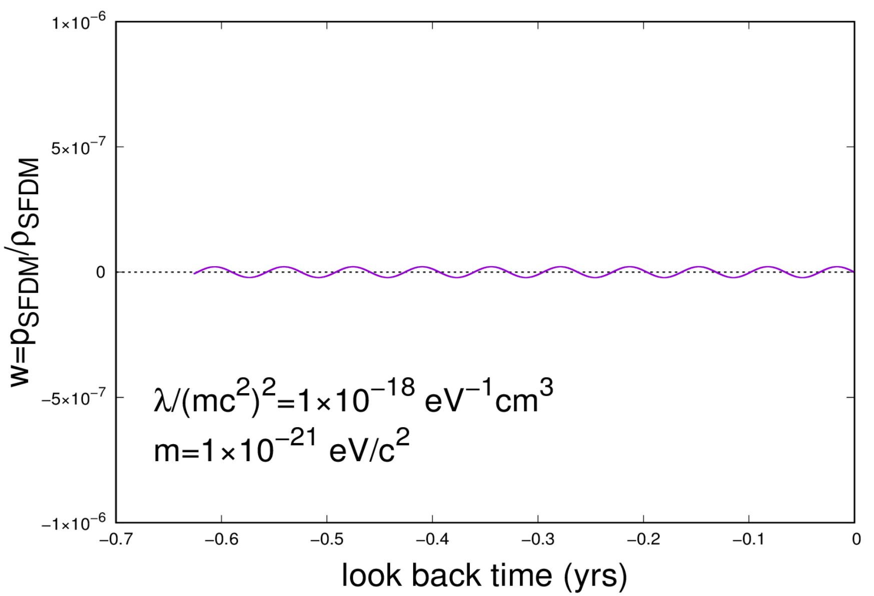
**Real field:**

w oscillates between -1 and 1




**Complex field (large-charge limit):**

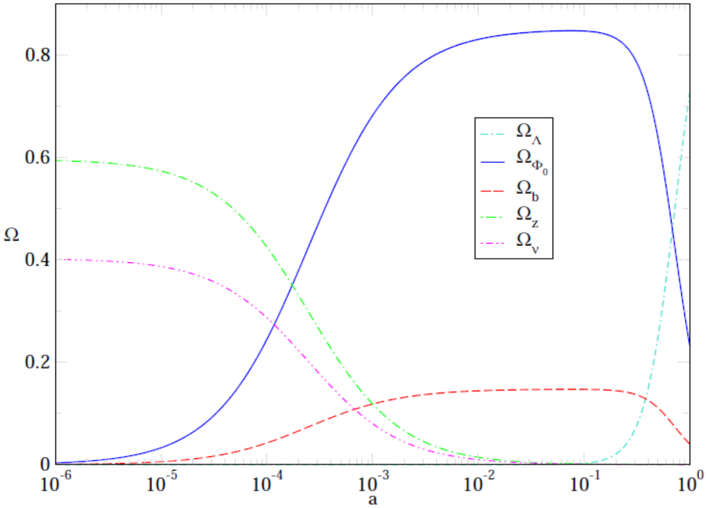
w oscillates between -c and +c where c <<< 1




***impacts structure formation, down to pulsar-timing signals within the Milky Way !***

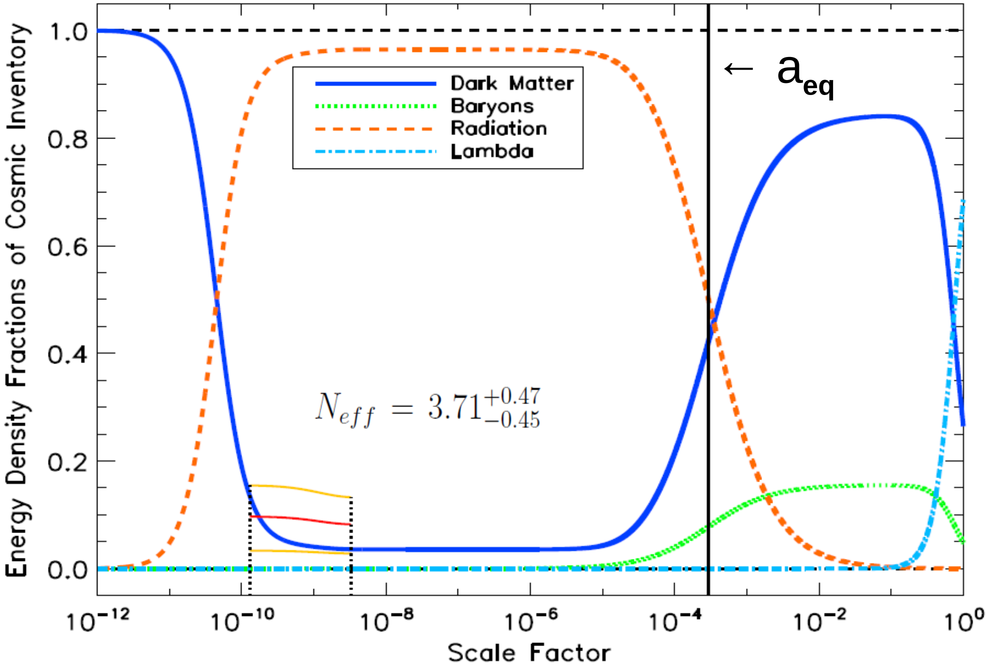
# real vs. complex SFDM:  evolution of Ω's

Magaña, Matos (2012)




$m = 10^{-22}$ eV

Li,TRD,Shapiro (1310.6061)




$$(m, \lambda)_{\text{fiducial}} = (3 \times 10^{-21}\ \text{eV}/\text{c}^2, 1.8 \times 10^{-59}\ \text{eV cm}^3)$$

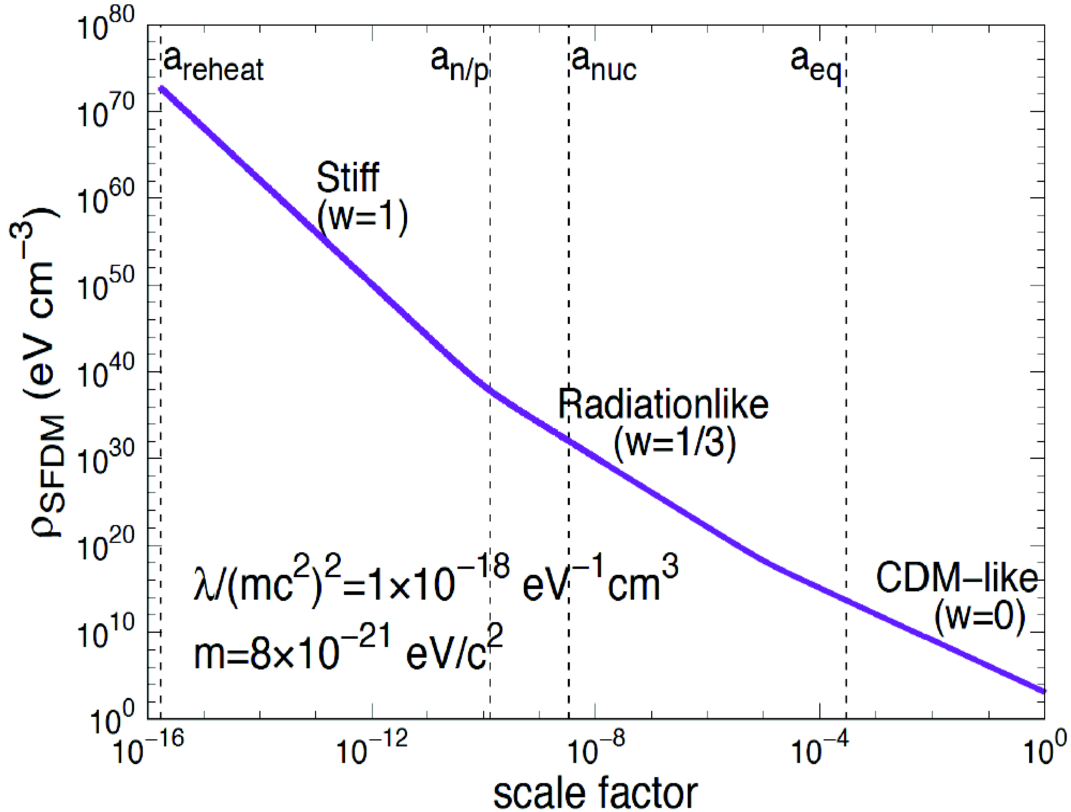
# cSFDM with repulsive SI has 3 phases:

*EOS: $(p/\rho)_{SFDM} = w(t)$*

**(1) Early: w = 1**
(stiff EOS)
**(2) Intermediate: w = 1/3**
(radiationlike, if positive SI)
**(3) Late: w = 0**
(non-relativistic matter)

→ ***change of standard expansion history !***

$\Omega_{SFDM} \rightarrow 1$ at early times




***Early Universe dominated by stiff cSFDM !***
→ ***implies additional $N_{eff}$ during (1) and (2) !***
→ ***amplifies primordial GWs from inflation during (1) !***

# ΛSFDM Model (2014) + GW (2017)

**2014:** take the same cosmic inventory as the basic ΛCDM model, except that

CDM is replaced by SFDM → **ΛSFDM**

**2017:** add stochastic GW background (**SGWB**) from inflation self-consistently to it

$\Omega_m = \Omega_b + \Omega_c$

Cosmological parameters from **Planck 2013/2015**

(assume massless SM neutrinos)

$\Omega_\Lambda = 1 - \Omega_m - \Omega_r$ (2014)

$\Omega_\Lambda = 1 - \Omega_m - \Omega_r - \Omega_{GW}$ (2017)

- SFDM particle parameters: m, $\lambda/(mc^2)^2$

$$\lambda/(mc^2)^2 = 1\times10^{-18}\,\mathrm{eV^{-1}cm^3} \implies l_{SI} \approx 0.8kpc$$

$$\mathcal{L} = \frac{\hbar^2}{2m} g^{\mu\nu}\partial_\mu\psi^*\partial_\nu\psi - \frac{1}{2}mc^2|\psi|^2 - \frac{\lambda}{2}|\psi|^4,$$

- Global U(1) symmetry ⟹ Charge (particle number density) conservation

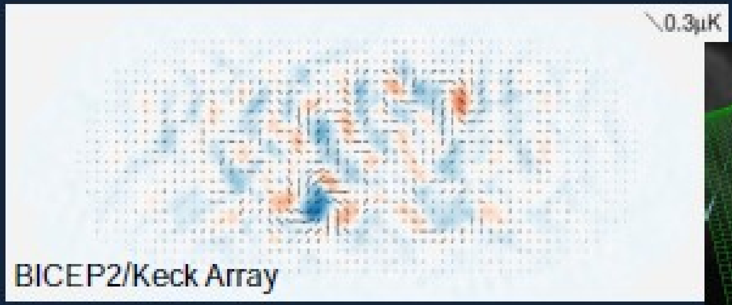
$$Q \equiv n - \bar{n} = \rho_{SFDM,0}/(mc^2)$$

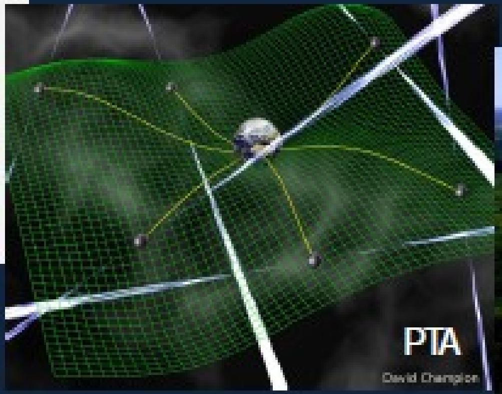
- Tensor-to-scalar ratio: $r = A_T/A_S$
- Reheating temperature: $T_{reheat}$

$$H_{\rm inf} = \frac{\pi M_{\rm pl}}{\hbar}\sqrt{rA_s}$$

*inflationary paradigm*

# Stochastic Gravitational-Wave Background from *Inflation*





## Single-field slow-roll inflation

- $r > 0.001$
- Consistency relation $n_t = -r/8$

**Subhorizon inflationary SGWB energy density spectrum:**

$$\Omega_{GW}(k,a) = \frac{\Delta^2_{h,init}(k)}{12}\left(\frac{kc}{aH}\right)^2 T_h(k,a), \qquad \Delta^2_{h,init}(k) = A_T(k/k_*)^{n_t}$$

# $\rho_{GW}$ (t): Tensor Mode Perturbations in the ΛSFDM Universe

Tensor mode equation of motion in Fourier space:

$$h_k''(\tau) + 2\frac{a'(\tau)}{a(\tau)}h_k'(\tau) + k^2 h_k(\tau) = 0$$

GW spectrum vs. k at scale factor a(t):

$$\Omega_{\mathrm{GW}}(k,a) \equiv \frac{\mathrm{d}\Omega_{\mathrm{GW}}(a)}{\mathrm{d}\ln k} = \frac{1}{\rho_{\mathrm{crit}}(a)}\frac{\mathrm{d}\rho_{\mathrm{GW}}(a)}{\mathrm{d}\ln k}$$
$$= \frac{\Delta_h^2(k,a)c^2}{24a^2H^2(a)}\left(\left|\frac{h_k'(a(\tau))}{h_k(a(\tau))}\right|^2 + k^2\right)$$

conformal time: $d\tau \equiv dt / a(t)$

- In subhorizon limit, different modes contribute to $\rho_{GW}$ (t) according to the expansion phase during which they re-entered the horizon, how many e-foldings elapse in each phase since horizon crossing, and the initial power spectrum: $\Delta_{h,\mathrm{init}}^2(k) \simeq k^0$

w = 0 (reheating era) ←→ $\Omega_{\mathrm{GW}}^{\mathrm{m}}(k,\tau) \simeq \frac{\Delta_{h,\mathrm{init}}^2(k)}{24}\cdot\frac{9}{4}\frac{1}{(k\tau)^2}$, Red tilt

w = 1 (stiff-SFDM-dominated) era ←→ $\Omega_{\mathrm{GW}}^{\mathrm{stiff}}(k,\tau) \simeq \frac{\Delta_{h,\mathrm{init}}^2(k)}{24}\cdot\frac{8}{\pi}k\tau$, Blue tilt

w = 1/3 (radiation-dominated era) ←→ $\Omega_{\mathrm{GW}}^{\mathrm{rad}}(k,\tau) \simeq \frac{\Delta_{h,\mathrm{init}}^2(k)}{24}$.

# Holistic Evolution of the ΛSFDM Universe

- Friedmann equation

$$H^2(t) \equiv \left(\frac{\mathrm{d}a/\mathrm{d}t}{a}\right)^2 = \begin{cases} H_{\inf}^2, & a < a_{\inf}, \\ H_{\inf}^2 \left(\frac{a_{\inf}}{a(t)}\right)^3, & a_{\inf} < a < a_{\text{reheat}}, \\ \frac{8\pi G}{3c^2}\left[\rho_r(t) + \rho_b(t) + \rho_\Lambda(t) + \rho_{\text{SFDM}}(t) + \rho_{\text{GW}}(t)\right], & a > a_{\text{reheat}}, \end{cases}$$

SGWB contribution to the expansion history *self-consistently* included

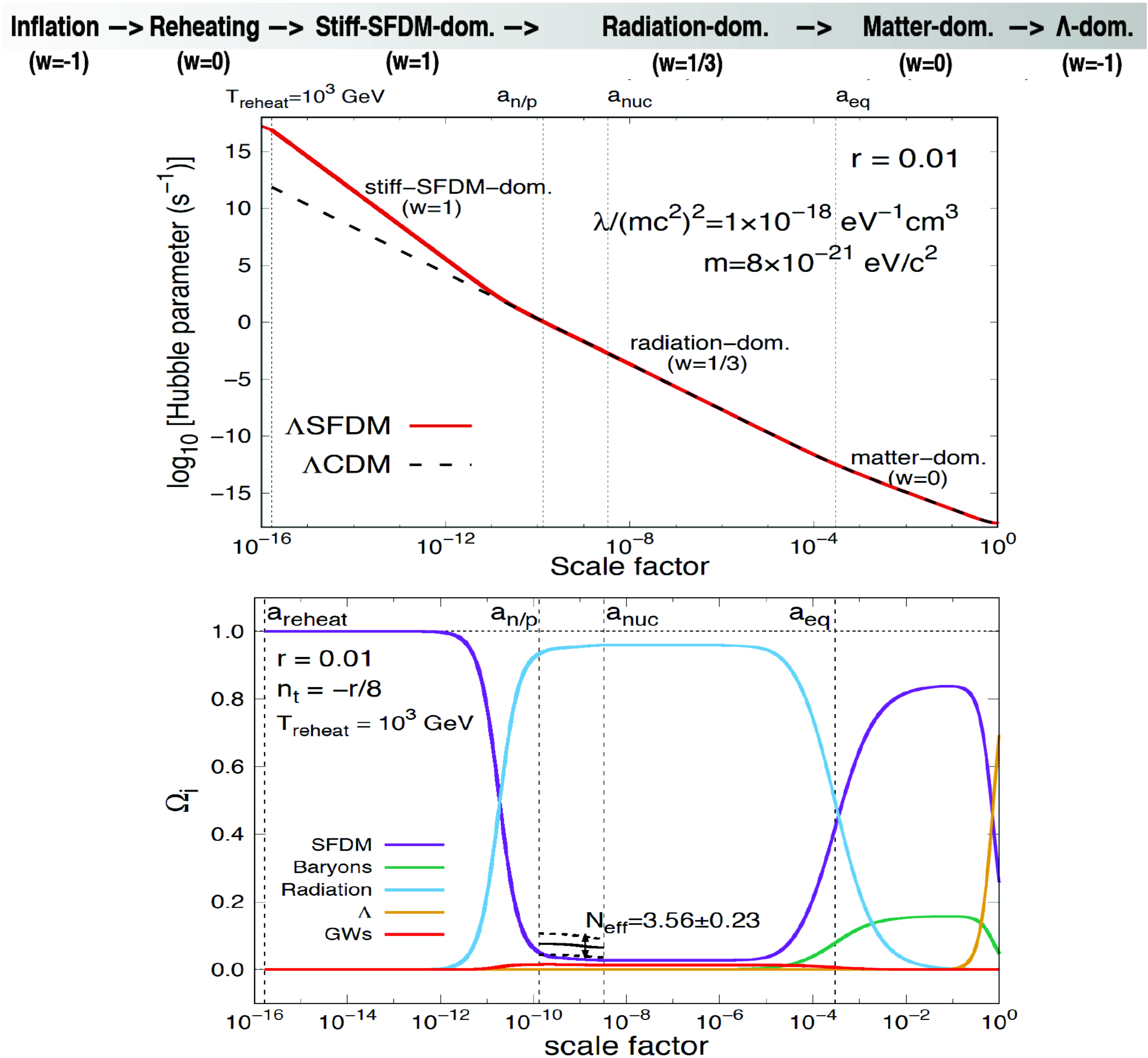
$$\Omega_{\text{GW}}(k,a) \equiv \frac{\mathrm{d}\Omega_{\text{GW}}(a)}{\mathrm{d}\ln k} = \frac{1}{\rho_{\text{crit}}(a)}\frac{\mathrm{d}\rho_{\text{GW}}(a)}{\mathrm{d}\ln k}$$
$$= \frac{\Delta_h^2(k,a)c^2}{24a^2H^2(a)}\left(\left|\frac{h_k'(a(\tau))}{h_k(a(\tau))}\right|^2 + k^2\right)$$

conformal time: $d\tau \equiv dt/a(t)$

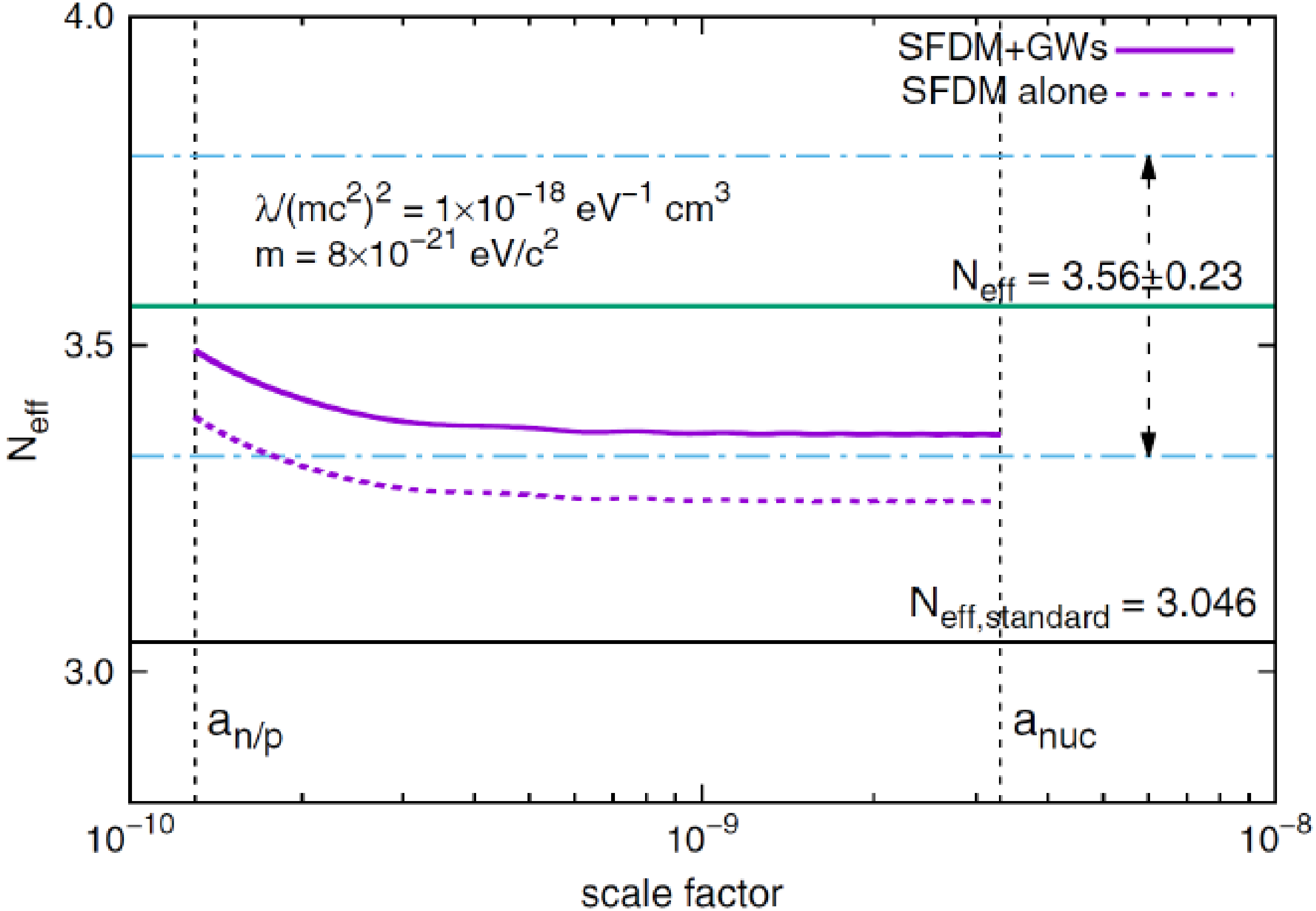
- Klein-Gordon Equation

$$\frac{\hbar^2}{2mc^2}\ddot{\psi} + 3\frac{\hbar^2}{2mc^2}\frac{\dot{a}}{a}\dot{\psi} + \frac{1}{2}mc^2\psi + \lambda|\psi|^2\psi = 0,$$

# ΛSFDM+Inflation: the Universe has 6 eras



# $N_{eff}$ during BBN



# ΛSFDM + SGWB

Limiting the duration of the stiff phase after reheating and before BBN constrains SFDM parameters via their contribution to $N_{\mathbf{eff}}$

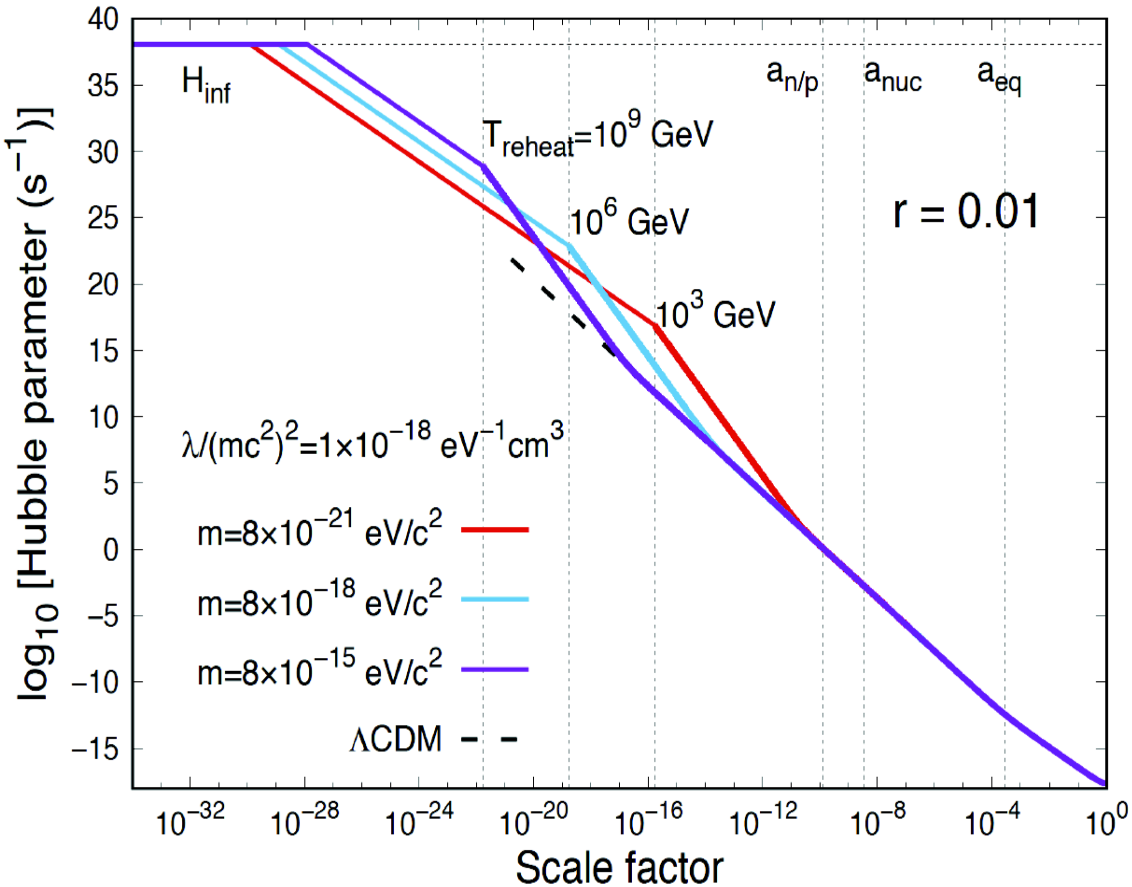
- for given r:
  the smaller the DM mass,
  the later must
  reheating occur

- Matter-radiation equality:

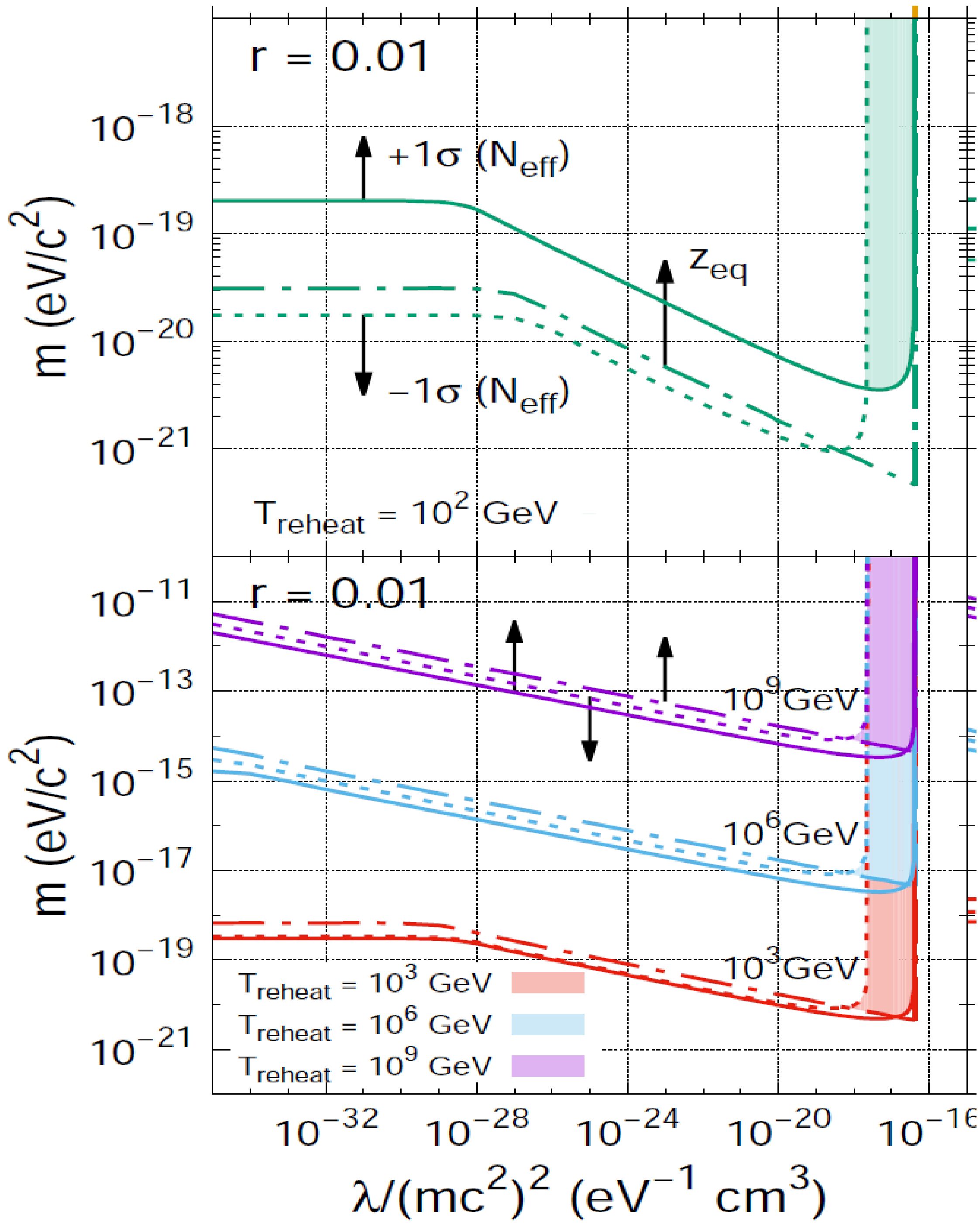
$$1 + z_{\rm eq} \equiv \frac{1}{a_{\rm eq}} = \frac{\Omega_b h^2 + \Omega_c h^2}{\Omega_r h^2 + \Omega_{\rm GW} h^2}$$

- $N_{\mathbf{eff}}$ during BBN:

$$\frac{\Delta N_{\rm eff,BBN}(a)}{N_{\rm eff,standard}} = \frac{\Omega_{\rm SFDM}(a) + \Omega_{\rm GW}(a)}{\Omega_\nu(a)}$$



## Constraints from $z_{eq}$ and BBN on the SFDM parameters with GW background included




Zeq = 3365 +/- 44
(68% C.L.)

Neff, BBN = 3.56 +/- 0.23
(68% C.L.)

# Constraints from $z_{eq}$ and BBN on the SFDM parameters with GW background included

$$2.3\times10^{-18}\,\mathrm{eV}^{-1}\,\mathrm{cm}^3 \leq \frac{\lambda}{(\mathrm{mc}^2)^2} \leq 4.1\times10^{-17}\,\mathrm{eV}^{-1}\,\mathrm{cm}^3,$$

$$m_{\min} \simeq (5\times10^{-21}\ \mathrm{eV}/c^2)\times\begin{cases}\dfrac{T_{\mathrm{reheat}}}{10^3\ \mathrm{GeV}}\sqrt{\dfrac{r}{0.01}}, & T_{\mathrm{reheat}} \gtrsim 10^3\ \mathrm{GeV},\\[2ex] 1, & T_{\mathrm{reheat}} < 10^3\ \mathrm{GeV}.\end{cases}$$

# Cosmological Constraints on the SFDM Particle Parameters

- Matter-radiation equality: $z_{eq}$

$$1 + z_{\rm eq} \equiv \frac{1}{a_{\rm eq}} = \frac{\Omega_b h^2 + \Omega_c h^2}{\Omega_r h^2 + \Omega_{\rm GW} h^2},$$

- Effective number of neutrino species at BBN: $N_{eff}$

$$\frac{\Delta N_{\rm eff,BBN}(a)}{N_{\rm eff,standard}} = \frac{\Omega_{\rm SFDM}(a) + \Omega_{\rm GW}(a)}{\Omega_\nu(a)},$$

- SGWB measured by laser interferometers:

$\Omega_{GW}(f)$ at a=1

# $\rho_{GW}$ (t): Tensor Mode Perturbations in the ΛSFDM Universe

Tensor mode equation of motion in Fourier space:

$$h_k''(\tau) + 2\frac{a'(\tau)}{a(\tau)}h_k'(\tau) + k^2 h_k(\tau) = 0$$

GW spectrum vs. k at scale factor a(t):

$$\Omega_{\rm GW}(k,a) \equiv \frac{d\Omega_{\rm GW}(a)}{d\ln k} = \frac{1}{\rho_{\rm crit}(a)}\frac{d\rho_{\rm GW}(a)}{d\ln k} = \frac{\Delta_h^2(k,a)c^2}{24a^2H^2(a)}\left(\left|\frac{h_k'(a(\tau))}{h_k(a(\tau))}\right|^2 + k^2\right)$$

conformal time: $d\tau \equiv dt / a(t)$

- In subhorizon limit, different modes contribute to $\rho_{GW}$ (t) according to the expansion phase during which they re-entered the horizon, how many e-foldings elapse in each phase since horizon crossing, and the initial power spectrum: $\Delta^2_{h,\rm init}(k) \simeq k^0$

w = 0 (reheating era) ↔ $\Omega^{\rm m}_{\rm GW}(k,\tau) \simeq \frac{\Delta^2_{h,\rm init}(k)}{24}\cdot\frac{9}{4}\frac{1}{(k\tau)^2}$, Red tilt

w = 1 (stiff-SFDM-dominated) era ↔ $\Omega^{\rm stiff}_{\rm GW}(k,\tau) \simeq \frac{\Delta^2_{h,\rm init}(k)}{24}\cdot\frac{8}{\pi}k\tau$, Blue tilt

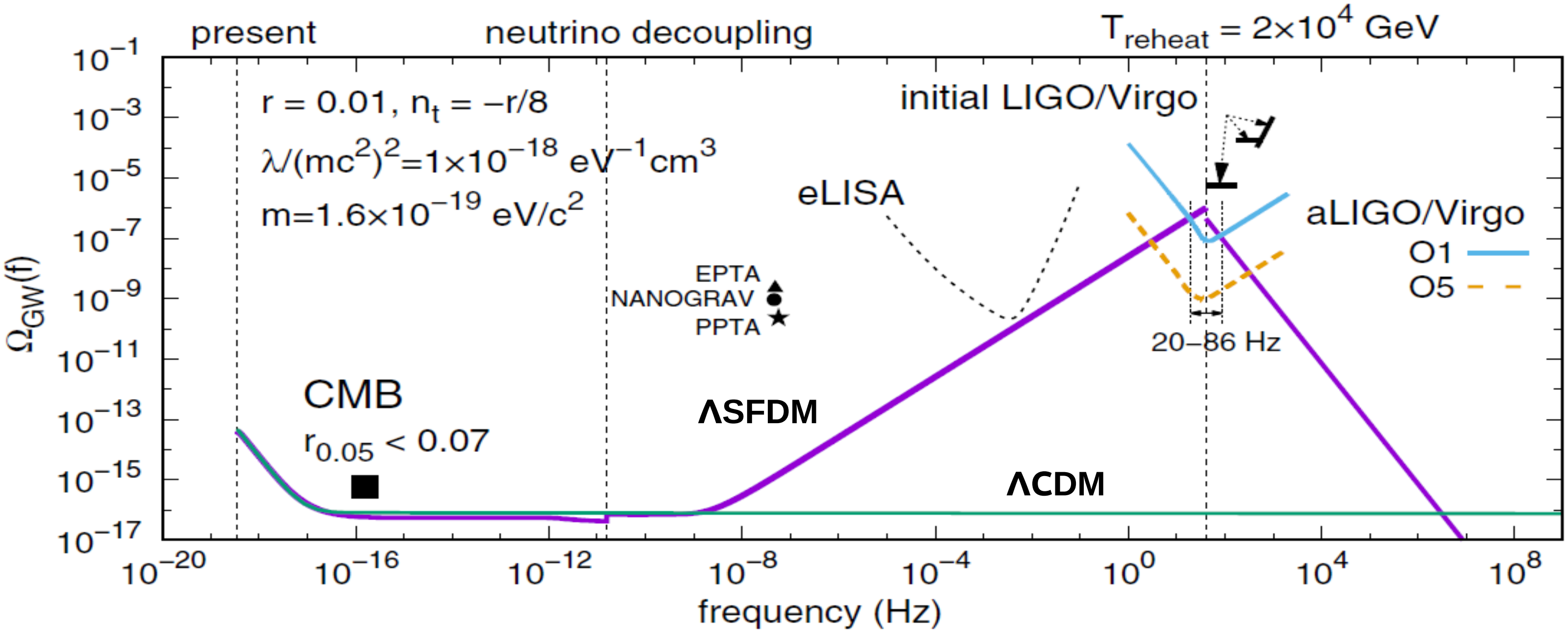
w = 1/3 (radiation-dominated era) ↔ $\Omega^{\rm rad}_{\rm GW}(k,\tau) \simeq \frac{\Delta^2_{h,\rm init}(k)}{24}$.

# ΛSFDM + SGWB:
# enhanced signal of inflationary SGWB due to DM !

**<u>Stiff-SFDM-dominated era</u> amplifies SGWB from (standard) inflation:**

**can be measured/constrained by GW laser interferometers !**

**Case 2**




**ΛSFDM predicts 2-parameter broken power-law spectrum at high frequencies:**

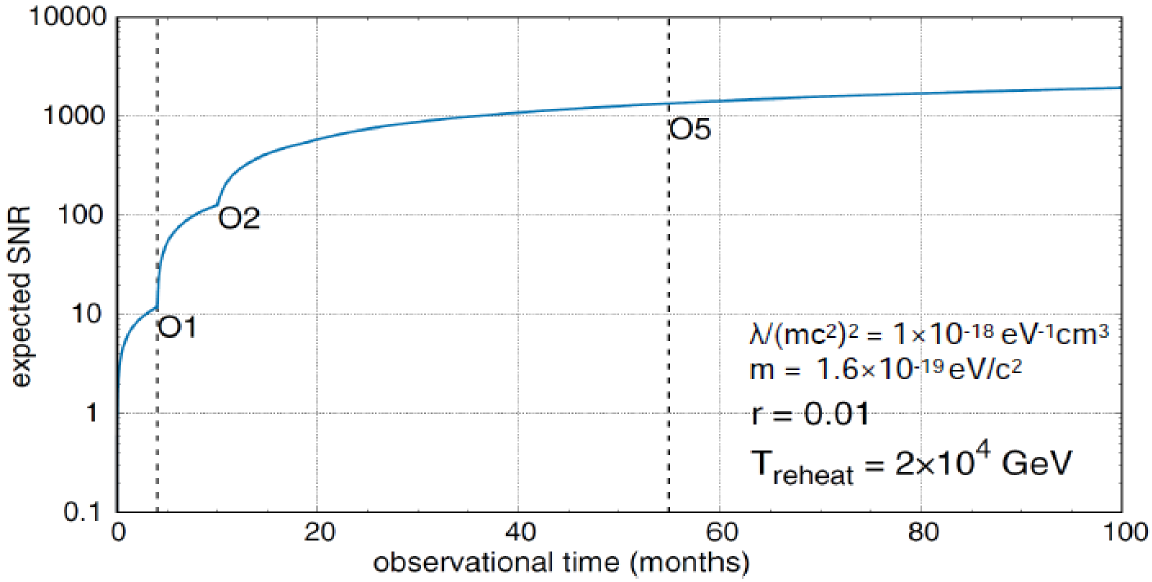
$$\Omega_{GW}(f) = \Omega_{GW,peak} \times \begin{cases} f/f_{peak}, & f \le f_{peak} \\ \frac{9\pi}{64}(f/f_{peak})^{-2}, & f > f_{peak} \end{cases}$$

# $\Lambda$SFDM + SGWB:
# enhanced signal of inflationary SGWB due to DM !

**<u>Stiff-SFDM-dominated era</u> amplifies SGWB from (standard) inflation:**

**can be measured/constrained by GW laser interferometers !**

## Case 2



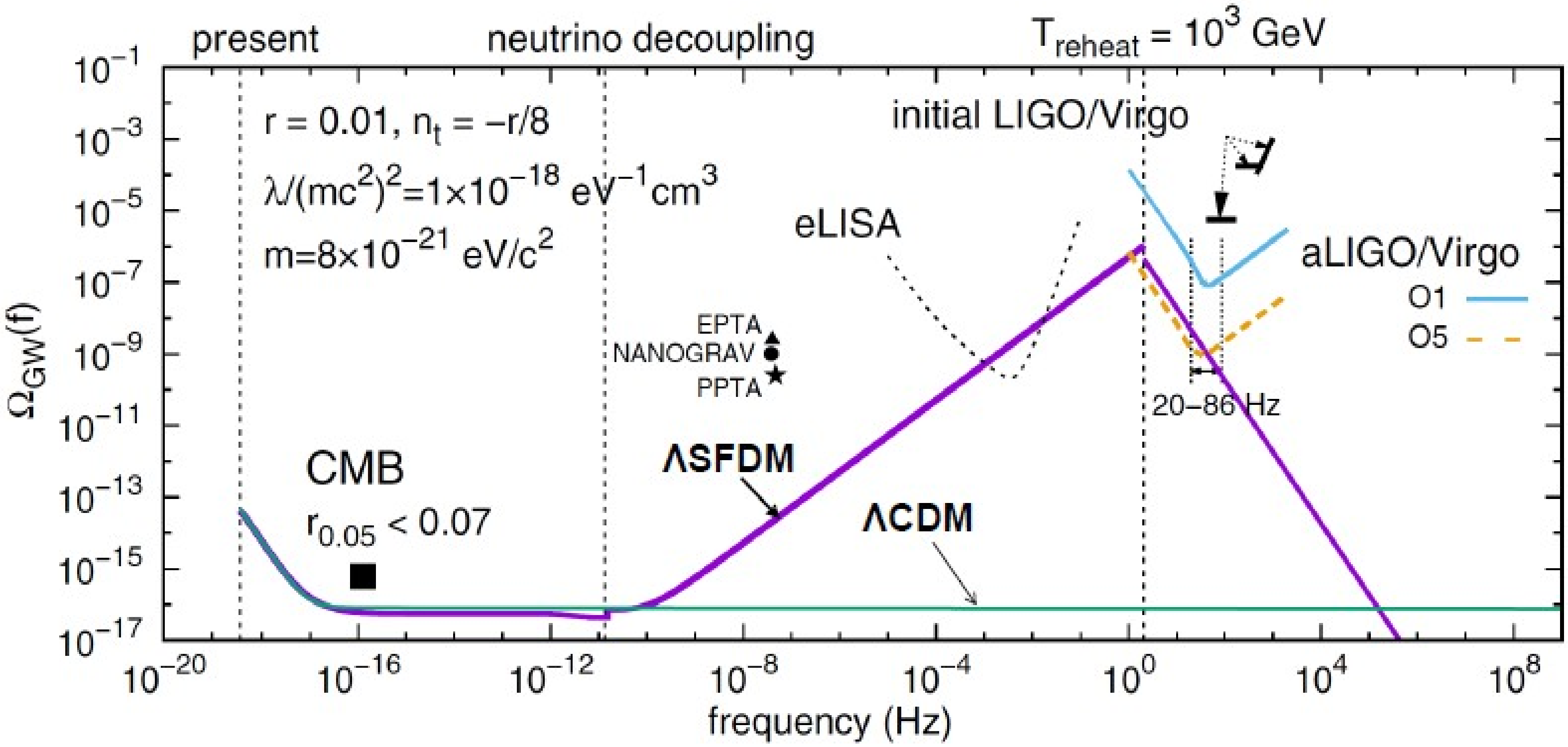

**Upper limit from LIGO O1 data excludes case 2 at 95% CL**

→ ***<u>The Age of DM Search/Constraints by GW Detection has begun !</u>***

# ΛSFDM + SGWB:

**<u>Stiff-SFDM-dominated era</u> amplifies SGWB from (standard) inflation: can be measured/constrained by GW laser interferometers !**

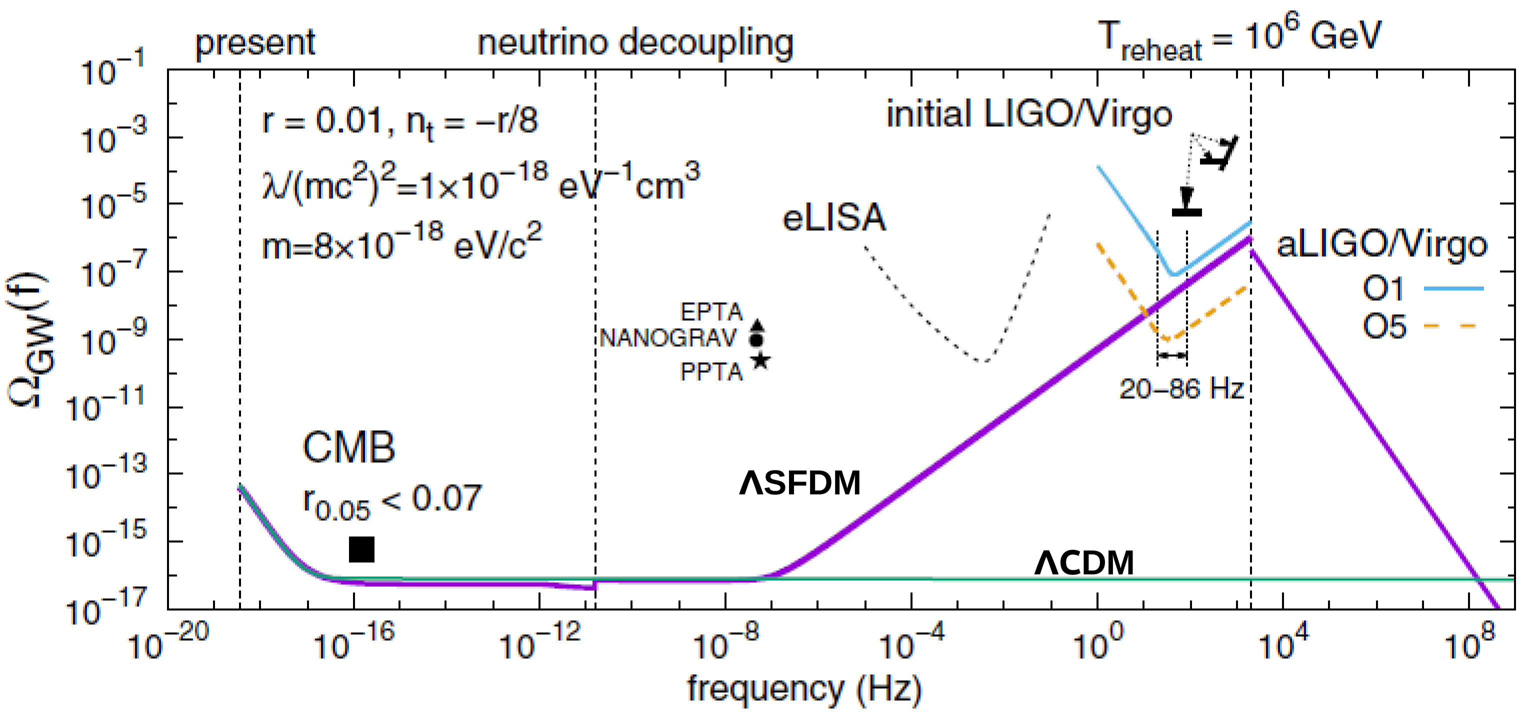
## Case 1



# ΛSFDM + SGWB:

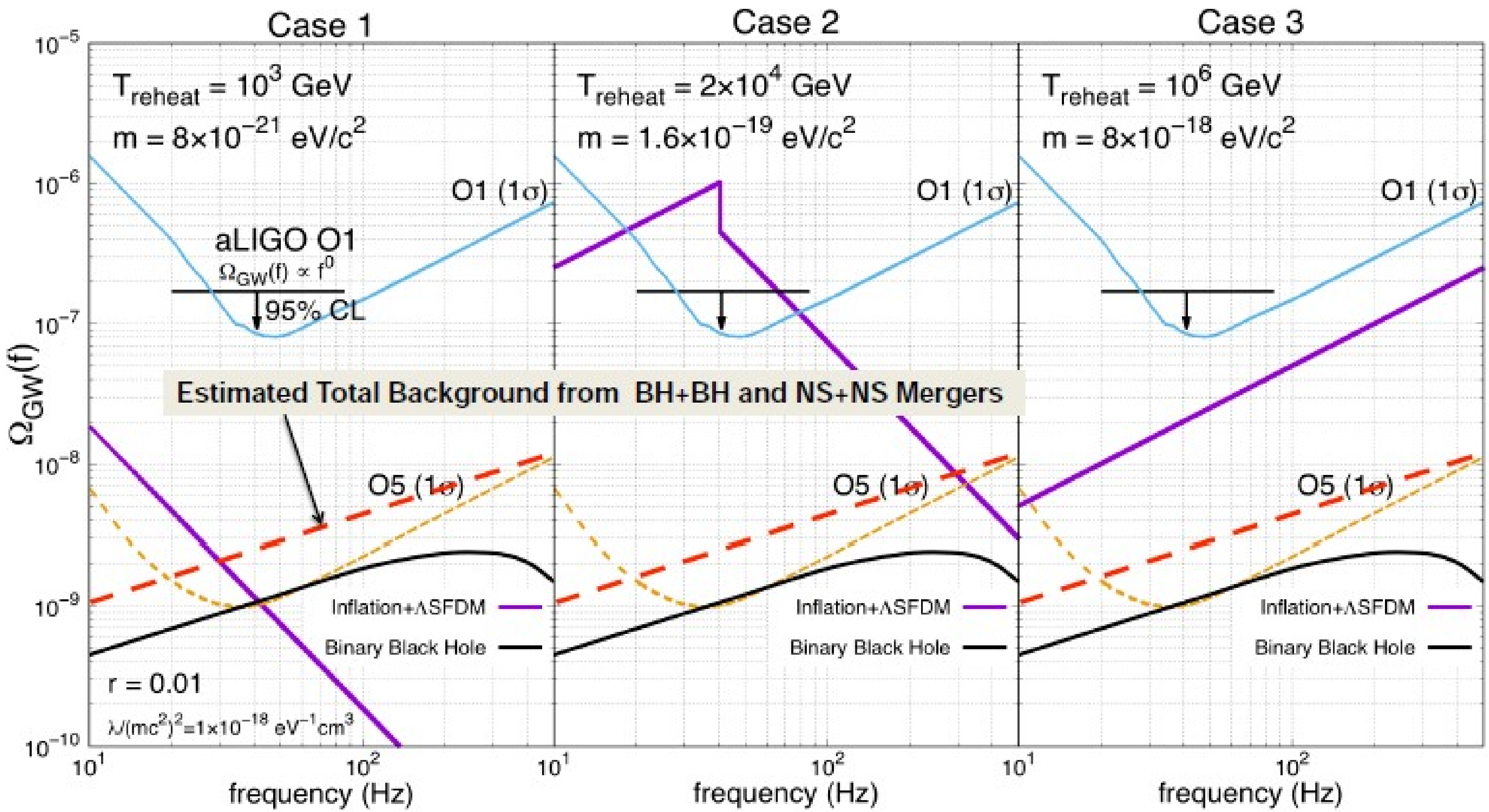
**<u>Stiff-SFDM-dominated era</u> amplifies SGWB from (standard) inflation: can be measured/constrained by GW laser interferometers !**

## Case 3



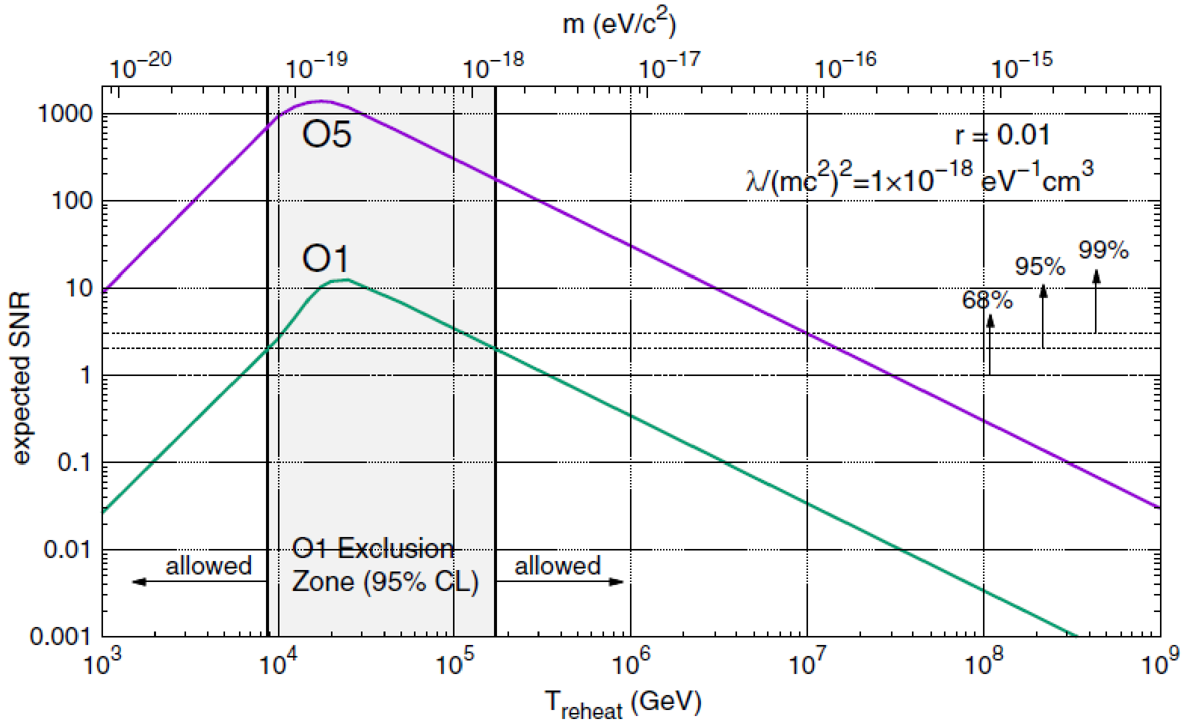
# Stiff-SFDM-dominated era amplifies SGWB from inflation

**SGWB's from (SFDM + Inflation) vs. (Unresolved BH + BH and NS + NS Binary Mergers)**



# ΛSFDM + SGWB:

**Limits from O1 of LIGO (1612.02029):**




| LIGO run | Epoch | $T_{\rm reheat}/\mathrm{GeV}$ (SNR > 2) | $m/(\mathrm{eV}/c^2)$ (SNR > 2) |
|---|---|---|---|
| O1 | 2015–2016 | $(8.75\times10^3, 1.7\times10^5)$ | $(7\times10^{-20}, 1.36\times10^{-18})$ |
| O5 | 2020–2022 | $(5\times10^2, 1.5\times10^7)$ | $(4\times10^{-21}, 10^{-16})$ |

***A detection of the inflationary SGWB is possible by looking for signals:***

a wide range of SFDM particle parameters and reheat temperatures can be already tested by **aLIGO/VIRGO limits on the SGWB**:

- some models (e.g. „Case 2“) are already ruled out from O1 limits
- the newest O2 limit does not exclude „Case 1“ and „Case 3“, but other models are ruled out, which may push beyond the allowed limit
- even more models will be tested by the time of O5

→ **current(!) GW laser interferometer experiments can already constrain DM models !**


Bohua Li, Tanja Rindler-Daller, Paul R. Shapiro 2014, PRD, 89, 083536 (arXiv: 1310.6061)

Bohua Li, Paul R. Shapiro, Tanja Rindler-Daller 2017, PRD, 96,063505 (arXiv: 1611.07961)

# Conclusions

- SFDM candidates may resolve small-scale problems of CDM *structure formation*

- However, deviations from CDM are also possible on large scales:

  - non-standard expansion histories before and after BBN

  - manifest field oscillations distinguish SFDM from CDM, and amplitudes differ between real and complex scalar-fields

- As a result: SFDM model parameters are constrained by the CMB, BBN, stochastic grav.wave background from inflation, large-scale structure, pulsar-timing, etc.

- Some of these constraints are already tighter than those inferred from small-scale structure

**Li, Shapiro, Rindler-Daller 2017 PRD, 96, 063505 (arXiv:1611.07961)**
**Li, Rindler-Daller, Shapiro 2014 PRD, 89, 083536 (arXiv: 1310.6061)**