$\bar{T}$: A New Cosmological Parameter?

based on Yoo, Mitsou, Dirian, Durrer 2019 PRD 100, 063510

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CMB Temperatures $\bar{T}$ and $\langle T \rangle^{\mathrm{obs}}$

- $\bar{T}$: **background CMB temperature** (cos. parameter)
  - determine **background** evolution
  - defined in background universe, ***unique*** number
  - influence **perturbation** evolution

- $\langle T \rangle^{\mathrm{obs}}$: **observed CMB temperature** (from FIRAS)
  - **angle average** CMB temperature over all sky
  - uncertainty in $\langle T \rangle^{\mathrm{obs}}$:
    - COBE FIRAS 1996: $\langle T \rangle^{\mathrm{obs}} = 2.728 \pm 0.004\ \mathrm{K}$ (0.15%)
    - + WMAP 2009: $\langle T \rangle^{\mathrm{obs}} = 2.7255 \pm 5.7 \cdot 10^{-4}\ \mathrm{K}$ (0.021%)

Are They Same?

- **Ergodic theorem?**
  - **standard practice:** $\bar{T} \equiv \langle T \rangle^{\mathrm{obs}}$
  - **Euclidean average becomes ensemble average**
  - **average over all observer positions:** ***impossible!***

- **observations:** $T^{\mathrm{obs}}(\hat{n}) := \bar{T}[1 + \Theta(\hat{n})]$
  - **angle average:** $$\langle T \rangle^{\mathrm{obs}} = \int \frac{d^2\hat{n}}{4\pi}\, T^{\mathrm{obs}}(\hat{n}) = \bar{T}\left(1 + \Theta_0\right) \neq \bar{T}$$
  - **monopole: non-zero, fluctuate in space**
  - **monopole in observation:** ***set zero***
  - ***even with future CMB experiment*** $\langle T \rangle^{\mathrm{obs}} \neq \bar{T}$

Standard Data Analysis

- **CMB observations:** $T^{\rm obs}(\hat{n}) := \sum T^{\rm obs}_{lm} Y_{lm}(\hat{n})$
  - **dimensionful power spectrum** $D^{\rm obs}_l := \sum_m |T^{\rm obs}_{lm}|^2/(2l+1)$
- **theory predictions:** $\Theta(\hat{n}) := \sum a_{lm} Y_{lm}(\hat{n})$
  - **dimensionless power spectrum** $C_l := \langle |a_{lm}|^2 \rangle$
  - **conversion:** $T_{lm} \equiv \bar{T} a_{lm}\,, \quad D_l \equiv \bar{T}^2 C_l$
- **standard (*imprecise*) practice:** $\bar{T} \equiv \langle T \rangle^{\rm obs}$
  - **background evolution is *different! if*** $\Theta_0 \neq 0$
  - **in practice compare**

$$\frac{T^{\rm obs}_{lm}}{\langle T \rangle^{\rm obs}} = \frac{a_{lm}}{1+\Theta_0}\,, \qquad C^{\rm biased}_l := \left\langle \frac{|a_{lm}|^2}{(1+\Theta_0)^2} \right\rangle = C_l \left(1 + \frac{3}{4\pi} C_0 + \cdots \right)$$

Problems in Standard Practice

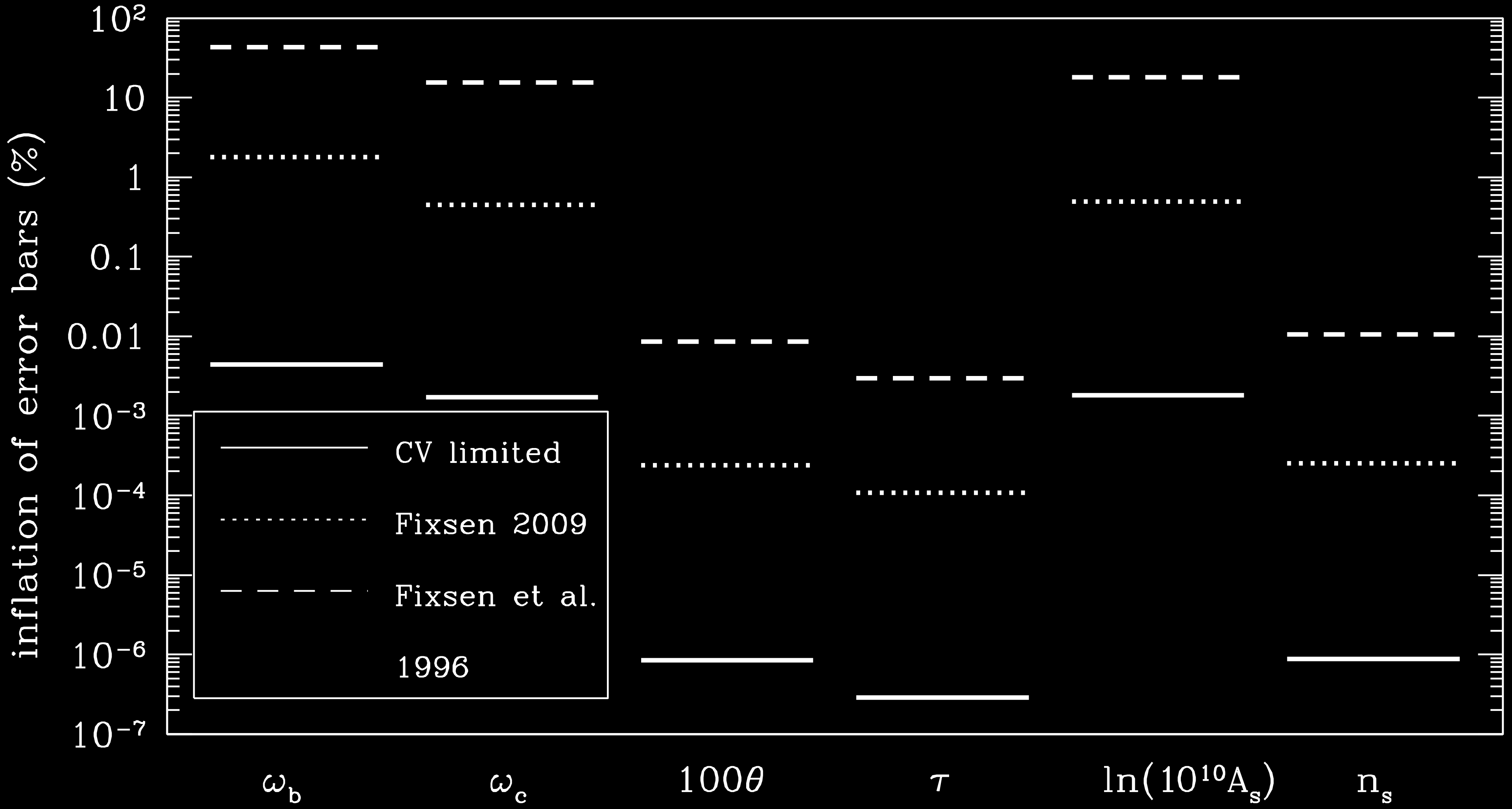
- **impact of standard practice:** $\bar{T} \equiv \langle T \rangle^{\rm obs}$
  - ***underestimation of error bars***
  - ***systematic biases in the best-fit parameters***

- **underestimation of error bars:** $\sigma_{\rm true} \gtrsim \sigma_{\rm std}$
  - **any models have *one less degree* of freedom:** $\bar{T}$
  - **error bars are always smaller than true error bars**

- **systematic biases:** $p_i^{\rm best} := p_i^{\rm true} + \delta p_i \ , \quad \delta p_i \neq 0$
  - ***bias*: in proportion to *monopole* at our position**
  - **monopole at our position: unknown**

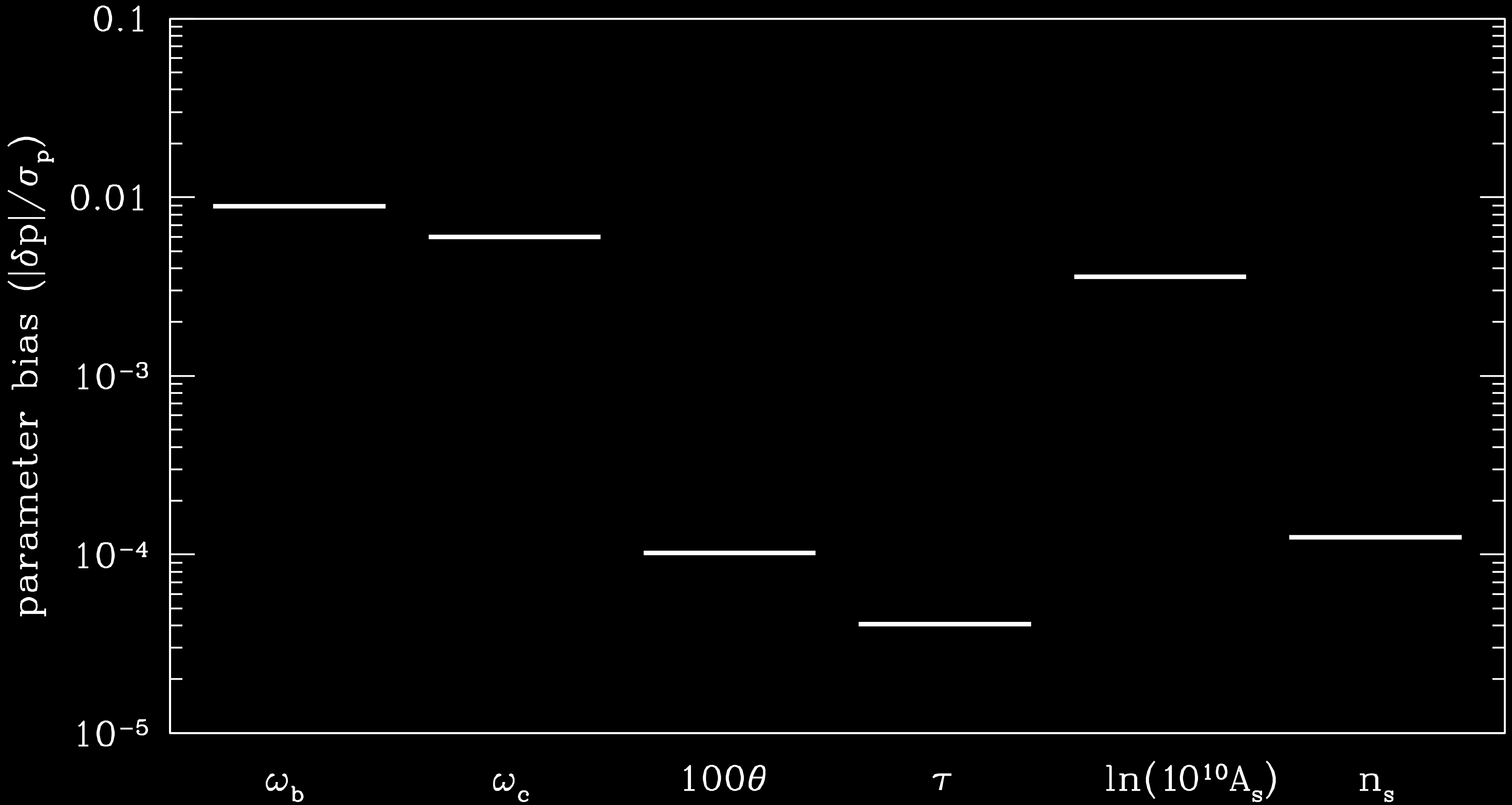
Inflation of Error Bars

- **true error bars are larger:** $\sigma_{\rm true} \gtrsim \sigma_{\rm std}$
  - **depend on uncertainty in** $\langle T \rangle^{\rm obs}$ ($\bar{T}$ **will be unknown**)



Systematic Biases

- **best-fit parameters are biased:** $\Theta_0 \sim 10^{-5}$ **(1-σ)**
  - **e.g., baryon density 1%, but *irreducible***



Conclusion

- **systematic errors:** $\langle T \rangle^{\rm obs} \neq \bar{T}$
  - **underestimation of error bars**
  - **systematic biases in the best-fit parameters**

- **significance of the systematic errors**
  - **smaller than the error bars from *Planck***
  - **monople power** $C_0 = 10^{-9}\ , \quad \Theta_0 \simeq 10^{-5}$ **(at 1-σ)**
  - **always present in any analysis**
- ***proper way* to analyze: simple!**
  - **include T as an extra cosmological parameter**

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Monopole & Gauge Choice

- **gauge dependence**: $T^{\rm obs}(\hat{n}) = \bar{T}(t)\left[1 + \Theta(\hat{n})\right]$
  - **hypersurface is unspecified,** $\Theta(\hat{n})$ **is** ***gauge dependent***
  - **coordinate transformation** $\tilde{t} = t + \xi^t$

  $$\bar{T}(\tilde{t}) = \bar{T}(t) + \dot{\bar{T}}(t)\xi^t \qquad \tilde{\Theta} = \Theta + H\xi^t$$

- **coordinate independent reference:**
  - **observed redshift is observable, redshift = 0 for CMB**
  - **time coordinate at redshift zero:** ***bg age of Universe***

  $$\bar{t}_o = \int_0^\infty \frac{dz}{(1+z)H(z)}$$

- **gauge-invariant monopole**:
  - **observer time coordinate:** $t_o := \bar{t}_o + \delta t_o$
  - $\bar{T} := \bar{T}(\bar{t}_o) \qquad T^{\rm obs}(\hat{n}) = \bar{T}\left[1 + \Theta - H\delta t_o\right]$
  - **time lapse vanish in synchronous gauge:** $\delta t_o = 0$

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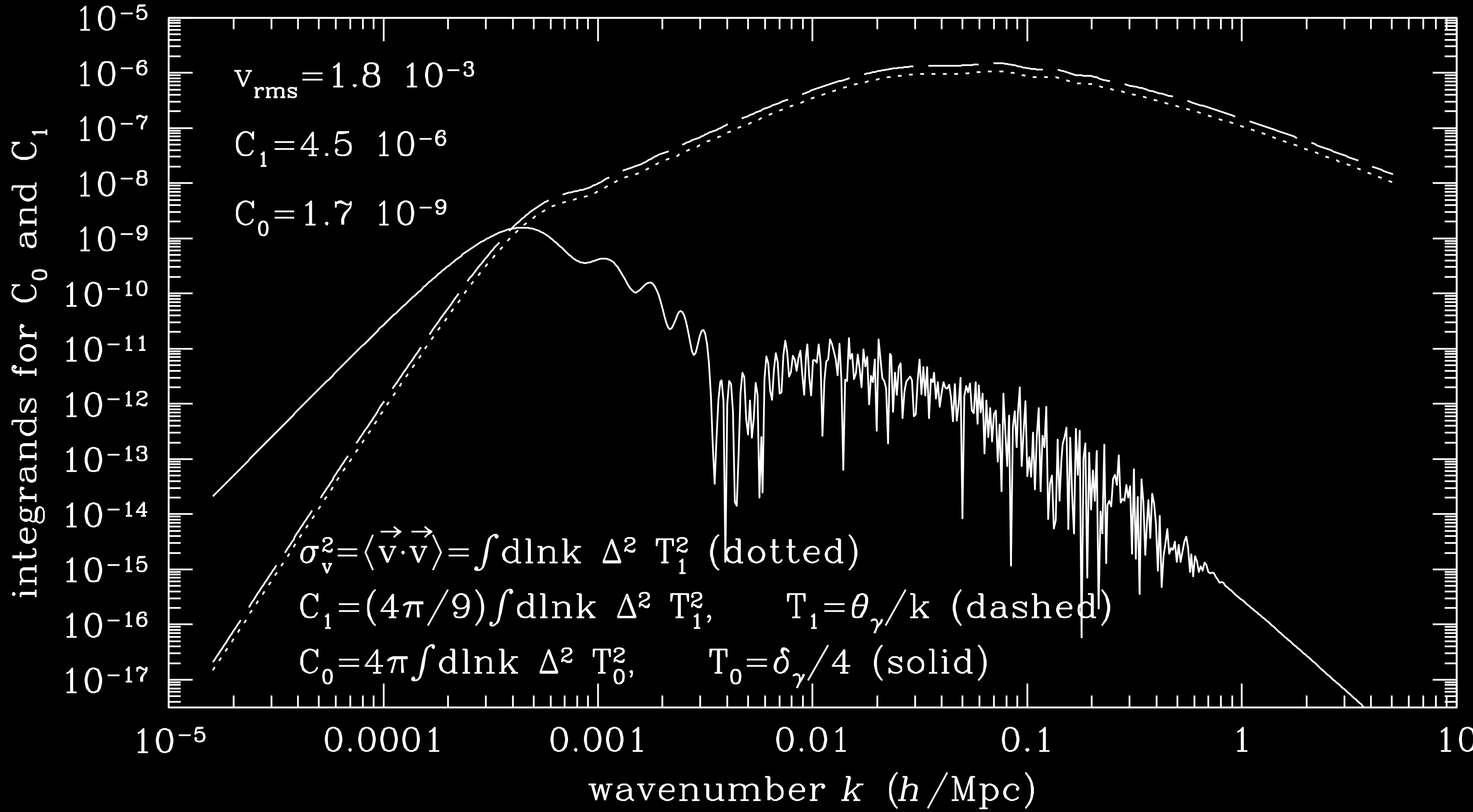
Monopole & Dipole



Baumgartner & Yoo, in preparation

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