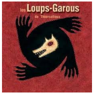
Winding mode calculation of the effective potential in extra-dimensional theories

(New method for gauge-higgs unification models)

COT Corentin

Institut de Physique des 2 Infinis de Lyon
Université Claude Bernard de Lyon



Journées de Rencontre des Jeunes Chercheurs, 2019

Outline

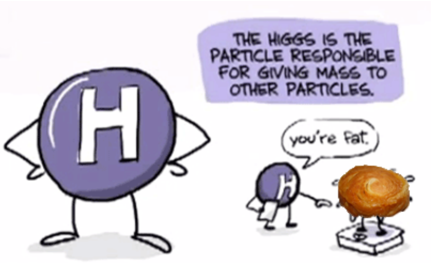
1. Introduction
   - The Higgs boson
   - The Hierarchy problem
   - Gauge-Higgs Unification
   - Validity of the model
2. The winding mode method
   - Propagators modes decomposition
   - The one-loop effective potential
   - Winding modes decomposition
   - Winding mode resummation
3. Results
   - $SU(2)$ and $SU(3)$ results
   - $SU(5)$ results
   - Future work
4. Bibliography and appendices

Outline

1. Introduction
   - The Higgs boson
   - The Hierarchy problem
   - Gauge-Higgs Unification
   - Validity of the model
2. The winding mode method
   - Propagators modes decomposition
   - The one-loop effective potential
   - Winding modes decomposition
   - Winding mode resummation
3. Results
   - $SU(2)$ and $SU(3)$ results
   - $SU(5)$ results
   - Future work
4. Bibliography and appendices

The Higgs boson

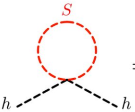
- Predicted in 1964 independantly by Brout, Englert, Higgs, Hagen, Guralnik and Kibble
- Discovered in 2012 by ATLAS and CMS detector at the LHC.
- Permit to explain particles mass and electroweak symmetry breaking.



- The quantum contributions to its mass (125.18 GeV) are a mystery $\rightarrow$ Hierarchy problem

What is the Hierarchy problem ?

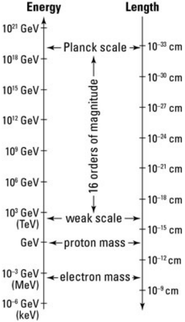
$$\mathcal{L} \supset \lambda_S |H|^2 S^2$$



$$= \frac{\lambda_S}{16\pi^2}\left[\Lambda_{\rm UV}^2 - 2m_S^2 \ln\left(\frac{\Lambda_{\rm UV}}{m_S}\right) + \cdots\right]$$

this is **not** the hierarchy problem
the regulator is not physical

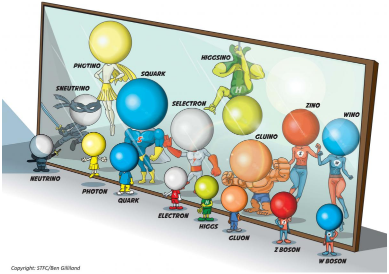
this is the hierarchy problem
the Higgs mass is quadratically sensitive to the mass of any new particles that couple to it

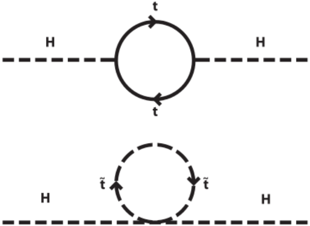


- SM particles = Contributions proportionnal to $ln\left(\frac{\Lambda_{UV}}{m}\right)$
- Heavy BSM particles = Big contributions to the Higgs mass
- We don't know where the SM stops $\rightarrow \Lambda_{UV}$ value ?

Famous solutions to the Hierarchy problem

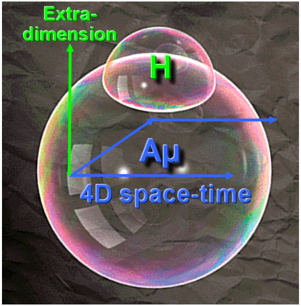
- Anthropy principle : Life can only appear in a perfect Universe $\rightarrow$ Every big radiative term cancel with one another miraculously (Fine-tunning)
- Supersymmetry : permit to protect the Higgs mass from power-law radiatives corrections $\rightarrow$ Where are sparticles ?





Extra-dimension solution : Gauge-Higgs Unification (GHU)

- What if the Higgs was part of a 5D gauge boson ?
- Higgs $\subset$ gauge boson $\rightarrow$ mass protected by gauge symmetry



Where is the 5th dimension ?

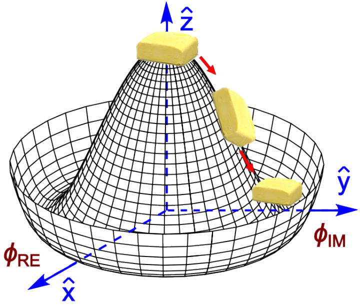
- Fifth dimension is compactified as an S^1/Z_2 orbifold
- Radius of compactification $\frac{1}{R} \approx 5$ TeV $\rightarrow$ So small we cannot see it yet
- S^1/Z_2 = Circle with fixed points y = 0 and y = πR

Where is the 5th dimension ? (This cake is a lie !)

- Fifth dimension is compactified as an S^1/Z_2 orbifold
- Radius of compactification $\frac{1}{R} \approx 5$ TeV $\rightarrow$ So small we cannot see it yet
- S^1/Z_2 = Circle with fixed points y = 0 and y = πR

The effective potential ?

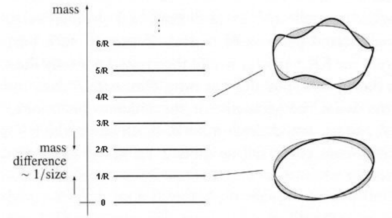
- V_{eff} permits to show symmetry breaking and calculate the broken generators bosons mass in a theory Higgs mass.
- At low energy, the vacuum will lie at a minimum. If this minimum is not (0,0), there is a symmetry breaking.
- Example : The mexican hat for $\phi \to e^{i\alpha}\phi$ ($U(1)$) symmetry in the Higgs potential.



Outline

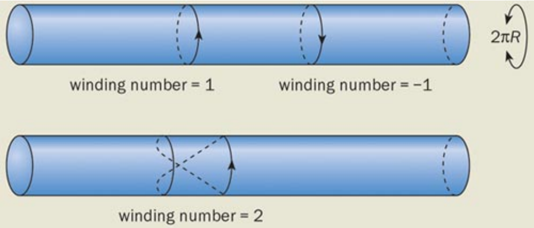
The Kaluza-Klein modes

- Propagator $\equiv$ "probability amplitude for a particle to travel to travel from one place to another" (Wikipedia)
- A 5th-dimension propagator can be decomposed as Fourier modes called Kaluza-Klein (KK) modes.
- Propagator in momentum-space of the n-th KK mode : $\frac{1}{p^2 - \frac{(n-\delta_0)^2}{R^2}}$ with $\frac{(n-\delta_0)^2}{R^2}$ its mass.
- Problem $\rightarrow$ All terms have divergencies and the generated Higgs mass at one-loop is too low.



The winding modes

- Another decomposition is with winding arround the compactified dimension.
- Propagator in position-space of the n-th winding mode :
  $$\frac{e^{i\sqrt{p^2-\left(\frac{nw-\delta_0}{R}\right)^2}|y-y'|}}{2\sqrt{p^2-\left(\frac{nw-\delta_0}{R}\right)^2}}$$
- Advantage $\rightarrow$ All the divergences are contained in the winding mode zero.



Equations ...

🤮 We now begin equations ... 🤮 Sorry ...

One-loop effective potential in gauge-higgs unification

- STEP 1 : Write the V_{eff} expression
- We focus on the gauge boson contribution to the one-loop V_{eff} here.
- For $SU(2) \to U(1)$ symmetry breaking, we have :

$$V^{g}_{eff} = -\frac{i}{2\pi R} \int \frac{dp^4}{(2\pi R)^4} \frac{3}{2} \sum_{n_{KK}=-\infty}^{+\infty} [\ln\left(-p^2 + \frac{n^2}{R^2}\right) + \ln\left(-p^2 + \left(\frac{n-\alpha}{R}\right)^2\right)]$$

- Most interesting fact $\to$ V_{eff} only have terms of the form

$$\int \frac{dp^4}{(2\pi)^4} \sum_{n_{KK}=-\infty}^{+\infty} \ln\left(-p^2 + \left(\frac{n-\delta_0}{R}\right)^2\right)$$

The KK modes propagator

- STEP 2 : Logarithm term → KK modes propagator

The previous term can be seen as the KK modes propagator :

$$\sum_{n_{KK}=-\infty}^{+\infty} \ln\left(-p^2 + \left(\frac{n-\delta_0}{R}\right)^2\right) = \sum_{n_{KK}=-\infty}^{+\infty} \int dp^2 \frac{1}{p^2 - \left(\frac{n-\delta_0}{R}\right)^2}$$

$$= \int dp^2 \sum_{n_{KK}=-\infty}^{+\infty} \frac{1}{p^2 - \left(\frac{n-\delta_0}{R}\right)^2} = \int dp^2\, \tilde{G}_{KK}(p, \delta_0)$$

Where $\tilde{G}_{KK}(p, \delta_0)$ represent the full KK modes propagator with a mass of $\frac{\delta_0}{R}$ for the zero-mode.

Winding mode transformation

- STEP 3 : Full KK modes propagator → Full winding modes propagator

$$\tilde{G}_{KK}(p,\delta_0) \rightarrow \int_0^{\pi R} dy \sum_{n_w=0}^{+\infty} [G_w(p,2n_w\pi R,\delta_0) \pm G_w(p,2y+2n_w\pi R,\delta_0)]$$

where $G_w(p,|y-y'|,\delta_0) = \frac{e^{i\chi|y-y'|}}{2\chi}$ and $\chi = \sqrt{p^2 - \frac{\delta_0^2}{R^2}}$.

Regularization and summation

- STEP 4 : Regularization procedure
- All divergent terms are contained in $n_w = 0$ so we can cut it $\rightarrow$ Regularization procedure
- STEP 5,6,7..., 99 : After integrating on y, on p^2 , Wick rotating $(i\chi \rightarrow -\chi_E)$ and summing on n_w, we finally obtain :

$$-\frac{i}{2\pi R}\int \frac{dp^4}{(2)^4}\frac{3}{2}\sum_{n_{KK}=-\infty}^{+\infty} \ln\left(-p^2+\left(\frac{n-\delta_0}{R}\right)^2\right)$$

$$= \frac{3}{256}\int_0^{+\infty} dp_E \frac{p_E^3}{\pi^3 R}\left[1-e^{-2\pi R\chi_E} \mp Ei(-2\pi R\chi_E)\right] = f(\delta_0)$$

where $\mathrm{Ei}(x) = \int_{-x}^{+\infty}\frac{e^{-t}}{t}$ and $\chi_E = \sqrt{p_E^2+\delta_0^2}$.

Effective potential expressed with f

- FINAL STEP : We have a simple expression of Veff in terms of the f function.

We can finally express V_{eff} in $SU(2)$ as :

$$V_{eff}^{g+gh} = f(0) + f(\alpha)$$

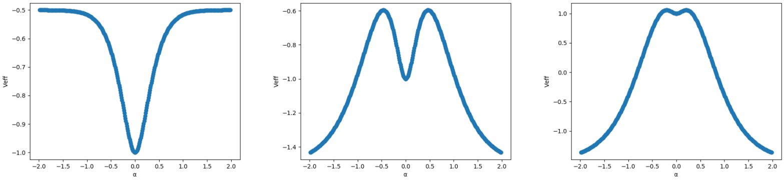
We can also compute the fermion contribution, with N_f the number of fermions

$$V_{eff}^{f} = -\frac{4N_f}{3} f\left(\frac{\alpha}{2}\right)$$

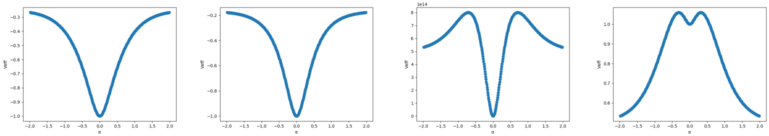
Outline

$SU(2)$ and $SU(3)$ results

- With all different particles content, no other minimum than $\alpha = 0$
- $SU(2) \rightarrow U(1)$ (Nf = 0, 1 and 2)

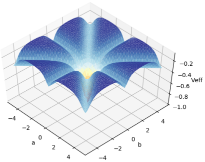


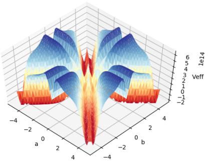
- $SU(3) \rightarrow SU(2) \times U(1)$ (Nf = 0, 1, 2 and 3)

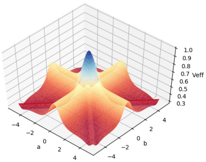


General cases in $SU(5)$

- Possible to play with the number of fermions in the 5-representation (N_f^5), fermions in the 10-representation (N_f^{10}) and the number of scalars (N_s) in the theory
- For general cases of (N_f^5, N_f^{10}, N_s), no other minimum than (0,0).







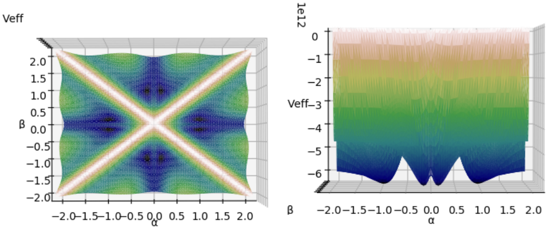
- $(N_f^5, N_f^{10}, N_s) = (1\ ,0,\ 0)\ ,\ (0,\ 2,\ 0)\ ,\ (3,\ 3,\ 1)$

The most interesting case : the camel case (two humps)

- For a particle content of $(N_f^5, N_f^{10}, N_s) = (N_f^5,\ 3,\ 3 + 2\ N_f^5)$, V_{eff} has a relative simple form :

$$V_{eff}(\alpha, \beta) = -2\left[f\left(\frac{\alpha+\beta}{2}\right) + f\left(\frac{\alpha-\beta}{2}\right)\right] + f(\alpha) + f(\beta)$$

- In this case, there are 8 non-trivial minima at $(\alpha, \beta) = (0.95, 0.15)$



Higgs mass in the camel case

Possible to compute the broken generators bosons masses (the Higgs) using the two eigenvalues of the Hessian matrix, λ_1 and λ_2, at the minimum $(0.95, 0.15)$ with :

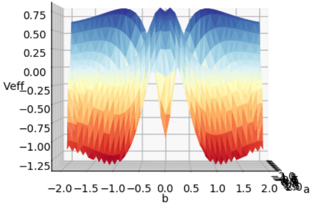
$$m_{1/2} = (gR)^2 \lambda_{1/2}$$

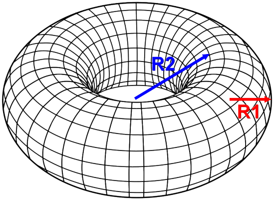
- For $\frac{1}{R} = 5$ TeV and $g = g_{GHU} \approx 0.1$, we finally find $m_1 = 71.07$ GeV and $m_2 = 87.12$ GeV $\rightarrow$ Same order of the Higgs mass
- R can be slightly different and we are just at one-loop $\rightarrow$ Possible to generate exactly the Higgs boson mass
- First calculation in GHU where the masses generated are of the same order of the Higgs mass !

Work next-to-leading order

This method can be applied to multiple situations :

- All the SU(N) representation groups ✓
- Supersymmetric models with SU(N) representation ✓
- Different groups representation than SU(N) ☐
- More than one compactified extra-dimension ☐



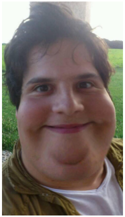


Soon, soon , soon !

Thanks to you, you and you too !

Thanks to my parents without whom I couldn't be here with you !

Thanks to my roommate Aurélien who handle my bad jokes and my snoring !



And obviously thank you for listening ! Let's start the questions !

Outline

1. Introduction
   - The Higgs boson
   - The Hierarchy problem
   - Gauge-Higgs Unification
   - Validity of the model
2. The winding mode method
   - Propagators modes decomposition
   - The one-loop effective potential
   - Winding modes decomposition
   - Winding mode resummation
3. Results
   - $SU(2)$ and $SU(3)$ results
   - $SU(5)$ results
   - Future work
4. Bibliography and appendices

Bibliography

Sidney R. Coleman and Erick J. Weinberg
Radiative Corrections as the Origin of Spontaneous Symmetry Breaking
Phys. Rev., D7:1888-1910, 1973.

R. Jackiw.
Functional evaluation of the effective potential.
Phys. Rev., D9:1686, 1974.

Martin Puchwein and Zoltan Kunszt.
Radiative corrections with 5-D mixed position / momentum space propagators.
Annals Phys., 311:288-313, 2004.

Leandro Da Rold.
Radiative corrections in 5-D and 6-D expanding in winding modes.
Phys. Rev., D69:105015

Yutaka Hosotani.
Dynamical Mass Generation by Compact Extra Dimensions.
Phys. Lett., 126B:309-313, 1983.

Bibliography

Yutaka Hosotani.
Dynamics of Nonintegrable Phases and Gauge Symmetry Breaking.
Annals Phys., 190:233, 1989.

Naoyuki Haba, Masatomi Harada, Yutaka Hosotani, and Yoshiharu Kawamura.
Dynamical rearrangement of gauge symmetry on the orbifold S1 / Z(2).
Nucl. Phys., B657:169-213, 2003.

Naoyuki Haba, Yutaka Hosotani, Yoshiharu Kawamura, and Toshifumi Yamashita.
Dynamical symmetry breaking in gauge Higgs unification on orbifold
Phys. Rev., D70:015010, 2004.

Masahiro Kubo, C. S. Lim, and Hiroyuki Yamashita.
The Hosotani mechanism in bulk gauge theories with an orbifold extra space S**1 / Z(2).
Mod. Phys. Lett., A17:2249-2264, 2002.

Appendix 1

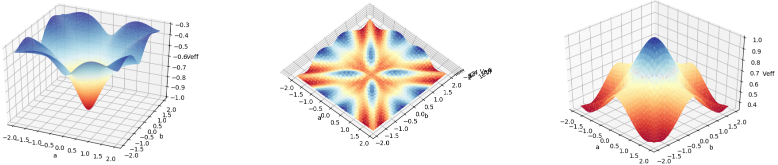
$$V_{eff}^{SU(2)} = f(0) + f(\alpha) - \frac{4N_f}{3} f\left(\frac{\alpha}{2}\right)$$

$$V_{eff}^{SU(3)} = f(0) + f(\alpha) + 2f\left(\frac{\alpha}{2}\right) - \frac{2N_f}{3}\left[f(0) + 2f\left(\frac{\alpha}{2}\right)\right]$$

Appendix 2

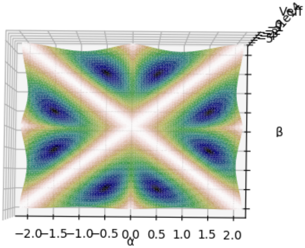
$$
\begin{aligned}
V_{eff}^{SU(5)} &= 2f(0) + 2f\left(\frac{\alpha+\beta}{2}\right) + 2f\left(\frac{\alpha-\beta}{2}\right) \\
&+ 2f\left(\frac{\alpha}{2}\right) + 2f\left(\frac{\beta}{2}\right) + f(\alpha) + f(\beta) \\
&- \frac{(2N_f^5 - N_s)}{3}[2f(0) + 2f\left(\frac{\alpha}{2}\right) + 2f\left(\frac{\beta}{2}\right)] \\
&- \frac{2N_f^{10}}{3}[2f(0) + 2f\left(\frac{\alpha}{2}\right) + 2f\left(\frac{\beta}{2}\right) \\
&+ 2f\left(\frac{\alpha+\beta}{2}\right) + 2f\left(\frac{\alpha-\beta}{2}\right)]
\end{aligned}
$$

Appendix 3



Effective potential obtained in the (1,0,0), (0,2,0) and (3,3,1) case

Appendix 4



Effective potential obtained in the camel case using KK modes