

IRSN

INSTITUT
DE RADIOPROTECTION
ET DE SÛRETÉ NUCLÉAIRE

Faire avancer la sûreté nucléaire

Learning to unmix in gamma-ray spectrometry

Journées Machine Learning et Physique Nucléaire

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Institut de recherche
sur les lois fondamentales
de l'Univers

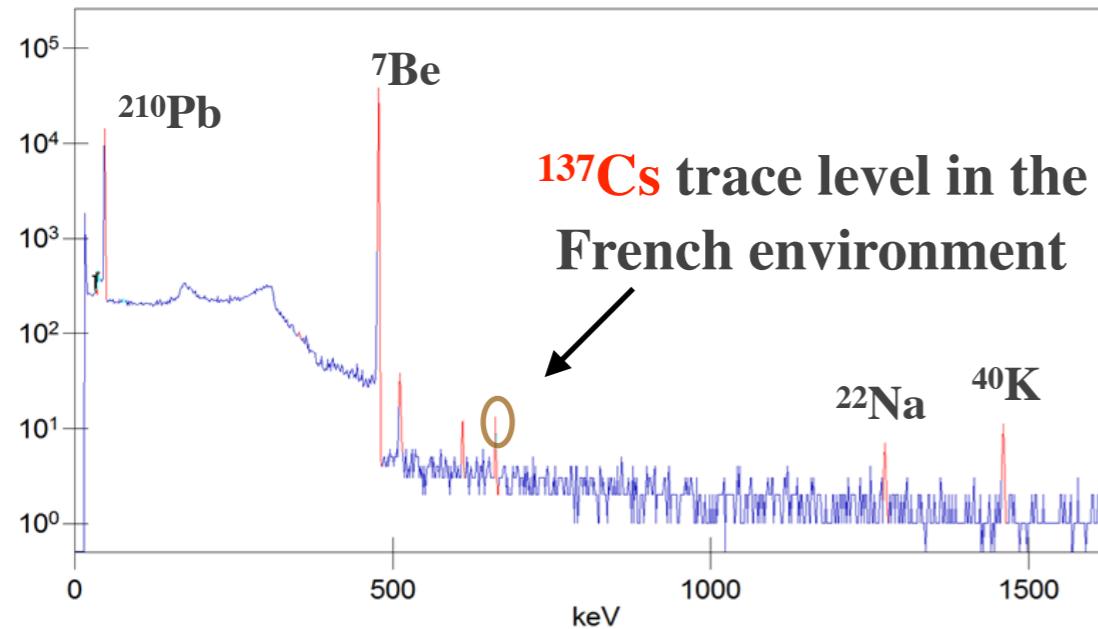


Background

IRSN/LMRE

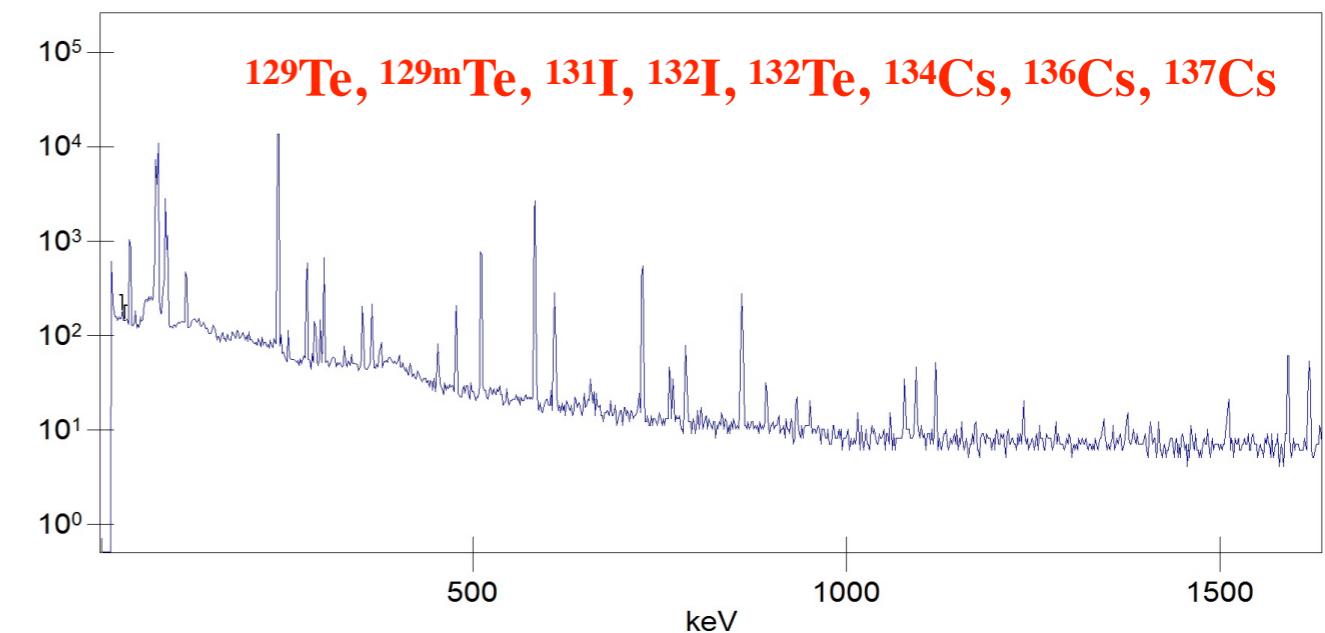
- Radioactivity determination in environmental samples
 - Measurements with Gamma-ray spectrometry
 - Naturally occurring radionuclides — Cosmogenic: ^{7}Be ... Telluric: ^{40}K ...
 - Artificial radionuclides

Routine measurement



Environmental monitoring

Measurement during the Fukushima accident



Emergency preparedness

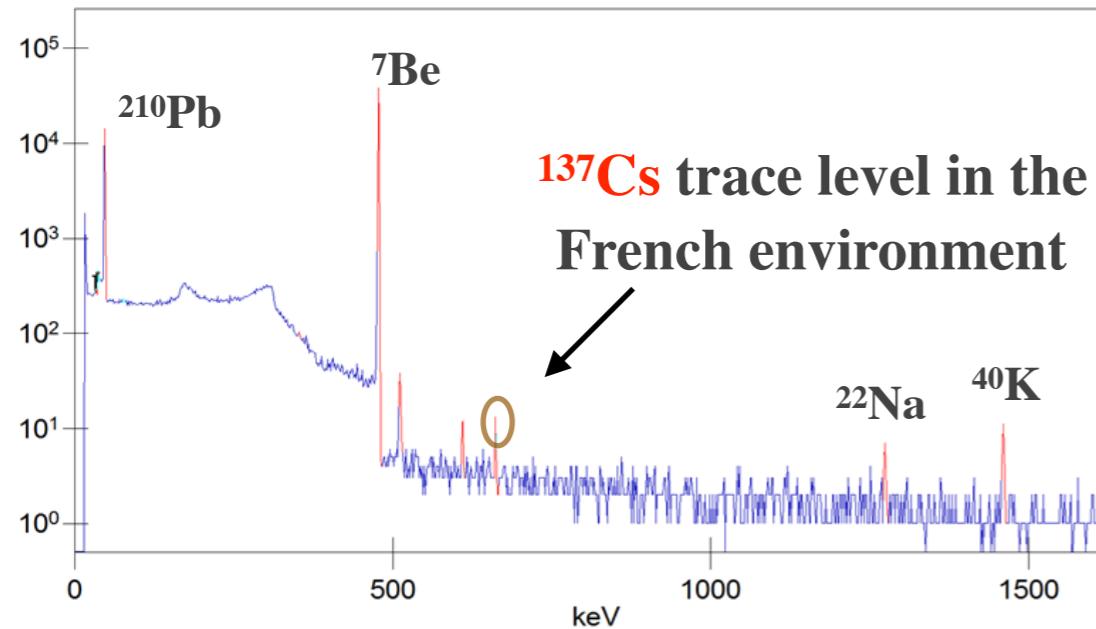
→ improve the spectral analysis: more accurate/sensitive

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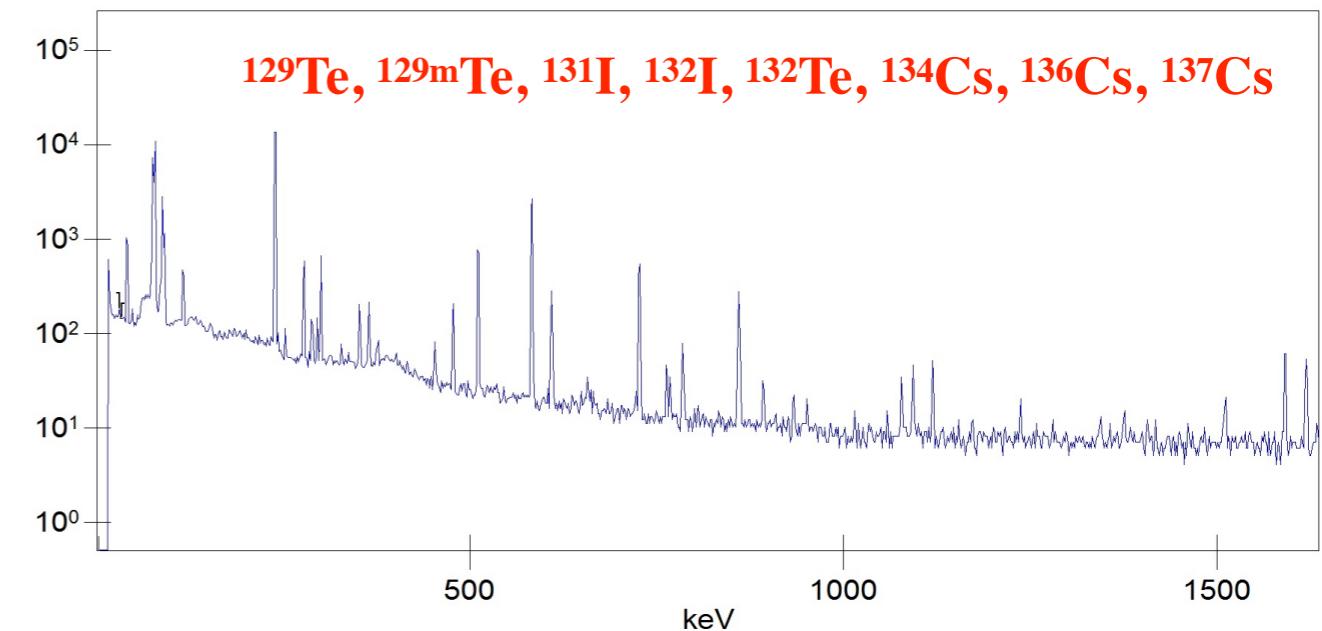
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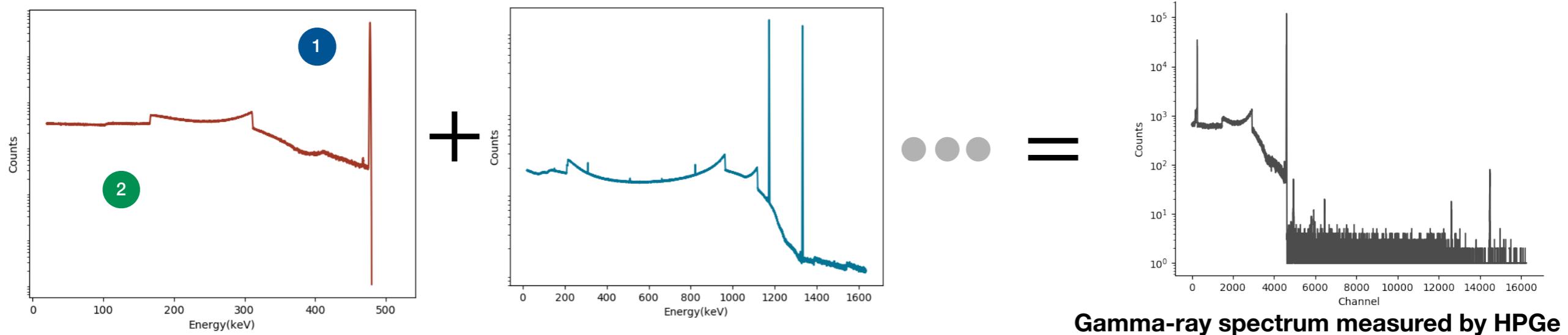


Emergency preparedness

→ improve the spectral analysis: more accurate/sensitive

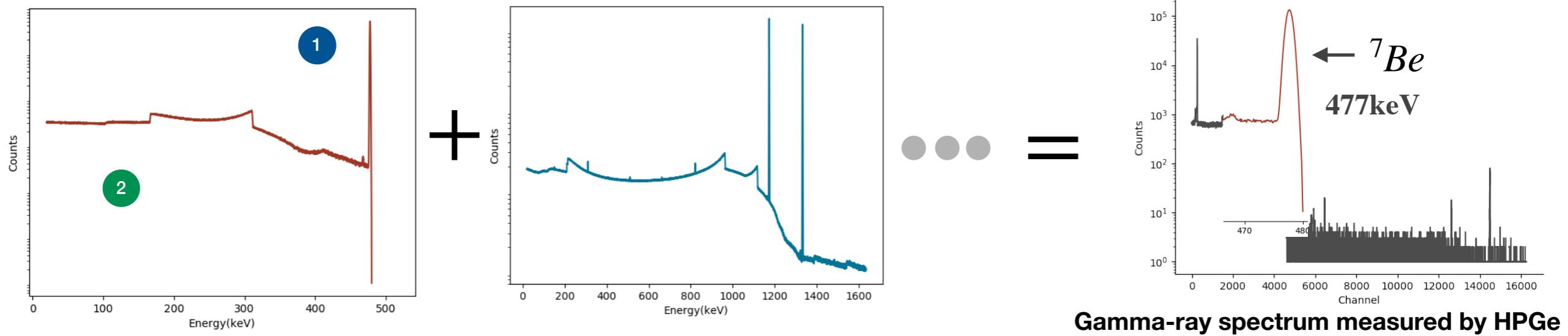
Gamma-ray spectrum

- Photon spectrum: ① full-energy peak + ② Compton continuum
- A radionuclide emits gamma photons w.r.t decay schema
- Gamma-ray spectrum = \sum (individual spectra of radionuclides)



Gamma-ray spectrum

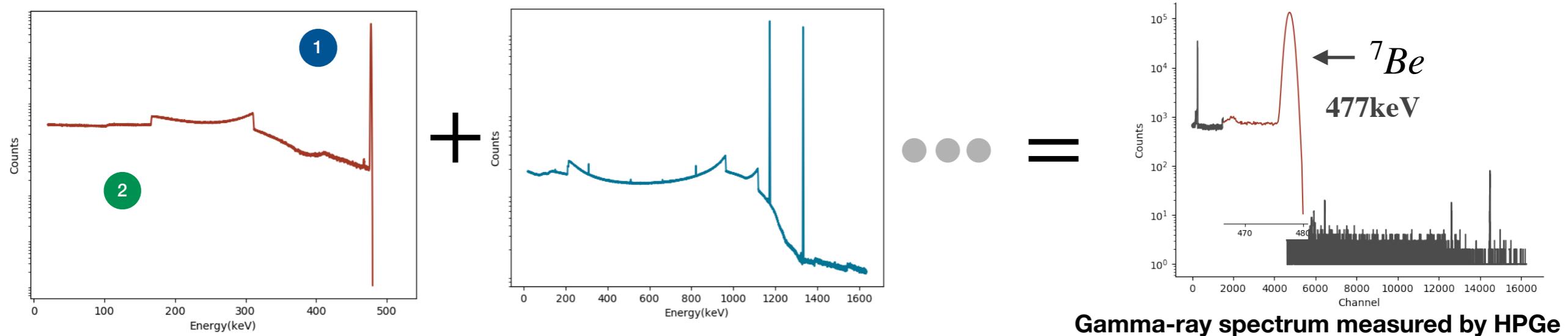
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Standard peak-fitting -> Analysis based on peaks

Gamma-ray spectrum

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Standard peak-fitting -> Analysis based on peaks

Limitations of method

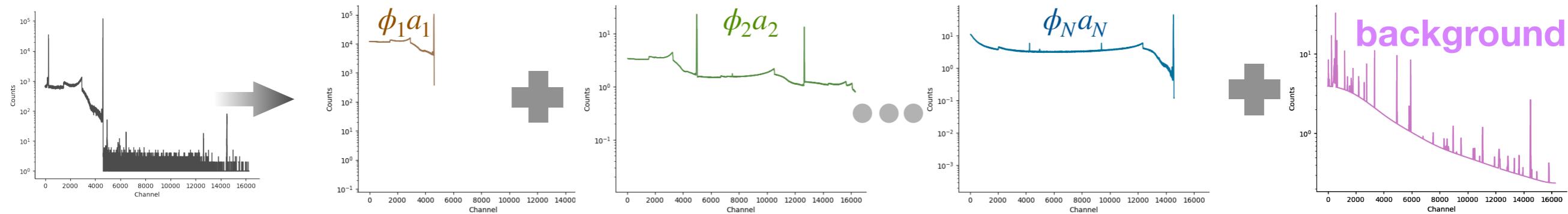
- a. Interference between individual spectra of radionuclides
- b. Limited performance with assumption Gaussian statistics

Proposed solutions

- a. Full-spectrum analysis: peaks + Compton continuum
- b. Poisson statistics of the detection process

Spectral unmixing

Measured spectrum-> spectral signatures of radionuclides



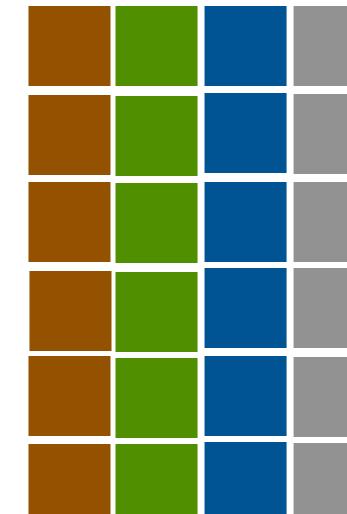
Estimation of activities: $x \sim \text{Poisson}(\Phi a + b)$



Measured spectrum

$$x = [x_1, \dots, x_M]$$

M : number of channels
 N : number of radionuclides



Spectral signatures

$$\Phi \in \mathbb{R}^{M \times N}$$



Mixing weights

$$a \in \mathbb{R}^{N \times 1}$$



Background spectrum

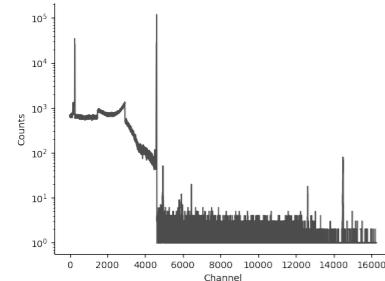
$$b \in \mathbb{R}^M$$

Regularized inverse problem:

$$\hat{a} \in \operatorname{argmin}_a f(a) + \sum_i g_i(a)$$

data fidelity Regularizations

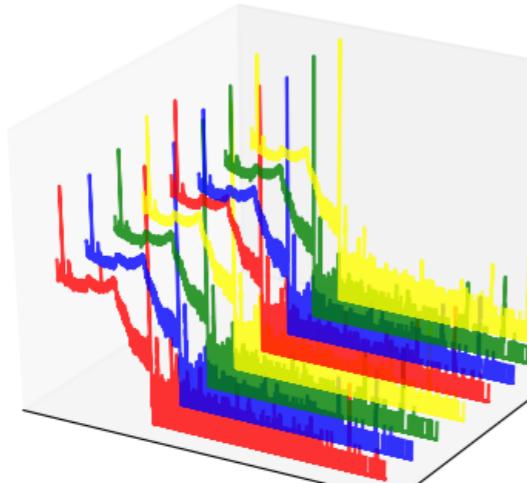
Estimation of a : Poisson neg-log-likelihood + regularization



$$\min_a \Phi a + b - x \odot \log(\Phi a + b) + \lambda \mathcal{J}(a) \quad (1)$$

- Spectral unmixing with **positivity constraint** (*Xu et al., 2019, IEEE TNS revised*)
- Sparse spectral unmixing- **number of active radionuclides** (*Xu et al., 2019, ARI in press*)

Challenge: more accurate estimation -> extraction of information from the archive of past measurements.



→ Learn the **regularization**: $\lambda \mathcal{J}(a)$

Routine measurements in the environment (aerosol filter samples) are composed of:

- **^{137}Cs** – Artificial radionuclide
- **$^{7}\text{Be}, ^{22}\text{Na}, ^{40}\text{K}, ^{210}\text{Pb}$** – Natural radionuclides



Activities do not vary to a large extent

Regularized inverse problem: $\hat{a} \in \operatorname{argmin}_a f(a) + \lambda \mathcal{J}(a)$

Spectral unmixing

- Interpretable priors
 - ✓ Physical constraint (e.g. non-negativity)
- Iterative algorithms (e.g. FBS)



unrolling methods

- Recursive neural network
 - ✓ Learned priors
data-driven prior from training data
- Learning to unmix

Existing methods - e.g. Neumann Networks (*Gilton et al., 2019*)

Least squares data fidelity term: $\min_a \frac{1}{2} \|x - \Phi a - b\|_2^2 + \lambda \mathcal{J}(a)$

Proposed method - Unrolled ADMM Poisson measurements (*Bobin et al., 2019*)

Poisson based data fidelity term: $\min_a \Phi a + b - x \odot \log(\Phi a + b) + \lambda \mathcal{J}(a)$

Neumann network architecture (*Gilton et al., 2019*)

Regularized least squares optimization: $\min_{\mathbf{a}} \frac{1}{2} \|\mathbf{x} - \Phi \mathbf{a} - \mathbf{b}\|_2^2 + \lambda \mathcal{J}(\mathbf{a})$

Inverse mapping: $\hat{\mathbf{a}} = (\Phi^T \Phi + J)^{-1} \Phi^T (\mathbf{x} - \mathbf{b})$, $J = \nabla \lambda \mathcal{J}$

Neumann network: $\tilde{\mathbf{a}}^{(0)} = \Phi^T (\mathbf{x} - \mathbf{b}) \rightarrow \tilde{\mathbf{a}}^{(j)} = (I - \eta \Phi^T \Phi - \eta J) \tilde{\mathbf{a}}^{(j-1)}$

From: $\tilde{\mathbf{a}}^{(0)} = \Phi^T (\mathbf{x} - \mathbf{b})$ to a reconstruction of : $\hat{\mathbf{a}} = \sum_{j=0}^B \tilde{\mathbf{a}}^j$

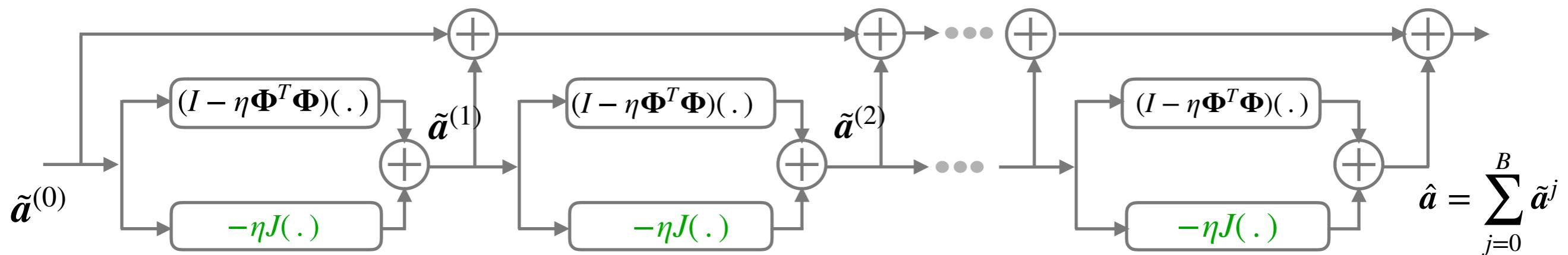
Neumann network architecture (*Gilton et al., 2019*)

Regularized least squares optimization: $\min_a \frac{1}{2} \|x - \Phi a - b\|_2^2 + \lambda J(a)$

Inverse mapping: $\hat{a} = (\Phi^T \Phi + J)^{-1} \Phi^T (x - b)$, $J = \nabla \lambda J$

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Successive operator
in each block

B = number of blocks

J - the trained neural network
 η - trained scale parameter

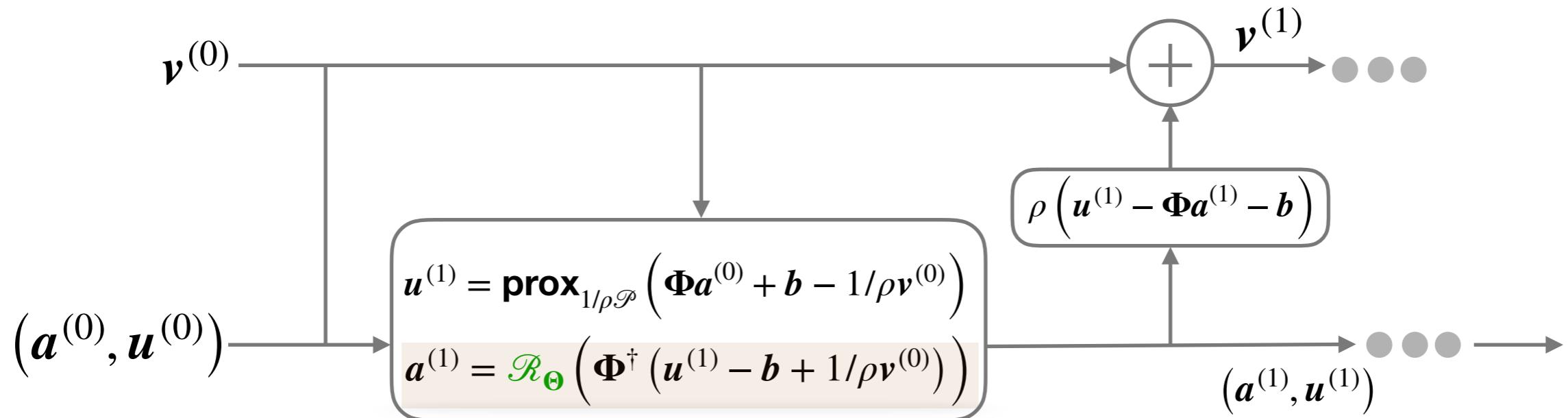
Implementation: $\min \sum \|\hat{a} - a^*\|_2^2$ Multilayer perceptron, activation functions = ReLU

Regularized Poisson based optimization:

$$\min_{\mathbf{a}} \Phi \mathbf{a} + \mathbf{b} - \mathbf{x} \odot \log(\Phi \mathbf{a} + \mathbf{b}) + \lambda \mathcal{J}(\mathbf{a})$$

Iteration steps of the learned ADMM

- Update of $\mathbf{u}^{(k+1)}$: $\mathbf{u}^{(k+1)} = \operatorname{argmin}_{\mathbf{u}} \mathbf{u} - \mathbf{x} \odot \log(\mathbf{u}) + \frac{\rho}{2} \|\mathbf{u} - (\Phi \mathbf{a}^{(k)} + \mathbf{b} - 1/\rho \mathbf{v}^{(k)})\|_2^2$
- Update of \mathbf{a} : $\mathbf{a}^{(k+1)} = \operatorname{argmin}_{\mathbf{a}} \mathcal{J}_{\Theta}(\mathbf{a}) + \frac{\rho}{2} \|\mathbf{u}^{(k+1)} - \mathbf{b} - \Phi \mathbf{a} + 1/\rho \mathbf{v}^{(k)}\|_2^2$
- Update of \mathbf{v} (dual variable): $\mathbf{v}^{(k+1)} = \mathbf{v}^{(k)} + \rho \left(\mathbf{u}^{(k+1)} - \Phi \mathbf{a}^{(k+1)} - \mathbf{b} \right)$



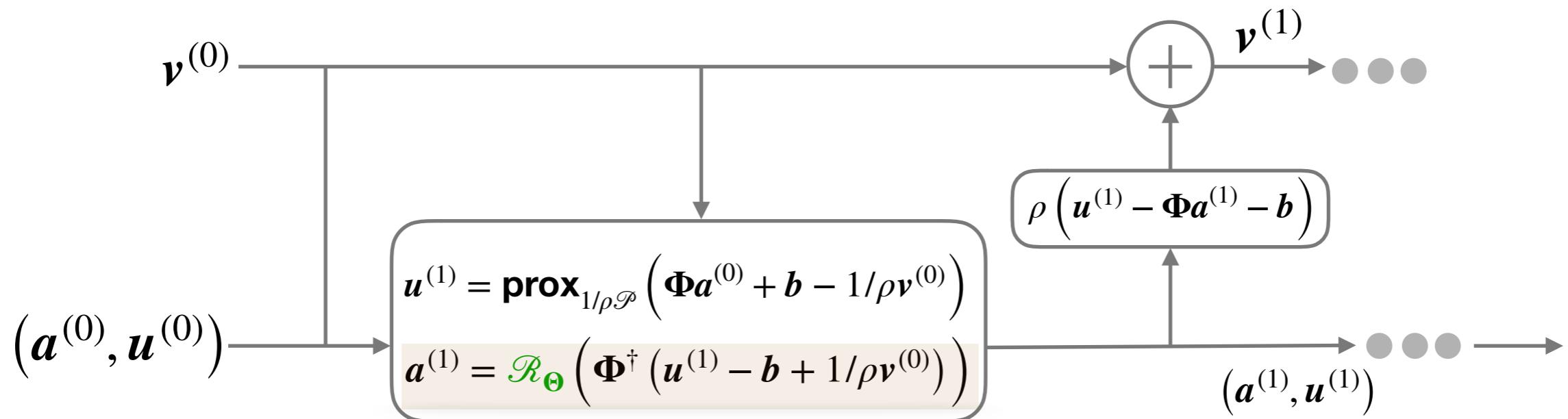
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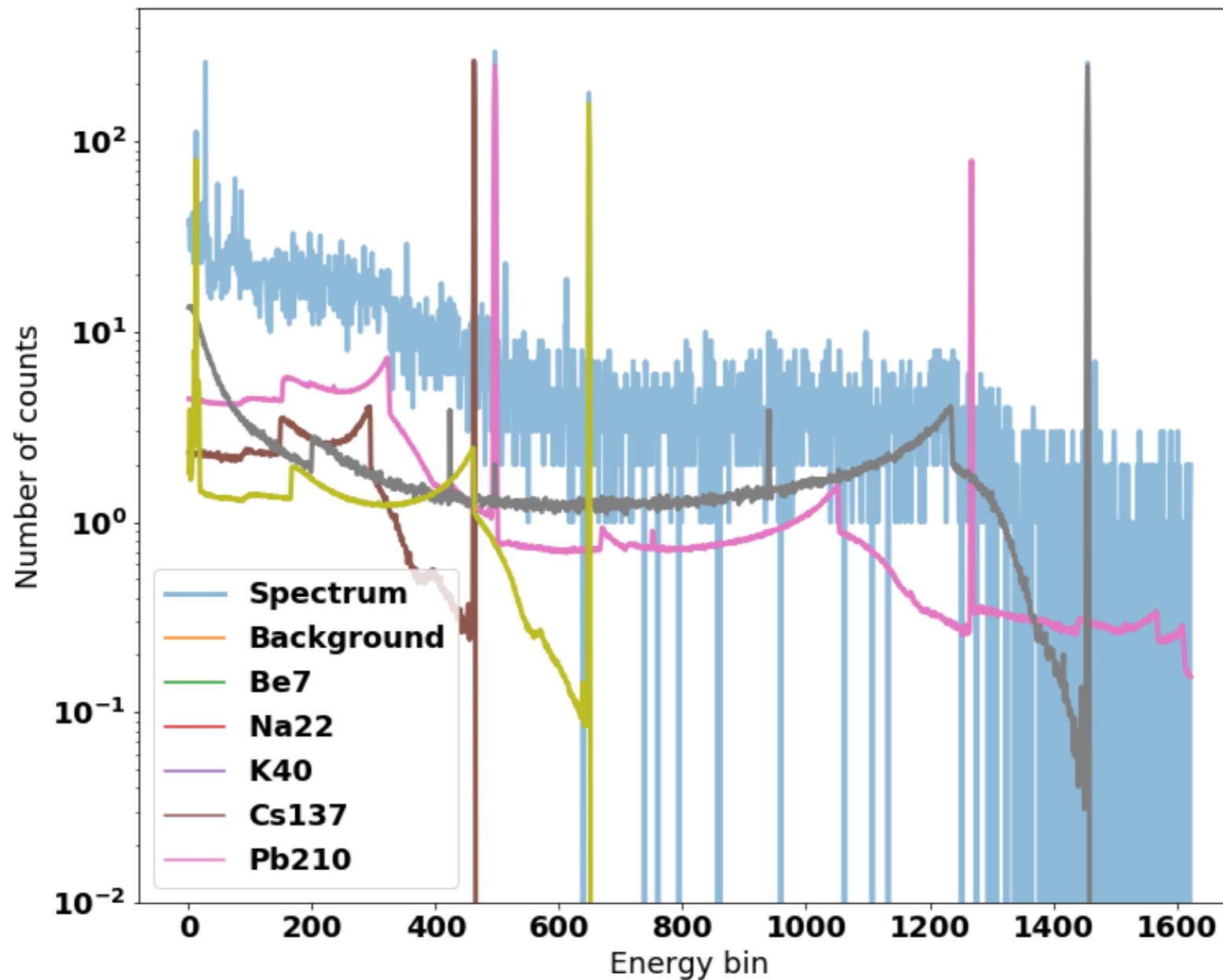
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Results on simulated data



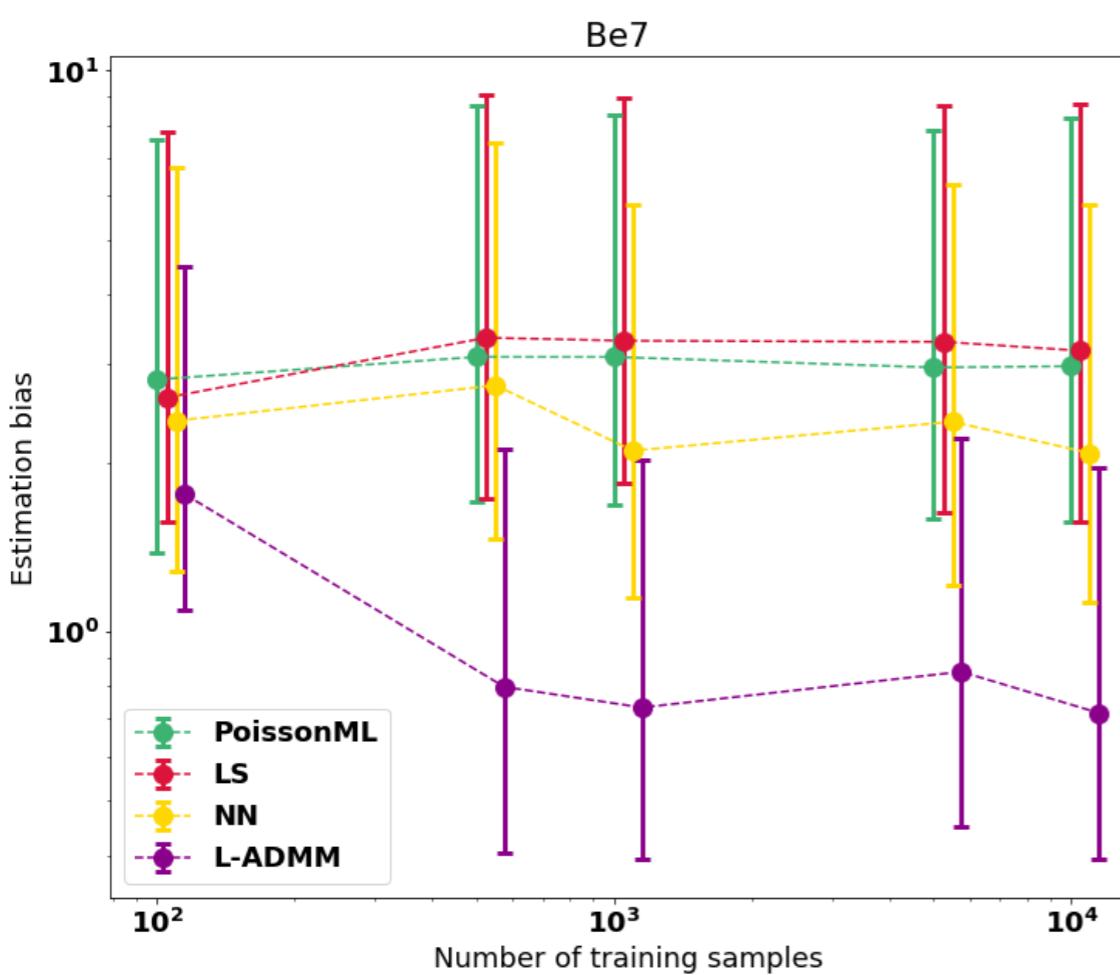
Example of a single spectrum

Results on simulated data

Iterative algorithms:

- PoissonML = non-negativity+Poisson
- LS = non-negativity+ least-squares

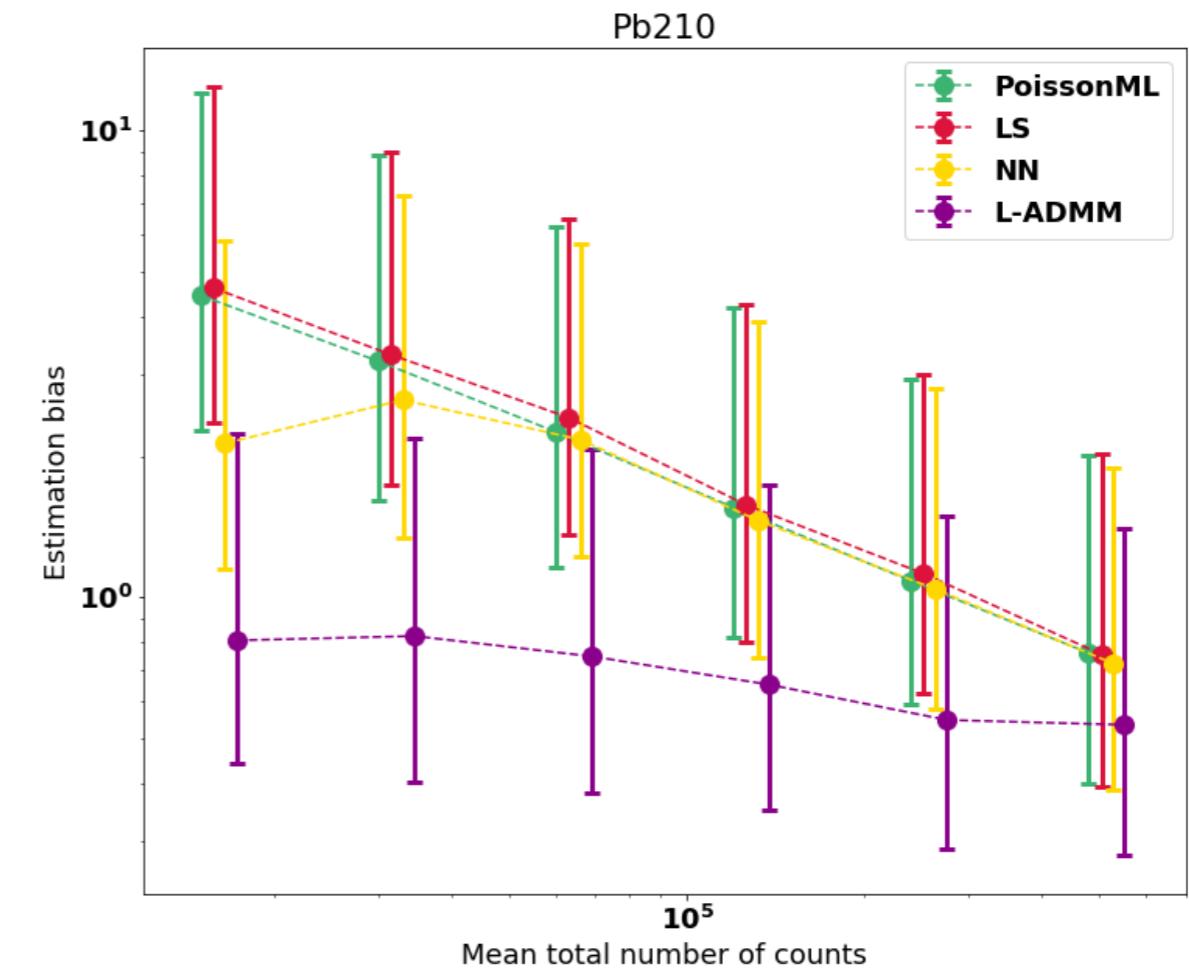
Relative estimation bias of ^{7}Be w.r.t training set size



Learn to unmix:

- NN = Neumann Network
- L-ADMM = Learned ADMM

Relative estimation bias of ^{210}Pb w.r.t total number of counts



- ✓ Spectral unmixing + Learned regularization
 - More accurate estimation with extraction of information from the archive

→ Applying the algorithm to real data:

- Routine measurements of aerosol samples
- > Learn prior of natural radionuclides' activities.

→ Automation of spectral unmixing

- Learn the knowledge given by human post-analysis
- > Standard spectral analysis needs an additional step of expertise, which can be learned with past measurements.
- Rapid detection of anomaly events