$T\bar{T}$ deformed CFT as a non-critical string

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Workshop on TTbar deformations

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1+1 CFT with a T$\bar{\text{T}}$ deformation

$$S = S_{\text{CFT}} - \mu \int dx d\tau \; T\bar{T}$$

[1] F. A. Smirnov and A. B. Zamolodchikov, "On space of integrable quantum field theories," 1608.05499.

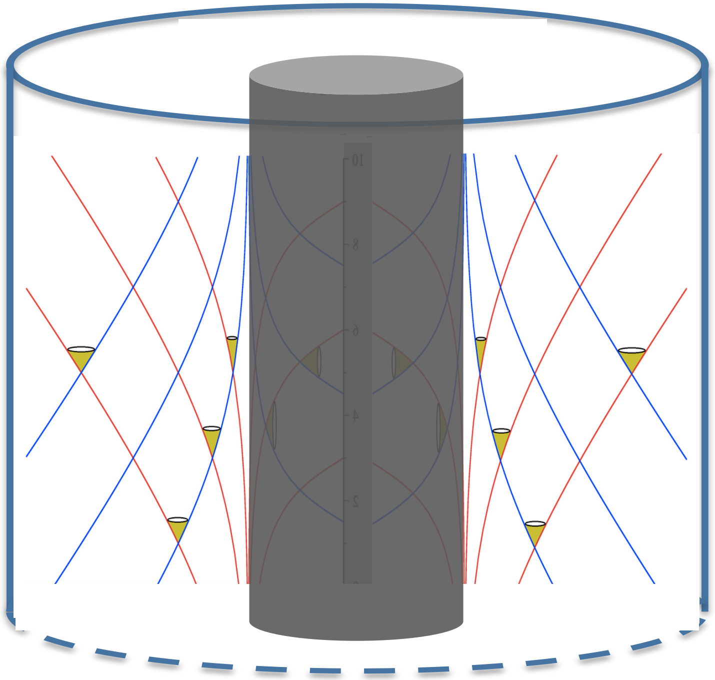
[2] A. Cavaglià, S. Negro, I. M. Szécsényi, and R. Tateo, "$T\bar{T}$-deformed 2D Quantum Field Theories," *JHEP* **10** (2016) 112, 1608.05534.

[3] S. Dubovsky, R. Flauger, and V. Gorbenko, "Solving the Simplest Theory of Quantum Gravity," *JHEP* **09** (2012) 133, 1205.6805.

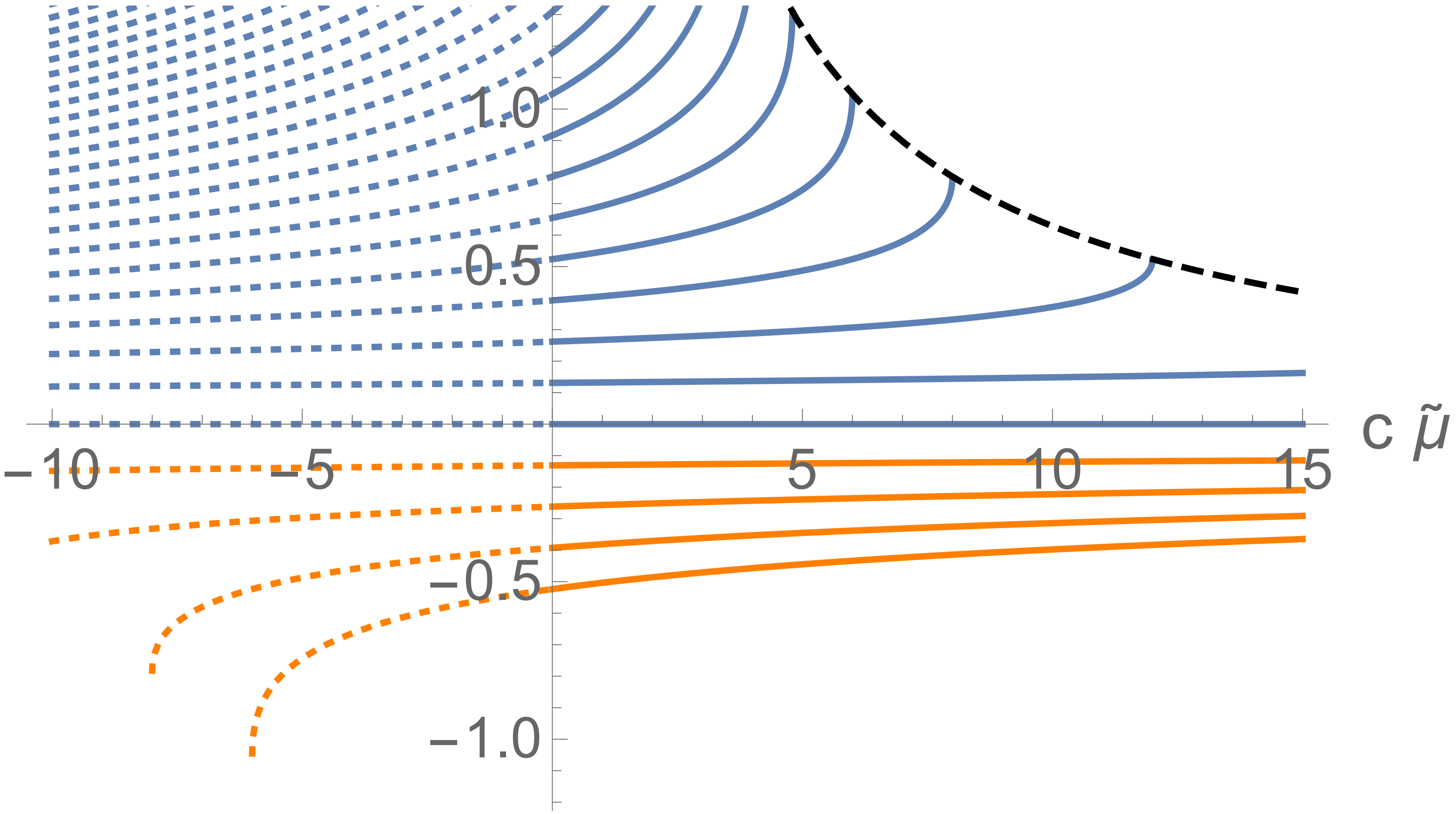
$T\bar{T}$ Energy Spectrum = Spectrum of Black Hole in a Box

$$E(R) = \frac{R}{4\mu}\left[1 - \sqrt{1 - \frac{\mu M}{R^2} + \frac{\mu^2 J^2}{4R^4}}\right]$$

$$\mu = \frac{16\pi G}{r_c^2}$$



[18] J. D. Brown, J. Creighton, and R. B. Mann, "Temperature, energy and heat capacity of asymptotically anti-de Sitter black holes," *Phys. Rev.* **D50** (1994) 6394–6403, gr-qc/9405007.



$$E(R) = \frac{R}{4\mu}\left[1 - \sqrt{1 - \frac{\mu M}{R^2} + \frac{\mu^2 J^2}{4R^4}}\right]$$

Total energy of a black hole with radial cut-off

$T\bar{T}$ = *Equivalent to Nambu-Goto string theory*

$$S_{\rm QFT} = S_{\rm CFT} + \frac{1}{\mu}\int d^2x\, \partial_{\rm v} X^+ \partial_{\rm u} X^-$$

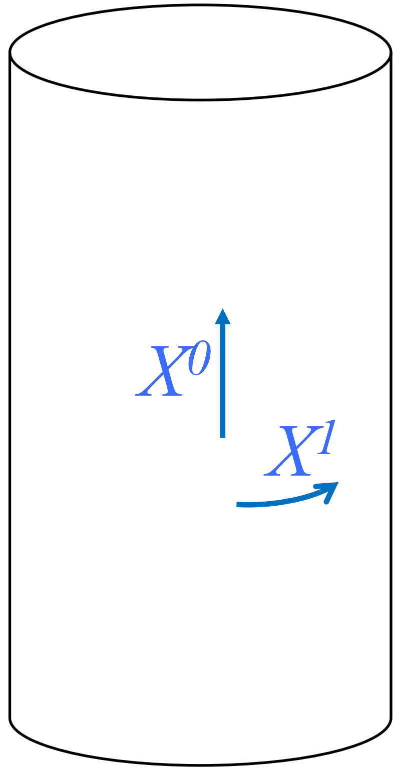
$$-\partial_u X^+ \partial_u X^- + \mu T^{\rm CFT}_{uu} = 0\,,$$

$$\partial_+ X^+ = \partial_- X^- = 1$$

$$X_1(e^{2\pi i}z, e^{-2\pi i}\bar{z}) = X_1(z,\bar{z}) + 2\pi R.$$

Target space = cylinder

$$X^- = x^- + \int^{x^+} dx^+ T^{\text{CFT}}_{++},$$
$$X^+ = x^+ + \int^{x^-} dx^- T^{\text{CFT}}_{--}.$$



LC gauge coordinates have operator valued periodicity:

$$(x^+, x^-) \simeq (x^+ + 2\pi R - P_-,\ x^- + 2\pi R - P_+)$$

$$P_+ = \oint dx^+ T^{\text{CFT}}_{++}, \qquad P_- = \oint dx^- T^{\text{CFT}}_{--}$$

$T\bar{T}$ for general c = non-critical string with worldsheet action

$$S = \int d^2z \left(\frac{1}{\mu} \partial X^- \bar{\partial} X^+ + \kappa \hat{R} \log(\partial X^+ \bar{\partial} X^-) \right)$$

=> preserves 2D Poincare symmetry

Stress tensor gets modified to

$$T = \partial X^+ \partial X^- + \kappa \partial^2 \log \partial X^+$$

$$\bar{T} = \bar{\partial} X^+ \bar{\partial} X^- + \kappa \bar{\partial}^2 \log \bar{\partial} X^-$$

satisfies Virasoro algebra with $c = 24\,(1+\kappa)$

An equivalent formulation starts from the dilaton gravity action

$$S = \int d^2x \sqrt{g}\left(\Phi R + \mu\right) + \kappa S_L(g) + S_{CFT}$$

Classical solution can be parametrized in terms of free fields X^+ and X^- via:

$$ds^2 = \partial_u X^+ \partial_v X^- du dv$$

$$\Phi = -\mu X^+ X^- + \omega^+(u) + \omega^-(v)$$

$$\partial_u \omega^+ = \widetilde{X}^- \partial_u X^+ - \frac{\hat{\kappa}}{2} \partial_u \log(\partial_u X^+)$$

Spectrum

$$S_X = \int d^2z\,(G_{ab} + B_{ab})\,\partial X^a \bar{\partial} X^b$$

$$p_{0L} = p_{0R} = \frac{\mathcal{E}}{2} + \frac{BR}{2}, \qquad p_{1L} = \frac{J}{2R} + \frac{R}{2}, \quad p_{1R} = \frac{J}{2R} - \frac{R}{2}$$

$$-2p_0^2 + p_{1L}^2 + p_{1R}^2 + \Delta_L + \Delta_R - 2\kappa - 2 = 0$$

$$\Delta_R - \Delta_L \;=\; p_{1L}^2 - p_{1R}^2 \;=\; J$$

$$\mathcal{E} = R\left(-B + \sqrt{1 + \frac{2E}{R} + \frac{J^2}{R^4}}\right)$$

$$-\kappa\,\partial^2 \log\big(\partial X^+(z)\big)\; e^{ip\cdot X(0)} = -\kappa\,\partial^2 :\log\Big(\frac{p_-}{z} + \partial X^+(z)\Big)\; e^{ip\cdot X(0)}:$$

$$\simeq\; -\frac{\kappa}{z^2}\, e^{ip\cdot X(0)} \;+\; \text{regular}$$

Partition function

$$\lambda = \frac{1}{4\pi^2 R^2}.$$

$$Z(\alpha,\beta) = \sum_n e^{i\alpha J_n - \beta \mathcal{E}_n}$$

$$\Lambda = 2\pi R\beta = 4\pi^2 R^2 \sigma_2,$$

$$Z(\sigma,\bar\sigma,\lambda) = \frac{\sigma_2}{\pi\lambda}\int_{\mathbf{F}} \frac{d^2\tau}{\tau_2^2} \sum_w e^{-S_{\rm cl}(\Lambda,\sigma,\tau,w)}\ Z_{\rm CFT}(\tau,\bar\tau)$$

$$= \frac{\sigma_2}{\pi\lambda}\int_{\mathbf{H}} \frac{d^2\tau}{\tau_2^2} \exp\Big(-\frac{\Lambda}{\tau_2\sigma_2}|\sigma-\tau|^2\Big) Z_{\rm CFT}(\tau,\bar\tau)$$

$$(\sigma,\Lambda,\lambda) \to \Big(\frac{a\sigma+b}{c\sigma+d},\Lambda,\frac{\lambda}{|c\sigma+d|^2}\Big)$$

Target space = cylinder

$$X_1(e^{2\pi i}z, e^{-2\pi i}\bar{z}) = X_1(z,\bar{z}) + 2\pi R.$$

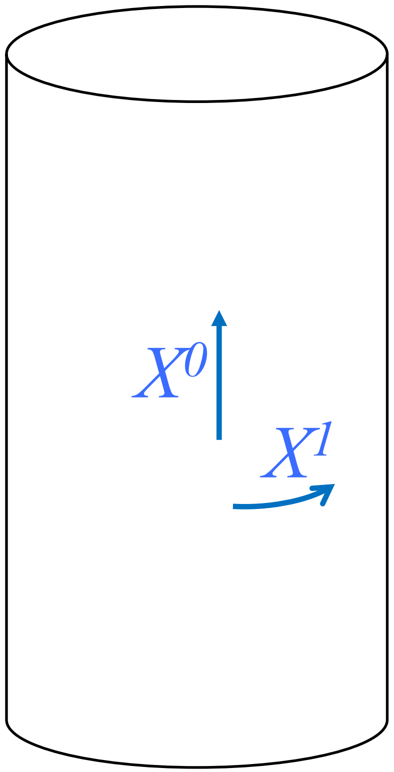
BRST charge

$$Q = \oint dz\left(c\left(T_{\rm CFT} + T_X + \frac{1}{2}T_{\rm gh}\right)\right)$$

Physical states = BRST cohomology

$$Q_{\rm brst}|{\rm phys}\rangle = 0, \qquad |{\rm phys}\rangle \simeq |{\rm phys}\rangle + Q_{\rm brst}|*\rangle$$

$$[Q_{\rm brst}, \mathcal{O}_{\rm phys}] = 0, \qquad \mathcal{O}_{\rm phys} \simeq \mathcal{O}_{\rm phys} + [Q_{\rm brst}, *]$$



How do we recover the stress-tensor?

$$\mathcal{L}_n = \oint dz \; (\partial X^- + \hat{\kappa} p_+ \partial \log \partial X^+) e^{ip_+ X^+(z)} \qquad p_+ = \frac{n}{R}$$

These are a generalization of the DDF operators of critical string theory.

They satisfy the Virasoro algebra with central charge c

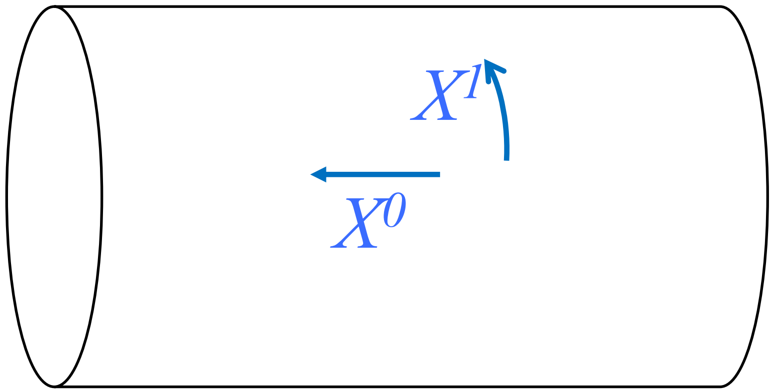
$$[\mathcal{L}_n, \mathcal{L}_m] = (n-m)\mathcal{L}_{n+m} + \frac{c}{12}(n^3 - n)\delta_{nm}$$

and can thus be identified with the stress-tensor of the deformed CFT.

This is the key result used in the old proof of the no ghost theorem.

Target space = cylinder

$$X_1(e^{2\pi i}z, e^{-2\pi i}\bar{z}) = X_1(z, \bar{z}) + 2\pi R.$$



BRST charge

$$Q = \oint dz \left(c \left(T_{\mathrm{CFT}} + T_X + \frac{1}{2} T_{\mathrm{gh}} \right) \right)$$

Physical states = BRST cohomology

$$Q_{\mathrm{brst}} |\mathrm{phys}\rangle = 0, \qquad |\mathrm{phys}\rangle \simeq |\mathrm{phys}\rangle + Q_{\mathrm{brst}} |*\rangle$$

$$[Q_{\mathrm{brst}}, \mathcal{O}_{\mathrm{phys}}] = 0, \qquad \mathcal{O}_{\mathrm{phys}} \simeq \mathcal{O}_{\mathrm{phys}} + [Q_{\mathrm{brst}}, *]$$

We would like to define correlation functions, such as the 2-point function in

$\langle\, \hat{\mathcal{O}}_h(x)\, \hat{\mathcal{O}}_h(y)\, \rangle$		$\langle\, \hat{\mathcal{O}}_h(p)\, \hat{\mathcal{O}}_h(-p)\, \rangle$
position space	or	momentum space

So what are the appropriate physical observables? Here is a first guess:

$$\hat{\mathcal{O}}_h(p)|0\rangle = |\, \boldsymbol{h}\, , p\,\rangle \;=\; c(0)\bar{c}(0)\,\mathcal{O}_h\,(0)\, e^{ip\cdot X(0)}\,|\,0\,\rangle,$$

$$\Updownarrow \qquad\qquad \text{with} \quad p^2 + h = \frac{c}{24}$$

$$\hat{\mathcal{O}}_h(p) \;=\; \int d^2z\, \mathcal{O}_h(z,\bar{z})\, e^{ip\cdot X}$$

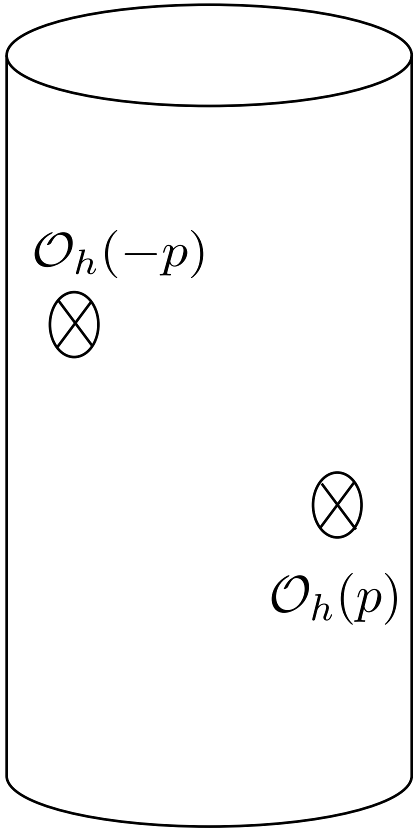
=> Only on-shell amplitudes! How can we obtain off-shell correlation functions?

Here are a concrete proposal: use boundary states! `D-branes'

Ishibashi states w/ fixed momentum …

$$\widehat{\mathcal{O}}_h(p)|0\rangle = \|h\rangle\rangle_{\rm cft}\ \|p\rangle\rangle_X\ \|0\rangle\rangle_{\rm gh}$$

$$(L_n - \bar{L}_{-n})\|h\rangle\rangle_{\rm cft} = (L^X_n - \bar{L}^X_{-n})\|p\rangle\rangle_X = (c_n - \bar{c}_{-n})\|0\rangle\rangle_{\rm gh} = 0$$

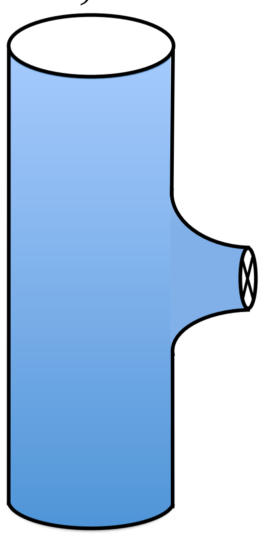


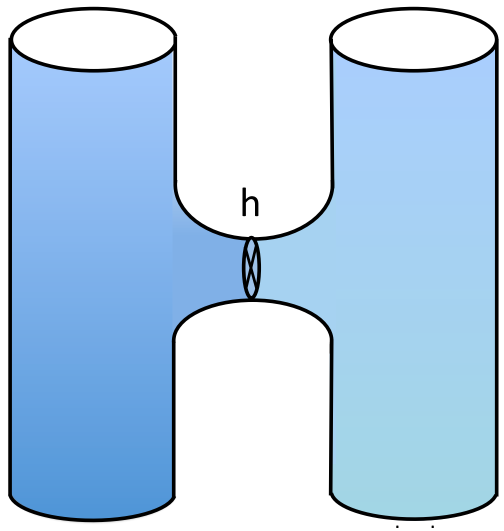
… or cross cap states:

$$\widetilde{\mathcal{O}}_h(p)|0\rangle = \|h\rangle\rangle_\otimes\ \|p\rangle\rangle_\otimes\ \|0\rangle\rangle_\otimes$$

$$(L_n - \widetilde{L}_{-n})\|h\rangle\rangle_\otimes = (L^X_n - \widetilde{L}^X_{-n})\|p\rangle\rangle_\otimes = (c_n - \widetilde{c}_{-n})\|0\rangle\rangle_\otimes = 0$$

To compute the matrix element of a cross-cap operator, it is useful to consider the CFT on the `Schottky double':

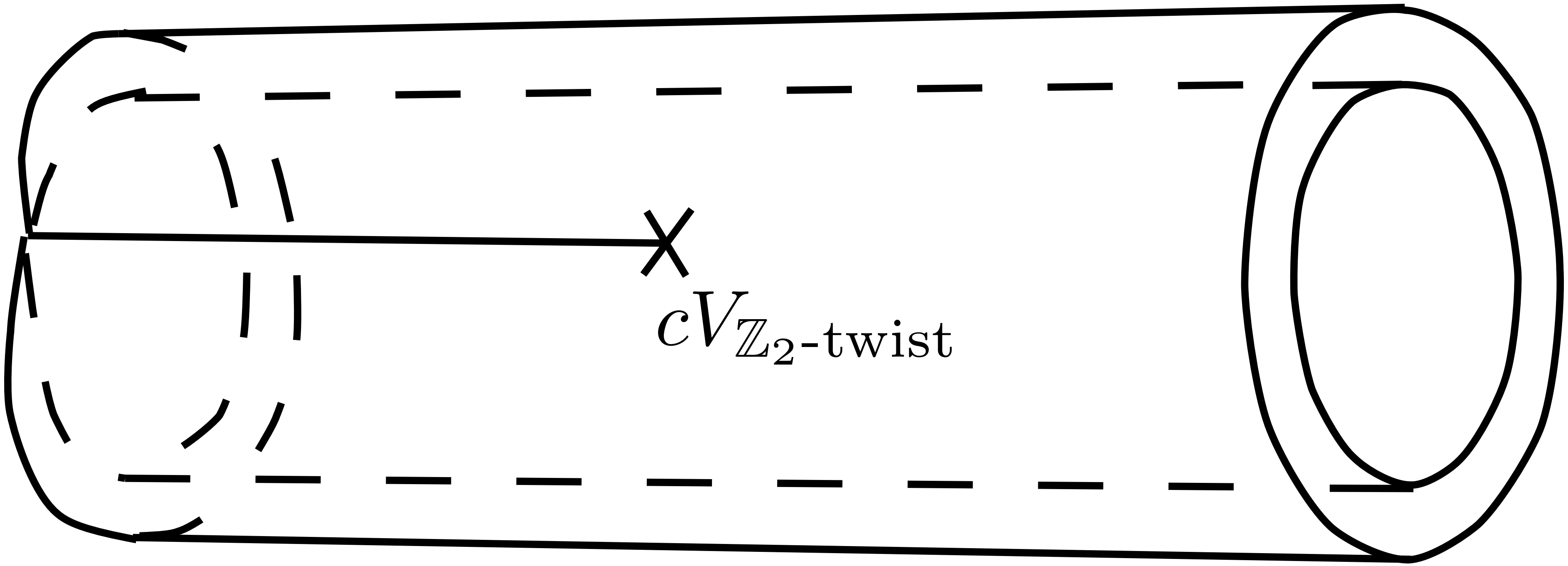




$$\langle \mathcal{E}', J' | \widetilde{\mathcal{O}}_h | \mathcal{E}, J \rangle = \int d\rho \left\langle e^{ik_1 X(\rho)} e^{ik_2 X(0)} e^{ik_2 X(1)} e^{ik_4 X(\infty)} \right\rangle \left\langle V_{\Delta'_L}(\rho) V_{\Delta_L}(0) \mathrm{P}_h V_{\Delta_R}(1) V_{\Delta'_R}(\infty) \right\rangle$$

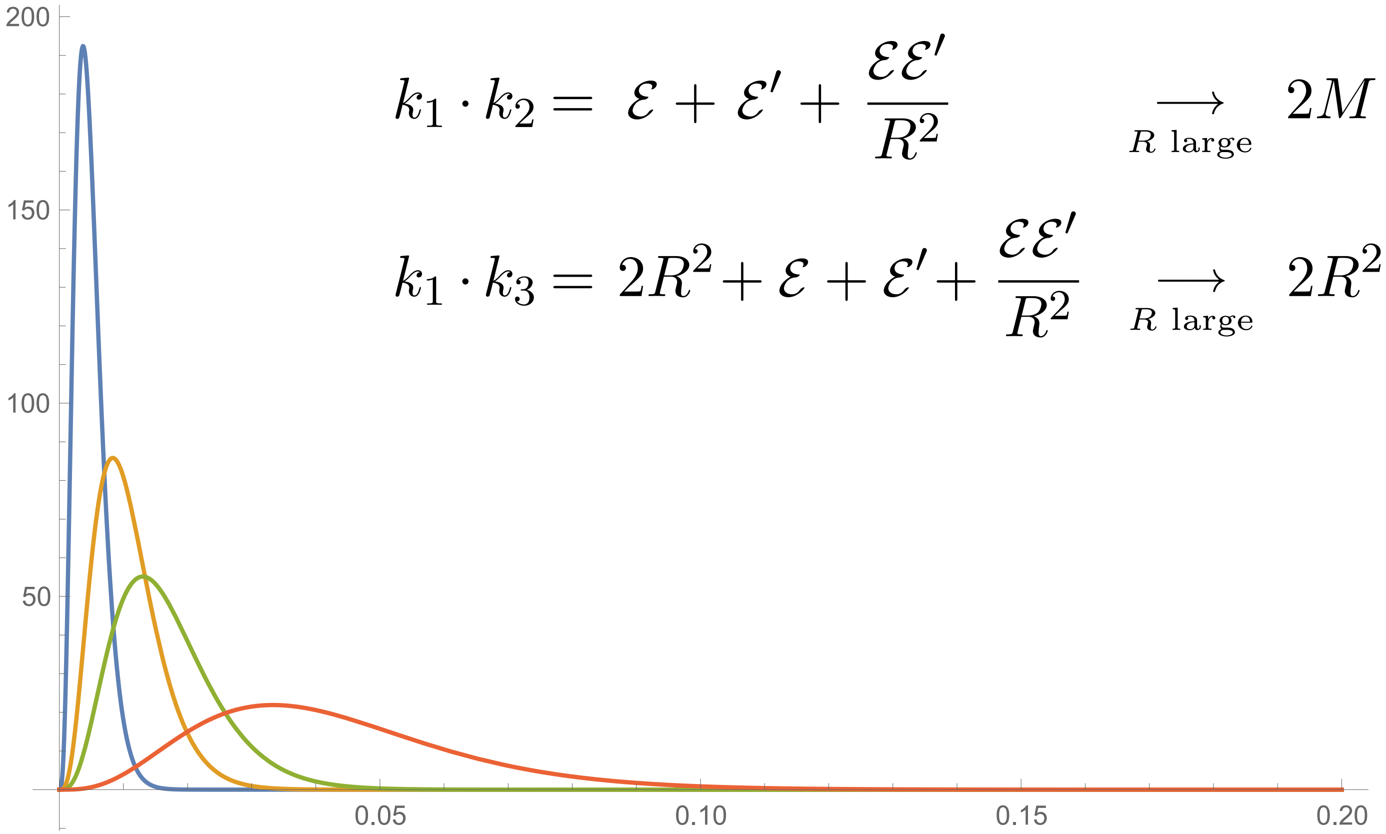
$$= \mathcal{N} \int d\rho \, \rho^{k_1 \cdot k_2} (1-\rho)^{k_1 \cdot k_3} \left\langle V_{\Delta'_L}(\rho) V_{\Delta_L}(0) \mathrm{P}_h V_{\Delta_R}(1) V_{\Delta'_R}(\infty) \right\rangle$$

chiral



$cV_{\mathbb{Z}_2\text{-twist}}$

$$\langle \mathcal{E}', J' | \widetilde{\mathcal{O}}_h | \mathcal{E}, J \rangle = \mathcal{N} \int d\rho \, \rho^{k_1 \cdot k_2} \, (1-\rho)^{k_1 \cdot k_3} \left\langle V_{\Delta'_L}(\rho) \, V_{\Delta_L}(0) \, \mathrm{P}_h \, V_{\Delta_R}(1) \, V_{\Delta'_R}(\infty) \right\rangle$$



$$k_1 \cdot k_2 = \mathcal{E} + \mathcal{E}' + \frac{\mathcal{E}\mathcal{E}'}{R^2} \quad \xrightarrow[R \text{ large}]{} \quad 2M$$

$$k_1 \cdot k_3 = 2R^2 + \mathcal{E} + \mathcal{E}' + \frac{\mathcal{E}\mathcal{E}'}{R^2} \quad \xrightarrow[R \text{ large}]{} \quad 2R^2$$

$$\langle \mathcal{E}', J' | \widetilde{\mathcal{O}}_h | \mathcal{E}, J \rangle = \mathcal{N} \int d\rho \, \rho^{k_1 \cdot k_2} \, (1-\rho)^{k_1 \cdot k_3} \, \big\langle V_{\Delta'_L}(\rho) \, V_{\Delta_L}(0) \, \mathrm{P}_h \, V_{\Delta_R}(1) \, V_{\Delta'_R}(\infty) \big\rangle$$

$$k_1 \cdot k_2 = \mathcal{E} + \mathcal{E}' + \frac{\mathcal{E}\mathcal{E}'}{R^2} \quad \xrightarrow[R \text{ large}]{} \quad 2M$$

$$k_1 \cdot k_3 = 2R^2 + \mathcal{E} + \mathcal{E}' + \frac{\mathcal{E}\mathcal{E}'}{R^2} \quad \xrightarrow[R \text{ large}]{} \quad 2R^2$$

$$\langle \mathcal{E}', J' | \widetilde{\mathcal{O}}_h | \mathcal{E}, J \rangle \simeq \big\langle V_{\Delta'_L}(\rho_c) \, V_{\Delta_L}(0) \, \mathrm{P}_h \, V_{\Delta_R}(1) \, V_{\Delta'_R}(\infty) \big\rangle.$$

→ identifies matrix element with square root of a conformal block!

It is convenient to introduce polar coordinates (ρ, t, ϕ) and write

$$f_{\omega\ell}(t,\varphi,\rho) = e^{-i\omega t} e^{i\ell\varphi} f_{\omega\ell}(\rho)$$

We wish to compare this matrix element with the classical mode function

$$f_{\omega\ell}(\rho) = \rho^h (1-\rho)^{\frac{i\omega}{2r_+}} {}_2F_1\left(h+\frac{i(\omega+\ell)}{2r_+}, h+\frac{i(\omega-\ell)}{2r_+}, 2h; \rho\right)$$

in the dual BTZ black hole. We conjecture that the two are in fact equal: