The two-body potential in modified gravity

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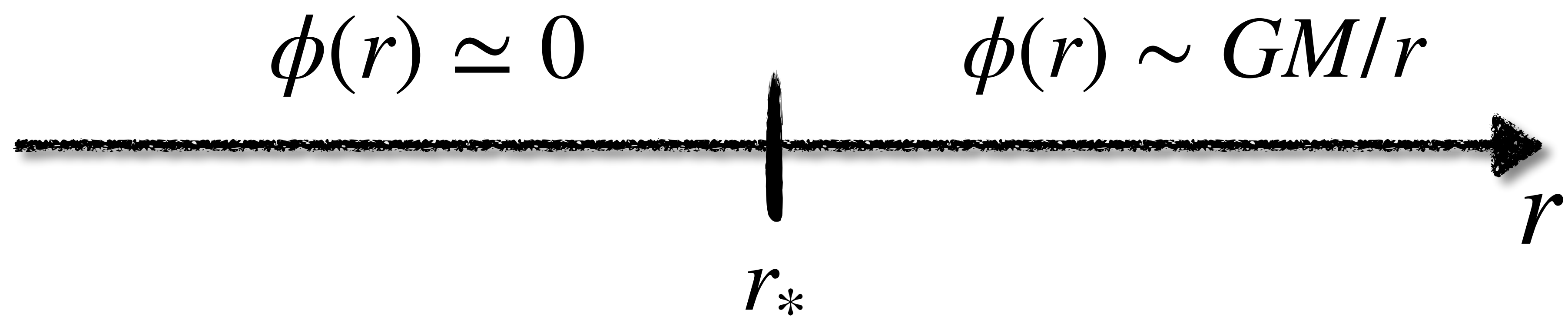
Dark energy meeting, IAP
20/05/2019

Motivation

Modified gravity should incorporate screening in order to recover GR on solar system scales



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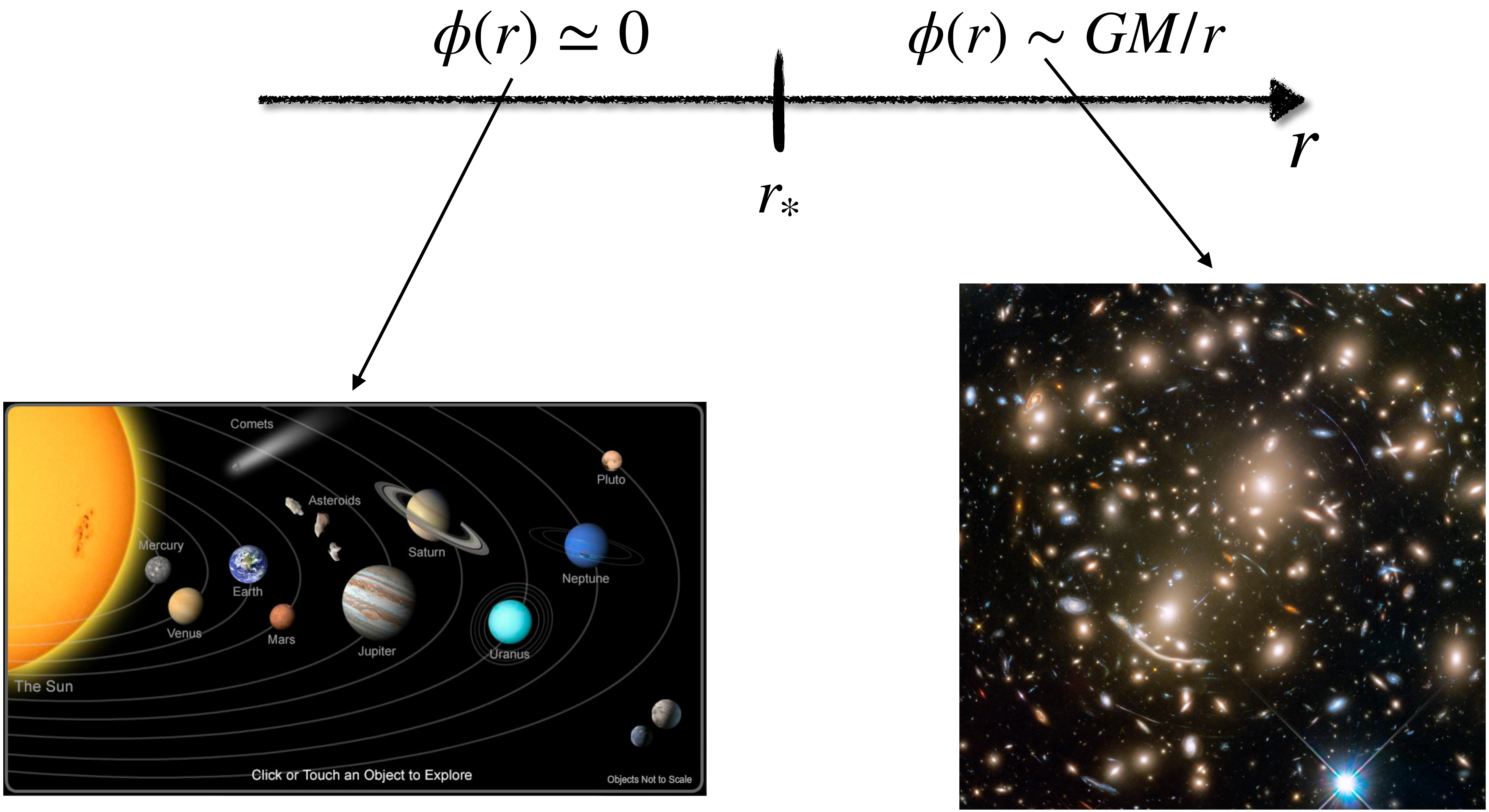

Spherically symmetric screening

Take the simplest screening : $$S = \int d^4x \left[-\frac{(\partial\phi)^2}{2} - \frac{1}{4\Lambda^4}(\partial\phi)^4 + \frac{\phi T}{M_P} \right]$$

(K-Mouflage)

Spherically symmetric screening

Take the simplest screening :

(K-Mouflage)

$$\tilde{S} = \int dt d^3x \left[-\frac{1}{2}(\nabla\tilde{\phi})^2 - \frac{1}{4}(\nabla\tilde{\phi})^4 + \tilde{\phi}\tilde{T} \right]$$

$$\phi' + (\phi')^3 = \frac{M}{r^2}$$

Spherically symmetric screening

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(K-Mouflage)

$$\phi' + (\phi')^3 = \frac{M}{r^2}$$

$$\Rightarrow \phi(r) = -\frac{M}{r}\,{}_3F_2\left(\frac{1}{4},\frac{1}{3},\frac{2}{3};\frac{5}{4},\frac{3}{2};-\frac{27M^2}{4r^4}\right)$$

$\phi(r) = 3(Mr)^{1/3} + \ldots$ $\qquad\qquad$ $\phi(r) = -\frac{M}{r} + \frac{M^3}{5r^5} + \ldots$

$$r_* = \sqrt{M}$$

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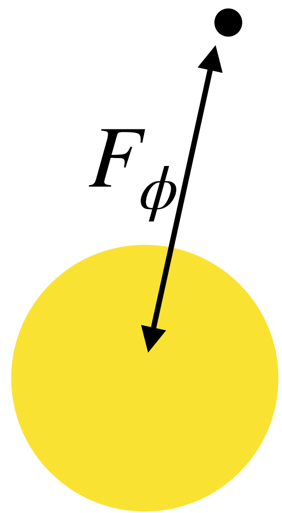
Modified gravity should incorporate screening in order to recover GR on solar system scales

Still small-scale tests of GR are very precise !



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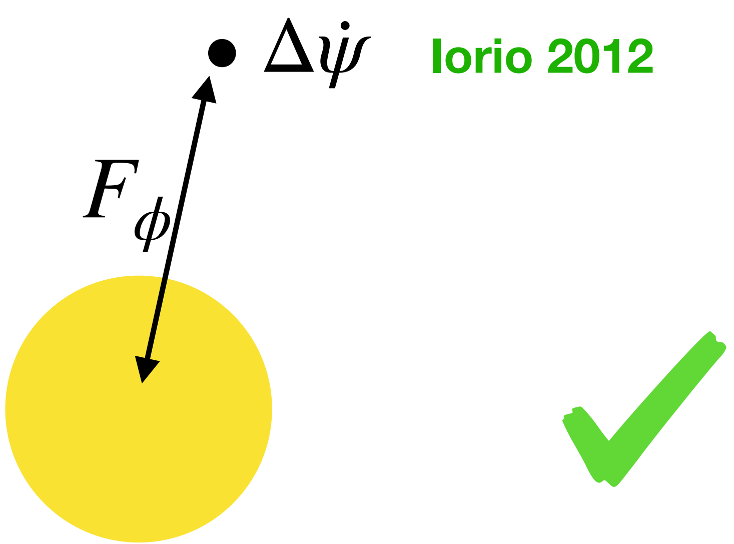

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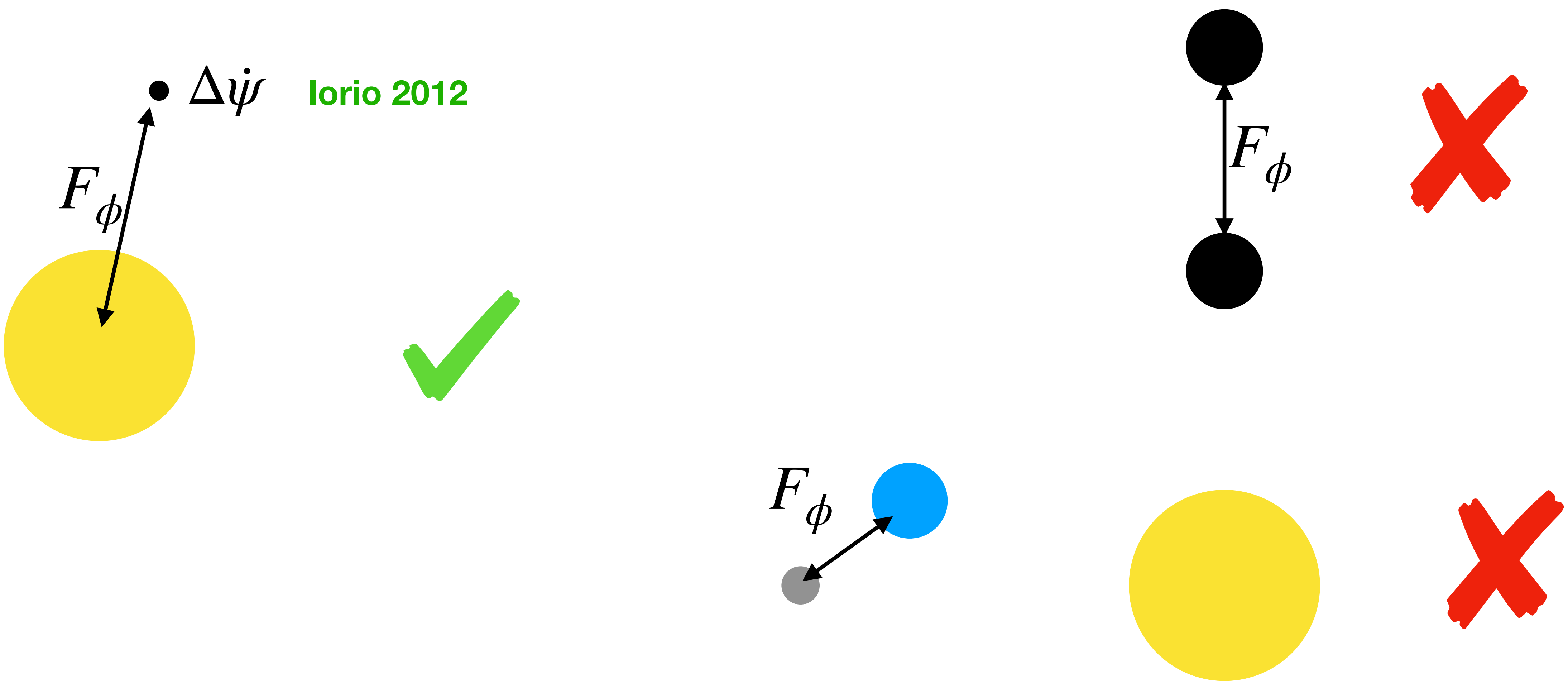
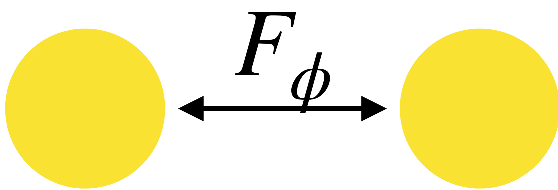


The two-body problem : outside



$$\tilde{S} = \int dt d^3x \left[-\frac{1}{2}(\nabla\tilde{\phi})^2 - \frac{1}{4}(\nabla\tilde{\phi})^4 + \tilde{\phi}\tilde{T} \right]$$

$$\tilde{T} = -m_1\delta^3(\mathbf{x} - \mathbf{x}_1) - m_2\delta^3(\mathbf{x} - \mathbf{x}_2)$$

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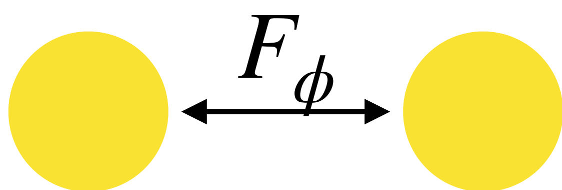
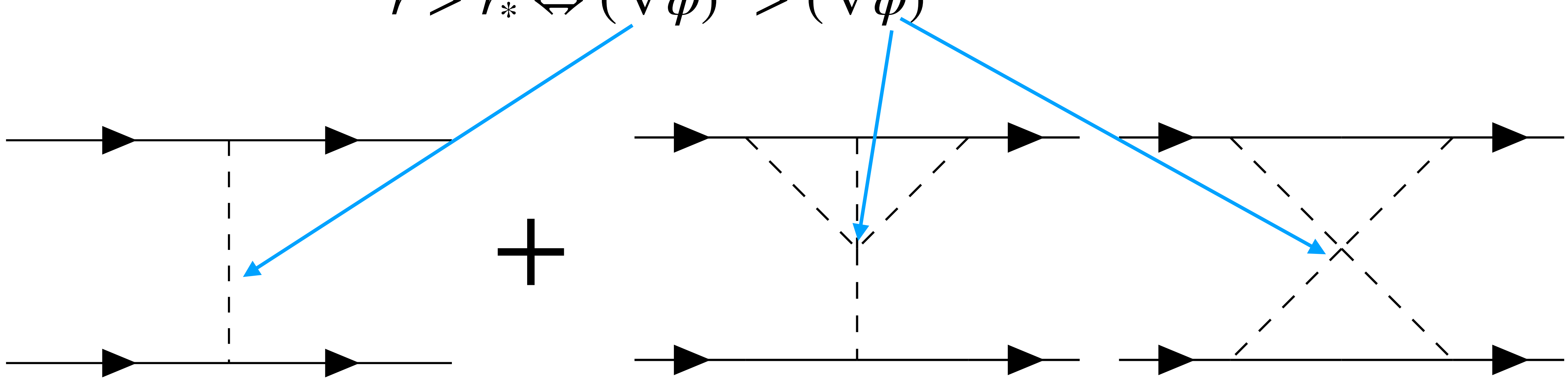


$$\tilde{S} = \int dt d^3x \left[-\frac{1}{2}(\nabla \tilde{\phi})^2 - \frac{1}{4}(\nabla \tilde{\phi})^4 + \tilde{\phi}\tilde{T} \right]$$

$$e^{iS_{\mathrm{cl}}[\mathbf{x}_1,\mathbf{x}_2]} = \int \mathscr{D}[\phi] e^{iS[\mathbf{x}_1,\mathbf{x}_2,\phi]}$$

$$r > r_* \Leftrightarrow (\nabla \phi)^2 > (\nabla \phi)^4$$

$S_{\mathrm{cl}} =$



(a) (b)

$+ \ldots$

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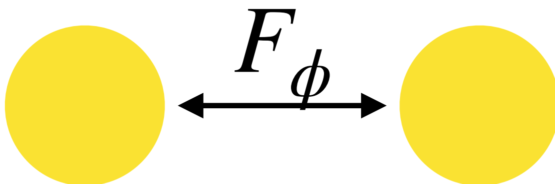
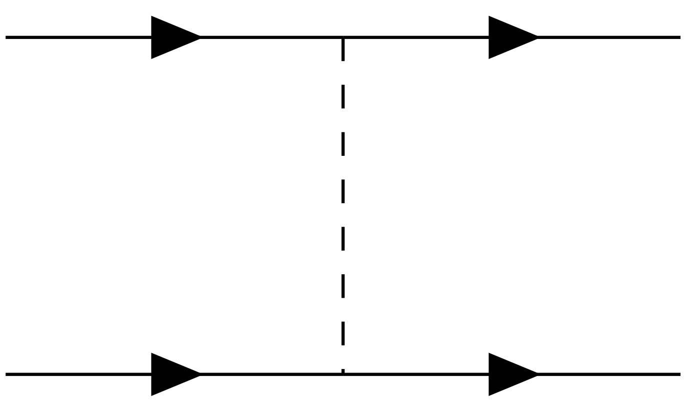
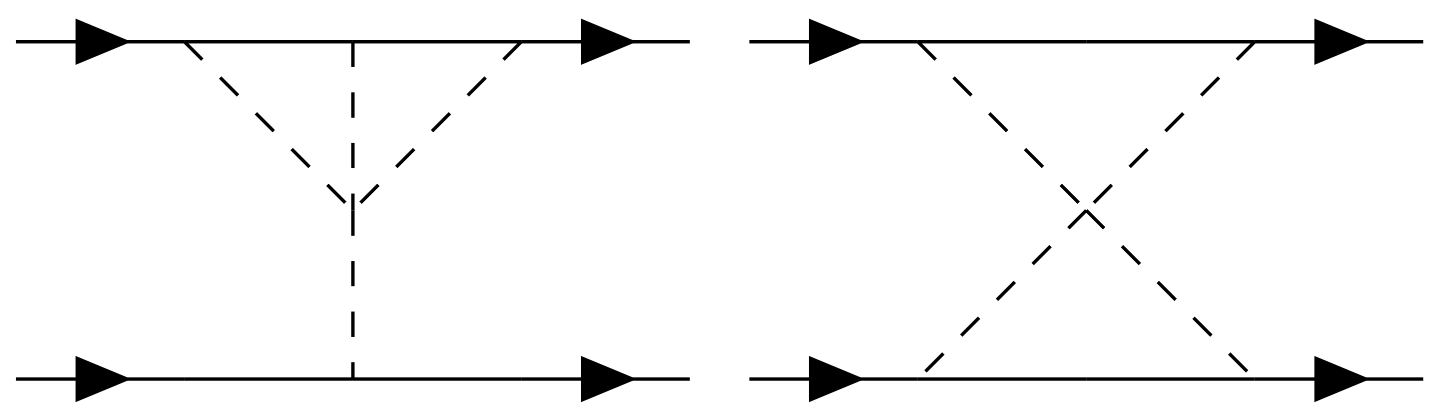


$$\tilde{S} = \int dt d^3x \left[-\frac{1}{2}(\nabla \tilde{\phi})^2 - \frac{1}{4}(\nabla \tilde{\phi})^4 + \tilde{\phi}\tilde{T} \right]$$

$$\int dtE = -S_{\rm cl} \Rightarrow \qquad E = -\frac{m_1 m_2}{r} + \frac{m_1 m_2 (m_1^2 + m_2^2)}{5r^5} + \dots$$





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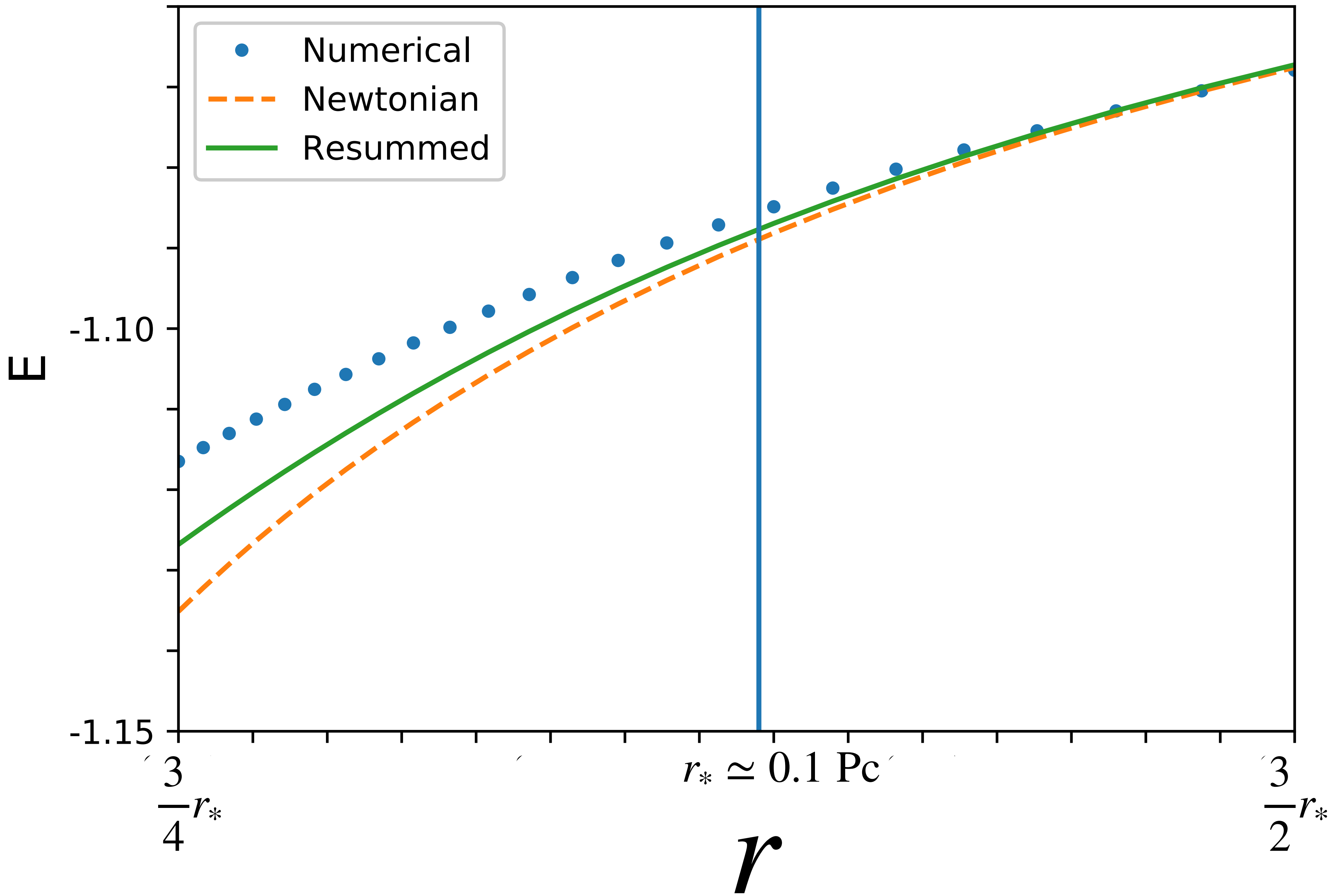
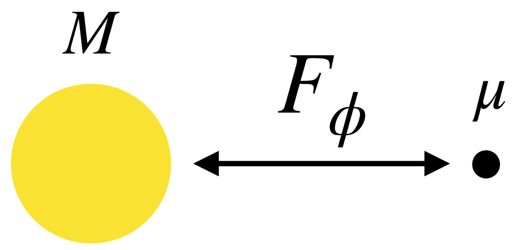


Effective One-Body (EOB) : outside



$$E_{\mathrm{tm}} = \mu\left(-\frac{M}{r} + \frac{M^3}{5r^5} + \dots\right)$$

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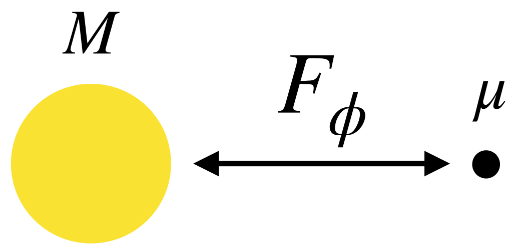
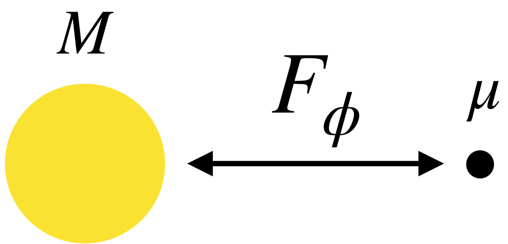


$$E_{\rm tm} = \mu\left(-\frac{M}{r} + \frac{M^3}{5r^5} + \dots\right)$$

$$E_N = -\mu\frac{M}{r}$$

$$\frac{E_{\rm tm}}{E_N} - 1 = -\frac{M^2}{5r^4} + \dots \ll 1$$

Effective One-Body (EOB) : outside



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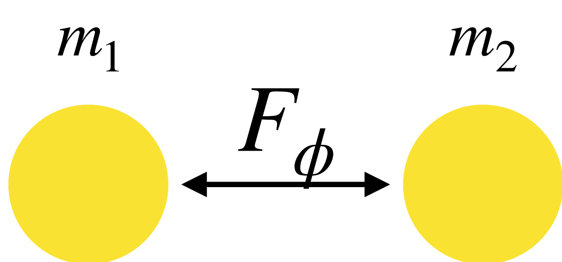

$$E = -\frac{m_1 m_2}{r} + \frac{m_1 m_2 (m_1^2 + m_2^2)}{5r^5} + \ldots$$

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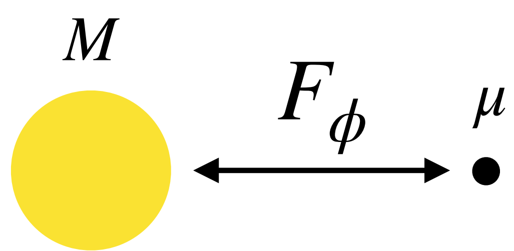

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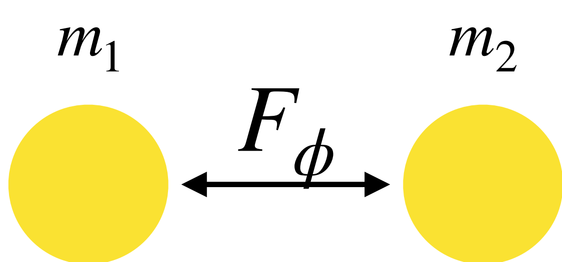

$$E = -\frac{m_1 m_2}{r} + \frac{m_1 m_2 (m_1^2 + m_2^2)}{5r^5} + \dots$$

$$= \mu \left(-\frac{M}{r} + \frac{M^3}{5r^5}(x^2 + (1-x)^2) + \dots \right)$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$M = m_1 + m_2$$

$$x = \frac{m_1}{m_1 + m_2}$$

The two-body energy is a deformation of the test-mass energy

Damour & Buonanno 99

Energy map outside

Idea : resum nonlinearities by using only E_{tm}

$$E_{\mathrm{tm}} = \mu\left(-\frac{M}{r} + \frac{M^3}{5r^5} + \ldots\right) \quad E = \mu\left(-\frac{M}{r} + \frac{M^3}{5r^5}(x^2 + (1-x)^2) + \ldots\right)$$

Energy map outside

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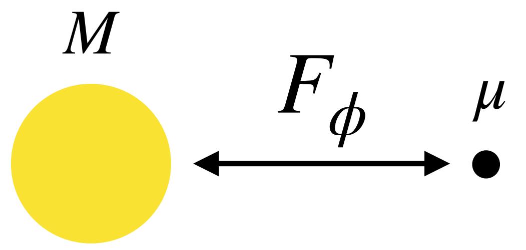
$$E_{\rm tm} = \mu\left(-\frac{M}{r} + \frac{M^3}{5r^5} + \ldots\right) \quad E = \mu\left(-\frac{M}{r} + \frac{M^3}{5r^5}(x^2 + (1-x)^2) + \ldots\right)$$

$$\frac{E}{E_{\rm tm}} = 1 + \frac{M^2}{5r^4}\left(1 - x^2 - (1-x)^2\right) + \ldots$$

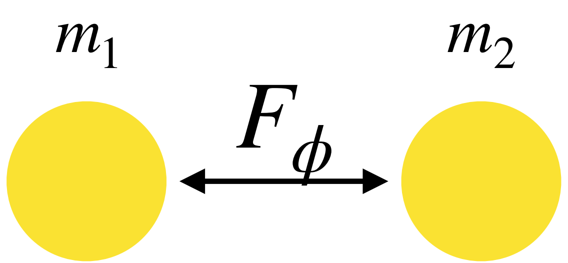
$$\frac{E}{E_{\rm tm}} = a_0 + a_1\left(\frac{E_{\rm tm}}{E_N} - 1\right) + a_2\left(\frac{E_{\rm tm}}{E_N} - 1\right)^2 + \ldots$$

$\frac{E_{\rm tm}}{E_N} - 1 = -\frac{M^2}{5r^4} + \ldots \ll 1$ is itself a resummed expansion in $\frac{1}{r}$

Inside the nonlinear radius



$$E_{\mathrm{tm}} = 3\mu\,(Mr)^{1/3} + \ldots$$



?

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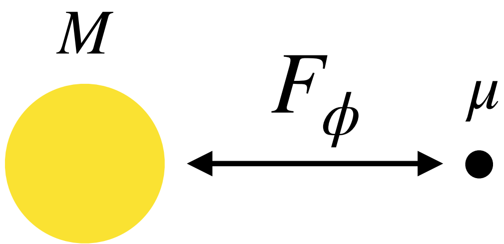
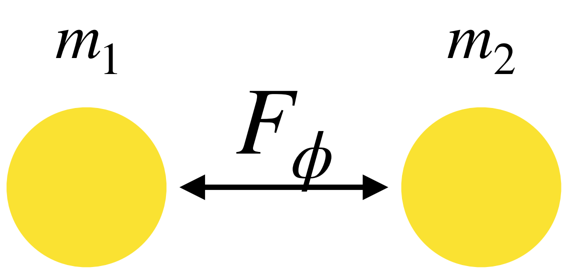


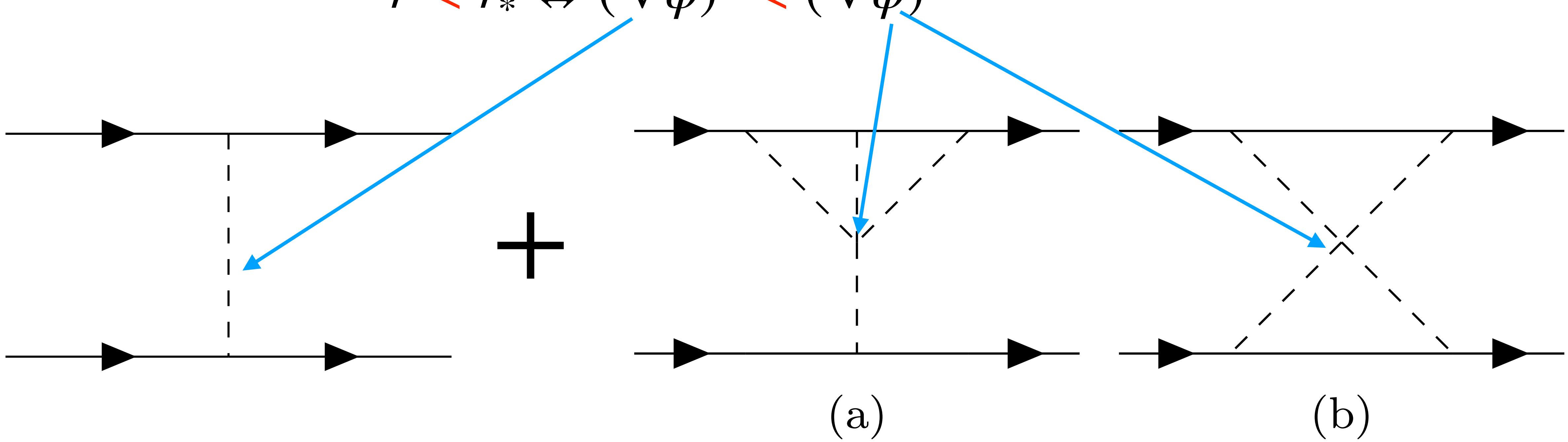
$$E_{\rm tm} = 3\mu\,(Mr)^{1/3} + \ldots$$



?

$$r < r_* \Leftrightarrow (\nabla\phi)^2 < (\nabla\phi)^4$$

$S_{\rm cl} =$



$+\ldots$

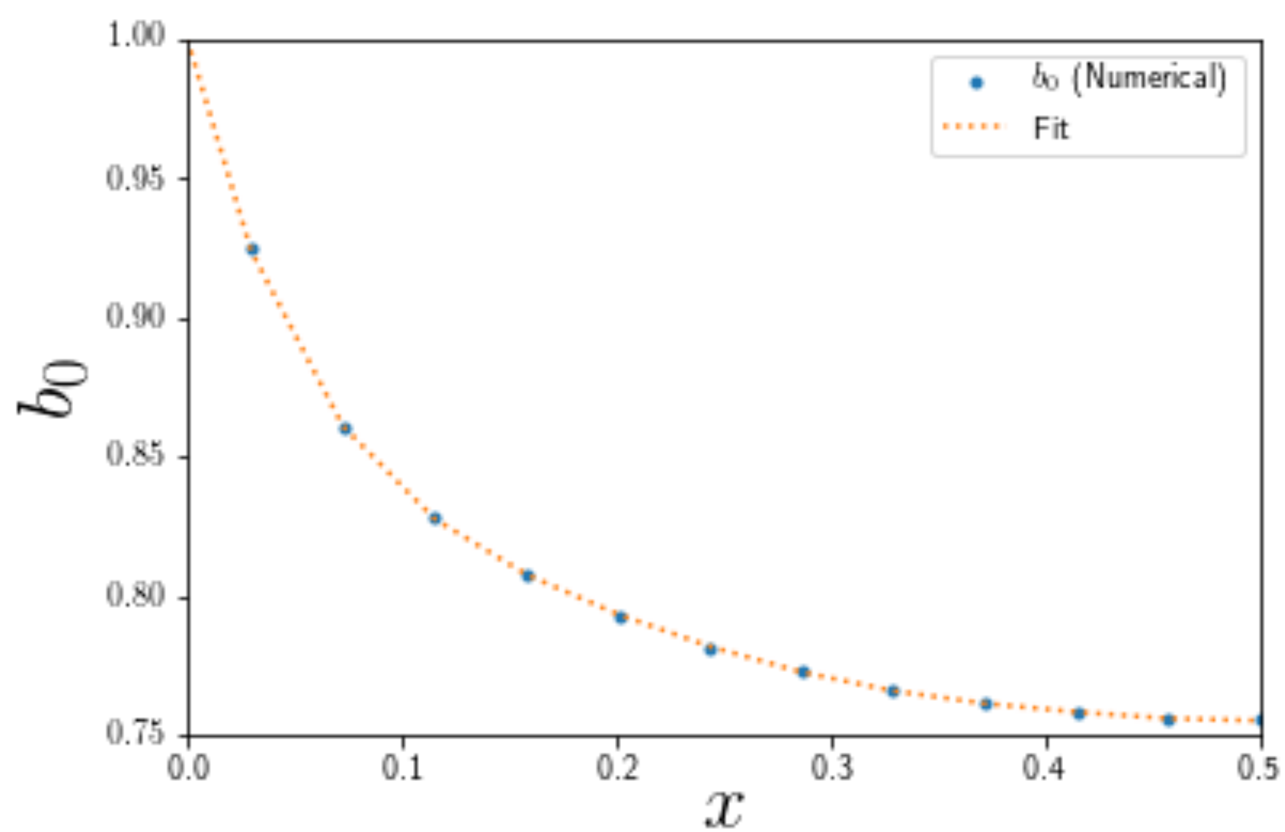
Diverges

Inside the nonlinear radius

Idea: Postulate that the energy map is valid inside

$$\frac{E}{E_{\rm tm}} = b_0 + b_1\left(\frac{E_{\rm tm}}{E_{\rm ref}} - 1\right) + b_2\left(\frac{E_{\rm tm}}{E_{\rm ref}} - 1\right)^2 + \dots$$

One cannot compute the b_i's, but one can get them with a numerical simulation !

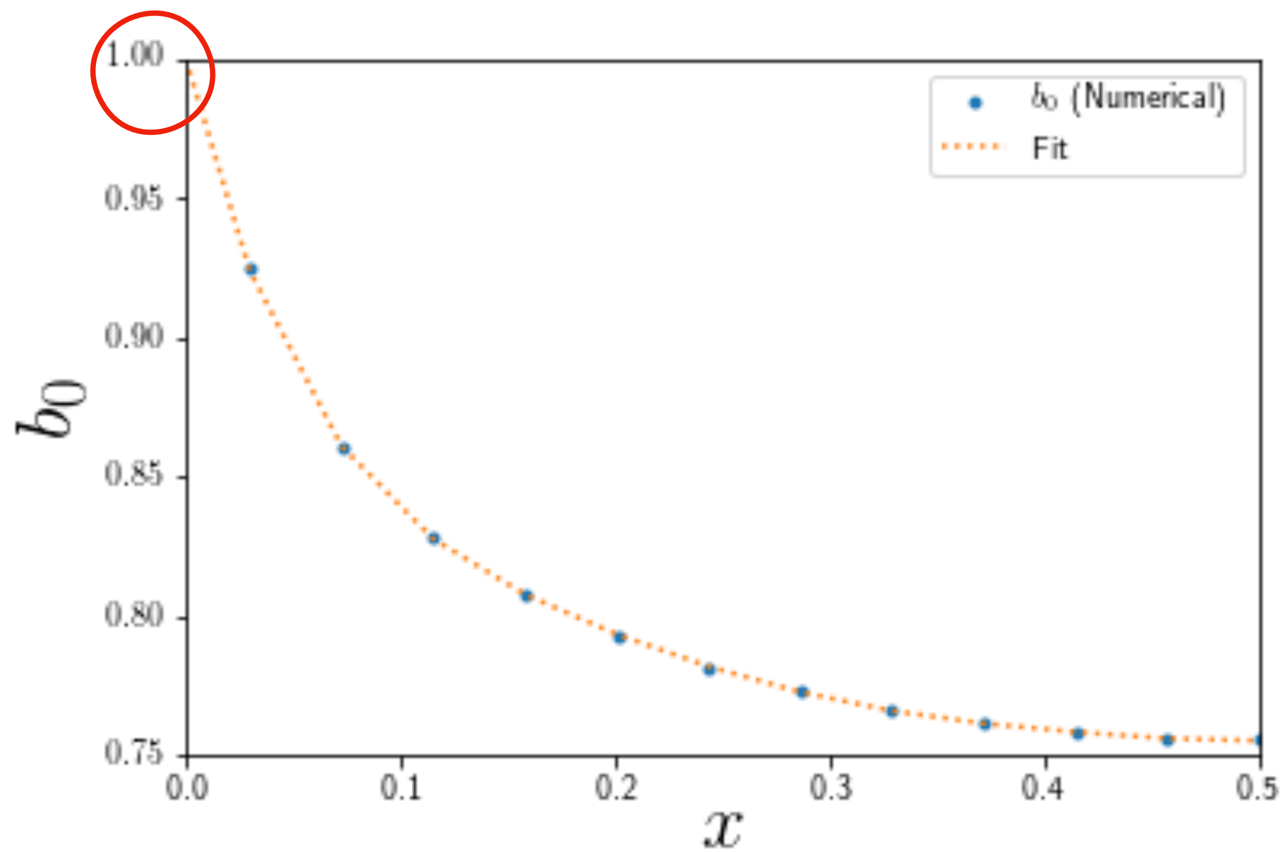


EP violation

The Moon is a test-mass :

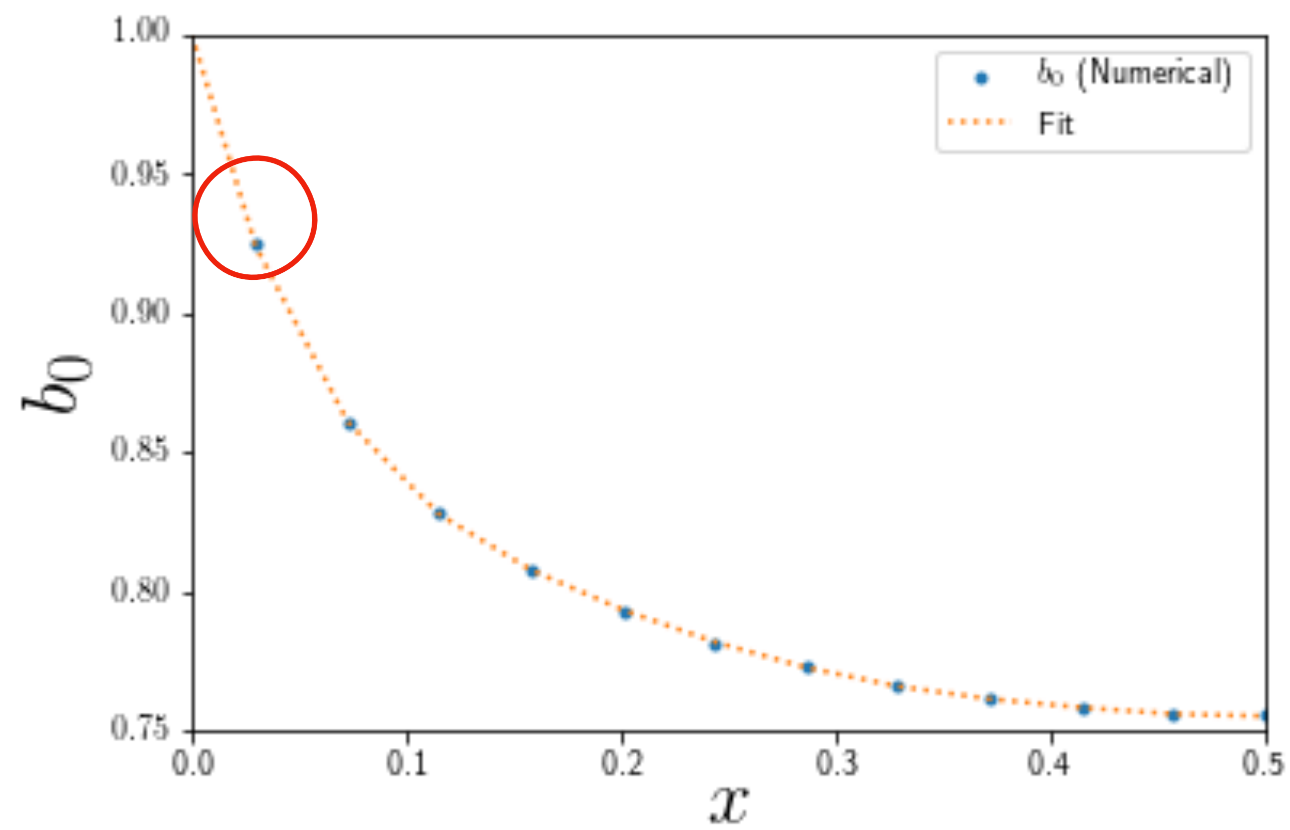
$$\vec{F}_M \simeq m_M \vec{\nabla} \phi_S(r) \quad \text{and} \quad \vec{F}_M = m_M \vec{a}_M$$

$$\Rightarrow \quad \vec{a}_M = \vec{\nabla} \phi_S(r)$$ does not depend on m_M



EP violation

The Earth is not a test-mass :



$\vec{F}_E \simeq m_E b_0(x_{SE}) \vec{\nabla} \phi_S(r)$ and $\vec{F}_E = m_E \vec{a}_E$

$\Rightarrow \quad \vec{a}_E = b_0(x_{SE}) \vec{\nabla} \phi_S(r)$ depends on x_{SE} !!

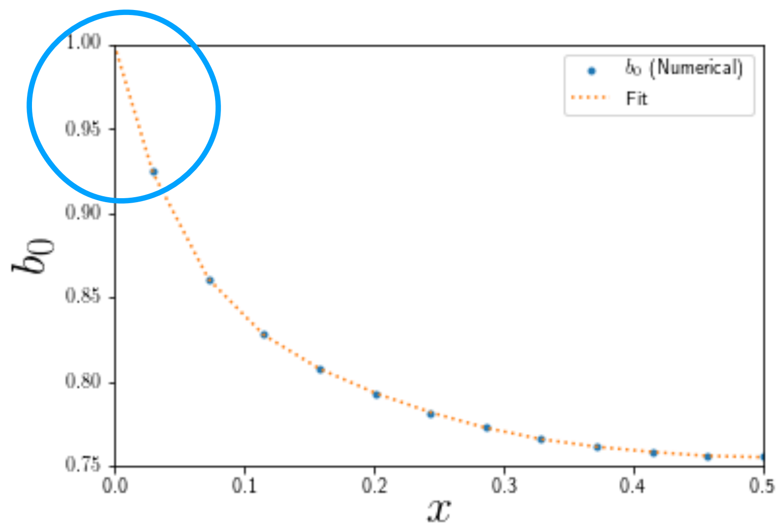
The Sun-Earth-Moon system

$$\delta r_{EM} \simeq 3 \times 10^{12} \left| (b_0(x_{SE}) - 1) \left(\frac{r}{r_*} \right)^n \right| \text{ cm}$$

$b_0(x) \simeq 1 - \kappa x$

This gives a constraint :

$$\kappa x_{SE} \left(\frac{r}{r_*} \right)^n \lesssim 10^{-13}$$



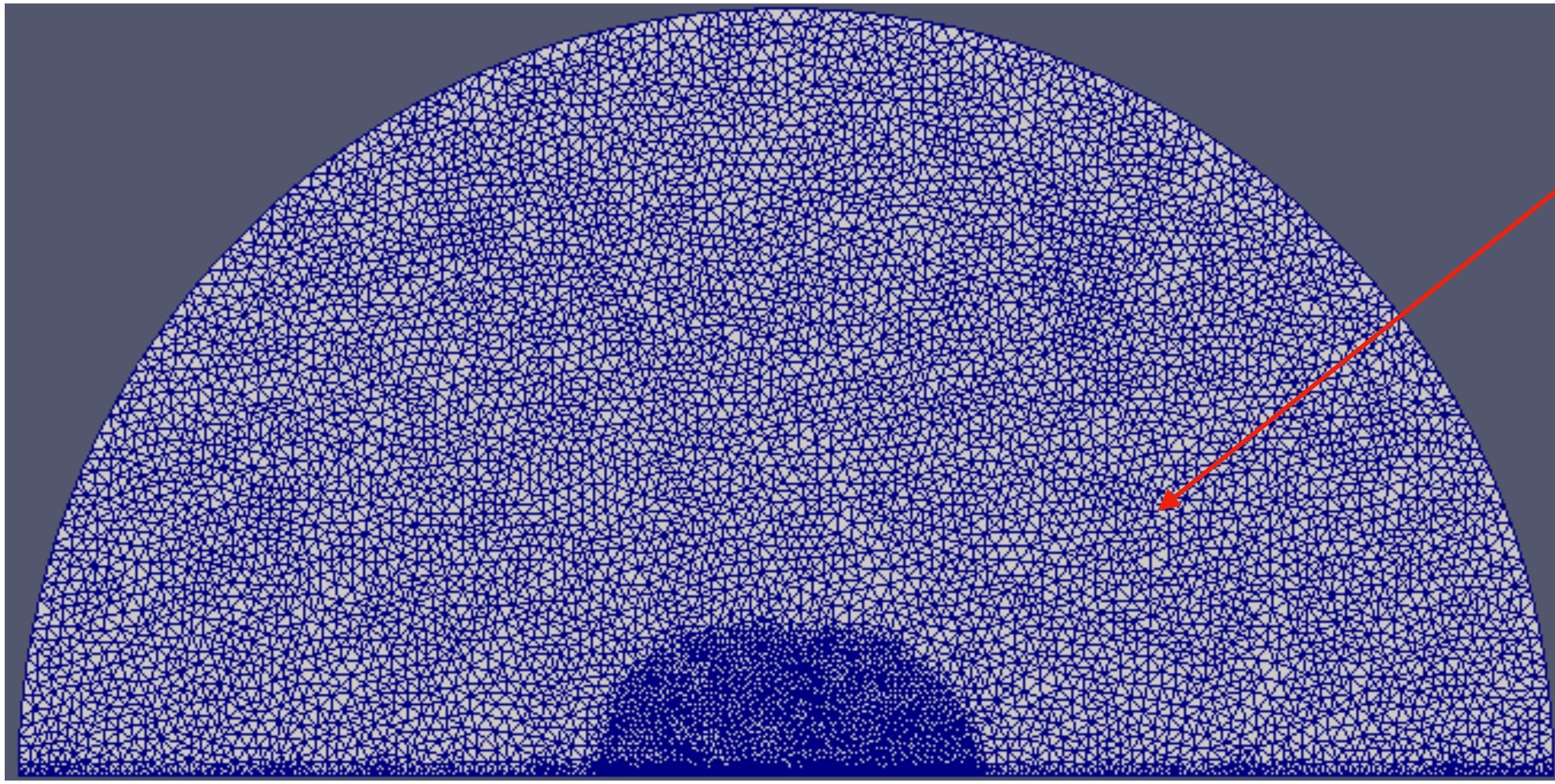
Since $x_{\rm SE} \simeq 10^{-6}$, the perihelion constraint is better :

$$\left(\frac{r}{r_*} \right)^n \lesssim 10^{-11}$$

Conclusions

- The two-body potential is easily expressed in the EOB formalism
- EP violation in the Sun-Earth-Moon system is weaker than anomalous perihelion precession in the P(X) model
- Future direction : conservative and dissipative dynamics of inspiralling compact objects

Finite Elements simulation



Basis function on the grid

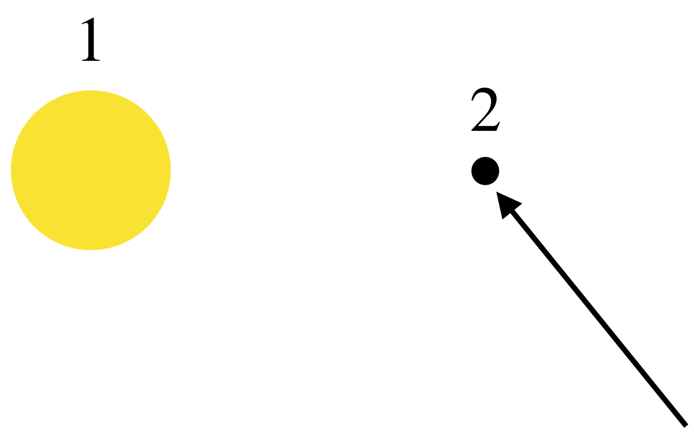
$$\phi = \sum_j c_j \psi_j$$

$$\nabla \cdot \left(\nabla \phi + (\nabla \phi)^2 \nabla \phi \right) = -T \quad \Leftrightarrow \quad \int d^3x \left((1 + (\nabla \phi)^2) \nabla \phi \cdot \nabla \psi_j - T \psi_j \right) = 0$$

Solve this matrix equation by LU decomposition

Then $$E = \int d^3x \left(\frac{(\nabla \phi)^2}{2} + \frac{(\nabla \phi)^4}{4} \right) + 4\pi m_1 \phi(\mathbf{x}_1) + 4\pi m_2 \phi(\mathbf{x}_2)$$

Superposing nonlinear solutions



$\phi = \phi_1(\mathbf{x} - \mathbf{x}_1) + \psi$ with $\phi_1(r) = 3(m_1 r)^{1/3}$

Sufficiently close to 2, we should expect : $\psi(\mathbf{x}) \simeq 3(m_2 |\mathbf{x} - \mathbf{x}_2|)^{1/3}$

Expand the action $S = \int dt d^3x \left[-\frac{1}{2}(\nabla\phi)^2 - \frac{1}{4}(\nabla\phi)^4 + \phi T \right]$

$$\Rightarrow S = S[\phi_1] + \int dt d^3x \left[-\frac{1}{2}(\nabla\phi_1)^2(\nabla\psi)^2 - (\nabla\phi_1 \cdot \nabla\psi)^2 - (\nabla\psi)^2 \nabla\phi_1 \cdot \nabla\psi - \frac{1}{4}(\nabla\psi)^4 + \psi T_1 \right]$$

If the last term dominates over the other, we recover the original action !

Superposing nonlinear solutions

$$\phi = \phi_1(\mathbf{x} - \mathbf{x}_1) + \psi$$ with $$\phi_1(r) = 3(m_1 r)^{1/3}$$

$$\psi(\mathbf{x}) \simeq 3(m_2|\mathbf{x} - \mathbf{x}_2|)^{1/3}$$

$$S = S[\phi_1] + \int dt d^3x \left[-\frac{1}{2}(\nabla\phi_1)^2(\nabla\psi)^2 - (\nabla\phi_1 \cdot \nabla\psi)^2 - (\nabla\psi)^2 \nabla\phi_1 \cdot \nabla\psi - \frac{1}{4}(\nabla\psi)^4 + \psi T_1 \right]$$

Ratio of the cubic and quartic terms :

$$\frac{(\nabla\psi)^2 \nabla\phi_1 \cdot \nabla\psi}{(\nabla\psi)^4} \le \frac{|\nabla\phi_1|}{|\nabla\psi|} \simeq \left(\frac{m_1}{m_2} \left(\frac{r_2}{r_1} \right)^2 \right)^{1/3}$$

(Recall that $r_* = \sqrt{M}$)

Condition to superpose the nonlinear solutions :

$$\frac{r_2}{r_{*,2}} \ll \frac{r_1}{r_{*,1}}$$