Cosmological cancellation of the vacuum energy density

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(with Ph. Brax)

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1- MOTIVATION

- Dark Energy or Modified-Gravity are motivated by the current acceleration of the Universe.

Apart from its current value, the main clue we have is that it is so small:

"OLD COSMOLOGICAL CONSTANT PROBLEM"

$\Rightarrow$ This could point to the solution !

- Many DE or MG models have been ruled out. The remaining ones do not match data much better than LCDM.

It would be good if they could be at least useful for something:

$\Rightarrow$ solve/address the cosmological constant problem !

“Old cosmological constant problem”: it is very sensitive to details of UV physics and gets huge contributions that arise during the radiation era:

A primordial tuning is not enough !

The problem of the vacuum energy should always have been at the fore of cosmological research. Indeed one can learn a great deal from its examination:

- ✓ The result seems to depend on an arbitrary UV cut off.
- ✓ The result should take into account not only the electron but all the known particles.
- ✓ The sole contribution from the proton is larger than the energy at the formation of the elements (Big Bang Nucleosynthesis) preventing one from understanding the Universe’s dynamics since then.

The first two points have been gradually understood since the 1950’s with the advent of modern Quantum Field Theory:

$$\rho_\Lambda = \Lambda m_{\rm Pl}^2 + \rho_{\rm transition} + \sum (2j+1)(-1)^{2j} \frac{m_j^4}{64\pi^2} \ln \frac{\mu^2}{m_j^2}$$

Cosmological constant

Phase transitions: QCD, electroweak …

Vacuum quantum fluctuations

“Self-tuning”: add extra fields that cancel the vacuum energy.

Weinberg’s theorem: this is impossible without fine tuning, under general assumptions !

- Lagrangian: $\mathcal{L}[g, \varphi_i]$.
- translationally invariant vacuum: $g_{\mu\nu}, \varphi_i = \text{constant.}$

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SUSY Large extra dimensions

string-theory scenarios

degravitation

superfluid, Lorentz-violating theory

self-tuning scalar-tensor models

mirror universe

sequestering

II- DEFINITION OF THE MODEL

A) Action S

We wish to recover GR

Keep standard E.H. and matter actions, but with a conformal coupling between Jordan and Einstein metrics

$$S = S_{\rm EH} + S_{\rm m} + S_{\varphi,\lambda}$$

$$S_{\rm EH} = \int d^4x \sqrt{-g}\frac{M_{\rm Pl}^2}{2}R, \qquad S_{\rm m} = \int d^4x \sqrt{-\tilde{g}}\tilde{\mathcal{L}}_{\rm m}(\psi_{\rm m}^{(i)}, \tilde{g}_{\mu\nu}), \qquad \tilde{g}_{\mu\nu} = A^2(\varphi) g_{\mu\nu}, \quad A(\varphi) > 0.$$

$$S_{\varphi,\lambda} = \int d^4x \sqrt{-g}\left[\mathcal{M}^3 A^4(\varphi)\lambda + \mathcal{M}^4 K(\varphi; X, Y, Z)\right], \qquad X = -\frac{\partial^\mu\lambda\partial_\mu\lambda}{2\mathcal{M}^4}, \quad Y = -\frac{\partial^\mu\varphi\partial_\mu\lambda}{\mathcal{M}^4}, \quad Z = -\frac{\partial^\mu\varphi\partial_\mu\varphi}{2\mathcal{M}^4}.$$

Idea: φ plays the role of a Lagrange multiplier that enforces the cancellation of the vacuum energy arising from the matter sector by the second field λ

This may be guessed from the symmetry:

using $\sqrt{-\tilde{g}} = A^4\sqrt{-g}$

$$\tilde{\mathcal{L}}_{\rm m} \to \tilde{\mathcal{L}}_{\rm m} - \tilde{V}_{\rm vac}, \quad \lambda \to \lambda + \tilde{V}_{\rm vac}/\mathcal{M}^3, \quad S \to S,$$

To make the cancellation apply for any $\tilde{V}_{\rm vac}$ the action does not depend on λ, only on its derivatives.

B) Equations of motion

Friedmann equations:

includes a possible cosmological constant

$$3M_{\rm Pl}^2\mathcal{H}^2 \;=\; a^2A^4(\tilde{V}_{\rm vac}+\tilde{\rho}+\tilde{\rho}_\gamma-\mathcal{M}^3\lambda)-a^2\mathcal{M}^4K+\frac{\partial K}{\partial X}\lambda'^2+2\frac{\partial K}{\partial Y}\lambda'\varphi'+\frac{\partial K}{\partial Z}\varphi'^2$$

$$M_{\rm Pl}^2(\mathcal{H}^2+2\mathcal{H}')=a^2A^4(\tilde{V}_{\rm vac}-\tilde{\rho}_\gamma/3-\mathcal{M}^3\lambda)-a^2\mathcal{M}^4K,$$

Equations of motion of the two scalar fields:

$$\mathcal{M}^4\frac{\partial K}{\partial\varphi}-a^{-4}\partial_\tau\left[a^2\left(\frac{\partial K}{\partial Y}\lambda'+\frac{\partial K}{\partial Z}\varphi'\right)\right]=4A^3\frac{dA}{d\varphi}(\tilde{V}_{\rm vac}+\tilde{\rho}/4-\mathcal{M}^3\lambda)$$

$$a^{-4}\partial_\tau\left[a^2\left(\frac{\partial K}{\partial X}\lambda'+\frac{\partial K}{\partial Y}\varphi'\right)\right]=\mathcal{M}^3A^4$$

C) Cancellation mechanism in the radiation era

Take $\tilde{\rho}=0,\quad \tilde{V}_{\rm vac}=\text{constant}.$ and $K=0,\ \frac{\partial K}{\partial\varphi}=0,\ \text{and}\ \frac{\partial K}{\partial Z}=0\ \text{when}\ \lambda'=0.$

Then we have the constant cancellation solution: $\lambda=\tilde{V}_{\rm vac}/\mathcal{M}^3,\quad \lambda'=0,$

and we recover the usual radiation-era Friedmann equations:

$$3M_{\rm Pl}^2\mathcal{H}^2=a^2\rho_\gamma,\quad M_{\rm Pl}^2(\mathcal{H}^2+2\mathcal{H}')=-a^2\rho_\gamma/3,$$

Thus, φ acts as a Lagrange multiplier, which enforces the cancellation by λ

$\Rightarrow$ dynamical cancellation mechanism.

D) How did we evade Weinberg's no-go theorem ?

λ - eq. of motion was: $$a^{-4}\partial_\tau\left[a^2\left(\frac{\partial K}{\partial X}\lambda' + \frac{\partial K}{\partial Y}\varphi'\right)\right] = \mathcal{M}^3 A^4$$

If we look for static solutions in Minkowski: $\Rightarrow$ $A = 0$

$\Rightarrow$ the matter action vanishes ! (Weinberg's result)

This is avoided by a time-dependent background φ that evolves on cosmological timescale.

Thus, this solution of the cosmological constant/vacuum energy problem is tied to the cosmological framework: time-dependent FLRW and background fields (instead of static Minkowski and backgrounds).

This way out (time dependence) is shared by other self-tuning models.
Alternatives can be to introduce spatial dependence for backgrounds,
or Lorentz-violating theories.

III- RADIATION ERA

A) Explicit model

$$A(\varphi) = A_\star e^{\nu_A \varphi}, \quad A_\star > 0, \qquad K(\varphi; X, Y, Z) = K_X e^{\nu_X \varphi} X^\gamma, \quad \gamma > 0, \qquad \sigma \equiv \frac{\nu_X}{4\nu_A}$$

By symmetry, only the difference $\bar{\lambda} = \lambda - \Omega_{\mathrm{vac}0}$, enters.

Eqs. of motion are nonlinear but we can obtain simple solutions: $\varphi = \varphi_\star + \mu_\varphi \eta, \quad \bar{\lambda} = \lambda_\star e^{\mu_\lambda \eta}.$

We require:

- $\bar{\lambda}$ decays (so that cancellation occurs)
- $\bar{\lambda}$ and K decay faster than radiation density
- $\tilde{a} = Aa$ Jordan-frame scale factor grows with time and a
- the solution is stable

This gives the allowed range of parameters:

$$0 < \gamma \leq \frac{5}{14}: \quad \frac{1 - \gamma + \sqrt{1 - 3\gamma^2}}{2} < \sigma < 1 - \frac{\gamma}{2},$$
$$\frac{5}{14} \leq \gamma < \frac{2}{5}: \quad 2\gamma < \sigma < 1 - \frac{\gamma}{2}.$$

φ and A are constant for: $\mu_\varphi = 0 \text{ when } \sigma = \frac{5}{2} - 5\gamma, \quad \frac{1}{3} < \gamma < \frac{5}{14}.$

(Weinberg: there is a nonzero running of $\bar{\lambda}$)

B) Matter phase transitions

The vacuum energy density is expected to jump at phase transitions during the radiation era:

$$T_{\rm EW} = 100\,{\rm GeV} \quad {\rm and} \quad T_{\rm QCD} = 200\,{\rm MeV}. \qquad\qquad \Delta\tilde{V}_{\rm vac} = -\alpha_{\rm pt}\tilde{\rho}_\gamma,$$

We take: $\alpha_{\rm EW} = 0.1 \quad {\rm and} \quad \alpha_{\rm QCD} = 0.1.$

Thus, even if the total vacuum energy density is zero at very early times, these jumps generate an effective cosmological constant that is much greater than the observed value at z=0:

standard cosmological constant problem

The cancellation mechanism must respond to these jumps and adjust to the new vacuum energy densities.

We take: $\gamma \simeq \frac{1}{3} \quad {\rm and} \quad \sigma \simeq \frac{5}{6},$

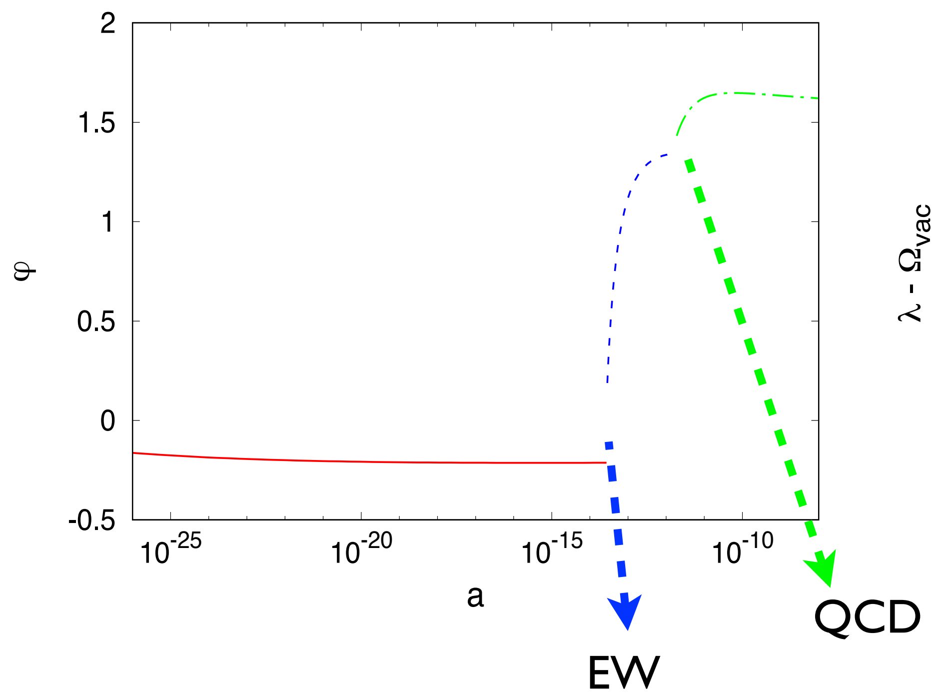
$\bar{\lambda}$ decays only slightly faster than radiation

φ and A are almost constant

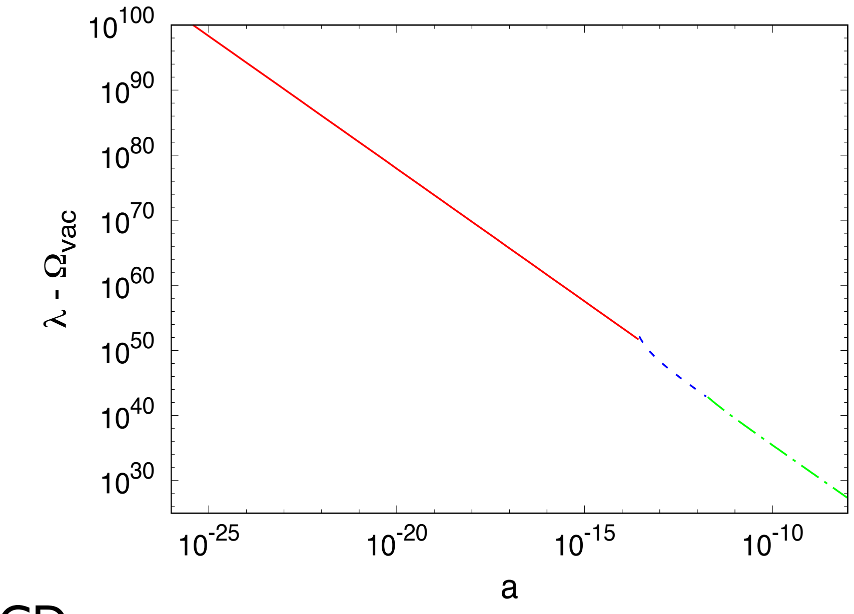
so that H(z) is not too much modified at BBN $\qquad \left|\frac{d\ln A}{d\ln a}\right| \lesssim 10^{-2} \quad {\rm at} \quad T_{\rm BBN} \sim 1\,{\rm MeV},$

$$\bar{\lambda}_2 = \bar{\lambda}_1 - \Delta\Omega_{\rm vac}.$$

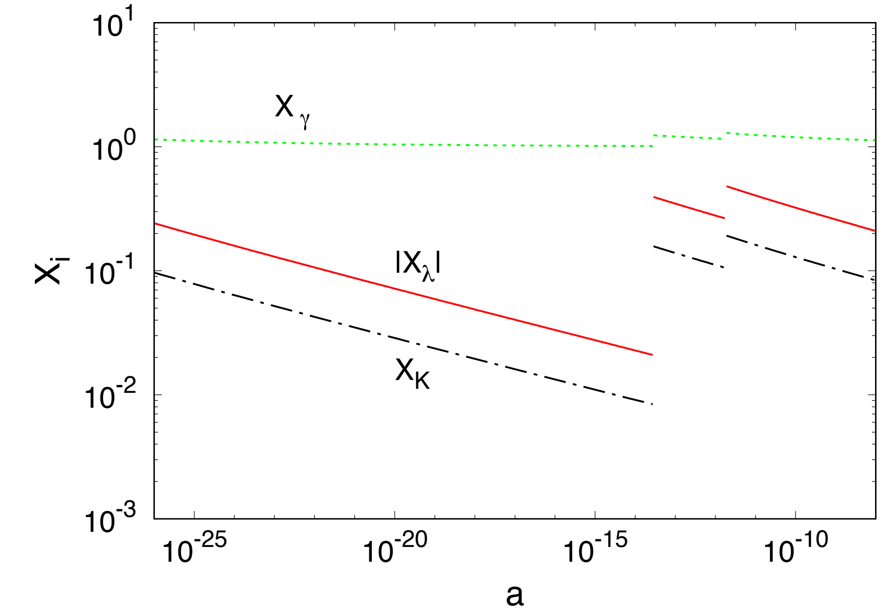
φ jumps but remains stable



$\bar{\lambda}$ jumps and resumes its decay



scalar and vacuum energy densities remain subdominant and decay

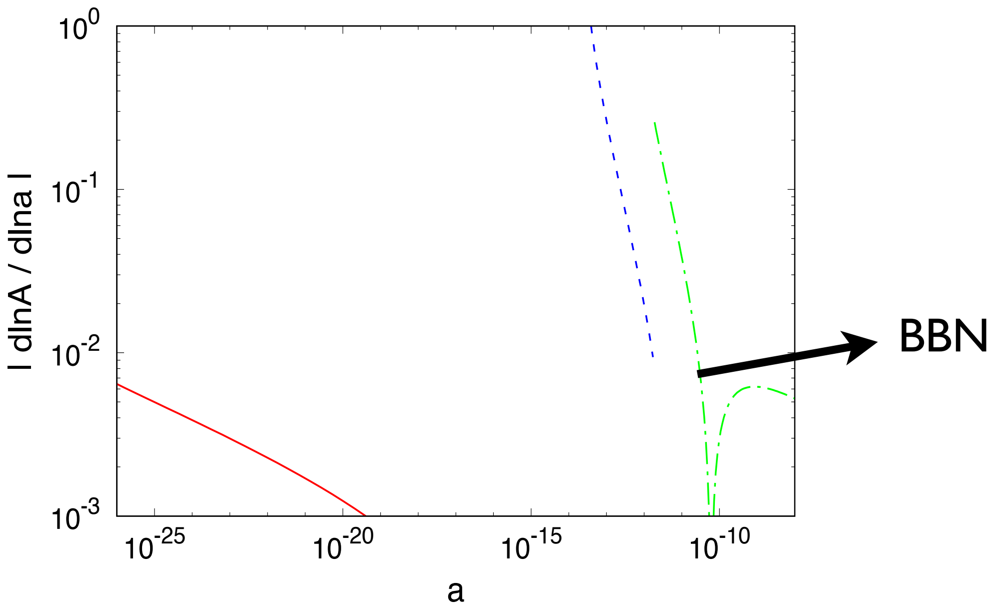


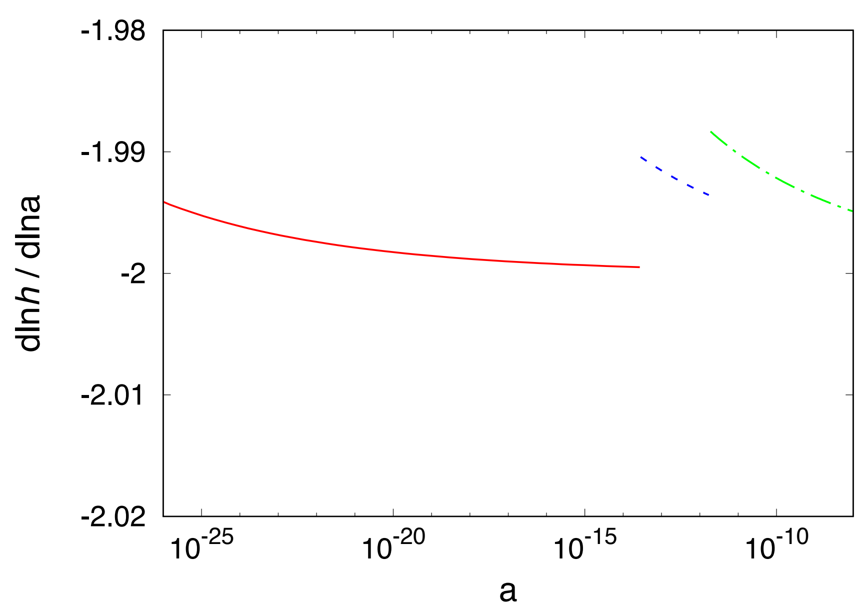
$$X_\lambda = \frac{A^4(\Omega_{\rm vac0} - \lambda)}{\hbar^2}, \quad X = \frac{A\Omega_0}{a^3\hbar^2}, \quad X_\gamma = \frac{\Omega_{\gamma 0}}{a^4\hbar^2}$$

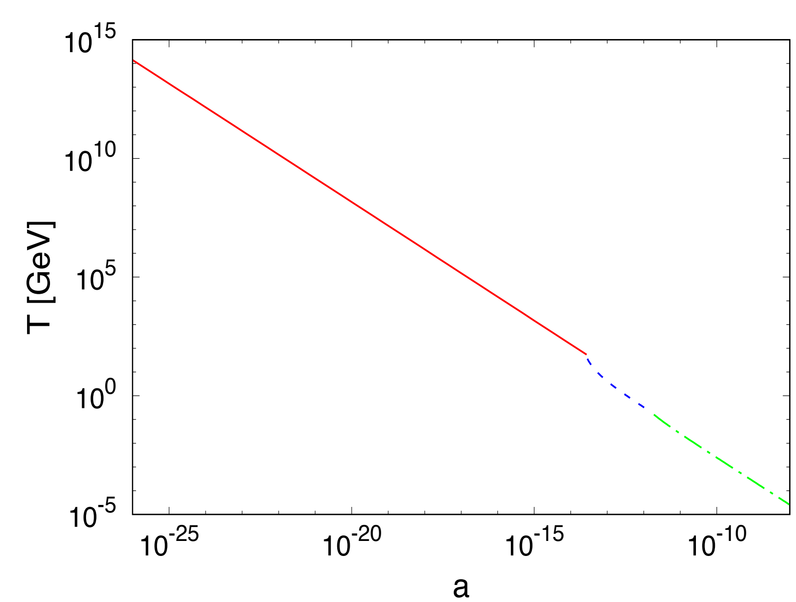
$$X_K = \frac{(2\gamma - 1)K}{\hbar^2}. \qquad ($$

At each phase transition, φ and $\bar{\lambda}$ jump.

But $\bar{\lambda}$ resumes its decay: the cancellation mechanism cancels the new vacuum energy density.







IV- MATTER ERA

A) Difficulties with the coupling to matter

φ -eq.: $$\mathcal{M}^4 \frac{\partial K}{\partial \varphi} - a^{-4}\partial_\tau \left[a^2 \left(\frac{\partial K}{\partial Y}\lambda' + \frac{\partial K}{\partial Z}\varphi' \right) \right] = 4A^3 \frac{dA}{d\varphi}(\tilde{V}_{\rm vac} + \tilde{\rho}/4 - \mathcal{M}^3\lambda)$$

λ cancels both the vacuum energy and 1/4 of the non-relativistic matter density because it is coupled to T^μ_μ . However, we should not cancel $\tilde{\rho}/4$ during the matter era !

This difficulty is common to some other self-tuning models. It arises from the difficulty to distinguish between the vacuum and matter densities at a given time.

This is solved in an elegant manner in the sequestering model by the use of global variables:

$$\Lambda = \frac{\int d^4x\sqrt{-g}\, T^\mu_\mu/4}{\int d^4x\sqrt{-g}}$$

The spacetime integral is dominated by late times, where the only density left is the low-energy vacuum energy density. This selects the vacuum energy density !

This is not possible in our dynamical framework, where we only know about T^μ_μ at each time-step, not about the distant future.

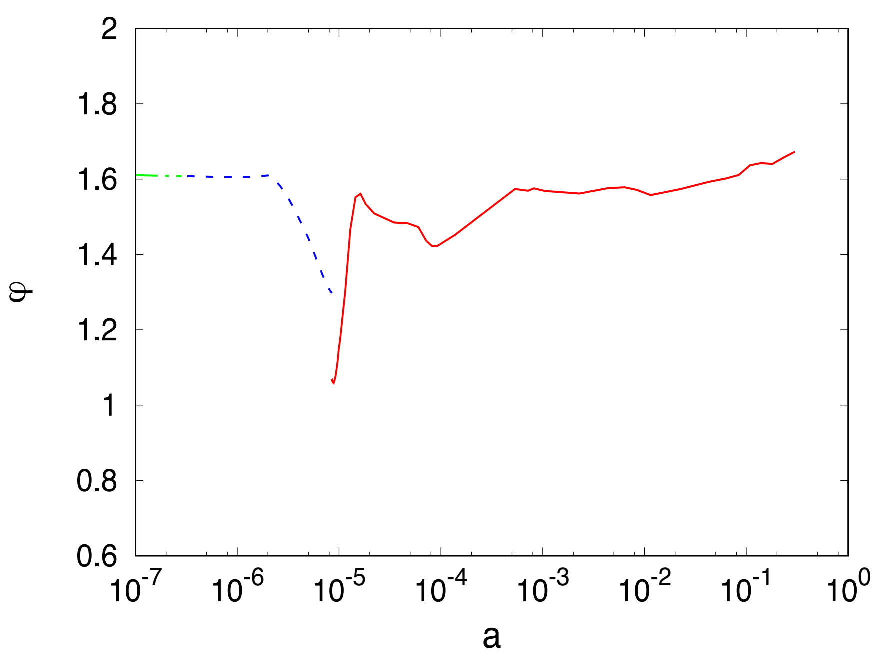
To face this coupling we consider solutions where λ scales as a constant fraction of the matter density.

B) Explicit implementation

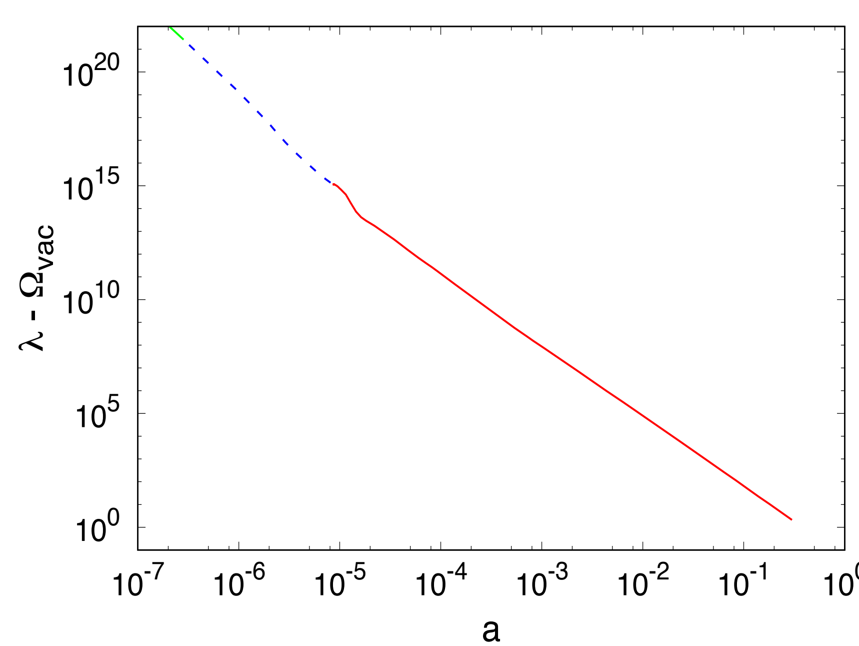
We look for stable solutions: $\varphi = \varphi_\star, \quad \bar{\lambda} = \lambda_\star e^{-3\eta}.$ $\gamma = \frac{1}{3}, \quad \frac{1-\sqrt{145}}{24} < \sigma < \frac{5}{6}$

- $\ast$ γ does not change
- $\ast$ σ goes down from 5/6 to -0.4
- $\ast$ the scalar relative density is 13% $\quad X_\lambda + X_K = \frac{1}{6\sigma - 5},$

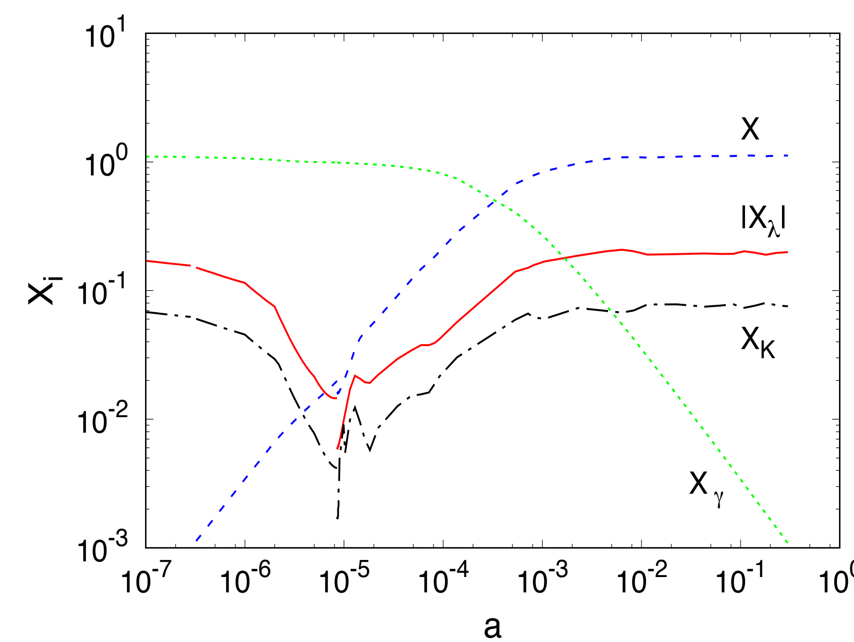
some tuning to keep φ almost constant



$\bar{\lambda}$ keeps decaying, along with ρ



crossover between scalar and matter densities



V- DARK ENERGY ERA

The end of the cancellation mechanism provides in a natural fashion a dark energy era.

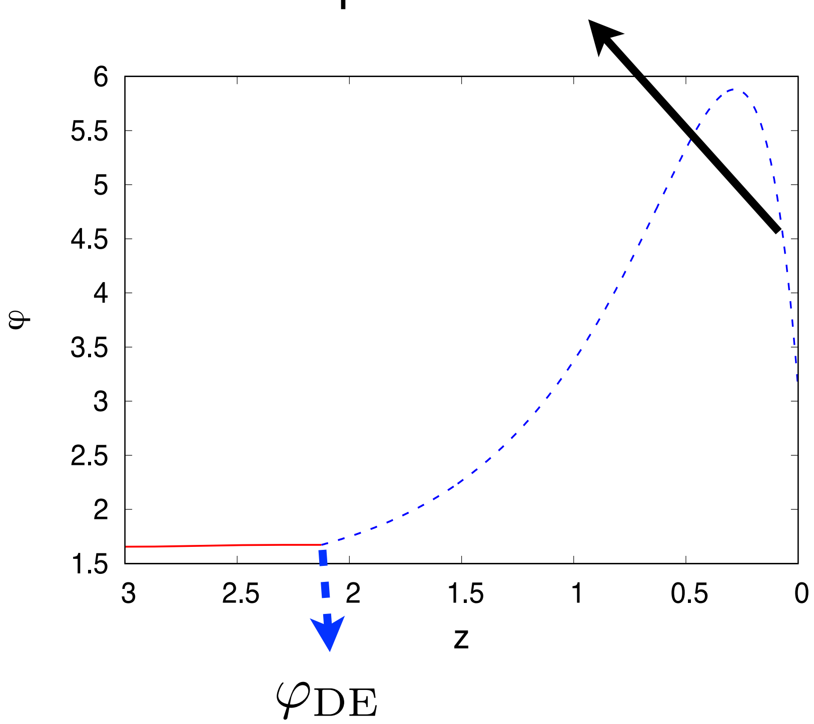
Because the cancellation mechanism arises from the conformal coupling $A(\varphi)$ it naturally stops when:

$A(\varphi)$ is constant or $\dfrac{dA}{d\varphi} \simeq 0$

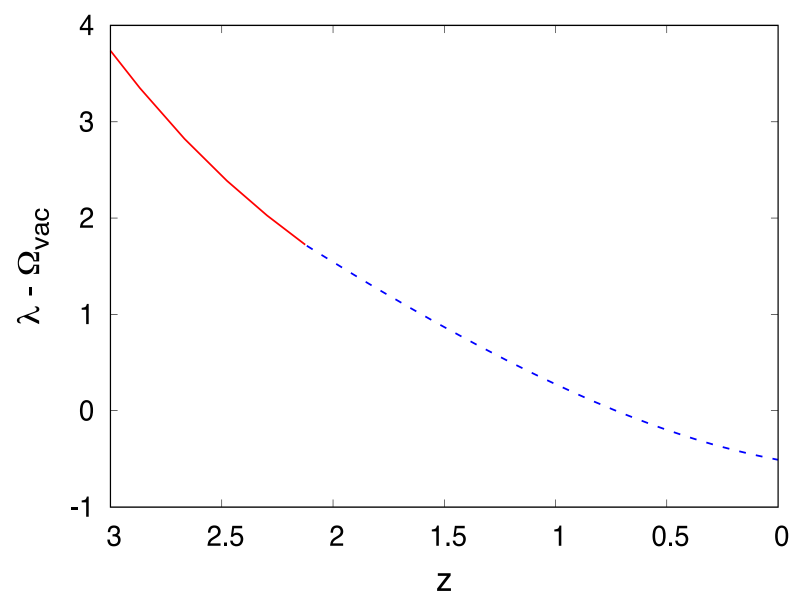
This is possible because φ plays the role of a clock.

$$\varphi > \varphi_{\rm DE}: \quad A(\varphi) = A_\star = 1. \qquad K(\varphi; X, Y, Z) = K_X X^\gamma + K_Y Y, \qquad \gamma = 1/4$$

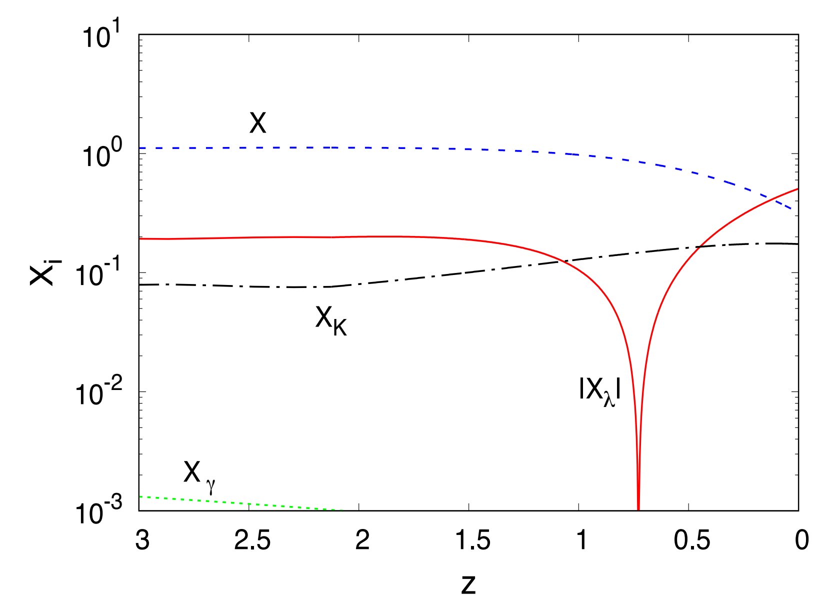
In this example DE will end



λ stops decaying to zero and plays the role of quintessence



crossover between matter and scalar_densities



VI- INFLATION ERA

A) Accelerated expansion

As for DE, accelerated expansion occurs for constant $A(\varphi)$

$$\varphi < \varphi_{\rm I}: \quad A(\varphi) = A_{\rm I}.$$

$$K(\varphi; X, Y, Z) = K_X X + K_Y Y,$$ (standard kinetic function)

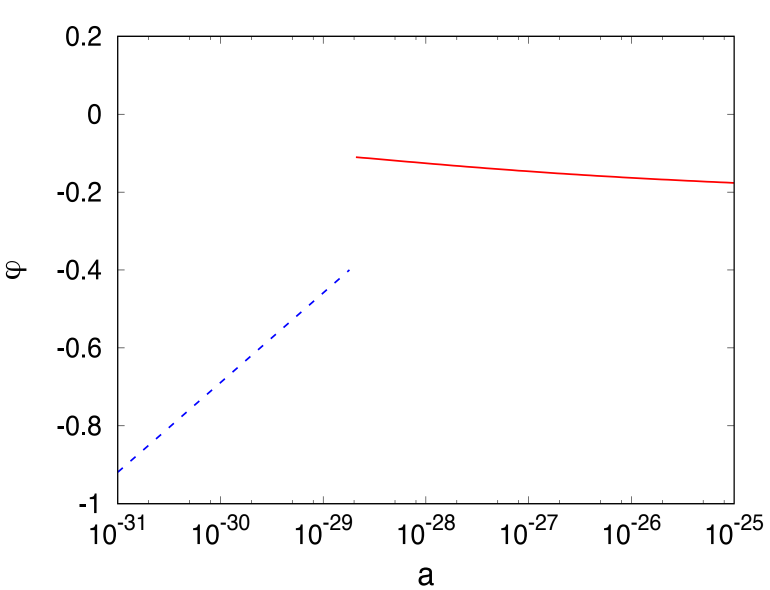
Constant-λ solution: $\lambda = \lambda_{\rm I}, \quad \bar{\lambda} = \lambda_{\rm I} - \Omega_{\rm vacI}, \quad \varphi = \varphi_I + \frac{A_{\rm I}^4}{3 K_Y \hbar_{\rm I}^2}(\eta - \eta_{\rm I}),$

B) End of the accelerated expansion and early radiation era

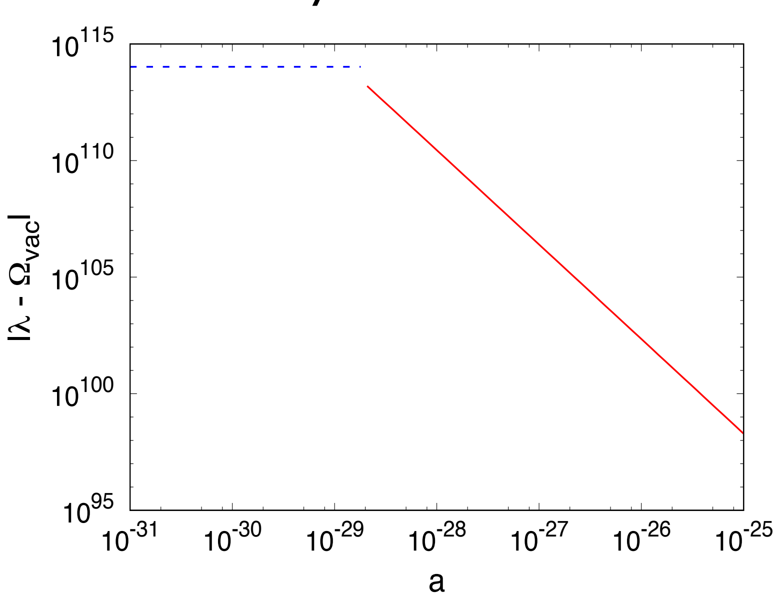
We assume a transition with a drop of the vacuum energy and a transfer to the radiation density:

$\Delta\Omega_{\rm vac} = -\alpha_{\rm I} \frac{\hbar_{\rm I}^2}{A_{\rm I}^4},$ while the conformal coupling and kinetic function take their radiation-era form.

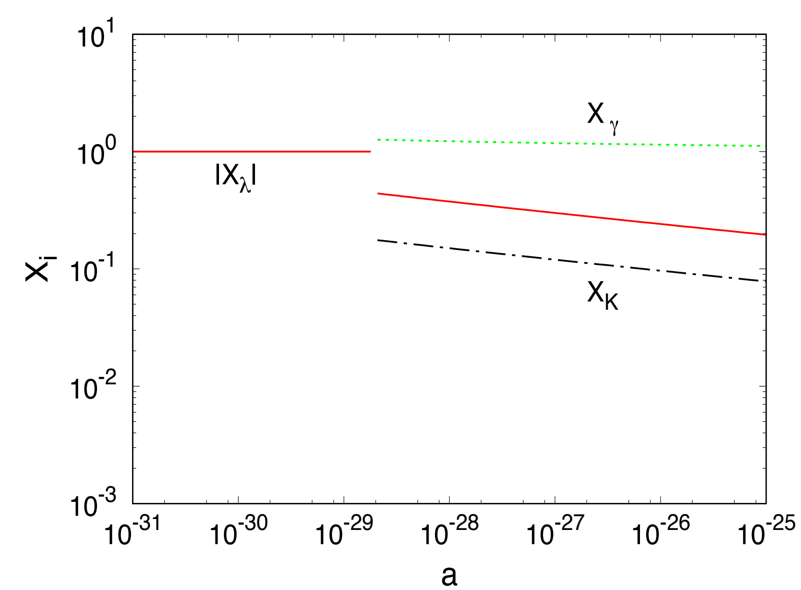
φ is a clock



$\bar{\lambda}$ drives the inflation and next decays in the radiation era



crossover between scalar and radiation densities



VII- CONCLUSION

- It is possible to build simple self-tuning models that provide a dynamical cancellation of the vacuum energy density

- This does not "explain" the value of the observed cosmological constant, but it can solve the "old cosmological constant problem".

- For these dynamical models, a delicate issue is to cancel the vacuum and only the vacuum ! (How to distinguish it from the matter density ?)

- An interesting feature is to link together:
  - "old cosmological constant problem"
  - DE
  - Inflation

→ hope to see something in cosmological data !

- This explicit model is probably not the final answer ! Need to overcome some tuning (matter era transition). Also need to check perturbations.

Sequestering model

Kaloper & Padilla (2014), PRD 90, 084023 (+....)

Padilla 1502.05296

$$S=\int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R-\lambda^4\mathcal{L}_m(\lambda^{-2}g^{\mu\nu},\Psi)-\Lambda\right]+\sigma\left(\frac{\Lambda}{\lambda^4\mu^4}\right)$$

Λ and λ are global variables

$\sqrt{-g}\lambda^4\mathcal{L}_m(\lambda^{-2}g^{\mu\nu},\Psi)=\sqrt{-\tilde{g}}\mathcal{L}_m(\tilde{g}^{\mu\nu},\Psi)$ with $\tilde{g}_{\mu\nu}=\lambda^2 g_{\mu\nu}$

→ constant conformal mapping between Jordan and Einstein metrics

Equations of motion with respect to Λ, λ and $g_{\mu\nu}$

$$\begin{aligned}\frac{1}{\lambda^4\mu^4}\sigma'\left(\frac{\Lambda}{\lambda^4\mu^4}\right)&=\int d^4x\sqrt{-g}\\ \frac{4\Lambda}{\lambda^4\mu^4}\sigma'\left(\frac{\Lambda}{\lambda^4\mu^4}\right)&=\int d^4x\sqrt{-g}\lambda^4\tilde{T}^\alpha_\alpha\\ M_{pl}^2G^\mu_\nu&=-\Lambda\delta^\mu_\nu+\lambda^4\tilde{T}^\mu_\nu\end{aligned}$$

This gives: $\Lambda=\frac{1}{4}\langle T^\alpha_\alpha\rangle=\frac{\int d^4x\sqrt{-g}\,T^\alpha_\alpha/4}{\int d^4x\sqrt{-g}}$ and hence:

$M_{pl}^2G^\mu_\nu=T^\mu_\nu-\frac{1}{4}\delta^\mu_\nu\langle T^\alpha_\alpha\rangle=\tau^\mu_\nu-\frac{1}{4}\delta^\mu_\nu\langle\tau^\alpha_\alpha\rangle$ with: $T^\mu_\nu=-V_{vac}\delta^\mu_\nu+\tau^\mu_\nu,$

τ^μ_ν are the local excitations above the vacuum.

The vacuum entirely drops out from the Einstein equations, to all loop orders.

There remains a small residual cosmological constant, but it is smaller than the current dark energy density:

$$\Lambda_{\rm eff} = \frac{1}{4}\langle \tau^\alpha_\alpha \rangle$$

$$\int d^4x\sqrt{-g} = a^4_{\max}, \qquad \langle \tau^\alpha_\alpha \rangle \sim \rho_{\rm turn}$$

$$\rho_{\rm turn} \sim M^2_{pl} H^2_{\rm turn} \lesssim \rho_c \sim M^2_{pl} H^2_0.$$

Spacetime volume is finite, Universe is closed k>0.

Transient dark energy before collapse:

$$V = m^3\phi$$

$$t_{\rm collapse} \sim \sqrt{\frac{M_{pl}}{m^3}}$$