Rotating black holes in higher order gravity theories

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Based on work with
Marco Crisostomi, Ruth Gregory and Nikos Stergioulas
arXiv:1903.05519
Dark energy workshop

GR is a unique metric theory in 4 dimensions

- Theoretical consistency: In $D=4$ dimensions, consider $\mathcal{L}=\mathcal{L}(\mathcal{M},g,\nabla g,\nabla\nabla g)$ where ∇ is a Levi-Civita connection. Then Lovelock's theorem in $D=4$ states that GR with cosmological constant is the unique metric theory emerging from,

$$S_{(4)}=\int_{\mathcal{M}} d^4x\sqrt{-g^{(4)}}\left[-2\Lambda+R+\alpha\hat{G}\right]$$

giving,

  - Equations of motion of 2^{nd}-order (Ostrogradski no-go theorem 1850!)
  - given by a symmetric two-tensor, $G_{\mu\nu}+\Lambda g_{\mu\nu}$
  - and admitting Bianchi identities.

GR is the unique massless-tensorial 4 dimensional theory of gravity.

- The Gauss-Bonnet term is a topological invariant: It does not contribute to the field equations in $D=4$
- This is no longer true for a connexion which is not Levi-Civita [Jimenez, Heisenberg, Koivisto]

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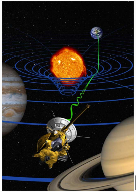
Observational data

- Experimental consistency:
  -Excellent agreement with solar system tests and strong gravity tests on binary pulsars
  -Recent data from the EHT compatible with GR for a supermassive black hole
  -Observational breakthrough GW170817: Non local, 40Mpc and strong gravity test from a coalescing binary of neutron stars. $c_T = 1 \pm 10^{-15}$

Time delay of light  Planetary tajectories 

 Neutron star binary

Galileons/Horndeski [Horndeski 1973]

- Important constraints on scalar tensor [Creminelli, Vernizzi, Ezquiaga, Zumalacaregui,...] albeit strong coupling issues [DeRham, Melville]
- Here, we will concentrate on $c_T = 1$ theories DHOST or EST [Crisostomi, Koyama, Langlois, Noui, Vernizzi,..] and obtain rotating black hole solutions
- The $c_T = 1$ theories (gravitons propagate at the speed of light) are disformal versions of Horndeski theories
- The theory under scrutiny has unique characteristics. It is far closer to GR than any version of Hordenski

$c_T = 1$ theories and their relation to Horndeski

Shift-symmetric scalar tensor theory $c_T = 1$ minimally coupled to matter is parametrized by K, A_3, G

$$\mathcal{L} = K(X) + G(X)R + A_3\phi^\mu\phi_{\mu\nu}\phi^\nu\Box\phi + A_4\phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu + A_5(\phi^\mu\phi_{\mu\nu}\phi^\nu)^2\,,$$

- Dependence on $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ guarantees shift symmetry
- $K(X)$ contains the cosmological constant and kinetic terms to lowest order
- the operators A_4, A_5 are fixed with respect to A_3, G
- Related to Horndeski via a transformation

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(X)g_{\mu\nu} + D(X)\nabla_\mu\phi\nabla_\nu\phi$$

  for given C and D.
- One can start with a $c_T \neq 1$ Horndeski theory and map it to a $c_T = 1$ theory for a specific function D.
- a disformal D changes speed of gravitons unlike C
- D is related to A_3 while G is related to C (conformal transformation)
- Horndeski frame is simply not the physical frame but can be used in order to find solutions to $c_T = 1$

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Finding exact solutions

Can we find exact solutions?

- Cosmological solutions: self-accelerating, self tuning, cosmological
- spherical symmetry: black holes, neutron stars, solitons, regular black holes...
- stationary solutions: black holes with rotation?

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An example of spherical symmetry [Babichev, CC, GEFarèse, Lehébel]

- The Horndeski theory

$$S = \int d^4x \sqrt{-g}\,[\zeta R - 2\Lambda - \eta X + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi]\,,$$

  is not in the physical frame.

- $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$
- Solution [Babichev, CC]: $f = h = 1 - \frac{\mu}{r} + \frac{\eta}{3\beta}r^2$, $\phi = qt \pm \frac{q}{h}\sqrt{1-h}$ with $\Lambda_{\rm eff} = -\zeta\eta/\beta$.
- The physical frame ($c_T = 1$) is :

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{\beta}{\zeta + \frac{\beta}{2}X}\varphi_\mu\varphi_\nu .$$

- The disformed metric is a black hole. $X = -q^2$ is constant! Solution is an Einstein metric. ϕ is regular at the event horizon but not at the cosmological horizon
- In Horndeski theory there are numerous solutions [Lehébel] some of which are not Einstein metrics X remains constant

$$f(r) = h(r)(1+\kappa r^2), \qquad h(r) = 1 + \alpha - \alpha\frac{\arctan(r\sqrt{\kappa})}{r\sqrt{\kappa}}$$

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Going beyond spherical symmetry? [cc, Crisostomi, Gregory, Stergioulas]

Consider an Einstein metric, $R_{\mu\nu} = \Lambda g_{\mu\nu}$ and $X = X_0$ constant. When is this solution to the field equations?

$$A_3(X_0) = 0, \qquad (K_X + 4\Lambda G_X)|_{X_0} = 0$$

where $\Lambda = -K/(2G)|_{X_0}$ (self-tuning condition)

- Any theory parametrized by A_3 having a zero at some value is enough to guarantee a solution.
- The real question though is what is the scalar such that X is constant?
- Note that if we take $Y_a = \partial_a \phi$ then the derivative of $X = Y_a Y_b g^{ab} = X_0$ is simply $a^b = Y^a \nabla_a Y^b = 0$
- Hence acceleration zero hence ϕ is related to a geodesic congruence in the given spacetime.
- the scalar field ϕ is the Hamilton-Jacobi potential S where $\frac{\partial S}{\partial \lambda} = g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu}$

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The example of Carter's solution (de Sitter-Kerr)

- Rotating black hole Einstein metric

$$ds^2 = -\frac{\Delta_r}{\Xi^2\rho^2}\left[dt - a\sin^2\theta d\varphi\right]^2 + \rho^2\left(\frac{dr^2}{\Delta_r} + \frac{d\theta^2}{\Delta_\theta}\right)$$
$$+ \frac{\Delta_\theta \sin^2\theta}{\Xi^2\rho^2}\left[a\,dt - \left(r^2 + a^2\right) d\varphi\right]^2 ,$$
$$\Delta_r = \left(1 - \frac{r^2}{\ell^2}\right)\left(r^2 + a^2\right) - 2Mr\,, \quad \Xi = 1 + \frac{a^2}{\ell^2}\,,$$
$$\Delta_\theta = 1 + \frac{a^2}{\ell^2}\cos^2\theta\,, \qquad \rho^2 = r^2 + a^2\cos^2\theta\,,$$

- Here we have parameters $a, M, \Lambda = 3/l^2$ which describe a black hole with an inner, outer event and cosmological horizon for $\Lambda > 0$.
- To evaluate the HJ potential for geodesics we need to know the inverse metric and solve a first order differential equation.

The example of Carter's solution (de Sitter-Kerr)

- The Hamilton Jacobi potential reads [Carter],

$$\mathcal{S} = -E\,t + L_z\varphi + S(r,\theta)\,,$$

since ∂_t and ∂_ϕ are Killing vectors and is separable $S(r,\theta) = S_r(r) + S_\theta(\theta)$!

$$S_r = \pm\int \frac{\sqrt{R}}{\Delta_r}dr\,,\qquad S_\theta = \pm\int \frac{\sqrt{\Theta}}{\Delta_\theta}d\theta\,,$$

$$\begin{aligned} R &= \Xi^2\left[E\left(r^2+a^2\right) - a\,L_z\right]^2 \\ &- \Delta_r\left[\mathcal{Q} + \Xi^2\left(a\,E - L_z\right)^2 + m^2 r^2\right]\,, \end{aligned} \tag{1}$$

$$\begin{aligned} \Theta &= -\Xi^2\sin^2\theta\left(a\,E - \frac{L_z}{\sin^2\theta}\right)^2 \\ &+ \Delta_\theta\left[\mathcal{Q} + \Xi^2\left(a\,E - L_z\right)^2 - m^2 a^2\cos^2\theta\right]\,. \end{aligned} \tag{2}$$

- This is the starting point for evaluating geodesics of spacetime.
- Note we have $E, m, L_z, \mathcal{Q}$ parametrising the Energy at infinity, rest mass, angular momentum and Carter's separation constant.
- Here we need ϕ regular in all the permitted domain of the coordinates. We clearly need that Θ and R are positive functions.
- This leads to $L_z = 0$ and fixes Carter's constant $\mathcal{Q} + \Xi^2 a^2 E^2 = m^2 a^2$,
- Now we can take $\phi = \mathcal{S}$

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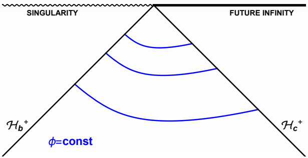
$$\Theta = a^2m^2\sin^2\theta\left(\Delta_\theta - \eta^2\right)\,, R = m^2(r^2+a^2)\left(\eta^2(r^2+a^2) - \Delta_r\right)$$

  where $\eta = \frac{\Xi E}{m} \in [\eta_c, 1]$.
- $\eta = 1$ in the $\Lambda = 0$ solution (Kerr).
- Once we have $\Lambda > 0$ and increasing we have $\eta_c < 1$ and decreasing
- η_c is such that R has a double zero at some $r_{EH} < r_0 < r_{CH}$
- Note that we have two branches of solutions. One which is regular at the event horizon and one which is regular at the cosmological horizon.
- Fixing $\eta = \eta_c$ we have a regular solution everywhere by joining the two branches at $r = r_0$!



Conclusions

- Although solution is stealth, perturbations defining quasi normal modes and resulting phenomenology will be different.
- Can obtain any GR vacuum solution with well defined hair in such theories
- One can use this stealth solution to construct numerically other non GR solutions by relaxation techniques
- For $c_T = 1$ the only $X = \textit{constant}$ solutions are Einstein spaces. If we expect solutions to have asymptotically X constant then in this theory all solutions are asymptotically Einstain spaces.